DIFFERENTIAL EQUATION

I.sc maths golu sir 8292654087

* Those eq^n which Involve dependent or Independent Varable and its derivative are called Differential eq^n

Ex

$$x\frac{dy}{dx}+y=0$$

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^2$$

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}=K\,d^2y/dx^2$$

$$x\,dz/dx+y\frac{dz}{dy}=2z$$

$$y\,dx+x\,dy+z\,dx=0$$

⇒ There are three Types of differential eq^n

(I) ordinary diff eq^n :- Those diff eq^n which Involve ordinary derivative (d/dx) are called ordinary diff eq^n.

Ex

$$x\frac{dy}{dx}+y=0$$

$$d^2y/dx^2-5\,dy/dx+6y=x^2$$

(II) Partial diff eq^n :- Those diff eq^n which Involve partial derivative $(d/dx, d/dy)$ are called partial diff eq^n

$$x\frac{dz}{dx}+y\frac{dz}{dy}=2z$$

I.sc maths golu sir 8292654087

(iii) Total diff eqⁿ:- Those diff eqⁿ which Invoive total derivative ($\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}$) are called Total diff eqⁿ

$$ydx + xdy + zdz = 0$$

⇒ If number of Independent varable is equal to number of dependent varable in a diff eqⁿ then we say that ordinary diff eqⁿ.

⇒ If number of Independent varable is grater then number of dependent varable Then we say that partial diff eqⁿ.

⇒ If number of Independent varable is less then no of dependant varable Then we say that total diff eqⁿ

* order of diff eqⁿ:- The order of the higest order deravative Involve in diff eqⁿ are called order of differential equation

I.sc maths golu sir 8292654087

* Degree of diff eqn: Degree of the height order derivative when the eqn are made free from radical of any are called degree of diff eqn

Ex $x\frac{dy}{dx}+y=0$, D = 1 O = 1

$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=0$ D = 1 O = 2

Q find the order & degree of the diff eqn

(1) $2\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^4+5=0$ D = 1 O = 2

(11) $d^2y/dx^2-5\,dy/dx+2y=0$ D = 2 O = 1

⇒ Solution of diff eqn:- Those function which satisfy the diff eqn are called Solution of diff eqn

(1) $x\frac{dy}{dx}+y=0$

$x\frac{dy}{dx}=-y$

$\frac{dy}{y}=-\frac{dx}{x}$

$\log y=-\log x+\log c$

$\log\frac{y}{c}=-x$ $=$ $y=cx$

Ex2 $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ --- (1)

$y = c_1e^{2x} + c_2e^{3x}$

$y_1 = 2c_1e^{2x} + 3c_2e^{3x}$

$y_2 = 4c_1e^{2x} + 9c_2e^{3x}$

Put value of y, y_1, y_2 in eqn (1)

~~$c_1e^{2x} + c_2e^{3x} - 10c_1e^{2x} - 15c_2e^{3x} + 24c_1e^{2x} +$~~

$4c_1e^{2x} + 9c_2e^{3x} - 10c_1e^{2x} - 15c_2e^{3x} + 6c_1e^{2x} + 6c_2e^{3x}$

$= 10c_1e^{2x} + 15c_2e^{3x} - 10c_1e^{2x} - 15c_2e^{3x} = 0$

$y = c_1e^{2x} + c_2e^{3x}$ is soln

⇒ There are Three Types of soln of diff eqn

(1) General Solution:- Those solution in which no of arbitrary constant is equal to order of diff eqn are called General soln of diff eqn

Ex $xy = c$ is general soln of the diff eqn $x\frac{dy}{dx} + y = 0$

$y = c_1e^{2x} + c_2e^{3x}$ is general soln of the diff eqn $d^2y/dx^2 - 5\,dy/dx + 6y = 0$

(2) Particular solⁿ:- Those solution which from general solution by assigining some particular value of arbitrary constant are called particular solⁿ

Ex $xy = 5$ is particular solⁿ of diff eq $x\frac{dy}{dx} + y = 0$

$y = 5e^{2x} + 6e^{3x}$ is particular solⁿ of diff eqⁿ

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

(3) Singular solⁿ:- Those solⁿ in which arbitrary constant is venish is called singular solⁿ

Ex: (i) $xy = 0$ is singular solⁿ of the diff eqⁿ $x\,dy/dx + y = 0$

(ii) $y^2 = 4ax$ is singular solⁿ of $x(dy/dx)^2 - y\,dy/dx + a = 0$

* formation of diff eqⁿ:-

Let $f(x, y, c_1, c_2, c_3 \ldots c_n) = 0$ be the eqⁿ of curve where $c_1, c_2 \ldots c_n$ is arbitrary constant

*

* Differentitan the given eqn of n^{th} times (no of diff is equals to no of arbitusary const)

We get $(n+1)$ no of eqn. Now we eliminate $c_1\, c_2 \,--\, c_n$

i.e arbitrary constant with the help of these $(n+1)$ eqn we get a relation b/w x, y & its derivitive it is the required diff eqn.

Q1 find diff eqn for parabola $y^2 = 4ax$

$$y^2 = 4ax \quad --- (1)$$

diff both side w.r.to x

$$2y\frac{dy}{dx} = 4a$$

put $4a = 2y\,dy/dx$ in eqn (1),

$$y^{\cancel{2}} = \cancel{2y}\,dy/dx \cdot x$$

$$y = 2\,dy/dx \cdot x$$

$$2x\,dy/dx - y = 0$$

Scanned by CamScanner

Q find diff eqn for following curves

(1) $y^2 = 4a(x+a)$

$y^2 = 4ax + 4a^2$ — — — (1)

diff both side w r to x

$2y\, dy/dx = 4a$

$a = \frac{y}{2} dy/dx$

put $a = y/2\, dy/dx$ in eqn (1)

$y^2 = 4 \times y/2\, dy/dx \cdot x + 4 \times \frac{y^2}{4} (dy/dx)^2$

$y^2 = y(2x\, dy/dx) + y(dy/dx)^2$

$y(dy/dx)^2 + 2x\, dy/dx - y = 0$

(2) $y = c(x-c)^2$ — — (1)

diff both side w r to x

$\frac{dy}{dx} = 2c(x-c)$

$(x-c)c = \frac{1}{2} y_1$

put $(x-c)c$ in eqn (1)

$y = \frac{1}{2} y_1 (x-c)$

$\frac{2y}{y_1} = x-c$

$c = x - \frac{2y}{y_1}$

put $c = x - 2y/y_1$ in eq (1)

$$y = (x - 2y/y_1)(x - x + 2y/y_1)^2$$

$$y = (x - 2y/y_1)\left(\frac{4y^2}{y_1^2}\right)$$

$$y = \frac{4xy^2}{y_1^2} - \frac{8y^3}{y_1^3}$$

$$yy_1^3 = 4xy^2y_1 - 8y^3$$

$$yy_1^3 - 4xy^2y_1 + 8y^3 = 0$$

$$y\left(\frac{dy}{dx}\right)^3 - 4xy^2\frac{dy}{dx} + 8y^3 = 0$$

$$\left(\frac{dy}{dx}\right)^3 - 4xy\,dy/dx + 8y^2 = 0$$

(iii) $y = a\sin x + b\cos x$

diff both side w r to x

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Again

$$\frac{d^2y}{dx^2} = -a\sin x - b\cos x$$

$$\frac{d^2y}{dx^2} = -(y)$$

$$\frac{d^2y}{dx^2} + y = 0$$

H.W

Q $y = A\cos 3x + B\sin 3x$

Q $y = Ae^{2x} + Be^{-2x}$

Q $xy = ae^{x} + be^{-x}$

Q $xy = Ae^{x} + Be^{-x}$

Q $y = a\cos(mx+b)$

Q $Ax^2 + By^2 = 1$ is sol^n $xy\,d^2y/dx^2 + x\left(\frac{dy}{dx}\right)^2 = y\,dy/dx$

Q $y = Ae^{mx} + Be^{-mx}$

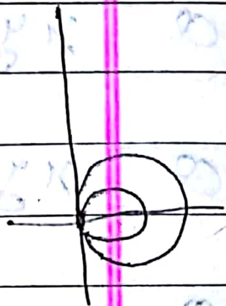
Q $y = Ae^{-x}\cos 2x$ is sol^n of $d^2y/dx^2 + 2dy/dx + 5y = 0$

Q $x = a\cos(\omega + \varepsilon)$

Q $x = a\cos(\omega t + \varepsilon) + \lambda\frac{\sin pt}{\omega^2 - p^2}$ is sol^n of $\frac{d^2x}{dt^2} + \cancel{\cos}\ \omega^2 x = \lambda\sin pt$

Q find the diff eqⁿ of all circle passing through the origen and having their center on x-axis

Let $(a,0)$ be the center of circle eqⁿ of circle is



$$(x-a)^2+(y-0)^2=r^2 \quad \text{---(1)}$$

Since (1) pass through $(0,0)$

$$a^2=r^2$$

eqⁿ (1)

$$(x-a)^2+y^2=a^2 \quad \text{--- (11)}$$

Diff w r to x

$$2(x-a)+2y\,dy/dx=0$$

$$2y\,dy/dx=2(a-x)$$

$$y\,dy/dx=(a-x)$$

$$a=x+y\,dy/dx$$

Ac put vale in eq (11)

$$(x-x+y\,dy/dx)^2+y^2=(x+y\,dy/dx)^2$$

$$y^2\left(\frac{dy}{dx}\right)^2+y^2=x^2+y^2\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}$$

$$y^2-x^2-2xy\frac{dy}{dx}=0$$

⇒ Differential eqn of first order and first degree can be expressed in the from of $Mdx + Ndy = 0$
where M & N are function of x, y and constant

⇒ Some method for solving differential eqn of first order and first degree

(1) Variable Separable method : Let Mdx and Ndy be the diff eqn of First degree and first order. Where M & N are the function of x & y and constant

If the eqn can be put in the from of $f_1(x)dx + g_1(y)dy = 0$
where $f_1(x)$ is function of x (constant y) and $g_1(x)$ is function of y (constant x)

Then we say that Varable is separable and its soln is obtained by
$$\int f_1(x)dx + \int g_1(y)dy = \text{constant}$$

Q. Solve the following eqn

(I) $x(y^2+1)dx + y(x^2+1)dy = 0$

$$dy(x^2+1)y = -x(y^2+1)dx$$

$$\frac{2y\,dy}{y^2+1} = \frac{-2x\,dx}{x^2+1}$$

$$\int \frac{2y\,dy}{y^2+1} = \int \frac{-2x\,dx}{x^2+1}$$

$$\log(y^2+1) = -\log(x^2+1) + \log c$$

$$\log(y^2+1) + \log(x^2+1) = \log c$$

$$\log\left((x^2+1)(y^2+1)\right) = \log c$$

$$(x^2+1)\cdot(y^2+1) = c$$

(II) $x(1+y^2)dx - y(1+x^2)dy = 0$

$$x(1+y^2)dx = y(1+x^2)dy$$

$$\frac{2x\,dx}{x^2+1} = \frac{2y\,dy}{y^2+1}$$

$$\log(x^2+1) = \log(y^2+1) + \log c$$

$$\log(x^2+1) - \log(y^2+1) = \log c$$

$$\frac{x^2+1}{y^2+1} = c$$

Q $(1+x^2)dy = (1+y^2)dx$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

$$\int\frac{dy}{1+y^2} = \int\frac{dx}{1+x^2}$$

$$\tan^{-1}(y) = \tan^{-1}(x) + \tan^{-1}(c)$$

$$\tan^{-1}(y) - \tan^{-1}(x) = \tan^{-1}c$$

$$\tan^{-1}\left(\frac{y-x}{1+yx}\right) = \tan^{-1}c$$

$$\tan^{-1}\left(\frac{y-x}{1+yx}\right) = \tan^{-1}c$$

$$\boxed{c = \frac{y-x}{1+yx}}$$

Q $\sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0$

$$\sqrt{1-x^2}\,dy = -\sqrt{1-y^2}\,dx$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{-dx}{\sqrt{1-x^2}}$$

