Q-1 find the Nash Equilibrium by finding best response of the players.

1 \ 2	L	C	R
T	1,1	a,2	0,0
M	2,2	2,1	1,1
B	0,0	1,1	2,0

Shivendr Srivastava

142402019

Q-2

1 \ 2	L	M	R
T	7,2	2,7	3,6
B	2,7	7,2	4,5

(a) what is the number of supports in this game?

(b) find NE(s) of this game.

Q-3 construct an example having 2 players to show that a weably dominated action can be in an NE.

Answers

Test 1 (Jan-May 2025)

DS5618 : Multi-Agent Systems

Important note:

1. The question paper consists of 6 questions and a total of 50 marks.
2. Write a complete explanation of the steps used in getting the answers. Answers with incomplete explanation will receive half of the marks allocated to that question.
3. The computation in questions 5 and 6 depends on the correctness of your answer to question 4. If you doubt that your answer for question 4 is not correct, you can provide the answers for questions 5 and 6 for the *original game* (written before question 1). If you do not doubt your answer for question 4, then simply ignore this note and proceed answering question 5 and 6.

Answer questions 1-6 for a game of two players P_1 and P_2 with the set of actions and the utility matrix as follows:

	X	Y	Z
A	$(3,2)$	$(1,4)$	$(2,1)$
B	$(5,3)$	$(2,2)$	$(3,0)$
C	$(4,0)$	$(3,2)$	$(2,5)$

Question 1

Compute the Best Response set of each player for each action of the other player. $2 \times 6 = 12$ *Marks*

Question 2

Find all Pure Strategy Nash Equilibria of this game. 5 *Marks*

Question 3

State the two characteristics of a Mixed Strategy Nash Equilibrium. $5 \times 2 = 10$ *Marks*

Question 4

Use the iterative elimination method to eliminate the **strictly** dominated actions of the players. Write the remaining utility matrix.

5 *Marks*

Question 5

What is the number of possible combination of supports in the remaining game after Question 4 (or *original game*)? Write all those combination of support sets in the remaining game after Question 4 (or in the *original game*). .

$1 + 2 = 3$ *Marks*

Question 6

Find **all** Mixed-Strategy Nash Equilibria of the remaining game after Question 4. If you doubt your answer to question 4, then find **all** Mixed-Strategy Nash Equilibria of the *original game*. .

15 *Marks*

Q- Define the property "Independence of Irrelevant alternatives"
of a social choice function.
Does plurality with the breaking rule satisfies IIA? prove it.

Q- Define "Condorcet Consistency (CC)". which among these
SCFs does not satisfy CC? Explain your answers.

(a) Plurality with tie-breaking
(b) maximum
(c) Copeland

why wasted time in writing
questions again?

Q-(3) Construct a Preference Profile P of less than five voters
and three alternatives such that, Plurality (P) ≠
Borda (P) ≠ Veto (P) while all three use the
same tie-breaking rule.

Q-(4) Define 'strategy proofness'. Is borda with a
tie-breaking rule manipulable. Show it.

Test 2

(Jan-May 2025)

DS5618 : Multi-Agent Systems

Important note

1. The question paper contains 6 questions of 40 marks in total.
2. Write a complete explanation of the steps used in getting the answers. Answers with incomplete explanation will receive half of the marks allocated to that question.

Question 1

Consider PUBLIC FACILITY SELECTION PROBLEM. Construct a preference profile P with <u>**cardinal**</u> preferences of **four** voters over **three** alternatives $A = \{Bank, Hospital, School\}$, such that the alternative chosen from A by **Egalitarian** and **Utilitarian** rule is same.

10 *Marks*

Question 2

Define a **'Pareto efficient'** SCF. Is the following statement true?
"If an SCF f satisfies Pareto efficiency, then f is unanimous."

$4 + 1 = 5$ *Marks*

Question 3

Construct a preference profile P such that the scoring rule **'Copeland'** on P is manipulable.

5 *Marks*

Question 4

Define **'Monotonicity'** of an SCF. Is the following statement true?
"An SCF f is monotone, ***iff*** it is strategyproof."

$4 + 1 = 5$ *Marks*

Question 5

State the '**Gibbard-Satterthwaite**' theorem for Social Choice function. 3 *Marks*

Question 6

Define the property "**Independence of Irrelevant Alternatives (IIA)**" of a Social Welfare function. Does the **Borda** rule satisfy IIA? Prove your answer.

$4 + 2 + 5 = 12$ *Marks*

Department of Data Science, IIT Palakkad

DS5618 : Multi-Agent Systems (Jan-May 2025)

09:30-12:30 | **End Semester (28 April 2025)** | **Marks : 50**

Instructions

1. Write your answers neatly in Blue/Black ink. Make sure your answers are legible.

2. If you have to make any assumptions about unspecified things, write them clearly with justification.

3. Write the question number clearly for each answer.

4. There will be partial markings for the questions, so even if you cannot solve the entire problem be sincere with the steps.

5. Write a complete explanation of the steps used in getting the answers. Answers with incomplete explanation will receive half of the marks allocated to that question.

6. You are not expected to write the question on the answer sheet. It is not a wise decision to waste time on doing so.

6. **Be precise.**

1. **VCG Mechanism** (3+7 marks)
   The Vickrey–Clarke–Groves (VCG) mechanism is a truthful mechanism for achieving an efficiently optimal allocation whenever monetary transfers are available.
   (a) Write the equation of VCG mechanism $M^{VCG} = (f, p)$.
   (b) Prove that VCG mechanism is truthful.

2. **Plurality voting rule** (5+5 marks)
   The plurality electoral system is the oldest and the most frequently used voting system. It is used for legislative elections in India. Construct an example of the preference profile of the population and explain at least **TWO** drawbacks of applying the Plurality voting rule for elections.

3. **Mixed Strategy Nash Equilibrium** (5 marks)
   A Nash equilibrium is a situation where no player could gain by changing their own strategy (holding all other players' strategies fixed). Find a Mixed strategy Nash equilibrium in the following game.

	L	R
T	$(3,1)$	$(0,2)$
B	$(1,2)$	$(1,1)$
D	$(0,4)$	$(3,1)$

4. **Revenue Monotonicity** (10 marks)
   The revenue generated by a mechanism $M := (f, p)$ is equal to $\sum_{i \in N} p_i$.

A mechanism guarantees *"revenue monotonicity"* if the revenue monotonically increases with the increase in the number of players.

Construct an example to show that VCG **does not** guarantee revenue monotonicity, which means that adding a player to the game decreases the revenue generated by VCG mechanism.

(3+6+6 marks)

5. **Single-peaked domain**
Let $A = \{a_1, \ldots, a_m\}$ be the set of possible outcomes. Let $N = \{1, \ldots, n\}$ be the set of agents. The preference-relation of agent i is denoted by $\succ_i$. The peak element of $\succ_i$ in A is denoted by a_i^*.
The group N is said to have single-peaked preferences over A, if there exists an ordering $>$ of the outcomes such that, for every agent $i \in N$:

- $a_k < a_j \leq a_i^* \Rightarrow a_j \succ_i a_k$
- $a_k > a_j \geq a_i^* \Rightarrow a_j \succ_i a_k$

(a) Define the class of SCFs called *"Median voting rule"* for single single-peaked domain.

(b) Prove that the Median voting rule satisfies Strategyproofness.

(c) Consider a SCF that selects the *"mean"* peak instead of the "median" peak as outcome. Does the *"mean voting rule"* guarantee strategyproofness?

END