

Question #3(ii)

Method (I): Least Square

For system $Ax = b$, the least square formula is obtained from;

$$\min_x ||Ax - b||$$

The solution of which can be written with the normal equations,

$$x = (A^T A)^{-1} A^T b$$

Question #4

Solution

Given, there are 4 couples and 3 floor cafes.

Condition:

Exactly one customer has his/her coffee on the 1st floor.

Insights:

When one customer has his/her coffee on the 1st floor, the other customer will have options to have their coffee on the 2nd or 3rd floor only.

That means, Remaining 7 have options to choose one from the remaining 2nd and 3rd floor. I.e. 2^7 possibilities.

Now, number of possibilities in which exactly one customer is on the 1st floor is $8 * 2^7$

Total number of possibilities is 3^8 Since each customer has 3 options to choose.

Now,

probability that exactly one customer has his/her coffee on the 1st floor =

$$(8 * 2^7) / 3^8 = 1024 / 6561 = 0.15607$$

Question #5(i)

Solution

Given, $f(x) = \sqrt[3]{x}$

At $x = 0$,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \sqrt[3]{x} \approx 0$$

(Approaches to Zero)

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \sqrt[3]{x} \approx 0$$

(Approaches to Zero)

At $x = a$,

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^-} \sqrt[3]{x} \approx \sqrt[3]{a}$$

(Defined as long as **a** is defined)

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^+} \sqrt[3]{x} \approx \sqrt[3]{a}$$

(Defined as long as **a** is defined)

Hence, $f(x) = \sqrt[3]{x}$ is **continuous**.

Question #5(ii)

Solution

Given, $g(x) = -x^6 - x^4 - 13x$

We know,

A function $f(x)$ will be concave if it exhibits

$$f''(x) < 0$$

A function $f(x)$ will be convex if it exhibits

$$f''(x) > 0$$

We have,

$$g(x) = -x^6 - x^4 - 13x$$

$$g'(x) = -6x^5 - 4x^3 - 13$$

$$g''(x) = -30x^4 - 12x^2$$

Here, $g(x)$ contains negative coefficients of x^4 and x^2 .

Insights:

x^4 will never be negative and similarly, x^2 will never be negative.

So, The value of $g''(x)$ will always be less than 0.

Hence, $g(x)$ is a **concave function**.

Question #7

Solution:

Outliers in a dataset are a set of data that exhibits different behavior from the majority of data.

Generally, outliers are considered as noise and removed during preprocessing of the dataset.

We can remove outliers either by using interquartile range or z score method.

Interquartile range method is the 1st quartile – 3rd quartile

Z score method calculation by subtracting given individual data – mean of all the data and then dividing it by standard deviation.

References

- [Gaussian and Normal Random Variable](#)
- [Gaussian NB Mathematical Model](#)
- [Overdetermined System](#)
- [Couple and Caffe Problem](#)
- [Emojis Regex](#)