Chapter 2 — Fundamentals of Fracture Mechanics

1. Atomic Level View of Fracture

A material fractures when sufficient stress and work are applied at the atomic level to break interatomic bonds.

Bond energy:

$$E_b = \int_{\infty}^{r_0} \sigma \, d\alpha$$

Cohesive strength (theoretical strength):

$$\sigma_c = \left(\frac{E\,\gamma_s}{r_0} \right)^{1/2}$$

where E = modulus, γ_s = surface energy, r_0 = atomic spacing.
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2. Stress Concentration Due to Flaws

Real materials contain flaws ⇒ stress is magnified at crack tips.

Inglis solution (elliptical flaw, length $2a$, tip radius ρ):

$$\sigma_A = \sigma \left(1 + 2\sqrt{\frac{a}{\rho}} \right)$$

For very sharp cracks ($\rho \to 0$): $\sigma_A \to \infty$.
Thus, fracture occurs when local stress at tip exceeds cohesive stress:

$$\sigma_f = \left(\frac{2E\gamma_s}{\pi a} \right)^{1/2}$$

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3. Griffith Energy Balance (1920)

First Law of Thermodynamics: a crack grows only if total energy decreases.

Energy terms:

$$\text{Potential energy released (driving force):} \quad -\frac{d\Pi}{dA} = \frac{\pi \sigma^2 a}{E}$$

$$\text{Surface energy required (resisting force):} \quad \frac{dW_s}{dA} = 2\gamma_s$$

Equilibrium (fracture condition):

$$G = \frac{\pi\sigma^2 a}{E} = 2\gamma_s$$

Critical stress for fracture:

$$\sigma_f = \left(\frac{2E\gamma_s}{\pi a}\right)^{1/2}$$

Meaning: Larger cracks ⇒ lower fracture stress ⇒ brittle materials fail at much lower stresses than their theoretical strength.

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4. Griffith vs. Critical Stress Criterion

Griffith model (global balance):

- Crack propagates when released energy ≥ surface energy.
- Fracture stress independent of crack tip radius ρ.

Critical stress model (local criterion):

- Failure occurs when cohesive atomic stress exceeded at crack tip.
- Inglis stress:

$$\sigma_{\text{tip}} \sim \sigma\sqrt{\frac{a}{\rho}}$$

Reality: Material behavior is in between. Crack tip radius still matters (nucleation, microcracks).

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5. Modified Griffith Equation (Irwin & Orowan, 1950s)

Griffith works well for brittle solids (e.g. glass), but underestimates strength of metals (ignores plastic dissipation).

Irwin & Orowan added plastic work per unit area γ_p:

$$\sigma_f = \left(\frac{2E(\gamma_s + \gamma_p)}{\pi a}\right)^{1/2}$$

General form with fracture energy w_f:

$$\sigma_f = \left(\frac{2Ew_f}{\pi a}\right)^{1/2}$$

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6. Energy Release Rate (Irwin, 1956)

Define crack driving force:

$$G = -\frac{d\Pi}{dA}$$

Crack extends when $G = G_c$ (critical value = fracture toughness).

For a wide plate with crack length $2a$:

$$G = \frac{\pi\sigma^2 a}{E}$$

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7. Load vs. Displacement Control

Potential energy:

$$\Pi = U - F$$

where U = strain energy, F = work by external forces.

Load control:

$$G = \frac{P}{2B}\left(\frac{d\Delta}{da}\right)_P$$

Displacement control:

$$G = -\frac{\Delta}{2B}\left(\frac{dP}{da}\right)_\Delta$$

Compliance form (general):

$$C = \frac{\Delta}{P}, \quad G = \frac{P^2}{2B}\frac{dC}{da}$$

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8. Example: Double Cantilever Beam (DCB)

Beam theory:

$$\Delta = \frac{Pa^3}{3EI}, \quad I = \frac{Bh^3}{12}$$

Compliance:

$$C = \frac{\Delta}{P} = \frac{2a^3}{3EI}$$

Energy release rate:

$$G = \frac{P^2 a^2}{BEI} = \frac{12P^2 a^2}{B^2 h^3 E}$$

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9. The σ–G–K Triangle

Stress Form (Griffith):

$$\sigma_f = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

Energy Form (Irwin):

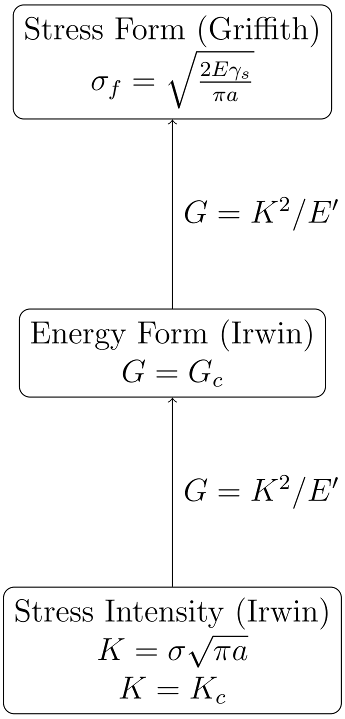
$$G = G_c$$

Stress Intensity Factor (Irwin):

$$K = \sigma\sqrt{\pi a}, \quad \text{fracture when } K = K_c$$

Link between forms:

$$G = \frac{K^2}{E'} \quad \text{with} \quad E' = \begin{cases} E & \text{(plane stress)} \\ \dfrac{E}{1-\nu^2} & \text{(plane strain)} \end{cases}$$



Conclusion: This triangle shows the equivalence of Griffith's energy criterion, Irwin's energy release rate, and the stress intensity approach.