Sudipt Nigam
2023AA05523
Sec 3

Assignment -I

1. Population Variance and population Standard Deviation.

x_i	f_i	$x_i f_i$	$x_i - \mu$	$(x_i - \mu)^2$	$(x_i - \mu)^2 \times f_i$
11	33	363	-8.98	80.64	2661.12
13	21	273	-6.98	48.72	1023.12
15	26	390	-4.98	24.8	644.8
17	34	578	-2.98	8.88	~~3101~~ 301.92
19	41	779	-0.98	+ 0.96	39.36
21	27	567	1.02	1.04	28.08
23	36	828	3.02	9.12	328.32
25	25	625	5.02	25.20	630
27	39	1053	7.02	49.28	1921.92
29	20	580	9.02	81.36	1627.2
	302	6036		330	9205.84

$N = \Sigma f_i = 302$

$\Sigma x_i \times f_i = 6036$

$$\mu = \frac{\Sigma x_i \cdot f_i}{\Sigma f_i} = \frac{6036}{302} = 19.98$$

Variance $$\sigma^2 = \frac{\Sigma (x_i - \mu)^2 \times f_i}{\Sigma f_i} = \frac{9205.84}{302} = \boxed{30.48}$$

Standard Deviation $\sigma = \sqrt{30.48} = \boxed{5.52}$

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2.

'PROBABILITY' - 11 letters

'STATISTICS' - 10 letters

Common letters to both words = 'A', 'I', 'T'

Let A be the event of choosing letter from 'PROBABILITY'
& B be the event of choosing letter from 'STATISTICS'

Since A & B are independent so

$$P(A \cap B) = P(A) \cdot P(B)$$

Probability of getting 'A' from 'PROBABILITY' $= \frac{1}{11}$

Probability of getting 'A' from 'STATISTICS' $= \frac{1}{10}$

Prob. of getting A from both $= \frac{1}{11} \times \frac{1}{10} = \frac{1}{110}$

Probability of getting 'I' from 'PROBABILITY' $= \frac{2}{11}$

Probability of getting 'I' from 'STATISTICS' $= \frac{2}{10}$

Prob. of getting 'I' from both $= \frac{2}{11} \times \frac{2}{10} = \frac{4}{110}$

Probability of getting 'T' from 'PROBABILITY' $= \frac{1}{11}$

Probability of getting 'T' from 'STATISTICS' $= \frac{3}{10}$

Prob. of getting 'T' from both $= \frac{1}{11} \times \frac{3}{10} = \frac{3}{110}$

Total probability of getting both letter common

$$P(\text{Common Letters}) = \frac{1}{110} + \frac{4}{110} + \frac{3}{110} = \frac{8}{110} = \boxed{\frac{4}{55}} = 0.073$$

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3. $P(X) = 0.25$

$P(Y) = 0.25$

$P(Z) = 0.35 \longrightarrow$ Out

$P(W) = 0.15$

New total after Z not participating $= .25 + .25 + .15$

$= 0.65$

Since proportion of probability remain same

so 0.65 is equivalent to 1 (Total probability)

New probability

~~P(X)~~

$P(X_{new}) = \dfrac{0.25 \times 1}{0.65} = \boxed{0.385}$

$P(Y_{new}) = \dfrac{0.25 \times 1}{0.65} = \boxed{0.385}$

$P(W_{new}) = \dfrac{0.15 \times 1}{0.65} = \boxed{0.230}$

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4.

Machine Name	No. of bolts/day	defective %
M1	2000	3%
M2	2500	4%
M3	4000	2.5%
	8500	

let D be the event of getting defective bolt.

$P(D|M_1) = 0.03$

$P(D|M_2) = 0.04$

$P(D|M_3) = 0.025$

Probability of picking the bolt from M_1 $P(M_1) = \frac{2000}{8500} = \frac{4}{17}$

" " " " " M_2 $P(M_2) = \frac{2500}{8500} = \frac{5}{17}$

" " " " " M_3 $P(M_3) = \frac{4000}{8500} = \frac{8}{17}$

Applying Baye's theorem

Probability of bolt from M_2 given its defective

$$P(M_2|D) = \frac{P(D|M_2) \cdot P(M_2)}{P(D|M_1) \cdot P(M_1) + P(D|M_2) \cdot P(M_2) + P(D|M_3) \cdot P(M_3)}$$

$$= \frac{0.04 \times 5/17}{0.03 \times 4/17 + 0.04 \times 5/17 + 0.025 \times 8/17}$$

$$\boxed{P(M_2|D) = \frac{5}{13}} = 0.384$$

5.

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Let E_1 is the event of getting 0 Reds ball from A
let $E2$ is the event of getting 1 Red ball from A
let $E3$ is the event of getting 2 Red ball from A
let $E4$ is the event of getting 3 Red ball from A

$$P(E_1) = \frac{{}^4C_3}{{}^8C_3} = \frac{1}{14}$$

$$P(E_2) = \frac{{}^4C_2 \cdot {}^4C_1}{{}^8C_3} = \frac{3}{7}$$

$$P(E_3) = \frac{{}^4C_1 \cdot {}^4C_2}{{}^8C_3} = \frac{3}{7}$$

$$P(E_4) = \frac{{}^4C_3}{{}^8C_3} = \frac{1}{14}$$

$$P(R|E_1) = \frac{4}{5} \qquad \frac{\text{no. of reb ball left in A}}{\text{Total no. of ball lef in A}}$$

$$P(R|E_2) = \frac{3}{5}$$

$$P(R|E_3) = \frac{2}{5}$$

$$P(R|E_4) = \frac{1}{5}$$

By theory of total probability we have

$$P(R) = P(E_1) \cdot P(R|E_1) + P(R|E_2) \cdot P(E_2) + P(R|E_3) \cdot P(E_3) + P(R|E_4)\, P(E_4)$$

$$= \frac{4}{5} \times \frac{1}{14} + \frac{3}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{3}{7} + \frac{1}{5} \times \frac{1}{14}$$

$$P(R) = \frac{35}{70} = \boxed{\frac{1}{2}} = 0.5$$