$$\int\frac{dy}{\sqrt{1-y^2}} = -\int\frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1}(y) = -\sin^{-1}(x) + \sin^{-1}c$$

$$\boxed{\sin^{-1}(y) + \sin^{-1}(x) = \sin^{-1}c}$$

I.sc maths golu sir 8292654087

Q $x^2\frac{dy}{dx}+y=1$

$$x^2\frac{dy}{dx}=1-y$$

$$x^2dy=(1-y)dx$$

$$\frac{dy}{1-y}=\frac{dx}{x^2}$$

$$\int\frac{dy}{1-y}=\int\frac{dx}{x^2}$$

$$-\log(1-y)=-\frac{1}{x}+c$$

$$\log(1-y)=\frac{1}{x}+c$$

$$c=\log(1-y)-\frac{1}{x}$$

$$\boxed{cx=x\log(1-y)-1}$$

Q $x\,dy/dx+y=0$

$$x\,dy/dx=-y$$

$$x\,dy=-y\,dx$$

$$\frac{dy}{y}=-\frac{dx}{x}$$

$$\int\frac{dy}{y}=-\int\frac{dx}{x}$$

I.sc maths golu sir 8292654087

$$\log y = -\log x + \log c$$

$$\log(y) + \log x = \log c$$

$$\log(yx) = \log c$$

$$yx = c$$

Q $y^2 \, dy/dx + y = 1$

$$y^2 \frac{dy}{dx} = 1 - y$$

$$y^2 dy = (1-y)dx$$

$$\frac{y^2 dy}{1-y} = dx$$

$$\frac{y^2 - 1 + 1}{-(y-1)} = dx$$

$$-\left(\frac{y^2-1}{y-1} + \frac{1}{y-1}\right)dy = dx$$

$$-\left(y+1 + \frac{1}{y-1}\right)dy = dx$$

$$-\int\left((y+1) + \frac{1}{(y-1)}\right)dy = dx$$

$$-\left(\frac{y^2}{2} + y + \log(y-1)\right) = x + c$$

$$\boxed{c = x + y + \frac{y^2}{2} + \log(y-1)}$$

Q $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$\frac{dy}{dx} = e^{x} \cdot e^{-y} + x^2 e^{-y}$

$\frac{dy}{dx} = e^{-y}(e^{x} + x^2)$

$\frac{dy}{e^{-y}} = (e^{x} + x^2)dx$

$e^{y}dy = (e^{x} + x^2)dx$

$\int e^{y}dy = \int (e^{x} + x^2)dx$

$e^{y} = e^{x} + \frac{x^3}{3} + c$

$\boxed{3e^{y} - 3e^{x} - x^3 = 3c}$

Q $\frac{dy}{dx} = e^{x+y} + x^2 e^{y}$

$\frac{dy}{dx} = e^{y}(e^{x} + x^2)$

$\frac{dy}{e^{y}} = (e^{x} + x^2)dx$

$-\frac{1}{e^{y}} = e^{x} + \frac{x^3}{3} + c$

$e^{x} + \frac{1}{e^{y}} + \frac{x^3}{3} + c = 0$

I.sc maths golu sir 8292654087

Q Solve the following diff eqn

(1) $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

(II) $x\frac{dy}{dx} - y = 2x^2 y$

(III) $a(x\,dy + 2y\,dx) = xy\,dy$

(IV) $y - x\frac{dy}{dx} = 2\left(y^2 + \frac{dy}{dx}\right)$

(V) $y\,dx - x\,dy = xy\,dx$

(VI) $y\,dx + x\,dy = xy\,dx$

(VII) $3e^x \tan y\,dx + (1-e^x)\sec^2 y\,dx$

(VIII) $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$

(IX) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

(X) $\log\left(\frac{dy}{dx}\right) = ax + by$

(XI) $\frac{dy}{dx}(1+x^2)/\tan^{-1} x + y = 0$

(XII) $\frac{dy}{dx} = \frac{y^2+y+1}{x^2+x+1}$

(13) $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$

(14) $x\sqrt{1-y^2}\,dx + y\sqrt{1-x^2}\,dy$

(15) $(2ax + x^2)\frac{dy}{dx} = (a^2 + 2ax)$

(16) $(x^2 - yx^2)\,dy + (y^2 + xy^2)\,dx = 0$

* Equation reduceeable from for Varable separable :-

There are certain equaction which cannot be directly put in the form of Varable separable but they can be reduce after certain Transfoma In the from of varable separable.

Q Solve $(x+y)^2 \frac{dy}{dx} = a^2$

put $x+y = z$

$$1 + \frac{dy}{dx} = z$$

$$\frac{dy}{dx} = \frac{dz}{d} - 1$$

Now
$$z^2\left(\frac{dz}{dx} - 1\right) = a^2$$

$$\frac{dz}{dx} - 1 = \frac{a^2}{z^2}$$

$$\frac{dz}{dx} = \frac{a^2}{z^2} + 1$$

$$\frac{dz}{dx} = \frac{a^2 + z^2}{z^2}$$

$$\frac{z^2 dz}{z^2 + a^2} = dx$$

$$\frac{z^2 + a^2 - a^2}{z^2 + a^2} dz = dx$$

$$\left(1 - \frac{a^2}{z^2+a^2}\right)dz = dx$$

$$\int dz - \int \frac{a^2}{z^2+a^2}dz = \int dx$$

$$z - \frac{a^2}{a}\tan^{-1}\left(\frac{z}{a}\right) = x + c$$

$$z - a\tan^{-1}\left(\frac{x+y}{a}\right) = x + c$$

$$\cancel{x}+y - a\tan^{-1}\left(\frac{x+y}{a}\right) = \cancel{x}+c$$

$$\boxed{y - a\tan^{-1}\left(\frac{x+y}{a}\right) = c}$$

Q $(x-y)^2\frac{dy}{dx} = a^2$

put $x-y = z$

$$1 - \frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dz}{dx}$$

Now $z^2\left(1-\frac{dz}{dx}\right) = a^2$

$$1 - \frac{dz}{dx} = \frac{a^2}{z^2}$$

$$\frac{dz}{dx} = 1 - \frac{a^2}{z^2}$$

$$\frac{dz}{dx} = \frac{z^2-a^2}{z^2}$$

$$\frac{z^2}{z^2-a^2}dz = dx$$

$$\left(\frac{z^2-a^2}{z^2-a^2}+\frac{a^2}{z^2-a^2}\right)dz = dx$$

$$\left(\frac{1}{1}+\frac{a^2}{z^2-a^2}\right)dz = dx$$

$$\int dz + \int \frac{a^2}{z^2-a^2} = \int dx$$

$$z + \frac{a^2}{2a}\log\left(\frac{z-a}{z+a}\right) = x+c$$

$$\cancel{x}+y+\frac{a}{2}\log\left(\frac{x-y-a}{x-y+a}\right) = \cancel{x}+c$$

$$\boxed{y+\frac{a}{2}\log\left(\frac{x-y-a}{x-y+a}\right) = c}$$

Q $\dfrac{dy}{dx} = (x+y)^2$

put $x+y = z$

$$1+\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx}-1$$

N. $\dfrac{dz}{dx}-1 = z^2$

$$\frac{dz}{dx} = z^2+1$$

$$\frac{dz}{z^2+1} = dx$$

$$\int \frac{dz}{z^2+1} = \int dx$$

$$\tan^{-1}(z) = x + c$$

$$\boxed{\tan^{-1}(x+y) - x = c}$$

Q $(y-x)\,dy/dx = a^2$

Put $y - x = z$

$$dy/dx - 1 = dz/dx$$

$$dy/dx = dz/dx + 1$$

$$z(dz/dx + 1) = a^2$$

$$\frac{dz}{dx} + 1 = \frac{a^2}{z}$$

$$\frac{dz}{dx} = \frac{a^2}{z} - 1$$

$$\frac{dz}{dx} = \frac{a^2 - z}{z}$$

$$\frac{z\,dz}{-(z-a^2)} = dx$$

$$\frac{z\,dz}{z-a^2} = -dx$$

$$\frac{z-a^2+a^2}{z-a^2}\,dz = -dx$$

$$\left(1+\frac{a^2}{z-a^2}\right)dz=-dx$$

$$\int 1\,dz+\int\frac{a^2}{z-a^2}dz=\int -dx$$

$$z+a^2\log(z-a^2)=-x+C$$

put $z=y-x$

$$y-\cancel{x}+a^2\log(y-x-a^2)=-\cancel{x}+c$$

$$y+a^2\log(y-x-a^2)=c$$

Q $\quad x\,dx+y\,dy+\dfrac{x\,dy-y\,dx}{x^2+y^2}=0 \quad —(1)$

put $\quad x=r\cos\theta$

$y=r\sin\theta$

$dx=-r\sin\theta+\cos\theta\, dr$

$dy=r\cos\theta+\sin\theta\, dr$

Now $\quad x\,dx+y\,dy=-r^2\cancel{\cos\theta\sin\theta}+r\cos^2\theta\, dr$

$+r^2\cancel{\sin\theta\cos\theta}+r\,dr\sin^2\theta$

$=r\,dr$

& $\quad x\,dy-y\,dx=r^2\cos^2\theta+r\,dr\,\cancel{r\cos\theta\sin\theta}$

$+r^2\sin^2\theta-r\,dr\,\cancel{r\cos\theta\sin\theta}$

$=r^2$

$x^2+y^2=r^2$

I.sc maths golu sir 8292654087

puting this value in eqn (1)

$$r\frac{dr}{d\theta} + \frac{r^2}{r^2} = 0$$

$$r\frac{dr}{d\theta} + 1 = 0$$

$$r\,dr = -d\theta$$

$$\frac{r^2}{2} = -\theta$$

$$r^2 + 2\theta = c$$

$$x^2 + y^2 + 2\tan^{-1}(y/x) = c$$

Q Solve the following diff eqn

(1) $(x+y+1)\frac{dy}{dx} = 1$

(2) $(x^2+2xy+y^2+1)\frac{dy}{dx} = x+y$

(3) $(x^2+2xy+y^2+1)\frac{dy}{dx} = 2(x+y)$

(4) $\left(\frac{x+y-a}{x+y-b}\right)\frac{dy}{dx} = \frac{x+y+a}{x+y+b}$

(5) $\frac{dy}{dx}+1 = e^{x-y}$

(6) $\frac{dy}{dx} = \sqrt{y-x}$

(7) $\frac{dy}{dx} = \sin(x+y)$

(8) $\cos(x+y)\frac{dy}{dx} = 1$

(9) $\frac{dy}{dx} = \sin(x+y)+\cos(x+y)$

(10) $x\,dx+y\,dy = \frac{a^2(x\,dy-y\,dx)}{x^2+y^2}$

(11) $x\,dx+y\,dy = \frac{y\,dx-x\,dy}{x^2+y^2}$

(12) $\frac{x\,dx+y\,dy}{x\,dy-y\,dx} = \sqrt{\frac{a^2-x^2-y^2}{x^2+y^2}}$

⇒ Homogeneous differential equation

* Homogeneous function:- Any function $f(x,y)$ is called Homogeneous if

$$f(tx,ty) = t^n f(x,y)$$

Then we say that it's homogeneous function of degree (n)

Ex:- $f(x,y) = x^2+xy+y^2$

put $x = tx$

$y = ty$

$$f(tx,ty) = t^2x^2 + t^2xy + t^2y^2$$

$$= t^2(x^2+xy+y^2)$$

$$f(tx,ty) = t^2 f(x,y)$$

$f(x,y)$ is homogeneous function of degree 2.

