# Lab Report 1

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## 1 Insertion Sort

**Algorithm 1:** Insertion Sort

**Data:** Array $A[1..n]$, Array Size $n$
**for** $i = 2..n$ **do**
  $Insert(A[1...i-1], i)$ `/* this function searches for an appropriate location` $j$ `to insert` $A[i]$ `in` $A[1...i-1]$ `so that` $A[1...i]$ `is sorted, and inserts` $A[i]$ `in the` $j^{th}$ `location */`
**end**

**Algorithm 2:** Insert

**Data:** Array $A[1..i]$, Index $i$
$j \leftarrow i - 1$;
$key \leftarrow A[i]$;
**while** $j \geq 1$ **do**
  `/* compare` $A[j]$ `and` $key$ `*/`
  **if** $A[j] > key$ **then**
    $A[j+1] \leftarrow A[j]$;
    $j \leftarrow j - 1$;
  **else**
    break;
  **end**
**end**
`/* insert` $A[i]$ `in the` $j^{th}$ `location */`
$A[j+1] \leftarrow key$;

## 2 Solutions

### 2.1 Problem 1

The possible locations that $A[i]$ may take in the $i^{th}$ iteration are $1...i$ where $1 \leq i \leq n$.

### 2.2 Problem 2

The number of comparisons performed by Insertion Sort to insert $A[i]$ in location $j$ is given by $i - j + 1$ or $i - j$, which is supported by comparing the observed values and the value of the expression for all values of $i$ and $j$ as shown in this table.

## 2.3 Problem 3

On running Insertion Sort for all possible permutations of 1, 2, 3, 4, we observe the following behaviour.

For $i = 1$, all $A[i]$ get placed at location $j = 1$, with probability 1.

For $i = 2$, we have

| Permutation | j=1 | j=2 |
|---|---|---|
| 1 2 3 4 | | 1 |
| 1 2 4 3 | | 1 |
| 1 3 2 4 | | 1 |
| 1 3 4 2 | | 1 |
| 1 4 2 3 | | 1 |
| 1 4 3 2 | | 1 |
| 2 1 3 4 | 1 | |
| 2 1 4 3 | 1 | |
| 2 3 1 4 | | 1 |
| 2 3 4 1 | | 1 |
| 2 4 1 3 | | 1 |
| 2 4 3 1 | | 1 |
| 3 1 2 4 | 1 | |
| 3 1 4 2 | 1 | |
| 3 2 1 4 | 1 | |
| 3 2 4 1 | 1 | |
| 3 4 1 2 | | 1 |
| 3 4 2 1 | | 1 |
| 4 1 2 3 | 1 | |
| 4 1 3 2 | 1 | |
| 4 2 1 3 | 1 | |
| 4 2 3 1 | 1 | |
| 4 3 1 2 | 1 | |
| 4 3 2 1 | 1 | |
| Occurences | 12 | 12 |

We observe that the probabilities of $A[i]$ to get placed in $j = 1$ and $j = 2$ are equal and given by $1/2$.

For $i = 3$, we have

| Permutation | j=1 | j=2 | j=3 |
|---|---|---|---|
| 1 2 3 4 | | | 1 |
| 1 2 4 3 | | | 1 |
| 1 3 2 4 | | 1 | |
| 1 3 4 2 | | | 1 |
| 1 4 2 3 | | 1 | |
| 1 4 3 2 | | 1 | |
| 2 1 3 4 | | | 1 |
| 2 1 4 3 | | | 1 |
| 2 3 1 4 | 1 | | |
| 2 3 4 1 | | | 1 |
| 2 4 1 3 | 1 | | |
| 2 4 3 1 | | 1 | |
| 3 1 2 4 | | 1 | |
| 3 1 4 2 | | | 1 |
| 3 2 1 4 | 1 | | |
| 3 2 4 1 | | | 1 |
| 3 4 1 2 | 1 | | |
| 3 4 2 1 | 1 | | |
| 4 1 2 3 | | 1 | |

| Permutation | j=1 | j=2 | j=3 |
|---|---|---|---|
| 4 1 3 2 | | 1 | |
| 4 2 1 3 | 1 | | |
| 4 2 3 1 | | 1 | |
| 4 3 1 2 | 1 | | |
| 4 3 2 1 | 1 | | |
| Occurences | 8 | 8 | 8 |

We observe that the probabilities of $A[i]$ to get placed in $j = 1$, $j = 2$ and $j = 3$ are equal and given by $1/3$.

