Additional - Exercise - 13.11442

Q

Q1 LED Bulbs fauty rate = 30%

Randomly select 6 LED.

(a) What is the probability of having 2 faulty LED in sample.

(b) Calculate the avg value of the process

(c) Calculate standard deviation.

Answer

(a) $P = P(\text{Faulty LED}) = 0.3$

$q = 1 - P(\text{Faulty LED}) = 0.7$

$n = 6$

$$P(X=2) = {}^6C_2\, p^2 q^4$$

$$= {}^6C_2\, (.3)^2 (.7)^4$$

$$= \frac{6!}{4!2!} (.09)(.49)(.49)$$

$$= (15)(.09)(.49)(.49)$$

$$= 0.325$$

(b) Avg value $= \mu = n \cdot p$

$= 6 \times .3$

$= 1.8$

(c) Std Standard deviation. $\sigma = \sqrt{n \cdot p \cdot q}$

$= \sqrt{6 \times .3 \times .7}$

$= \sqrt{1.26}$

$= 1.12$

Why binomial

Binomial is the summation of Bernoulli distribution. Probability of finding one LED's quality is bernoulli but we need to calculate probability of two LED's quality. Hence its Binomial.

Q.2 Gaurav attempts 8 question/day
Barkha attempts 12 question/day

G - success rate - 75%
B - success rate - 45%

(a) Probability of each of them solve 5 question correctly.

(b) Probability of Gaurav solving 4 and 6 questions correctly and barkha solving 4 and 6 qs correctly.

(c) Give pictorial representations.

$$\begin{cases} p_1 = P(\text{Gaurav 1 qs correction}) = 0.75 \\ q_1 = P(\text{Gaurav 1 qs incorrect}) = 0.25 \\ n_1 = 8 \end{cases}$$

$$\begin{cases} p_2 = P(\text{Barkha 1 qs correct}) = 0.45 \\ q_2 = P(\text{Barkha 1 qs incorrect}) = 0.55 \\ n_2 = 12 \end{cases}$$

Gaurav

(a) $P(X=5) = {}^{8}C_5 (p_1)^5 (q_1)^3$

$= {}^{8}C_5 (.75)^5 (0.25)^3$

$= 56 \times 0.237 \times 0.015$

$= 0.207$

Barkha

$P(X=5) = {}^{12}C_5 (p_2)^5 (q_2)^7$

$= {}^{12}C_5 (0.45)^5 (0.55)^7$

$= 792 \times 0.018 \times 0.0152$

$= 0.217$

(b) Cumrav

$$P(X=4) = {}^{8}C_4 (p_1)^4 (q_1)^4$$
$$= {}^{8}C_4 (0.75)^4 (0.25)^4$$
$$= 0.08$$

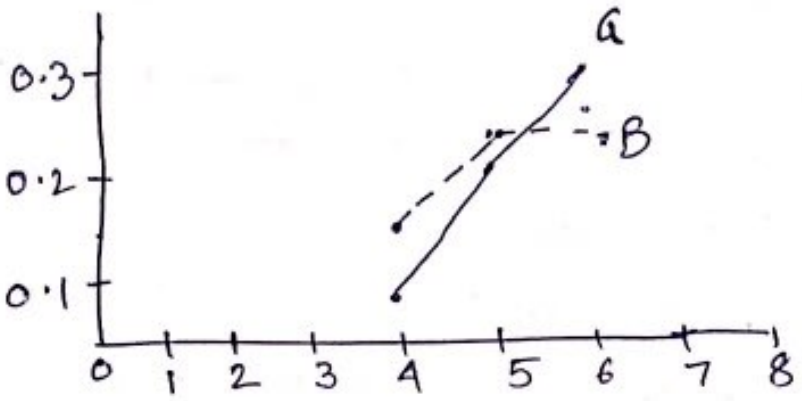
$$P(X=6) = {}^{8}C_6 (p_1)^6 (q_1)^2$$
$$= {}^{8}C_6 (0.75)^6 (0.25)^2$$
$$= 0.309$$

Barkha

$$P(X=4) = {}^{12}C_4 (p_2)^4 (q_2)^8$$
$$= {}^{12}C_4 (0.45)^4 (0.55)^8$$
$$= 0.169$$

$$P(X=6) = {}^{12}C_6 (p_2)^6 (q_2)^6$$
$$= {}^{12}C_6 (0.45)^6 (.55)^6$$
$$= 0.211$$

(c)



We infer that Barkha's probability of providing correct answer will decrease with the increase of n.