⇒* Homogeneous differential equation

Differential eq^n of the from

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$ where $f(x,y)$ & $g(x,y)$

are homogeneous function of same degree

Ex $dy/dx = \frac{x+y}{x-y}$ / $\frac{dy}{dx} = \frac{x^3-y^3}{x^2y}$

I.sc maths golu sir 8292654087

⇒ Rule for solving homogeneous differential eqn :-

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$ in given

homogeneous diff eqn

after this substution our eqn reduce in the from of varable separable solve then of replace $v = y/x$

we get required soln of given homogeneous diff eqn

Q $x^2y\,dx - (x^3+y^3)dy = 0$

$$x^2y\,dx = (x^3+y^3)dy$$

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3}$$

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now

$$V + x\frac{dV}{dx} = \frac{x^2 \cdot Vx}{x^3 + (Vx)^3}$$

$$x\frac{dV}{dx} = \frac{x^3 V}{x^3(1+V^3)} - V$$

$$x\frac{dV}{dx} = \frac{V}{1+V^3} - V$$

$$x\frac{dV}{dx} = \frac{\cancel{V} - \cancel{V} - V^4}{1+V^3}$$

$$\frac{(1+V^3)}{V^3}dV = -\frac{dx}{x}$$

$$\left(\frac{1}{V^4} + \frac{1}{V}\right)dV = -\frac{dx}{x}$$

intrigation both side

$$-\frac{1}{2V^2} + \log V = -\log x + C$$

$$\log x + \log V - \frac{1}{2V^2} = C$$

Eliminat V

$$\log x \cdot \frac{y}{x} + \frac{y}{x} - \frac{x^2}{2y^2} = C$$

$$\boxed{\log y + \frac{y}{x} - \frac{x^3}{2y^3} = C}$$

Q $(x^3+3xy^2)dx + (y^3+3x^2y)dy = 0$

$$(x^3+3xy^2)dx = -(y^3+3x^2y)dy$$

$$\frac{dy}{dx} = -\frac{x^3+3xy^2}{y^3+3x^2y}$$

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = -\frac{x^3+3xv^2x^2}{(vx)^3+3x^2vx}$$

$$v + x\frac{dv}{dx} = -\frac{\cancel{x^3}(1+3v^2)}{\cancel{x^3}(v^3+3v)}$$

$$x\,dv/dx = -\frac{(1+3v^2)}{v^3+3v} - v$$

$$= \frac{-1-3v^2-v^4-3v^2}{v^3+3v}$$

$$x\frac{dv}{dx} = -\frac{(1+6v^2+v^4)}{v^3+3v}$$

$$\frac{v^3+3v}{v^4+6v^2+1}dv = -\frac{dx}{x}$$

Intrigrate both side

$$\int\frac{v^3+3v}{v^4+6v^2+1}dv = \int\frac{-dx}{x}$$

$$\frac{1}{4}\log(v^4+6v^2+1) = -\log x + \log c$$

$$\log(v^4+6v^2+1) = 4\log\left(\frac{c}{x}\right)$$

$$\log(v^4+6v^2+1) = \log\left(\frac{c}{x}\right)^4$$

$$v^4+6v^2+1 = \left(\frac{c}{x}\right)^4$$

eliminat v

$$\frac{y^4}{x^4} + 6\frac{y^2}{x^2} + 1 = \frac{c^4}{x^4}$$

$$y^4 + 6x^2y^2 + x^4 = c^4$$

$$\boxed{y^4 + x^4 + 6x^2y^2 = c^4}$$

Q Solve the following differential eqn

(I) $(x^3-3xy^2)dx \mp (y^3-3x^2y)dy$

(II) $(x^3+y^3)dx = xy^2dy$

(III) $(x^2-y^2)dy/dx = 2xy$

(IV) $2xy\,dy = (x^2+y^2)dx$

(V) $(x^2+y^2)\frac{dy}{dx} = xy$

(VI) $y^2+x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

(VII) $x\,dy - y\,dx - \sqrt{x^2+y^2}\,dx = 0$

(8) $y\,dx - x\,dy = \sqrt{x^2-y^2}\,dx$

(9) $(x+y)\frac{dy}{dx} + (x-y) = 0$

(10) $\frac{dy}{dx} = \frac{y(x-2y)}{x(x-3y)}$

(11) $\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$

(12) $\frac{dy}{dx} + \frac{2y}{x} = \frac{y^3}{x^3}$

(13) $x^2\,dy + y(x+y)\,dx = 0$

(14) $(x^2-2xy)\,dy + (x^2-3xy+2y^2)\,dx =$

(15) $\frac{dy}{dx} = \frac{y}{x} + \tan^{-1}\left(\frac{y}{x}\right)$

(16) $x\cos\frac{y}{x}(y\,dx + x\,dy) = y\sin\frac{y}{x}\left(x\frac{dy}{dx} - y\,dx\right)$

(17) $(1+e^{x/y})\,dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$

(18) $(1+3e^{x/y})\,dx + 3e^{x/y}\left(\frac{x}{y}-1\right)dy = 0$

(19) $x^2\,dy/dx + xy = y^2$

(20) $(4x+3y)\,dx + (x-2y)\,dy = 0$

(21) $x\,dy/dx - y = x\sqrt{x^2+y^2}$

* Non Homogeneous differential equaction

Those differential eqⁿ which not Homogeneous are called non-Homogeneous differential eqⁿ

There are certain Types of non Homogeneous differential eqⁿ which can be reduce in the from of homogeneous differential eqⁿ

Type-1

$$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}, \quad \frac{a}{a'} \neq \frac{b}{b'}$$

Put $x = x'+h$ & $y = y'+k$

$dx = dx'$ & $dy = dy'$

Now

$$\frac{dy'}{dx'} = \frac{a(x'+h)+b(y'+k)+c}{a'(x'+h)+b'(y'+k)+c}$$

$$= \frac{ax'+by'+ah+bk+c}{a'x'+b'y'+a'h+b'k+c}$$

Chose h & k

$ah+bk+c=0 \quad \text{---} (i)$

$a'h+b'k+c=0 \quad \text{---} (ii)$

$$\frac{dy'}{dx'} = \frac{ax'+by'}{a'x'+b'y'}$$ It is homogeneous differential equation

Put $y' = vx'$

differentiate both side

$$\frac{dy'}{dx'} = v + x'\frac{dv}{dx'}$$

After this substitution our eqn reduce in the form of variable seperable in terms v & x'

Solve them,

we get soln in terms of v & x'

Now Replace v by $\frac{y'}{x'}$

& Replace $y' = y-k$

$x' = x-h$

whers h & k are soln of eqn (I) & (II)

I.sc maths golu sir 8292654087

Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

Sol

$\frac{1}{2} \neq \frac{2}{1}$

It is non Homogeneous eqn

put $x = x'+h \Rightarrow dx = dx'$

$y = y'+k \Rightarrow dy = dy'$

Now

$$\frac{dy'}{dx'} = \frac{x'+h+2(y'+k)-3}{2(x'+h)+y'+k-3}$$

$$\frac{dy'}{dx'} = \frac{x'+2y'+h+2k-3}{2x'+y'+2h+k-3}$$

$$h+2k-3 = 0 \quad \text{---(i)}$$
$$2h+k-3 = 0 \quad \text{---(ii)}$$

$$\frac{h}{-6+3} = \frac{k}{-6+3} = \frac{1}{1-4}$$

$$\frac{h}{-3} = \frac{k}{-3} = \frac{1}{-3}$$

$$h = 1 \;\&\; k = 1$$

Now

$$\frac{dy'}{dx'} = \frac{x'+2y'}{2x'+y'}$$

It is Homogeneous differential eqn

Now Put $y' = vx'$

$$\frac{dy'}{dx'} = v + x'\frac{dv}{dx'}$$

$$\therefore \quad v + x'\frac{dv}{dx'} = \frac{x' + 2vx'}{2x' + vx'}$$

$$v + x'\frac{dv}{dx'} = \frac{x'(1+2v)}{x'(2+v)}$$

$$x'\frac{dv}{dx'} = \frac{1+2v}{2+v} - v$$

$$x'\frac{dv}{dx'} = \frac{1+\cancel{2v}-\cancel{2v}-v^2}{2+v}$$

$$x'\frac{dv}{dx'} = \frac{1-v^2}{2+v}$$

$$\frac{2+v}{1-v^2}dv = \frac{dx'}{x'}$$

$$\int\left(\frac{2}{1-v^2} + \frac{v}{1-v^2}\right)dv = \frac{dx'}{x'}$$

$$\int\left(\frac{2}{1-v^2} + \frac{1}{-2}\,\frac{-2}{1-v^2}\right)dv = \frac{dx'}{x'}$$

$$\left(\frac{2}{1-v^2} - \frac{1}{2}\,\frac{-2}{1-v^2}\right)dv = \frac{dx'}{x'}$$

Intrigation both side

$$\int \frac{2}{1-v^2}\,dv - \frac{1}{2}\int \frac{-2}{1-v^2}\,dv = \int \frac{dx'}{x'}$$

$$\frac{2}{2}\log\left(\frac{1-v}{1+v}\right) - \frac{1}{2}\log(1-v^2) = \log(x') + c$$

$$\log\left(\frac{1-v}{1+v}\right) - \frac{1}{2}\log(1-v^2) = \log x' + \log c$$

$$\log\left(\frac{1-v}{1+v}\right)^2 - \log(1-v^2) = \log(x')^2 - \log c$$

$$\log(1-v)^2 - \log(1+v)^2 - \log(1-v^2) = \log x'^2 - \log c$$

$$\log(1-v)^2 - \left(\log\left(\frac{(1+v)^2}{1-v^2}\right)\right)$$

$$\log(1-v)^2 - \log\left(\frac{1+v}{1-v}\right) = \log x'^2 - \log c$$

$$\log(1-v)^2 - \log(1+v) + \log(1-v) = \log x'^2 - \log c$$

$$\log(1-v)^3 - \log(1+v) = \log x'^2 - \log c$$

$$\log\left(\frac{(1-v)^3}{1+v}\right) = \log\left(\frac{x'^2}{c}\right)$$

$$\frac{c}{x'^2} = \frac{1+v}{(1-v)^3}$$

Replace $v = \frac{y'}{x'}$

$$\frac{c}{x'^2} = \frac{1 + \frac{y'}{x'}}{\left(1 - \frac{y'}{x'}\right)^3}$$

& Replace $y' = y - k$ & $x' = x - h$

$y' = y$ & $x' = x$

$$\frac{c}{x^2} = \frac{1 + \frac{y}{x}}{\left(1 - \frac{y}{x}\right)^3}$$

Q Solve the following differential eqn

(i) $(x+y-3)dy = (y-x+1)dx$

(ii) $(x-y)dy = (x+y+1)dx$

(iii) $(3x+y-5)dy = 2(x+y-1)dx$

(iv) $(4x-y+3)dy + (2x-3y-1)dx = 0$

(v) $(6x-5y+4)dy = (2x-y+1)dx$

(vi) $(y-3x+3)dy = (2y-x-4)dx$

Type-II Differential equation is of the from

$$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'} \quad \text{where } \frac{a}{a'} = \frac{b}{b'} = k$$

Let $\frac{a}{a'} = \frac{b}{b'} = k$

$a = ka' \quad b = kb'$

Put value of a & b in the given eqn

$$\frac{dy}{dx} = \frac{ka'x + kb'y + c}{a'x+b'y+c'}$$

$$= \frac{k(a'x+b'y)+c}{a'x+b'y+c'}$$

Put $a'x + b'y = z$

$$\frac{dz}{dx} = a' + b'\frac{dy}{dx}$$

$$b'\frac{dy}{dx} = \frac{dz}{dx} - a'$$

$$\frac{dy}{dx} = \frac{1}{b'}\frac{dz}{dx} - \frac{a'}{b'} = \frac{kz+c}{z+c'}$$

It is the from of varable-sepparable in terms z & x solve then & replace z by $a'x+b'y$ we get required soln

Q $\frac{dy}{dx} = \frac{6x-2y+7}{3x-y+4}$

$\frac{6}{3} = \frac{2}{1}$

It is non homogeneous eqn

$\frac{dy}{dx} = \frac{2(3x-y)+7}{3x-y+4}$

Put $3x-y = z$

$3 - \frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dy}{dx} = 3 - \frac{dz}{dx}$