For $i = 4$, we have

| Permutation | j=1 | j=2 | j=3 | j=4 |
|---|---|---|---|---|
| 1 2 3 4 | | | | 1 |
| 1 2 4 3 | | | 1 | |
| 1 3 2 4 | | | | 1 |
| 1 3 4 2 | | 1 | | |
| 1 4 2 3 | | | 1 | |
| 1 4 3 2 | | 1 | | |
| 2 1 3 4 | | | | 1 |
| 2 1 4 3 | | | 1 | |
| 2 3 1 4 | | | | 1 |
| 2 3 4 1 | 1 | | | |
| 2 4 1 3 | | | 1 | |
| 2 4 3 1 | 1 | | | |
| 3 1 2 4 | | | | 1 |
| 3 1 4 2 | | 1 | | |
| 3 2 1 4 | | | | 1 |
| 3 2 4 1 | 1 | | | |
| 3 4 1 2 | | 1 | | |
| 3 4 2 1 | 1 | | | |
| 4 1 2 3 | | | 1 | |
| 4 1 3 2 | | 1 | | |
| 4 2 1 3 | | | 1 | |
| 4 2 3 1 | 1 | | | |
| 4 3 1 2 | | 1 | | |
| 4 3 2 1 | 1 | | | |
| Occurences | 6 | 6 | 6 | 6 |

We observe that the probabilities of $A[i]$ to get placed in $j = 1$, $j = 2$, $j = 3$ and $j = 4$ are equal and given by $1/4$.

We can thus say that the probability that $A[i]$ will be inserted at the $j^{th}$ location is $1/i$. ■

### 2.3.1 Part (a)

For every $i = 2..4$, we report the total number of comparisons done to insert $A[i]$ to the $j^{th}$ location and compute the average number of comparisons over all possible permutations. The observations are recorded in the table below.

| Permutation | i=2 | i=3 | i=4 | Total Comparisons |
|---|---|---|---|---|
| 1 2 3 4 | 1 | 1 | 1 | 3 |
| 1 2 4 3 | 1 | 1 | 2 | 4 |
| 1 3 2 4 | 1 | 2 | 1 | 4 |
| 1 3 4 2 | 1 | 1 | 3 | 5 |
| 1 4 2 3 | 1 | 2 | 2 | 5 |
| 1 4 3 2 | 1 | 2 | 3 | 6 |
| 2 1 3 4 | 1 | 1 | 1 | 3 |
| 2 1 4 3 | 1 | 1 | 2 | 4 |

| Permutation | i=2 | i=3 | i=4 | Total Comparisons |
|---|---|---|---|---|
| 2 3 1 4 | 1 | 2 | 1 | 4 |
| 2 3 4 1 | 1 | 1 | 3 | 5 |
| 2 4 1 3 | 1 | 2 | 2 | 5 |
| 2 4 3 1 | 1 | 2 | 3 | 6 |
| 3 1 2 4 | 1 | 2 | 1 | 4 |
| 3 1 4 2 | 1 | 1 | 3 | 5 |
| 3 2 1 4 | 1 | 2 | 1 | 4 |
| 3 2 4 1 | 1 | 1 | 3 | 5 |
| 3 4 1 2 | 1 | 2 | 3 | 6 |
| 3 4 2 1 | 1 | 2 | 3 | 6 |
| 4 1 2 3 | 1 | 2 | 2 | 5 |
| 4 1 3 2 | 1 | 2 | 3 | 6 |
| 4 2 1 3 | 1 | 2 | 2 | 5 |
| 4 2 3 1 | 1 | 2 | 3 | 6 |
| 4 3 1 2 | 1 | 2 | 3 | 6 |
| 4 3 2 1 | 1 | 2 | 3 | 6 |
| Sum | 24 | 40 | 54 | 118 |
| Average | 1 | 1.66666666666667 | 2.25 | 4.91666666666667 |

#### 2.3.2 Part (b)

We construct a 2D-table with $i$ in rows and $j$ in columns where each $P[i, j]$ represents the probability that $A[i]$ will be inserted in location $j$ in the $i^{th}$ iteration.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 24/24 | 0 | 0 | 0 |
| 2 | 12/24 | 12/24 | 0 | 0 |
| 3 | 8/24 | 8/24 | 8/24 | 0 |
| 4 | 6/24 | 6/24 | 6/24 | 6/24 |

#### 2.3.3 Part (c)

TODO.

### 2.4 Problem 4

TODO.

