Chapter 1 – Fracture Mechanics: Fundamentals & Design

1. Fracture Mechanics Approach to Design

A material is considered safe if the applied stress is less than the material strength.

- **Conventional design:** $\sigma_{applied} < \sigma_{yield/UTS}$ (no flaw considered).
- **Fracture mechanics:** Failure depends on flaw size and fracture toughness.

2. Approaches to Fracture Analysis

(a) Energy Criterion (Griffith)

- Crack grows if the available energy $\geq$ resistance.
- Material resistance = surface energy + plastic work + other dissipations.
- Energy release rate:

$$G = \frac{\pi \sigma^2 a}{E}, \quad a = \tfrac{1}{2} \text{ crack length}$$

- At fracture: $G = G_c$.

(b) Stress Intensity Approach (Irwin)

- Fracture occurs when stress intensity reaches a critical value.

$$K_I = \sigma \sqrt{\pi a}, \quad \text{fracture if } K_I = K_{IC}$$

- Relation to energy:

$$G = \frac{K^2}{E'}$$

where $E' = E$ (plane stress), $E' = \frac{E}{1-\nu^2}$ (plane strain).

3. Time-Dependent Crack Growth (Fatigue)

- Paris Law:

$$\frac{da}{dN} = C(\Delta K)^m$$

- ΔK: stress intensity range.
- Predict service life by integrating until flaw reaches critical size.

4. Effect of Material Properties on Fracture

- **Low toughness:** Brittle fracture (LEFM valid).
- **Intermediate toughness:** Nonlinear fracture mechanics.
- **High toughness:** Plastic collapse → Limit load analysis.

5. Dimensional Analysis in Fracture Mechanics

Buckingham Π-Theorem

- Any physical relation can be reduced to dimensionless groups.

$$u = f(W_1, W_2, \ldots, W_n) \quad \Rightarrow \quad \pi = F(\pi_1, \pi_2, \ldots, \pi_{n-m})$$

- Reduces parameters → reveals key controlling ratios.

Applications to cracked plates under remote stress σ^∞:

1. **Wide plate (semi-infinite):**

$$\frac{\sigma_{ij}}{\sigma^\infty} = F_1\left(\frac{E}{\sigma^\infty}, \frac{r}{a}, \nu, \theta\right)$$

2. **Finite width plate:**

$$\frac{\sigma_{ij}}{\sigma^\infty} = F_2\left(\frac{E}{\sigma^\infty}, \frac{r}{a}, \frac{a}{W}, \nu, \theta\right)$$

3. **With plastic zone:**

$$\frac{\sigma_{ij}}{\sigma^\infty} = F_3\left(\frac{E}{\sigma^\infty}, \frac{\sigma_y}{\sigma^\infty}, \frac{r}{a}, \frac{a}{W}, \frac{r_y}{a}, \nu, \theta\right)$$

Validity of LEFM: $r_y \ll a$ and $\sigma^\infty \ll \sigma_y$.

6. Material Behavior (Typical Cases)

- High-strength steels → Linear elastic (LEFM).
- Medium-strength steels → Elastic–plastic.
- Polymers → Viscoelastic / viscoplastic (depending on T).
- Ceramics → Brittle, LEFM valid.
- Metals at high T or high strain rate → Viscoplastic fracture.