Work Integrated Learning Programmes Division
M. Tech. in AIML

Assignment 1

AIMLC ZC416 - Mathematical Foundations for Machine Learning

Instructions

1. Use any programming language (other than Excel) of your choice. Attach only the relevant data in your submission and no need to submit the entire code.
2. By random entries, I mean a system generated random number. No marks would be awarded for deterministic entries.
3. This is not a group activity. Each student should do the problems and submit individually.
4. Assignments have to be handwritten and uploaded as a single pdf file with name BITSID.pdf
5. Submissions beyond 26th of Dec, 2023 19.00 hrs would not be graded.
6. Assignments sent via email / other electronic forms would not be accepted.
7. Copying is strictly prohibited. Adoption of unfair means would lead to disciplinary action.

Answer all the questions

Q1) Finding solutions of linear systems

a) Write a code taking as input a matrix $\boldsymbol{A}$ of size $m \times n$ and a vector $\boldsymbol{b}$ of size $m \times 1$, where m and n are arbitrarily large numbers and $m < n$, constructing the augmented matrix and performing

- REF, and
- RREF

without using any built-in functions. In case you encounter any division by 0, you can choose a different $\boldsymbol{A}$ and/or $\boldsymbol{b}$.

Deliverables: The code snippet showing the procedure for REF and RREF. (1 mark + 1 mark)

b) Identify the pivot and non-pivot columns and find the particular solution and solutions to $\boldsymbol{Ax} = \boldsymbol{0}$.

Deliverables: The code snippet showing the pivot and non-pivot columns, particular solution and the solutions to $\boldsymbol{Ax} = \boldsymbol{0}$ (1 mark)

c) Consider a random 5×7 matrix $\boldsymbol{A}$ and a suitable $\boldsymbol{b}$ and show the REF, RREF, pivot columns, non-pivot columns, the particular solution, the solutions to $\boldsymbol{Ax} = \mathbf{0}$, the general solution and verify the general solution.

Deliverables: The random matrix $\boldsymbol{A}$, vector $\boldsymbol{b}$ and other quantities mentioned in the question. $(1/4 \times 8 = 2$ marks)

Q2) Matrix decompositions

a) Consider a symmetric and positive definite matrix $\boldsymbol{A}$ of size $n \times n$. Write a code to construct an elementary matrix for every elementary row operation that is performed on $\boldsymbol{A}$ and using the theory explained in the class write the decomposition of $\boldsymbol{A}$ as LU, where L is a lower triangular matrix and U is an upper triangular matrix.

Deliverables: The code snippet showing the generation of elementary matrices for a given elementary row operation and getting L and U, and verification of $\boldsymbol{A} = LU$. (2 marks)

b) For the above problem, work out the Cholesky's decomposition.

Deliverables: The code snippet showing the generation of L and verification of $\boldsymbol{A} = LL^T$. (1 mark)

c) Given n linearly independent vectors in m dimensions (the corresponding matrix with these as columns is $\boldsymbol{A}$ of size $m \times n$), with $m > n$, read about the QR decomposition of a matrix and generate Q and R.

Deliverables: The code snippet generating Q and R. (1 mark)

d) Take a random 5 x 4 matrix having all its columns as linearly independent and its decompose into Q and R. What is your observation on the diagonal elements of R?

Deliverables: The random matrix, Q and R and your observation on the diagonal elements of R (1 mark)