## 3 Raw Data

| permutation | i | j | comparisons |
|---|---|---|---|
| 1 2 3 4 | 2 | 2 | 1 |
| 1 2 3 4 | 3 | 3 | 1 |
| 1 2 3 4 | 4 | 4 | 1 |
| 1 2 4 3 | 2 | 2 | 1 |
| 1 2 4 3 | 3 | 3 | 1 |
| 1 2 4 3 | 4 | 3 | 2 |
| 1 3 2 4 | 2 | 2 | 1 |
| 1 3 2 4 | 3 | 2 | 2 |
| 1 3 2 4 | 4 | 4 | 1 |
| 1 3 4 2 | 2 | 2 | 1 |
| 1 3 4 2 | 3 | 3 | 1 |
| 1 3 4 2 | 4 | 2 | 3 |
| 1 4 2 3 | 2 | 2 | 1 |
| 1 4 2 3 | 3 | 2 | 2 |
| 1 4 2 3 | 4 | 3 | 2 |
| 1 4 3 2 | 2 | 2 | 1 |
| 1 4 3 2 | 3 | 2 | 2 |

| permutation | i | j | comparisons |
|---|---|---|---|
| 1 4 3 2 | 4 | 2 | 3 |
| 2 1 3 4 | 2 | 1 | 1 |
| 2 1 3 4 | 3 | 3 | 1 |
| 2 1 3 4 | 4 | 4 | 1 |
| 2 1 4 3 | 2 | 1 | 1 |
| 2 1 4 3 | 3 | 3 | 1 |
| 2 1 4 3 | 4 | 3 | 2 |
| 2 3 1 4 | 2 | 2 | 1 |
| 2 3 1 4 | 3 | 1 | 2 |
| 2 3 1 4 | 4 | 4 | 1 |
| 2 3 4 1 | 2 | 2 | 1 |
| 2 3 4 1 | 3 | 3 | 1 |
| 2 3 4 1 | 4 | 1 | 3 |
| 2 4 1 3 | 2 | 2 | 1 |
| 2 4 1 3 | 3 | 1 | 2 |
| 2 4 1 3 | 4 | 3 | 2 |
| 2 4 3 1 | 2 | 2 | 1 |
| 2 4 3 1 | 3 | 2 | 2 |
| 2 4 3 1 | 4 | 1 | 3 |
| 3 1 2 4 | 2 | 1 | 1 |
| 3 1 2 4 | 3 | 2 | 2 |
| 3 1 2 4 | 4 | 4 | 1 |
| 3 1 4 2 | 2 | 1 | 1 |
| 3 1 4 2 | 3 | 3 | 1 |
| 3 1 4 2 | 4 | 2 | 3 |
| 3 2 1 4 | 2 | 1 | 1 |
| 3 2 1 4 | 3 | 1 | 2 |
| 3 2 1 4 | 4 | 4 | 1 |
| 3 2 4 1 | 2 | 1 | 1 |
| 3 2 4 1 | 3 | 3 | 1 |
| 3 2 4 1 | 4 | 1 | 3 |
| 3 4 1 2 | 2 | 2 | 1 |
| 3 4 1 2 | 3 | 1 | 2 |
| 3 4 1 2 | 4 | 2 | 3 |
| 3 4 2 1 | 2 | 2 | 1 |
| 3 4 2 1 | 3 | 1 | 2 |
| 3 4 2 1 | 4 | 1 | 3 |
| 4 1 2 3 | 2 | 1 | 1 |
| 4 1 2 3 | 3 | 2 | 2 |
| 4 1 2 3 | 4 | 3 | 2 |
| 4 1 3 2 | 2 | 1 | 1 |
| 4 1 3 2 | 3 | 2 | 2 |
| 4 1 3 2 | 4 | 2 | 3 |
| 4 2 1 3 | 2 | 1 | 1 |
| 4 2 1 3 | 3 | 1 | 2 |
| 4 2 1 3 | 4 | 3 | 2 |
| 4 2 3 1 | 2 | 1 | 1 |
| 4 2 3 1 | 3 | 2 | 2 |
| 4 2 3 1 | 4 | 1 | 3 |
| 4 3 1 2 | 2 | 1 | 1 |
| 4 3 1 2 | 3 | 1 | 2 |
| 4 3 1 2 | 4 | 2 | 3 |
| 4 3 2 1 | 2 | 1 | 1 |
| 4 3 2 1 | 3 | 1 | 2 |
| 4 3 2 1 | 4 | 1 | 3 |