Heat Conduction in a Solid Medium

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LES10A170 Applied Mathematics I

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1 Abstract

This report presents a comprehensive numerical analysis of transient and steady-state heat conduction in a cylindrical copper medium. In this project, Finite Difference Method and MATLAB ODE45 solver has been used. This study focuses on analysis of heat conduction equation to find out the distribution of temperature over a certain space and time. Computational method incorporates on key thermal properties of copper and applies it over a certain boundary and initial conditions. The results of transient temperature, steady state temperature and temperature gradients over radial and axial axis has been demonstrated in this report. The finings can be utilized in application of numerical approaches in engineering applications such as engineering design and thermal management of products.

2 Introduction

Heat conduction is one of the three fundamental mode of thermal energy transfer (convection and radiation being the remaining two). Thermal energy transfer basically occurs through transfer of kinetic energy of molecules or atoms of a relatively hot body to the relatively colder one. Practically, the transfer of energy occurs from the particle with higher energy to that of the lower one. Heat conduction is generally noticed in solid particles, but it can also be observed in liquids and gases too.

The Fourier law expresses heat conduction as:

$$qt = -\lambda \text{ grad } Tx \ tx$$

where qt stands for flow rate density, grad Tx (tx) denotes the temperature gradient of space co-ordinate x and the value of λ is considered constant, depending upon the material. (Ledoux M. et al, 2021)

This study focuses on numerically solving the heat conduction equation for a cylindrical copper medium subjected to specified boundary and initial conditions.

The Objectives of this project are:

1. To study how temperature changes over time and stabilizes in a copper cylinder.
2. To create visual representations showing how temperature evolves and reaches a steady state.
3. To better understand how heat transfers through cylindrical geometries.

3 Method

Numerical computational method and its visualization is the primary method adopted in order to calculate the scenario under provided conditions, analyze the process and showcase the results. The methods applied in this project are elaborated under:

3.1 Numerical Discretization

Numerical discretization methods are often employed in MATLAB to solve differential equations, to handle the system that consists of individual systems or to approximate the integrals. In this project, I considered a copper cylinder of radius 0.3 meters and 1 meter length. Divided it into nodes of grid, with 50 grids along axial and radial directions. These grids will ensure the temperature distribution will be represented accurately in two dimensions. The partial derivatives are approximated using finite difference formulae which will help replace the continuous variations with discrete changes between these grid points.

3.2 Solver Implementation

MATLAB ODE45 solver is used in this project, The temperature values for all of the grid points are kept in a single vector which is then updated iteratively at provided interval of time. In order to balance organized precision and computational demands, relative and absolute tolerances are kept in consideration.

3.3 Visualization Techniques

Transient temperature distribution, Steady-state temperature distribution and temperature gradient across radial and axial section of cylinder has been demonstrated through visualization of obtained results.

4 Results

4.1 Transient Temperature Distribution

The transient temperature results highlight how temperature changes over time as heat spreads throughout the cylinder. At first, there are sharp temperature differences, especially near the boundaries where the set temperature conditions have the most impact. As time passes, the heat distributes more evenly, and the temperatures inside the cylinder become more uniform. This progression is clearly shown in contour plots, which depicts the gradual smoothing of temperature differences with each time step. (Barton J, et al 2009)

