$$d_1 = u^1 + \sigma_1 \eta_1(0,1)$$
$$d_2 = u^2 + \sigma_2 \eta_2(0,1)$$

$$\longrightarrow d = \sum_{j=0}^{7} \psi_j(\xi_1, \xi_2, \eta_1, \eta_2)\, d_j = \underset{\begin{pmatrix} d_1 \\ d_2 \end{pmatrix}}{d_0} + 0 + 0 + 0 + 0 + 0 + \eta_1 \underset{\to d_6}{\begin{pmatrix} \sigma_1 \\ 0 \end{pmatrix}} + \eta_2 \underset{\to d_7}{\begin{pmatrix} 0 \\ \sigma_2 \end{pmatrix}}$$

$$u^f = \underbrace{1}_{\psi_0} \begin{pmatrix} u_0^1 \\ u_0^2 \\ u_0^3 \end{pmatrix} + \underbrace{\xi_1}_{\psi_1} \begin{pmatrix} u_1^1 \\ u_1^2 \\ u_1^3 \end{pmatrix} + \underbrace{\xi_2}_{\psi_2} \begin{pmatrix} u_2^1 \\ u_2^2 \\ u_2^3 \end{pmatrix} + \underbrace{(\xi_1^2 - 1)}_{\psi_3} \begin{pmatrix} u_3^1 \\ u_3^2 \\ u_3^3 \end{pmatrix} + \underbrace{(\xi_1 \xi_2)}_{\psi_4} \begin{pmatrix} u_4^1 \\ u_4^2 \\ u_4^3 \end{pmatrix}$$

$$+ \underbrace{(\xi_2^2 - 1)}_{\psi_5} \begin{pmatrix} u_5^1 \\ u_5^2 \\ u_5^3 \end{pmatrix} + \underbrace{\eta_1}_{\psi_6} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \underbrace{\eta_2}_{\psi_7} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \sum_{j=0}^{7} \psi_j(\xi_1, \xi_2, \eta_1, \eta_2)\, u_j^f \longrightarrow u^a = \sum_{j=0}^{7} \psi_j\, u_j^a$$

$$A^f = \begin{pmatrix} u_0 & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 \end{pmatrix}; \quad D = \begin{pmatrix} d_0 & 0 & 0 & 0 & 0 & 0 & d_7 & d_8 \end{pmatrix}$$

$$A^a = A^f + K(D - H A^f)$$

$$u_j^a = u_j^f + K(d_j - H u_j^f), \quad j = 0, \dots, 7$$

$$d_1 = u^1 + \sigma_1 \eta_1$$
$$d_2 = u^2 + \sigma_2 \eta_2$$

$$d = \underset{H}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$K = P^f H^T (H P^f H^T + T)^{-1}$$

$$T = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}; \quad P^f = \sum_{j=1}^{7} E(\psi_i^2)\, u_j u_j^T$$

$$P^f = \underset{1}{E(\psi_1^2)} u_1 u_1^T + \underset{1}{E(\psi_2^2)} u_2 u_2^T + E(\psi_3^2) u_3 u_3^T + E(\psi_4^2) u_4 u_4^T + E(\psi_5^2) u_5 u_5^T$$

$$+ \underset{1 = E(\eta_1^2)}{E(\psi_6^2)} u_6 u_6^T + \underset{1 = E(\eta_2^2)}{E(\psi_7^2)} u_7 u_7^T$$

$$E(\psi_3^2) = E[(\xi_1^2 - 1)^2] = 2,$$
$$E(\psi_4^2) = E[\xi_1^2 \xi_2^2] = E(\xi_1^2)\, E(\xi_2^2) = 1$$
$$E(\psi_5^2) = E[(\xi_2^2 - 1)^2] = 2$$

$$E(\psi_3 \psi_4) = 0$$
$$E[\xi_1^4] = E(\xi_1 \xi_1 \xi_1 \xi_1) = 3\, E(\xi_1^2)\, E(\xi_1^2) = 3\sigma^4$$
$$E(\xi_1 \xi_2 \xi_3 \xi_4) \to 3\sigma^4$$
$$= E(\xi_1 \xi_2)\, E(\xi_3 \xi_4) + E(\xi_1 \xi_2)\, E(\xi_2 \xi_4)$$
$$E(\xi_1 \xi_4)\, E(\xi_2 \xi_3)$$

$$u_0^a = u_0^f + K(d_0 - H u_0^f)$$
$$u_j^a = u_j^f + K(d_j - H u_j^f)$$

$$PH^T (HPH^T + \sigma^2)^{-1}$$
$$=$$

$$u_0^a = \begin{pmatrix} u_0^1 \\ u_0^2 \\ u_0^3 \end{pmatrix} + [K]_{3\times 1} (d_1 - u_0^1)$$

$$u_1^a = \begin{pmatrix} u_1^1 \\ u_1^2 \\ u_1^3 \end{pmatrix} + [K]_{3\times 1} (0 - u_1^1)$$

$$u_2^a = \begin{pmatrix} u_2^1 \\ u_2^2 \\ u_2^3 \end{pmatrix} + [K]_{3\times 1} (0 - u_2^1)$$

$$u_3^a = \begin{pmatrix} u_3^1 \\ \cancel{u_3^2}^{\,0} \\ u_3^3 \end{pmatrix} + [K] (0 - u_3^1)$$

$$u_4^a = \begin{pmatrix} 0 \end{pmatrix} + K (\sigma_1 - 0)$$

$$E\left((\xi^2 - 1)^2\right)$$
$$= E(\xi^4 - 2\xi^2 + 1)$$
$$= E(\xi^4) - 2E(\xi^2) + 1$$
$$= 3E(\xi^2) - 2E(\xi^2) + 1$$
$$= 3 - 2 + 1 = 2$$

$$u^a = u_0^a + \xi_1 u_1^a + \xi_2 u_2^a + 0 + \eta u_4^a$$

Sample: ξ_1, ξ_2, η

$$\bar{u}^a = u_0^a, \quad P^a = \underbrace{E(\xi_1^2)}_{1} u_1^a u_1^{aT} + \underbrace{E(\xi_2^2)}_{1} u_2^a u_2^{aT} + \underbrace{E(\eta^2)}_{1} u_4^a u_4^{aT}$$