Now

$3 - \frac{dz}{dx} = \frac{2z+7}{z+4}$

$\frac{dz}{dx} = 3 - \frac{2z+7}{z+4}$

$= \frac{3z+12-2z-7}{z+4}$

$\frac{dz}{dx} = \frac{z+5}{z+4}$

I.sc maths golu sir 8292654087

$$\frac{z+4}{z+5}\,dz = dx$$

$$\left(\frac{z+5}{z+5} - \frac{1}{z+5}\right)dz = dx$$

$$\left(1 - \frac{1}{z+5}\right)dz = dx$$

Intrigation both side

$$z - \log(z+5) = x + c$$

Now Replac z by $3x-y$

$$3x-y-(\log(3x-y+5)) = x+c$$

$$2x-y-\log(3x-y+5) = c$$

Q Solve following the differential eqn

(1) $\dfrac{dy}{dx} = \dfrac{4x+6y+5}{3y+2x+4}$

(2) $(x+y+1)dy - (2x+2y-1)dx = 0$

(3) $(2x+3y-5)dy + (2x+3y-1)dx = 0$

(4) $(2x+4y+1)dy - (x+2y-1)dx = 0$

(5) $(2x+4y-3)dy = (x+2y-3)dx$

* Linear differential equation

& differential eq^n is of the form

$$\frac{dy}{dx} + Py = Q$$

where P & Q are function of x alone y (not y) are called linear diff eq^n

Its Solution is given by

$$y \cdot e^{\int P dx} = \int (Q e^{\int P dx}) dx + C$$

$$\Rightarrow \quad \frac{dy}{dx} + Py = Q \quad \text{---(1)}$$

mutiply both side of the eq^n by $e^{\int P dx}$

$$e^{\int P dx} \frac{dy}{dx} + e^{\int P dx} Py = Q e^{\int P dx}$$

$$\frac{d(y \cdot e^{\int P dx})}{dx} = Q e^{\int P dx}$$

$$d(y \cdot e^{\int P dx}) = Q e^{\int P dx} \cdot dx$$

Intrigation both side

$$\int d(y.e^{\int P dx}) = \int Q e^{\int P dx} dx$$

$$y e^{\int P dx} = \int Q e^{\int P dx} dx + C$$

Q

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

$$\cos^2 x\left(\frac{dy}{dx} + \frac{y}{\cos^2 x}\right) = \tan x$$

$$\frac{dy}{dx} + \sec^2 x\, y = \frac{\tan x}{\cos^2 x}$$

$$\frac{dy}{dx} + \sec^2 x . y = \tan x . \sec^2 x$$

It is Linear differential eqn

Nc

$$P = \sec^2 x \;\&\; Q\; \tan x . \sec^2 x$$

$$y . e^{\int P dx} = \int Q e^{\int P dx} dx + C$$

$$y . e^{\int \sec^2 x dx} = \int \tan x . \sec^2 x . e^{\int P dx} dx + C$$

$$y . e^{\tan x} = \int \tan x . \sec^2 x\, e^{\tan x} dx + c$$

$$y . e^{\tan x} = \int \tan x . \sec^2 x\, e^{\tan x} dx + c$$

I.sc maths golu sir 8292654087

Put $\tan x = z$

$\sec^2 x\,dx = dz$

$$y \cdot e^{z} = \int z \cdot e^{z}\,dz$$

$$y \cdot e^{z} = z e^{z} - \int e^{z}\,dz$$

$$y \cdot e^{z} = z \cdot e^{z} - e^{z}$$

$$y \cdot \cancel{e^{z}} = \cancel{e^{z}}(z-1)$$

$$y = z - 1 \quad + c$$

Replace z by $\tan x$

$$y = (\tan x - 1) + c$$

Solve the following differential eqn

(i) $\sec^2 x \, dy/dx + y = \cot x$

(ii) $(1+x^2)\, dy/dx + y = \tan^{-1} x$

(iii) $dy/dx + y/x = \sin x^2$

(iv) $dy/dx + y \cot x = 2\cos x$

(v) $x\dfrac{dy}{dx} = y - \cos\left(\dfrac{1}{x}\right)$

(vi) $\dfrac{dy}{dx} + y \sec x = \tan x$

* Linear differential eq^n in a another form

differential eq^n is of the from

$$\frac{dx}{dy} + Px = Q$$ where P & Q

are function of of y but not x are linear diff eq^n in terms of x and y and its sol^n is given by

$$x e^{\int P dy} = \int Q . e^{\int P dy} dy + c$$

Q $$(1+y^2)\frac{dx}{dy} = \tan^{-1}y - x$$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

It is linear differential eq^n

$$P = \frac{1}{1+y^2} \qquad Q = \frac{\tan^{-1}y}{1+y^2}$$

So

$$x . e^{\int P dy} = \int Q e^{\int P dy} dy + c$$

$$x . e^{\int \frac{1}{1+y^2} dy} = \int \frac{\tan^{-1}y}{1+y^2} e^{\int \frac{1}{1+y^2} dy} dy + c$$

$$x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} e^{\tan^{-1}y} dy$$

Put $\tan^{-1}y = z$

$$\frac{1}{1+y^2} dy = dz$$

$$x \cdot e^{z} = \int z \cdot e^{z} dz$$

$$x \cdot e^{z} = z e^{z} - e^{z} + c$$

$$x \cancel{e^{z}} = \cancel{e^{z}}(z-1) + c$$

$$x = (z-1) + c$$

Replace z by $\tan^{-1}y$

$$x = (\tan^{-1}y - 1) + c$$

(Q) $\frac{dy}{dx}(x+y+1) = 1$

(Q) $(x+3y+2)\frac{dy}{dx} = 1$

(Q) $y + \left(x - \frac{1}{y}\right)\frac{dy}{dx} + y = 0$

(Q) $(1+y^2)dx = (yx + y^3 + y)dy$

* Equation Reducible to Linear from BERNOULLI's Equations

differential eq^n is of the from

$$\frac{dy}{dx} + Py = Qy^n$$

where P & Q are function of x are called Bernoulli's equation

Bernoulli's eq^n is directly not in the form of linear diff eq^n but they can reduce in to the from of linear diff eq^n after Substitution

$$\frac{dy}{dx} + Py = Qy^n$$

$$\frac{1}{y^n}\frac{dy}{dx} + \frac{P.y}{y^n} = Q$$

$$\frac{1}{y^n}\frac{dy}{dx} + \frac{P}{y^{n-1}} = Q$$

Put $\frac{1}{y^{n-1}} = z$

$$(1-n)\frac{1}{y^n}\frac{dy}{dx} = \frac{dz}{dx}\frac{1}{1-n}$$

$$\frac{1}{1-n}\frac{dz}{dx} + Pz = Q$$

It is Linear differential eqn

$$\frac{dz}{dx} + (1-n)z \cdot P = Q(1-n)$$

It is Linear diff eqn in terms of z & x

Solve then and Replace z by $\frac{1}{y^{n-1}}$

we get required soln

Q $\frac{dy}{dx} + xy = x^3y^3$

$$\frac{1}{y^3}\frac{dy}{dx} + \frac{xy}{y^3} = x^3$$

$$\frac{1}{y^3}\frac{dy}{dx} + \frac{x}{y^2} = x^3$$

Put $\frac{1}{y^2} = z$

$$-2\frac{1}{y^3}\frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{1}{y^3}\frac{dy}{dx} = -\frac{1}{2}\frac{dz}{dx}$$

$$-\frac{1}{2}\frac{dz}{dx} + xz = x^3$$

$$\frac{dz}{dx} + -2x \cdot z = -2x^3$$

It is ~~diff~~ Linear diff eq^n

$P = -2x$ & $Q = -2x^3$

given sol^n

$$z \cdot e^{\int -2x dx} = \int -2x^3 e^{\int -2x dx} dx$$

$$z \cdot e^{-x^2} = \int -2x^3 e^{-x^2} dx$$

$$z \cdot e^{-x^2} = \int -2x \cdot x^2 \cdot e^{-x^2} dx$$

Put $-x^2 = z'$

$-2x dx = dz'$

$$z \cdot e^{z'} = \int -z' \cdot e^{z'} dz$$

$$z \cdot e^{z'} = -\int z' e^{z'} dz$$

$$z \cdot \cancel{e^{z'}} = -\cancel{e^{z'}}(z' - 1) + C$$

$$z = -(z' - 1) + C$$

Reple. $z' = y - x^2$

$$z = -(-x^2-1)+C$$

$$z = x^2+1+C$$

Replace z by $\frac{1}{y^2}$

$$\boxed{\frac{1}{y^2} = x^2+1+C}$$

Solve the following diff eqn

(1) $(x^3y^2+xy)dx = dy$

(2) $\frac{dy}{dx}+1 = e^{x+y}$

(3) $\frac{dy}{dx}+\frac{y}{x} = \frac{y^2}{x^2}$

(4) $\frac{dy}{dx}+\frac{y\log y}{x} = \frac{y}{x^2}(\log y)^2$

(5) $\frac{dy}{dx}+x\sin 2y = x^2\cos^2 y$

(1) $dx - xy(1+xy^2)dy = 0$

$$\frac{dy}{dx}+\frac{1}{x}\sin 2y = x^3\cos^2 y$$

* Exact Differential equaction

The differential equaction of first order and first degree can be writen in the form of

$$Mdx + Ndy = 0$$

If $\frac{d(M)}{dy} = \frac{d(N)}{dx}$

then we say that it is exact diff eq^n and it's sol^n is obtained in the following step

Step I $\int M dx$

Treating y as constant

Step II $\int N dy$

Which do not Involve x
(Treating x as constant)

Step III Step I + Step II = constant

Q Solve $(2x-y+1)dx + (2y-x-1)dy = 0$

$(2x-y+1)dx + (-x+2y-1)dy = 0$

It is the from of $Mdx + Ndy = 0$

$M = 2x-y+1$

$N = -x+2y-1$

$\frac{\delta M}{\delta x} = 2 \quad \frac{\delta N}{\delta y} = 2$

Step-I $\int M dx$

$= \int (2x-y+1)dx$

$= x^2 - yx + x$

Step II : $\int N dy$ whe x-term Not allow

$= \int (2y-1)dy$

$= y^2 - y$

Step III : Step I + Step II = const

$x^2 - yx + x + y^2 - y = c$

Q. $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$

It is the from $Mdx + Ndy = 0$

$$M = x^2 - 4xy - 2y^2$$

$$\frac{\delta M}{\delta y} = -4x - 4y$$

$$N = y^2 - 4xy - 2x^2$$

$$\frac{\delta N}{\delta x} = -4y - 4x$$

$$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

It is Exact diff eqn

Step-I $\int (x^2 - 4xy - 2y^2)dx + \int y^2 dy = c$

$$\frac{x^3}{3} - \frac{4yx^2}{2} - 2y^2x + \frac{y^3}{3} = c$$

$$\frac{x^3}{3} + \frac{y^3}{3} - 2yx^2 - 2y^2x = c$$

Solve the following differential eqn

(I) $(2x+3y-5)\frac{dy}{dx} + (3x+2y-5) = 0$

(II) $\frac{dy}{dx} = \frac{2x-y+1}{x+2y-3}$

(III) $(2x+3y-5)dy + (3x+2y-5)dx = 0$

(IV) $(4x+3y+1)dx + (3x+2y+1)dy = 0$

(V) $\frac{dy}{dx} = \frac{x-2y+5}{2x+y-1}$

(VI) $(ax+hy+g)dx + (hx+by+f)dy = 0$

* INTEGRATION FACTOR

Integration factor in special case

An equation which is not exat can be made exact by multiply same sutable funtion such ~~that~~ function are called I.F

⇒ Some Rules for finding I.F

Rule-1 If $Mdx+Ndy=0$ be a homogeneous differential equation and $Mx+Ny \neq 0$ then $\frac{1}{Mx+Ny}$ is an I.F

Q Solve $(x^2y-2xy^2)dx+(3x^2y-x^3)dy=0$ -1

It is homogeneous diff eq^n ~~the~~ from of $Mdx+Ndy$

$$M = x^2y-2xy^2$$

$$\frac{\delta M}{\delta y} = x^2-4xy$$

$$\frac{\delta N}{\delta x} = 3x^2y-x^3$$

$$= yx^3-\frac{x^4}{4}$$

$\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$

It is Not Exact diff eqn

$Mx + Ny = (x^2y - 2xy^2)x + y(3x^2y - x^3)$

$= \cancel{x^3y} - 2x^2y^2 + 3x^2y^2 - \cancel{x^3y}$

$= x^2y^2 \neq 0$

$\frac{1}{Mx+Ny} = \frac{1}{x^2y^2}$ is I.F

Multiply in eqn (1) by I.F

$\frac{x^2y - 2xy^2}{x^2y^2}dx + \frac{3x^2y - x^3}{x^2y^2}dy = 0$

$\left(\frac{1}{y} - \frac{2}{x}\right)dx + \left(\frac{3}{y} - \frac{x}{y^2}\right)dy = 0$

