$A = LU$

$[a_{ij}] = [l_{ij}][u_{ij}]$

where, $l_{ij} = 0$ if $j > i$ and $u_{ij} = 0$ if $i > j$

$$[a_{ij}] = \left[\sum_{k=1}^{N} l_{ik} u_{kj}\right]$$

if $i < j$

$$a_{ij} = \sum_{k=1}^{i} l_{ik} u_{kj} = \sum_{k=1}^{i-1} l_{ik} u_{kj} + l_{ii} u_{ij}$$

Sunday 20

for cholesky decomposition, $u_{ij} = l_{ij} = l_{ji}^{*} = l_{ji}$ ($\because$ real matrix)

$$\Rightarrow \left(a_{ij} - \sum_{k=1}^{i-1} l_{ik} l_{kj}\right) \frac{1}{l_{ii}} = l_{ij} \qquad (i < j)$$

if $i = j$, $\Rightarrow a_{ii} - \sum_{k=1}^{i-1} l_{ik} l_{ki} = l_{ii}^2$

$$\Rightarrow l_{ii} = \pm\sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik} l_{ki}}$$

$$\Rightarrow l_{ii} = \pm\sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2}$$