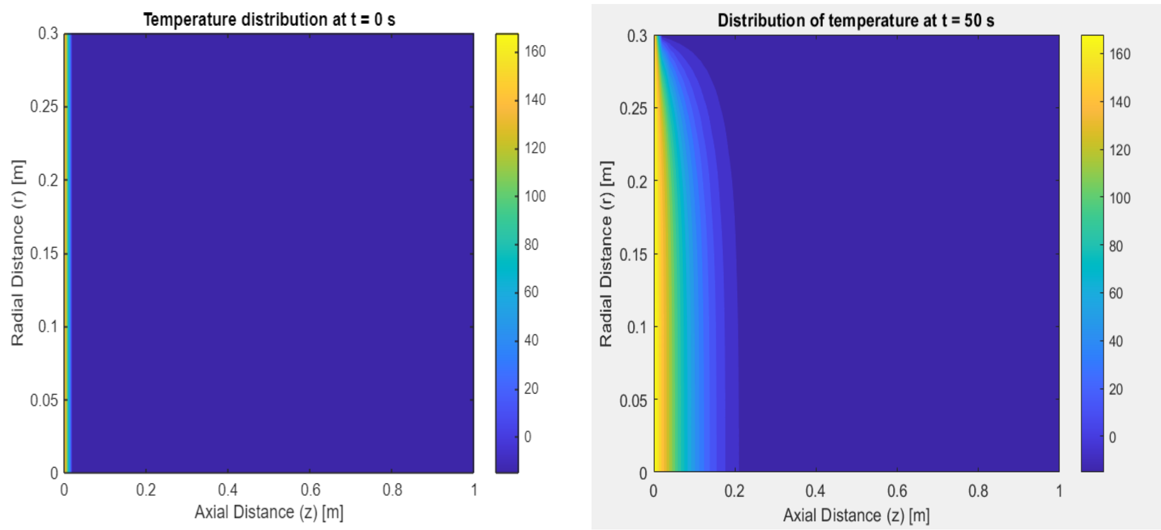

Figure: Transient Temperature Distribution around the cylinder with increased time.

4.2 Steady State Temperature Profile

At steady state, the temperature stops changing over time, indicating the system has stabilized. The results show a clear temperature drop along the length of the cylinder, from the heated end to the cooler, ambient end. The symmetry across the radius is consistent, as expected for a cylinder made of a uniform material. These patterns are effectively illustrated using 2D contour maps and 3D surface plots, offering a detailed view of the stable temperature gradients. (Okonechnikov, A. et al 2020)

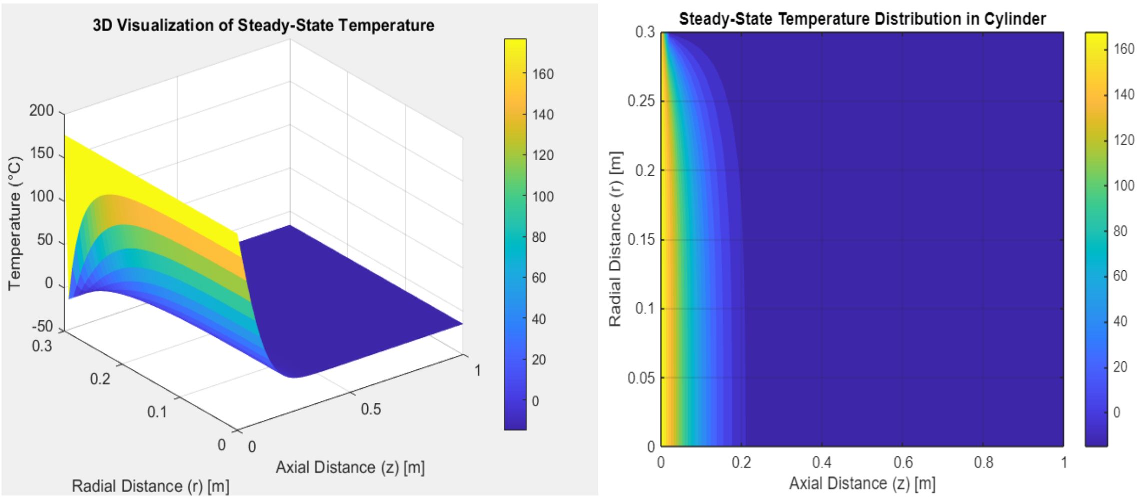

Figure: Steady-State Temperature Distribution in cylinder visualized in 3D

4.3 Temperature Gradients Profiles

Radial temperature profiles at different heights along the cylinder represents how temperature differs across the radius. Close to the heated end, the temperature differences across the radius are more noticeable due to the impact of the boundary condition. Further down the cylinder, in the middle and lower sections, the profiles become more uniform, showing less variation as the temperature nears the ambient level. These patterns help in understanding how heat moves both along the length and across the radius of the cylinder. (Fedetenkov G, et al 2022)