It is Exact differential eqn

given sol^n

$\int\left(\frac{1}{y} - \frac{2}{x}\right)dx + \int\frac{3}{y}dy - \int\frac{x}{y^2}dy = c$

$\frac{x}{y}\ \cancel{\log y} - 2\log x + 3\log y + \frac{x}{y} = c$

$\frac{x}{y}\ \cancel{\log y} + \log\left(\frac{y^3}{x^2}\right) = c$

Q Solve following diff eqn

$$x^2y\,dx-(x^3+y^3)dy=0$$

Rule-II

Let $Mdx+Ndy=0$ be a differential eqn of the eqn can be expressed in the form of $y f_1(xy)dx+x f_2(xy)dy=0$ and $Mx-Ny\neq 0$ then $\frac{1}{Mx-Ny}$ is an I.F

Q $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$

It is the from of $Mdx+Ndy=0$

$$M=xy^2+2x^2y^3$$

$$\frac{\delta M}{\delta y}=2xy+6x^2y^2$$

& $N=x^2y-x^3y^2$

$$\frac{\delta N}{\delta x}=2xy-3x^2y^2$$

$$\therefore \frac{\delta M}{\delta y}\neq\frac{\delta N}{\delta x}$$

It is not Exact diff eq

$$Mx - Ny = (xy^2 + 2x^2y^3)x - (x^2y - x^3y^2)y$$

$$= \cancel{x^2y^2} + 2x^3y^3 - \cancel{x^2y^2} + x^3y^3$$

$$= 3x^3y^3$$

$$\frac{1}{Mx - Ny} = \frac{1}{3x^3y^3} \text{ is I.F}$$

multiplying by in eq^n (1) by I.F

$$\frac{y(xy + 2x^2y^2)}{3x^3y^3} dx + \frac{x(xy - x^2y^2)}{3x^3y^3} dy = 0$$

$$= \left(\frac{1}{3x^2y} + \frac{2}{3x}\right)dx + \left(\frac{1}{3xy^2} - \frac{1}{3y}\right)dy = 0$$

It is Exact diff eq^n

It is given sol^n is

$$\int\left(\frac{1}{3x^2y} + \frac{2}{3x}\right)dx + \int -\frac{1}{3y}dy = c$$

$$\frac{1}{3y}\int\frac{1}{x^2}dx + \frac{2}{3}\int\frac{1}{x}dx - \frac{1}{3}\int\frac{1}{y}dy = c$$

$$-\frac{1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = -c$$

$$\frac{1}{3xy} - \frac{2}{3}\log x + \frac{1}{3}\log y = c$$

I.sc maths golu sir 8292654087

Solve the following differential equation

(I) $(xy\sin xy + \cos xy)y\,dx + (xy\sin xy - \cos xy)x\,dy = 0$

(II) $(x^4y^4 + x^2y^2 + xy)y\,dx + (x^4y^4 - x^2y^2 + xy)x\,dy$

Rule-III If $\frac{1}{N}\left(\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x}\right)$ is $f^n(x)$ alone (not y) then $e^{\int f(x)dx}$ is an I.F of $Mdx + Ndy = 0$

Rule IV If $\frac{1}{M}\left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right)$ is $f^n(y)$ alone (y) not (x) then $e^{\int f(y)dy}$ is an I.F of $Mdx + Ndy = 0$

(1) $(x^2+y^2+2x)dx + 2y\,dy = 0$

It is the form of $Mdx + Ndy = 0$

$M = x^2+y^2+2x$

$\frac{\delta M}{\delta y} = 2y$

& $N = 2y$

$\frac{\delta N}{\delta x} = 0$

$\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$

It is not Exact diff eqn

$\therefore \frac{1}{N}\left(\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x}\right)$

$\frac{1}{2y}(2y - 0) = 1$

$f(x) = 1$

$\therefore e^{\int 1 dx} = e^{x}$ is the an I.F

multiply in eqn (1) by I.F

$(x^2 + y^2 + 2x)e^{x}dx + 2ye^{x}dy$

It is exact diff eqn

It is given soln

$\therefore \int(x^2e^{x} + y^2e^{x} + 2xe^{x})dx + \int 0\, dy$

$\int x^2e^{x}dx + \int y^2e^{x}dx + \int 2xe^{x}dx$

$x^2\int e^{x} - \cancel{\int 2xe^{x}dx} + y^2e^{x} + \cancel{\int 2xe^{x}dx}$

$= x^2e^{x} + y^2e^{x} = c$

I.sc maths golu sir 8292654087

Solve the following diff eq^n

(I) $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)dy=0$

(II) $(3x^2y^4+2xy)dx+(2x^3y^3-x^2)dy=0$

8-MARKS

Orthogonal Trajectory

⇒ Trajectory of curve :- Let $f(x,y)=0$ be a given family of curve's. If $g(x,y)=0$ intersect the given family of curves under definite rule then we say that $g(x,y)=0$ is trajectory of given family of curve $f(x,y)=0$

⇒ Orthogonal Trajectory :- If $g(x,y)=0$ intersect the given family of curve $f(x,y)=0$ orthogonal ($\perp^{cr}$) or at Right angle) then we say that $g(x,y)=0$ is orthogonal Trajectory of curve $f(x,y)=0$

Rule for finding orthogonal Trajectory in Cartesian form

(I) Let $f(x,y,c)=0$ ---(1) be the given family of curve

(II) Differentiate the given family of curve w r to x & eliminate arbitrary constant C

(III) We get differential eqn of given family of curve i.e $\quad f\left(x, y, \frac{dy}{dx}\right) = 0 \quad$ — (ii)

(iv) Replace dy/dx by $-dx/dy$

We get differential equation of orthogonal Trajectory.

Solve then we get required equation of orthogonal Trajectory

Q1 find orthogonal Trajectory of family of circle $x^2+y^2=a^2$

Soln $\quad x^2+y^2=a^2$ — (1)

diff both side w r to x

$$2x+2y\frac{dy}{dx}=0$$

& $\quad 2y\frac{dy}{dx} = -2x$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Replace dy/dx by $-dx/dy$

$$-\frac{dx}{dy} = -\frac{x}{y}$$

$$\frac{dx}{x} = \frac{dy}{y}$$

intrigation both side

$$\log x = \log y + \log c$$

$$\boxed{x = yc}$$

Q2 find the orthogonal Trajectory of the family of parabola $y^2 = 4ax$

S $y^2 = 4ax$ --- (1)

diff both side w r to x

$$2y\frac{dy}{dx} = 4a$$

It value put in eqn (1)

$$y^{\cancel{2}} = 2\cancel{y}\frac{dy}{dx}\cdot x$$

$$y = 2\frac{dy}{dx}\cdot x$$

$$\frac{dy}{dx} = \frac{y}{2x}$$

Replace dy/dx by $-dx/dy$

$$-\frac{dx}{dy} = \frac{y}{2x}$$

$$-dx \cdot 2x = y \cdot dy$$

intrigation both side

$$-x^2 = \frac{y^2}{2} + c^2$$

$$\frac{y^2}{2} + x^2 = c^2$$

$$\boxed{x^2 + \frac{y^2}{2} = c^2}$$

Solve the following orthogonal Trajectory

(I) $y = ax^2$

(II) $y^2 = 4a(x+a)$

(III) $xy = c^2$

(IV) $x^2 - y^2 = a^2$

(V) $x^{2/3} + y^{2/3} = a^{2/3}$

(VI) $x^2 + y^2 = 2ax$

(VII) $x^2 + y^2 = ay$

(VIII) $x^2 + y^2 + 2gx + c = 0$

(IX) $3xy = x^3 - a^3$

(X) $ay^2 = x^3$

(XI) $\left(\frac{dy}{dx}\right)^2 = \frac{a}{x}$

(XII) $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$

* Orthogonal Trajectory in polar form

(I) Let $f(r, \theta, c) = 0$ be the eqn of curve in polar form

(II) Differentiate the given eqn and eliminate c, we get differential eqn of given family of curves

(III) Replace $r\frac{d\theta}{dr}$ by $-\frac{1}{r}\frac{dr}{d\theta}$

or $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$

We get diff eqn of orthogonal Trajectory of given family of Curve Solve then we get required eqn of orthogonal Trajectory

Q find orthogonal Trajectory of
$r\theta = a$

Sol

$$r\theta = a \quad \text{---} \quad (1)$$

taking log both side

$$\log(r\theta) = \log a$$

$$\log r + \log \theta = \log a$$

diff both side w r to θ

$$\frac{1}{r}\frac{dr}{d\theta} + \frac{1}{\theta} = 0$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\frac{1}{\theta}$$

$$\frac{dr}{d\theta} = -\frac{r}{\theta}$$

Replace $dr/d\theta$ by $-r^2 d\theta/dr$

$$-r^2 \, d\theta/dr = -r/\theta$$

$$\frac{r\,d\theta}{dr} = \frac{1}{\theta}$$

$$\theta\, d\theta = \frac{dr}{r}$$

(2)

Integration both side

$$\frac{\theta^2}{2} = \log r + \log c$$

$$\frac{\theta^2}{2} = \log r.c$$

$$\log e^{\theta^2/2} = \log(r.c)$$

$$\boxed{rC = e^{\theta^2/2}}$$

Q find orthogonal Trajectory of family of cardoide

$$r = a(1+\cos\theta)$$

Sol^n

$$r = a(1+\cos\theta) \quad --- \quad (1)$$

Taking log both side

$$\log(r) = \log\{a(1+\cos\theta)\}$$

$$\log(r) = \log a + \log(1+\cos\theta)$$

diff both side w r to θ

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta} \cdot -\sin\theta$$

$$\frac{dr}{d\theta} = \frac{-r\sin\theta}{1+\cos\theta}$$

$$\frac{dr}{d\theta} = -r\frac{\cancel{2}\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{\cancel{2}\cos^2\frac{\theta}{2}}$$

$$\frac{dr}{d\theta} = -r\tan\frac{\theta}{2}$$

Replace $dr/d\theta$ by $-r^2 d\theta/dr$

$-r^2 d\theta/dr = \cancel{r} \tan\frac{\theta}{2}$

$$\frac{r}{dr} = \frac{\tan\frac{\theta}{2}}{d\theta}$$

$$\frac{dr}{r} = \frac{d\theta}{\tan\theta/2}$$

Intrigation both side

$$\int \frac{dr}{r} = \int \frac{\cos\theta/2}{\sin\theta/2} d\theta$$

$\log r = 2\log(\sin\theta/2) + \log c$

$\log r = \log(\sin\theta/2)^2 + \log c$

$$\boxed{r = c(\sin\theta/2)^2}$$

Q. Solve the following orthogonal trajectory

(1) $r = a^\theta$

(2) $r = a\theta$

$r^n = a^n \cos n\theta$

$r = a(1-\cos\theta)$

$r^n \sin n\theta = a^n$

$1+\cos\theta = 2ar$

* Differential eqⁿ of first order but not of the first degree

differential eqⁿ is of the form

$$\left(\frac{dy}{dx}\right)^n + Q_1\left(\frac{dy}{dx}\right)^{n-1} + Q_2\left(\frac{dy}{dx}\right)^{n-2} + \cdots + Q_n = 0$$

Where $Q_1 \cdot Q_2 \cdots Q_n$ are function of x any y are constant are called differential eqⁿ of first order in n^{th} degree

Put $\frac{dy}{dx} = p$

i.e $\quad p^n + Q_1 p^{n-1} + Q_2 p^{n-2} + \cdots + Q_n = 0 \quad \text{--- (1)}$

There are two Types of differential equaction of first order in n^{th} degree

(1) If the L.H.S of eqⁿ (1) can be resolve into linear factor in terms of derivative Equation solve for p
If the L.H.S of eqⁿ (1) can be factorized in to linear factor in terms of p then we say that eqⁿ solvable for p

Ex $\quad p^2 - p(e^x + e^{-x}) + 1 = 0$

Let $p^n + q_1 p^{n-1} + q_2 p^{n-2} + \cdots + q_n = 0$ be a differential equation of first order in n^{th} degree then the eqn can be factorized into linear factor in terms of P.