Two main governing factor –

1. Correction rate
2. Number of question attempt / day

This is also binomial distribution as we are trying figure out multiple question's correction probability.

Q3 Customer's arrival rate – 72/hour

(a) What is the probability of 'K' customer coming in 4 minutes –

(a) 5 Customer

(b) not more than 3 customer

(c) more than 3 customer

(d) Provide pictorial graph.

Ans: It is poisson distribution as μ is given as we are looking for "how many" in a given time interval.

Customer's arrival rate:

60 min – 72

4 min – $\frac{72}{60} \times 4$

$= \frac{72}{15} = 4.8$

$\mu = 4.8$

$$P(X) = \frac{e^{-\mu} \mu^x}{x!}$$

(a) $P(X=5) = \frac{e^{-4.8} (4.8)^5}{5!}$

$= 0.174$

(b) $P(X \leq 3)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \frac{e^{-4.8}(4.8)^0}{0!} + \frac{e^{-4.8}(4.8)^1}{1!} + \frac{e^{-4.8}(4.8)^2}{2!} + \frac{e^{-4.8}(4.8)^3}{3!}$

$= 0.445$

(c) $P(X > 3)$

$= 1 - P(X \leq 3)$

$= 1 - 0.445$

$= 0.55.$

Q4 Efficiency = 77 words / min

Error rate = 6 Err / hour

(a) What's the probability that I will commit 2 error in a 455 word financial report.

(b) What happens no of words increases to 1000?

(c) What happens no of words decreases to 255?

(d) How is λ affected? PMF? Graph?

(a) Error rate = 6 Err | 60 min

$= \frac{6}{60}$ Err | 1 min

= ·1 Err/min

Efficiency = 77 words/min

No of words = 455

77 words → ·1 Err

255 words → $\frac{\cdot 1}{77} \times 455$ Err

$\mu = \boxed{0.59}$

$$P(X=2) = \frac{e^{-\mu}\mu^x}{x!} = \frac{e^{-0.59}(0.59)^2}{2!} = 0.096$$

(b) No of words = 1000

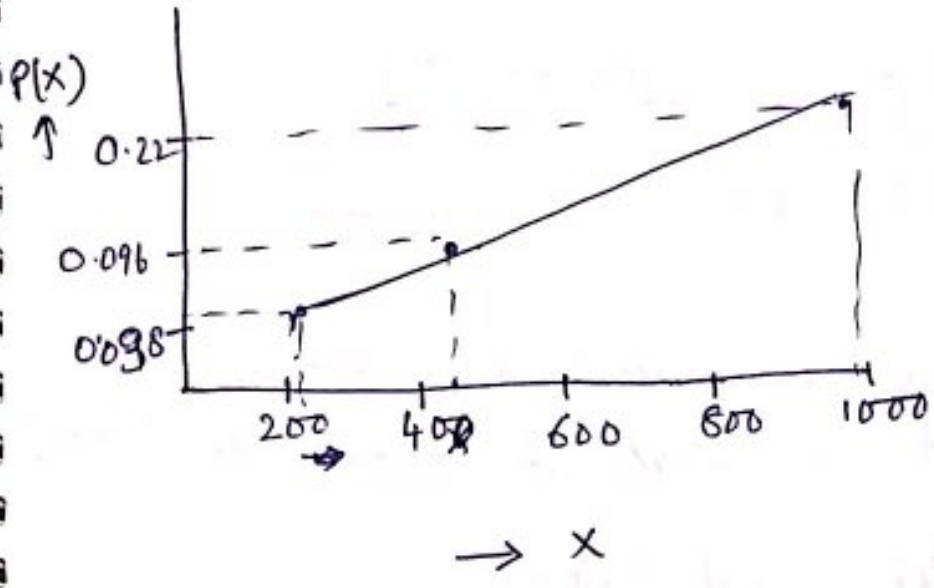
$$\mu = \frac{0.1}{77} \times 1000 = \frac{100}{77} = \boxed{1.27}$$