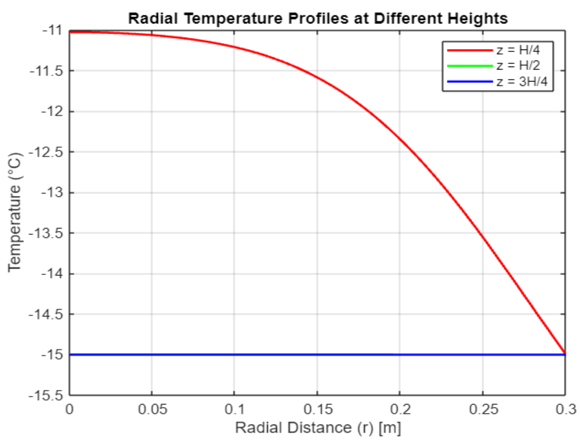

Figure: Radial Temperatures at different heights of the cylinder.

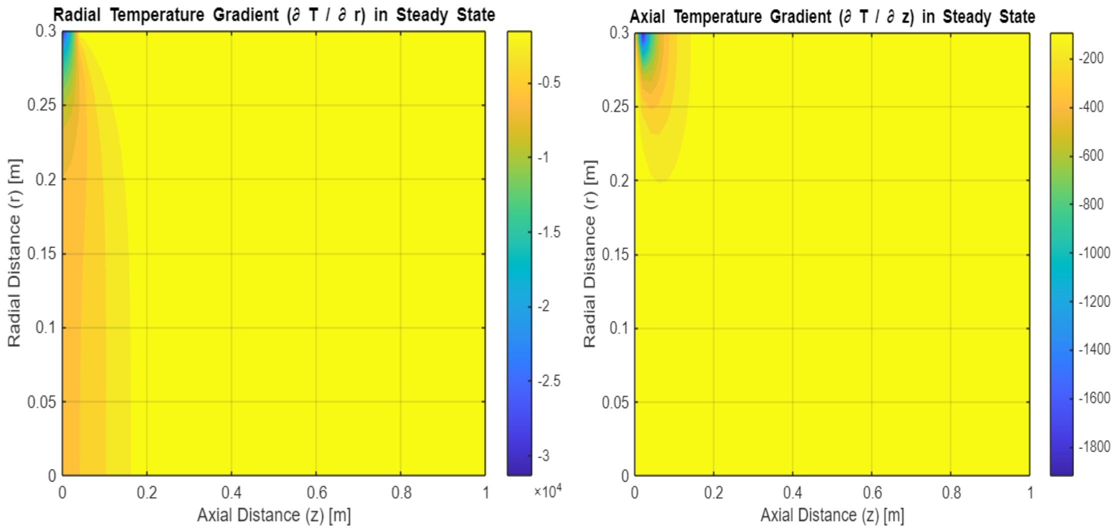

Figure: Temperature Gradient Analysis

5 Discussion

The results match what we expect based on the theory of heat transfer in uniform materials.

5.1 Key Observations

1. During the initial phase, heat spreads over time, and the rate of this process depends on how easily the material allows heat to flow through it.
2. Once the system settles, the temperature along with the material mainly changes in one direction, as expected under the given conditions.

5.2 Possible Applications

1. Creating more efficient heat exchangers.
2. Improving materials to manage heat better in electronics.

5.3 Limitations

1. The model assumes that the material's properties, like its ability to conduct heat, stay constant, which isn't always true.
2. The level of detail in the calculations depends on the grid resolution; more detail gives better results but takes more time and computing power.

6 References

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