Let $(P - A_1)(P - A_2)(P - A_3) \cdots (P - A_n)$ be its factor

i.e

$$(P - A_1)(P - A_2) \cdots (P - A_n) = 0$$

where $A_1, A_2, \ldots A_n$ is function of x any y or constant

i.e $P - A_1 = 0,\ P - A_2 = 0, \ldots P - A_n = 0$

Each factor is diff eqn of first order & first degree.

Solve them

$f_1(x, y, c_1) = 0,\ f_2(x, y, c_2) \ldots$

$f_n(x, y, c_n) = 0$ be its soln

& combining these soln we get required soln of given diff eqn.

I.sc maths golu sir 8292654087

(P) $p^2 - p(e^x + e^{-x}) + 1 = 0$

$p^2 - pe^x - pe^{-x} + 1 = 0$

$p(p - e^x) - e^{-x}(p - e^x) = 0$

$(p - e^x)(p - e^{-x}) = 0$

$p - e^x = 0 \quad \& \quad p - e^{-x} = 0$

$p = e^x \quad \& \quad p = e^{-x}$

$\frac{dy}{dx} = e^x \quad \& \quad \frac{dy}{dx} = e^{-x}$

$dy = e^x dx \quad \& \quad dy = e^{-x} dx$

$y = e^x + c \quad \& \quad y = -e^{-x} + c$

$y - e^x - c = 0 \quad \& \quad y + e^{-x} - c = 0$

$(y - e^x - c)(y + e^{-x} - c) = 0$

It is require eqn

Q1 $p^2 - 2p\cosh x + 1 = 0$ where $\cosh x = \frac{e^x + e^{-x}}{2}$

Q2 $p^2 + 2py\cot x = y^2$

Q3 $yp^2 - (y-x)p - x = 0$

(4) $x^2p^2 + xyp - 6y^2 = 0$

(5) $p^2 + 2px - 3x^2 = 0$

(6) $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$

(7) $xyp^2 - (x^2 - y^2)p - xy = 0$

(8) $p^2 + 2px + py + 2xy = 0$

(9) $p^2 - (a+b)p + ab = 0$

(10) $p(p+y) = y(x+y)$

(11) $p^2 + yp = x^2 + xy$

(12) $p(p^2 + xy) = p^2(x+y)$

(13) $p^2 - 7p + 12 = 0$

$p^2 - 9p + 18 = 0$

$3p^2y^3 - 2xyp + 4y^2 - x^2 = 0$

* Equaction Solvable for y

If the eqⁿ can be expressed in the form of $y = f(p.x)$ then we say that eqⁿ is solvable by y and it's solⁿ is given by

$y = f(p.x)$ --- (1)

diff w r to x

$$\frac{dy}{dx} = f'(p x)\cdot\frac{dp}{dx}$$

put $dy/dx = p$

$$p = f'(p.x)\frac{dp}{dx}$$

ie $F\left(p, x, \frac{dp}{dx}\right) = 0$ --- (II)

It is diff eqⁿ of first order & first degree in terms of p & x

Solve then we get a relation in terms of p & x

Let $\phi(p, x, c) = 0$ --- (III) we get solⁿ.

Eliminate p from (1) & (III) we get required solⁿ.

If p cannot be easly eliminate from eqⁿ (1) & (III) then $y = f(p.x)$ & $x = \phi(p.c)$ is required sol

Q Solve $y + px = x^4 p^2$

$$y + px = x^4 p^2 \quad \text{--- (1)}$$

diff w r to x

$$\frac{dy}{dx} + p + \frac{x\,dp}{dx} = 2x^4 p\frac{dp}{dx} + 4p^2x^3$$

put $dy/dx = p$

$$p + p + \frac{x\,dp}{dx} = 2x^4 p\frac{dp}{dx} + 4p^2x^3$$

$$2p - 4p^2x^3 = 2x^4p\frac{dp}{dx} - x\frac{dp}{dx}$$

$$\frac{dp}{dx} = \frac{2p - 4p^2x^3}{2x^4p - x}$$

$$\frac{dp}{dx} = \frac{2p(1 - 2px^3)}{-x(1 - 2px^3)}$$

$$\frac{dp}{2p} = -\frac{dx}{x}$$

Integrate both side

$$\frac{1}{2}\log p = -\log x + \log c$$

$$\log p^{1/2} + \log x = \log c$$

I.sc maths golu sir 8292654087

$\sqrt{p} \cdot x = c$

$p = \dfrac{c^2}{x^2}$ ——— (iii)

~~put~~ eliminat p in eq (i) & (iii)

$y + \dfrac{c^2}{x^2} = \dfrac{\cancel{x^4} c^4}{\cancel{x^4}}$

$y + \dfrac{c^2}{x^2} = c^4$

It is requir sol

Q $y = apx + bp^3$ - - - (1)

diff w r to x

$\dfrac{dy}{dx} = ap + ax\dfrac{dp}{dx} + 3bp^2\dfrac{dp}{dx}$

put $dy/dx = p$

$p = ap + ax\,dp/dx + 3b\,p^2\,dp/dx$

$p - ap = (ax + 3bp^2)\,dp/dx$

$\dfrac{dp}{dx} = \dfrac{ax + 3bp^2}{p(1-a)}$

$$\frac{dx}{dp} = \frac{ax}{p(1-a)} + \frac{3bp^2}{(1-a)p}$$

$$\frac{dp}{dx} - \frac{a}{p(1-a)}x = \frac{3b\cancel{p^2}p}{(1-a)\cancel{p}}$$

It is Linear diff eqⁿ

$$P = \frac{-a}{p(1-a)} \;\&\; Q = 3bp^2$$

$$\frac{dp}{dx} - \frac{3b}{(1-a)}p$$

$$\frac{dx}{dp} = \frac{a}{p(1-a)}x - \frac{3bp}{(1-a)}$$

It is Linear diff eqⁿ

$$P = \frac{-a}{p(1-a)} \;\&\; Q = \frac{3bp}{1-a}$$

It is given solⁿ

$$x\,e^{\int \frac{-a}{p(1-a)}dp} = \int \frac{3bp}{1-a}\, e^{\int \frac{-a}{p(1-a)}dp}\,dp$$

$$x\, e^{\frac{a}{a-1}\log p} = \int \frac{3bp}{1-a}\, e^{\frac{a}{a-1}\log p}\, dp$$

$$x \cdot p^{a/a-1} = \int \frac{3bp}{1-a}\, p^{a/a-1}\, dp$$

$$x \cdot p^{a/a-1} = \int \frac{3b}{1-a} \cdot p^{\frac{a}{a-1}+1}\, dp$$

$$x\, p^{a/a-1} = \frac{3b}{1-a} \int p^{\frac{2a-1}{a-1}}\, dp$$

$$x\, p^{a/a-1} = \frac{3b}{1-a} \cdot \frac{p^{\frac{2a-1}{a-1}+1}}{\frac{2a-1}{a-1}+1} + C$$

$$x\, p^{a/a-1} = \frac{3b}{\cancel{1-a}} \cdot \frac{p^{\frac{3a-2}{a-1}}}{\frac{3a-2}{\cancel{a-1}}} + C$$

$$x\, \cancel{p^{a/a-1}} = \frac{-3b}{3a-2}\, p^{\frac{3a-2}{a-1}} + C$$

$$x\, \cancel{p^{a/a-1}} = \frac{-3b}{3a-2} \cdot \cancel{p^{\frac{a}{a-1}}} \cdot p^{\frac{2a-2}{a-1}}$$

$$x = \frac{-3b}{3a-2} \cdot p^{2}$$

$$p^2 = \frac{x(2-3a)}{3b} \quad \text{---} \; (iii)$$

Eliminat p in eq (I) & (II)

$$\cancel{y = \frac{a\sqrt{3bx}}{\sqrt{\ }}}$$

$$y = \frac{ax\sqrt{2-3y}}{\sqrt{3b}} + \frac{b\,x^{3/2}(2-3y)^{3/2}}{(3b)^{3/2}}$$

Q Solve the following diff eq

(I) $y = 2px + p^4x^2$

(II) $xp^2 - 2yp - x = 0$

(III) $p^3x - p^2y - 1 = 0$

(IV) $y = x(p + p^3)$

(V) $y = x(p + p^2)$

(VI) $y = (1+p)x + ap^2$

(VII) $y = 2px - p^2$

(VIII) $yp = xp^2 + x^2$

(IX) $y = xp^2 + p$

(X) $y = apx + bp^3$

(XI) $y = apx + bp^2$

(XII) $y = px + p^3$

(XIII) $xp^2 - 2yp + x = 0$

(XIV) $y - 2px = f(xp^2)$

* Equaction Solvable for x

gf the eqⁿ can be expressed in the form of $x = f(P, y)$ then we say that eqn is solvable for x

Rule Let $x = f(P, y)$ — (1)
diff both side w r to x

$$\frac{dx}{dy} = f'(P y)\frac{dp}{dy}$$

put $\frac{dx}{dy} = \frac{1}{P}$

$$\frac{1}{P} = f'(P y)\frac{dp}{dy}$$

$$F\left(P, y\frac{dp}{dy}\right) = 0$$

It is diff eqⁿ of first order & first degree in terms of y & P.

Solve we get a relation in terms of P&y
Let $g(P, y, C) = 0$ — (II)
be its solⁿ

Now Eliminate p from eqn (i) & (ii) we get required soln

If p cannot be easily eliminated from (i) & (ii) then express it in the from of

$$x = f(p, y)$$
$$y = y_1(p, c)$$

It is require soln

Q Solve $y = 2px + p^2 y$

$$y = 2px + p^2 y \quad \text{--- (1)}$$

$$2px = y - p^2 y$$

$$x = \frac{y - p^2 y}{2p}$$

$$x = \frac{y}{2p} - \frac{1}{2} py$$

diff both side w r to y

$$\frac{dx}{dy} = \frac{2p - 2y\frac{dp}{dx}}{4p^2} - \frac{1}{2}p - \frac{1}{2}y\frac{dp}{dy}$$

$$\frac{dx}{dy} = \frac{p - y\frac{dp}{dx}}{2p^2} - \frac{1}{2}p - \frac{1}{2}y\frac{dp}{dy}$$

Put $\frac{dx}{dy} = \frac{1}{p}$

$$\frac{1}{p} = \frac{1}{2p} - \frac{y}{2p^2}\frac{dp}{dy} - \frac{p}{2} - \frac{1}{2}y\frac{dp}{dy}$$

$$\frac{1}{p} - \frac{1}{2p} + \frac{p}{2} = -\frac{dp}{dy}\left(\frac{y}{2p^2} - \frac{y}{2}\right)$$

$$\frac{2-1+p^2}{\cancel{2p}} = -y\frac{dp}{dy}\left(\frac{1-p^2}{\cancel{2p^2}}\right)$$

$$\cancel{(1+p^2)} = \frac{y\,\cancel{(1+p^2)}}{p}\frac{dp}{dy}$$

$$\frac{dy}{y} = \frac{dp}{p}$$

$$\log y = \log p + \log c$$

$$y = pc$$

$$p = y/c \text{ ----- (II)}$$

eliminate p from eq (I) & (II)

$$y = 2\cdot\frac{y}{c}\cdot x + \frac{y^2\cdot y}{c^2}$$

$$\boxed{y = \frac{2yx}{c} + \frac{y^3}{c^2}}$$

Solve the following diff. eq^n

Q1 $y = 2px + y^2 p^3$

Q2 $p^2 y - 2px = y$

Q3 $y = 2px + y^2 p^2$

Q4 $p^3 - 4xyp + 8y^2 = 0$

Q5 $4yp^2 - 2xp + y = 0$

Q6 $xp^2 - py = y$

Q7 $p^2 - py + x = 0$

Q8 $x = y - p^2$

Q9 $x = y + p^2$

Q $xp^2 = 1 + p^2$

$p^2 - 2px + y = 0$

$yp^2 - 2xp + y = 0$

$y^2 \log y = xyp + p^2$

* Clairaut's Equation:-

⇒ Clairaut's differential equation

differential eqn is of the form $y = px + f(p)$ where $f(p)$ is function of p are called clairaut's differential equation.