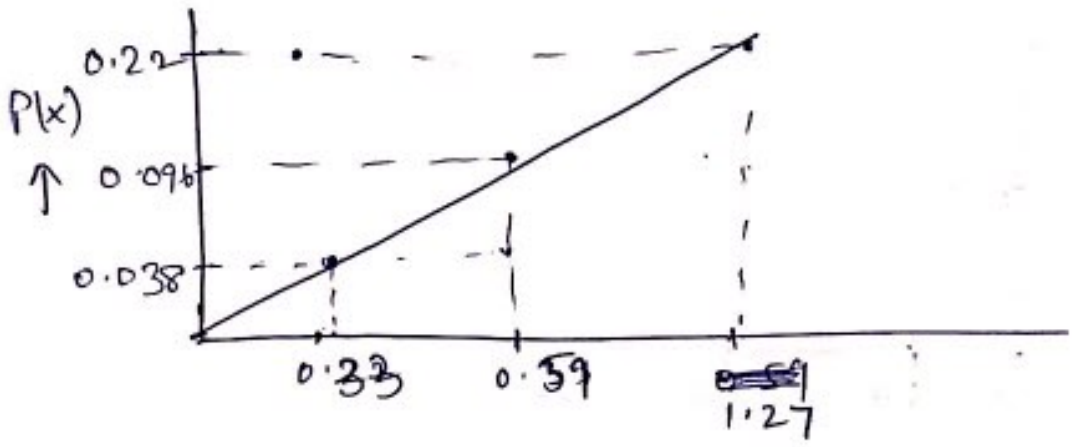
$$P(X=2) = \frac{e^{-\mu}\mu^x}{x!} = \frac{e^{-1.27}(1.27)^2}{2!} = 0.22$$

(c) No of words = 255

$$\mu = \frac{0.1}{77} \times 255 = \boxed{0.33}$$

$$P(X=2) = \frac{e^{-0.33}(0.33)^2}{2!} = 0.038$$





As we increase "no of words", the Error mean(μ) is increasing and also probability of Commiting the "fixed no of errors" is increasing.

Q5 The current measured in a copper wire is modelled by a continuous random variable X in milliamps.

$$X \text{ range} - [0, 20\,mA]$$

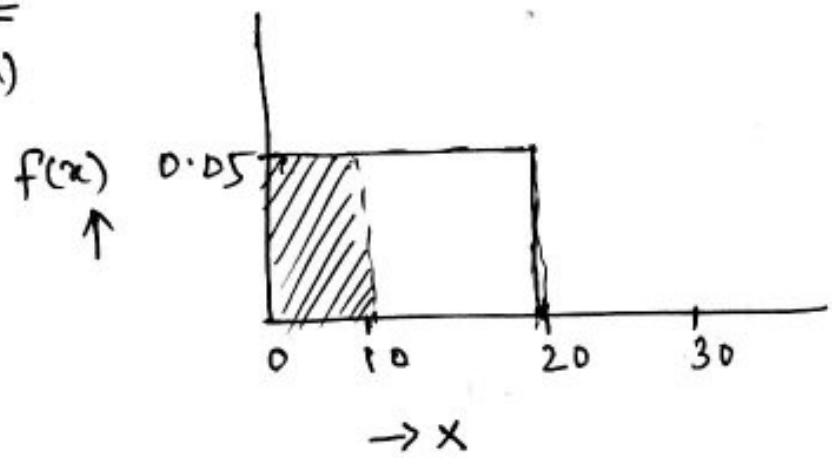
$$f(x) = 0.05 \text{ for } 0 \leq x \leq 20$$

(a) What is the probability that a current measure is less than 10mA?

(b) DTDN PDF and CDF.

Ans:

(a)



$$P(X < 10mA) = \int_0^{10} f(x)\,dx$$

$$= \int_0^{10} 0.05\,dx$$

$$= [0.05]_0^{10} = 0.5$$

(b) $P(X<5) = \int_0^5 0.05\,dx = 0.25$

$P(X<10) = \int_5^{10} 0.05\,dx = 0.5$

$P(X<15) = \int_{10}^{15} 0.05\,dx = 0.75$

$P(X<20) = \int_{15}^{20} 0.05\,dx = 1$