Its soln is given by

$$y = px + f'(p)$$

diff w r to x

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

Put $dy/dx = p$

$$p = p + (x + f'(p))\, dp/dx$$

$$(x + f'(p))\frac{dp}{dx} = 0$$

$$x + f'(p) = 0 \quad \& \quad \frac{dp}{dx} = 0$$

$$x = -f'(p) \quad \text{---(1)} \quad dp = dx \quad \cdots (2)$$

$$p = c$$

Now eliminat p from (I) & (II) we get

$$y = cx + f(c)$$

it's general sol^n

& Now eliminat p from (I) & (II) we get singular sol^n

Q find general & singular sol^n of the following eq^n

(1) $y = px + \frac{a}{p}$ — (I)

diff both side w r to x

$$\frac{dy}{dx} = p + x\frac{dp}{dx} - \frac{a}{p^2}\frac{dp}{dx}$$

put $\frac{dy}{dx} = p$

$$\cancel{p} = \cancel{p} + \left(x - \frac{a}{p^2}\right)\frac{dp}{dx}$$

$$\left(x - \frac{a}{p^2}\right)\frac{dp}{dx} = 0$$

$$\frac{dp}{dx} = 0 \quad \& \quad x = \frac{a}{p^2}$$

$$dp = 0\,dx \quad \& \quad p^2 = \frac{a}{x}$$

$p = c$ --(IV) & $p = \sqrt{a/x}$ -- (III)

eliminat p from (I) & (II)

$$y = cx + \frac{a}{c}$$

It is general solⁿ

& eliminate p from (I) & (III)

$$y = \sqrt{a/x} \cdot x + \frac{a}{\sqrt{a/x}}$$

$$y = \sqrt{ax} + \sqrt{a \cdot x}$$

$$y = 2\sqrt{ax}$$

Square both side

$$y^2 = 4ax$$

It is singular solⁿ

Q ~~Solve the following~~

Q Find the general & singular Solⁿ of the following equation

(I) $y = px + p - p^2$

(II) $y = px + \log p$

(III) $(y - px)(p-1) = p$

(IV) $y = px + \sin^{-1} p$

(V) $y = (x-a)p - p^2$

(VI) $y = p(x-p) + a/p$

(VII) $(y - px)(1+p^2) = a^2p^2$

(VIII) $(y - px)(p-1) = p^2 \log p$

(IX) $(y+1)p - xp^2 + 2 = 0$

(X) $(x-a)p^2 + (x-y)p - y = 0$

(XI) $y = px + a\sqrt{1+p^2}$

(XII) $y = px + \sqrt{a^2p^2 + b^2}$

* differential equaction higer order in 1st degree (with constent coffisient)

=> differential eqn is of the form

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_n y = X$$

are called higer order Linear diff eqn.

=> If $P_1, P_2, P_3 \dots P_n$ are varable then be say that it is higer order linear diff eqn with varable coffecient.

Ex

$$\frac{d^2 y}{dx^2} - 5x \frac{dy}{dx} + by = 0$$

$$\frac{d^2 y}{dx^2} - \frac{1}{2-x} \frac{dy}{dx} + y = x^2 + x + 1$$

=> If $P_1\ P_2\ P_3 \dots P_n$ are constant then we say that it is higer order Linear diff eqn with constant coffecient

Ex

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 5y = 0$$

⇒ If $X = 0$

i.e $$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + P_n y = 0$$

are called higer order homogeneous linear differential eqn

Ex $$\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

⇒ If $x \neq 0$

i.e $$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-1}} + \cdots + P_n y = X$$

are called higer order non-homogeneous linear diff eqn

ie $$\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + y = \sin 2x$$

$$\frac{d^2 y}{dx^2} + 4y = e^{2x} + \cos 3x$$

I.sc maths golu sir 8292654087

Q If u & v are two solution of the equation $\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_n y = 0$ — (1)

then $X = C_1 u + C_2 v$ are also solⁿ of its eqⁿ.

Let $\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_n y = 0$ — eq(1)

Let u & v be two solⁿ of eqⁿ (1)

we want to show that

$X = C_1 u + C_2 v$ is also solⁿ in eq(1)

Now $\frac{d^n (C_1 u + C_2 v)}{dx^n} + P_1 \frac{d^{n-1}(C_1 u + C_2 v)}{dx^{n-1}} + \cdots + P_n (C_1 u + \ldots = 0$

$= \left(\frac{d^n (C_1 u)}{dx^n} + \frac{d^n (C_2 v)}{dx^n}\right) + \left(P_1 \frac{d^{n-1}(C_1 u)}{dx^{n-1}} + P_1 \frac{d^{n-1}(C_2 v)}{dx^{n-1}}\right) + \cdots P_n C_1 u + P_n C_2 v = 0$

$= \left(\frac{d^n (C_1 u)}{dx^n} + P_1 \frac{d^{n-1}(C_1 u)}{dx^{n-1}} + \cdots + P_n C_1 u\right) + \left(\frac{d^n (C_2 v)}{dx^n} + P_1 \frac{d^{n-1}(C_2 v)}{dx^{n-1}} + \cdots P_n C_2 v\right) = 0$

$C_1 \left(\frac{d^n u}{dx^n} + P_1 \frac{d^{n-1}u}{dx^{n-1}} + \cdots P_n u\right) + C_2 \left(\frac{d^n v}{dx^n} + P_1 \frac{d^{n-1}v}{dx^{n-1}} + \cdots P_n v\right)$

$= 0$

~~$C_1u + C_2v = 0$~~

u & v are also solⁿ

~~$C_1u + C_2v = 0$~~

$$C_1 0 + C_2 0 = 0$$

$y = C_1u + C_2v$ also satisfy eqⁿ

$y = C_1u + C_2v$ is also solⁿ

⇒ If $y_1, y_2, y_3 \ldots y_n$ are the solⁿ of homogeneous part of higer order Linear differential eqⁿ with constant coffecient. then

$y = C_1y_1 + C_2y_2 + C_3y_3 + \ldots + C_ny_n$
are also solⁿ of homogeneous part.

⇒ If u is the solⁿ of the eqⁿ

$$\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \ldots + P_ny = 0 \quad (i)$$

& V is the solⁿ of

$$\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \ldots P_ny = X \quad (ii)$$

then $Y = u + v$ is soln of eqn (11)

$$\frac{d^n(u+v)}{dx^n} + P_1 \frac{d^{n-1}(u+v)}{dx^{n-1}} + \cdots + P_n(u+v) = X$$

$$\left(\frac{d^n u}{dx^n} + \frac{d^n v}{dx^n}\right) + P_1\left(\frac{d^{n-1}u}{dx^{n-1}} + \frac{d^{n-1}v}{dx^{n-1}}\right) + \cdots + P_n(u+v)$$

$$\left(\frac{d^n u}{dx^n} + P_1 \frac{d^{n-1}u}{dx^{n-1}} + \cdots + P_n u\right) + \left(\frac{d^n v}{dx^n} + P_1 \frac{d^{n-1}v}{dx^{n-1}} + \cdots P_n v\right) \quad \text{--- (11)}$$

$\because$ u is soln of eqn (1) & v is soln of (11)

$$= 0 + X = X$$

$Y = u + v$ is soln of eqn (11)

$\Rightarrow$ Some of soln of homogeneous parts & non-homogeneous parts are the soln of non-homogeneous parts

* Complementary function:- Solution of homogeneous parts of higer order Linear differential eq^n are called complementary function

⇒ Symmbulic Repersentation of higer order Linear diff eq^n with constant coffecient.

Let $$\frac{d^n y}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \dots + P_n y = X$$

$$\left(\frac{d^n}{dx^n} + P_1\frac{d^{n-1}}{dx^{n-1}} + P_2\frac{d^{n-2}}{dx^{n-2}} + \dots + P_n\right)y = x$$

Put $\frac{d}{dx} = D$

$$\left(D^n + P_1 D^{n-1} + P_2 D^{n-2} + \dots + P_n\right)y = X$$

$$f(D)\,y = x$$

Ex $$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$$

$$(D^2 - 5D + 6)\,y = e^{4x}$$

⇒ Auxiliary eqⁿ:-

Let $\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_ny = X$ --(1)

be differentiable eqⁿ of nth or degree put $X = 0$

i.e $\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_ny = 0$

$\left(\frac{d^n}{dx^n} + P_1\frac{d^{n-1}}{dx^{n-1}} + P_2\frac{d^{n-2}}{dx^{n-2}} + \cdots + P_n\right)y = 0$

put $\frac{d}{dx} = D$

$(D^n + P_1D^{n-1} + P_2D^{n-2} + \cdots P_n)y = 0$

or

put $D = m$

$(m^n + P_1m^{n-1} + P_2m^{n-2} + \cdots + P_n)y = 0$

It is Auxiliary eqⁿ

Ex $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

A.E

$m^2 - 5m + 6 = 0$

7 Rule for finding complimentary function (C.F)

Let $\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_n y = V$

The higer order linear diff eqn with constant coffecent

Its A.E is given by

$$m^n + P_1 m^{n-1} + P_2 m^{n-2} + \dots P_n = 0$$

Solve AE & find the Root of AE

(1) If The Root of AE are Real & non-Repeted (unequal)

Let $m = \alpha_1 \alpha_2 \dots \alpha_n$ be the root of A.E then its CF

$$y = C_1 e^{\alpha_1 x} + C_2 e^{\alpha_2 x} + \dots + C_n e^{\alpha_n x}$$

Q find C.F

$$\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$$

I.sc maths golu sir 8292654087

Its A.E

$$m^2 - 5m + 6 = 0$$

$$m^2 - 3m - 2m + 6 = 0$$

$$m(m-3) - 2(m-3) = 0$$

$$(m-2)(m-3) = 0$$

$$m = 2, 3,$$

The root of A.E is real & non-Rea

Its C.F is

$$\boxed{C.F = C_1 e^{2x} + C_2 e^{3x}}$$

(2) If the root of AE are the Real & Repeted (equal)

Let $\alpha_1 = \alpha_2 = \alpha_3 = \dots = \alpha_n$ $r \leq n$ be the root of A.F

then Its CF is given by

$$C.F = (C_0 + C_1 x) e^{\alpha x}$$

$$\cancel{y} = (C_0 + C_1 x + C_2 x^2 + C_n x^{n-1}) e^{\alpha x}$$

Ex $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

Its A·E is given by

$m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$m = 2, 2$

Its root of A·E is real & equal then Its C·F is given by

$$\boxed{y = (c_0 + c_1 x)e^{2x}}$$

(III) If the root of AE are imaginary & non-Repeted (unequal)

Let $m = (\alpha + i\beta), \& \alpha - i\beta$ is root of A E

then Its CF is given by

$y = c_1 e^{(\alpha+i\beta)x} + c_2 e^{(\alpha-i\beta)x}$

$= c_1 e^{\alpha x} \cdot e^{i\beta x} + c_2 e^{\alpha x} e^{-i\beta x}$

$= e^{\alpha x}(c_1 e^{i\beta x} + c_2 e^{-i\beta x})$

$e^{\alpha x}\{c_1(\cos\beta x + i\sin\beta x) + c_2(\cos\beta x - i\sin\beta x)$

$e^{\alpha x}\{C_1\cos\beta x + C_2\cos\beta x + i(C_1\sin\beta x - C_2\sin\beta x)$

$e^{\alpha x}\{(C_1+C_2)\cos\beta x + i(C_1-C_2)\sin\beta x)$

$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

(IV) If the root of A.E are imaginary & equal

Let $m = (\alpha \pm iB)(\alpha \pm iB)$ be the root of A E

then its C.F is given by

$y = e^{\alpha x}[(C_0 + C_1 x)\cos\beta x + (C_2 + C_3 x)\sin\beta x$

$\cancel{y = e^{\alpha x}[}$

Q $\frac{d^2y}{dx^2} - 4y = 0$

Its A.E

$m^2 = -4$

$m = \pm 2i$

It is the root of A.E are imaginary & unequal

Its C.F is given by

$\boxed{y = A\cos 2x + B\sin 2x}$

Q find the C.F of following diff eqn

Q1 $\frac{d^2y}{dx^2} + x^2 y = 0$

Q2 $\frac{d^2y}{dx^2} - m^2 y = 0$

Q3 $\frac{d^3y}{dx^3} - 8y = 0$

Q4 $(D^3 + 1)y = 0$

Q $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 0$

Q $2\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 18y = 0$

Q $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$

Q $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = 0$

Q $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

Q $(D^2 + 4D + 13)y = 0$

Q $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

Particular Integral

⇒ Solution of the non-homogeneous part of higer order linear diff eqn with constant co-efficient are called particular Integral (P.I)

⇒ Rule of finding particular Integral

Let the given diff eqn

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + P_n y = X \quad \text{--(i)}$$

which in symoblic form can be writt-

$$D^n y + P_1 D^{n-1} y + P_2 D^{n-2} y + \dots P_n y = X$$

$$(D^n + P_1 D^{n-1} + P_2 D^{n-2} + \dots + P_n) y = X$$

$$f(D) y = X$$

$$y = \frac{1}{f(D)} X$$

It is Particular Integral

Case-1 To find the P.I when $X = e^{\alpha x}$

$$f(D)Y = X$$
$$f(D)Y = e^{\alpha x}$$

$$Y = \frac{1}{f(D)} e^{\alpha x}$$ Replace D by α

$$\boxed{Y = \frac{1}{f(\alpha)} e^{\alpha x}}$$ where $f(\alpha) \neq 0$

Q $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}$

The A.E is

$$(D^2 + 3D + 2)Y = e^{2x}$$
$$Y = \frac{1}{D^2 + 3D + 2} e^{2x}$$

$$m^2 + 3m + 2 = 0$$
$$m^2 + 2m + m + 2 = 0$$
$$m(m+2) + 1(m+2) = 0$$
$$(m+2)(m+1) = 0$$
$$m = -1, -2 \quad Y = m e^{-x} + m_2 e$$

Now the given eqn written as

$$(D^2 + 3D + 2)Y = e^{2x}$$

It is P.I

$$Y = \frac{1}{D^2 + 3D + 2} e^{2x}$$

D Replace by 2

$$y = \frac{1}{(2)^2 + 3\cdot2 + 2} e^{2x}$$

$$= \frac{1}{4+6+2} e^{2x}$$

$$= \frac{1}{12} e^{2x}$$

the general solⁿ by

$$y = C.F + P.I$$

$$= (Ae^{-x} + Be^{-2x}) + \frac{1}{12} e^{2x}$$

If $X = e^{\alpha x}$ when $f(\alpha) = 0$

Then it P.I is

$$f(D)\, y = e^{\alpha x}$$

$$y = \frac{e^{\alpha x}}{f(D)}$$

D Replace D+α

$$\boxed{X = \frac{e^{\alpha x}}{f(D+\alpha)} x^0}$$

Solve the following diff eqⁿ

(1) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

I.sc maths golu sir 8292654087

caseII If $x = \cos\alpha x$ or $\sin\alpha x$

The P.I is

$$f(D)y = \cos\alpha x \text{ or } \sin\alpha x$$

$$y = \frac{1}{f(D)} \cos\alpha x \text{ or } \sin\alpha x$$

$$= \frac{1}{-\phi(\alpha^2)} \cos\alpha x \text{ or } \sin\alpha x$$

Q Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$

The Auxiliary equation is

$$m^2 + m + 1 = 0$$

$$m = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

$$m = \frac{-1 + \sqrt{3}i}{2} \quad m = \frac{-1 - \sqrt{3}i}{2}$$

$$C.F = e^{-x/2}\left[A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right]$$

& Now, $P.I = \frac{1}{D^2 + D + 1} \sin 2x$

$$= \frac{1}{-(2)^2 + D + 1} \sin 2x$$

$$\frac{1}{-4 + 1 + D} \sin 2x$$

I.sc maths golu sir 8292654087

$$\frac{1}{D-3}\sin 2x$$

$$= \frac{D+3}{D^2-9}\sin 2x$$

$$\frac{D+3}{-4-9}\sin 2x$$

$$= \frac{1}{13}(D+3)\sin 2x$$

$$-\frac{1}{13}\cos 2x.2 - \frac{1}{13}3\sin 2x$$

$$-\frac{1}{13}\left[2\cos 2x + 3\sin 2x\right]$$

The comple general eqn

$$y = C.F + P.I$$

$$= e^{-x/2}\left[A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right] + \frac{-1}{13}\left[2\cos 2x + 3\sin 2x\right]$$

Q $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10\sin x$

(2) $\frac{d^2y}{dx^2} + y = \cos 2x$

Q3 $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 9\cos 2x$

cor (iii) If $X = x^m$ positive integer

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \dots$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$$

Q $\frac{d^2y}{dx^2} - 4y = x^2$

The A.E

$$m^2 - 4 = 0$$
$$m^2 = 4$$
$$\therefore m = \pm 2$$

$$C.F = Ae^{2x} + Be^{-2x}$$

The $P.I = \frac{1}{D^2 - 4}x^2$

$$= \frac{1}{-(4 - D^2)}x^2$$

$$= -\frac{1}{4\left(1 - \left(\frac{D}{2}\right)^2\right)}x^2$$

$$= -\frac{1}{4}\left(1 - \left(\frac{D}{2}\right)^2\right)^{-1} \cdot x^2$$

$$-\frac{1}{4}\left(1+\frac{D^2}{4}+\right)x^2$$

$$-\frac{1}{4}\left(x^2+\frac{D^2x^2}{4}\right)$$

$$-\frac{1}{4}\left(x^2+\frac{1}{2}\right)$$

general Sol^n

$$P.I + C.F$$

$$\left(Ae^{2x}+Be^{-2x}\right)+\frac{-1}{4}\left(x^2+\frac{1}{2}\right)$$

Q $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^2+e^x$

Q $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^2+e^x+\cos 2x$

Q $\frac{d^2y}{dx^2}+4y=x^2$

Q $\frac{d^2y}{dx^2}-9y=x^2+x$

ans-(v) If $y = e^{ax}.V$ where V is any function of x

then P.I. is

$$f(D)\,y = e^{ax}.V$$

$$y = \frac{1}{f(D)} e^{ax}.V$$

Replace D by D+a

$$y = e^{ax} \frac{1}{f(D+a)} .V$$

$$= e^{ax} f(D+a)^{-n}.V$$

(1) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = xe^{2x}$

The A.E. is

$$m^2 - 4m + 4$$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

$$C.F = e^{2x}(C + C_1 x)$$

P.I =

$$y = \frac{1}{D^2 - 4D + 4} xe^{2x}$$

$$y = \frac{1}{(D-2)^2} x e^{2x}$$

Replace D by D+2

$$= e^{2x} \frac{1}{(D+2-2)^2} \cdot x$$

$$e^{2x} \frac{1}{D^2} x$$

$$e^{2x} \frac{1}{D^2} \cdot x$$

$$e^{2x} \cdot \frac{x^3}{6}$$

The general Sol

$$y = C.F + P.I$$

$$= e^{2x}(c_0 + c_1 x) + e^{2x} \frac{x^3}{6}$$

(1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{3x}$

(1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^{x}\cos x$

(1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{x}$

(1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^{x}\cos x$

VARIATION of PARAMETERS

Let $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

Let u & v are soln of

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$$

then

$$A_1u + B_1v = 0 \quad \text{---(I)}$$

$$A_1u_1 + B_1v_1 = R \quad \text{---(II)}$$

Solve (I) & (II),

we get A_1 & B_1,

$$A = \int A_1 dx \ \& \ B = \int B_1 dx$$

$Y = Au + Bu$ are soln of

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

Q $\frac{d^2y}{dx^2} + 4y = \tan 2x$

This A.E

$m^2 + 4 = 0$

$m^2 = -4$

$m = \pm 2i$

C.F is

CF

$y = A\cos 2x + B\sin 2x$

P.I is

Let $u = \cos 2x \quad v = \sin 2x$

$u_1 = -2\sin 2x \quad v_1 = 2\cos 2x$

No

$A_1 \cos 2x + B_1 \sin 2x = 0 \quad \text{—(i)}$

$-A_1 2\sin 2x + 2B_1 \cos 2x = \tan 2x \quad \text{—(ii)}$

eqn (i) $\times 2\sin 2x$ − (ii) $\times \cos 2x$

$A_1 2\sin 2x \cdot \cos 2x + B_1 2\sin^2 2x = 0$

$-A_1 2\sin 2x \cdot \cos 2x + 2B_1 \cos^2 2x = \tan 2x \cdot \cos 2x$

$2B_1 = \sin 2x$

$B_1 = \frac{\sin 2x}{2}$

Put in eqn (i)

$A_1 \cos 2x = -\frac{\sin^2 2x}{2}$

$$A_1 = -\frac{1}{2}\frac{\sin^2 2x}{\cos 2x}$$

$$A = \int -\frac{1}{2}\frac{\sin^2 2x}{\cos 2x}\,dx$$

$$A = \int -\frac{1}{2}(\sec 2x - \cos 2x)\,dx$$

$$A = -\frac{1}{2}\left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right]$$

$$= \frac{1}{4}\left[\sin 2x - \log(\sec 2x + \tan 2x)\right]$$

$$B = \int B_1\,dx$$

$$= \int \frac{\sin 2x}{2}\,dx$$

$$= -\frac{\cos 2x}{4}$$

$$y = Au + B\cdot v$$

$$= \frac{1}{4}\left[\sin 2x - \log(\sec 2x + \tan 2x)\right]\cos 2x$$

$$+ -\frac{\cos 2x}{4}\sin 2x$$

Q) $\frac{d^2y}{dx^2} + a^2y = \sec ax$

Its A E

$m^2 + a^2 = 0$

$m^2 = -a^2$

$m = \pm a$

C.F is

$y = [A\cos ax + B\sin ax]$

its P.I is

Let $u = \cos ax \quad v = \sin ax$

$u_1 = -\sin ax \cdot a \quad v_1 = \cos ax \cdot a$

$A_1 \cos ax + B_1 \sin ax = 0 \quad ---(1)$

$-A_1 \sin ax \cdot a + B_1 \cos ax = \sec ax \quad -(1)$

eqn (1) × a sin ax + II cos ax

$$\frac{\cancel{aA_1 \cos ax \cdot \sin ax} + B_1 \sin^2 ax \cdot a}{\cancel{-A_1 \sin ax \cdot \cos ax} + B_1 \cos^2 ax \cdot a} = 1$$

$aB_1 = 1$

$B_1 = \frac{1}{a}$

Put in eqn (1)

$A_1 \cos ax = -B_1 \cdot \sin ax$

$A_1 = \frac{1}{a} \sin ax \cdot \frac{1}{\cos ax}$

$A_1 = \frac{1}{a} \tan ax$

$A = \int A_1 dx = \int \frac{1}{a} \frac{\sin ax}{\cos ax}$

$= -\frac{1}{a} \log(\cos ax) + c$

$B = \frac{1}{a} x$

$X = Au + Bv$

$= -\frac{1}{a} \log \cos ax \cdot \cos x + \frac{1}{a} x \cdot \sin ax$

(1) $\frac{d^2y}{dx^2} + n^2 y = \sec nx$

$\frac{d^2y}{dx^2} + 9y = \sec 3x$

$\frac{d^2y}{dx^2} + 4y = \sec 2x$

$(D^2 + 1)y = \operatorname{cosec} x$

$\frac{d^2y}{dx^2} + y = \sec^2 x$

$\frac{d^2y}{dx^2} + 4y = \tan 2x$