Lab Report 1

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1 Insertion Sort

Algorithm 1: Insertion Sort

Data: Array $A[1...n]$, Array Size n
for $i = 2...n$ **do**
  $Insert(A[1...i-1], i)$ `/* this function searches for an appropriate location j to insert A[i] in A[1...i-1] so that A[1...i] is sorted, and inserts A[i] in the` j^{th} `location */`
end

Algorithm 2: Insert

Data: Array $A[1...i]$, Index i
$j \leftarrow i - 1$;
$key \leftarrow A[i]$;
while $j \geq 1$ **do**
  `/* compare` $A[j]$ `and` key `*/`
  if $A[j] > key$ **then**
    $A[j+1] \leftarrow A[j]$;
    $j \leftarrow j - 1$;
  else
    break;
  end
end
`/* insert` $A[i]$ `in the` j^{th} `location */`
$A[j+1] \leftarrow key$;

2 Solutions

2.1 Problem 1

The possible locations that $A[i]$ may take in the i^{th} iteration are $1...i$ where $1 \leq i \leq n$.

2.2 Problem 2

The number of comparisons performed by Insertion Sort to insert $A[i]$, where $i = 2...n$, in location j is given by the following expression, and is supported by the observed values and the value of the expression for all values of i and j as shown in this table. There are no comparisons for $i = 1$.

$$comps(i, j) = \begin{cases} i - j, & \text{if } j = 1 \\ i - j + 1, & \text{if } 1 < j \leq i \end{cases}$$

2.3 Problem 3

On running Insertion Sort for all possible permutations of 1, 2, 3, 4, we observe the following behaviour.

For $i = 1$, all $A[i]$ get placed at location $j = 1$, with probability 1.

For $i = 2$, we have

Permutation	j=1	j=2
1 2 3 4		1
1 2 4 3		1
1 3 2 4		1
1 3 4 2		1
1 4 2 3		1
1 4 3 2		1
2 1 3 4	1	
2 1 4 3	1	
2 3 1 4		1
2 3 4 1		1
2 4 1 3		1
2 4 3 1		1
3 1 2 4	1	
3 1 4 2	1	
3 2 1 4	1	
3 2 4 1	1	
3 4 1 2		1
3 4 2 1		1
4 1 2 3	1	
4 1 3 2	1	
4 2 1 3	1	
4 2 3 1	1	
4 3 1 2	1	
4 3 2 1	1	
Occurrences	12	12

We observe that the probabilities of $A[i]$ to get placed in $j = 1$ and $j = 2$ are equal and given by $1/2$.

For $i = 3$, we have

Permutation	j=1	j=2	j=3
1 2 3 4			1
1 2 4 3			1
1 3 2 4		1	
1 3 4 2			1
1 4 2 3		1	
1 4 3 2		1	
2 1 3 4			1
2 1 4 3			1
2 3 1 4	1		
2 3 4 1			1
2 4 1 3	1		
2 4 3 1		1	
3 1 2 4		1	
3 1 4 2			1
3 2 1 4	1		
3 2 4 1			1
3 4 1 2	1		
3 4 2 1	1		
4 1 2 3		1	

Permutation	j=1	j=2	j=3
4 1 3 2		1	
4 2 1 3	1		
4 2 3 1		1	
4 3 1 2	1		
4 3 2 1	1		
Occurrences	8	8	8

We observe that the probabilities of $A[i]$ to get placed in $j = 1$, $j = 2$ and $j = 3$ are equal and given by $1/3$.

For $i = 4$, we have

Permutation	j=1	j=2	j=3	j=4
1 2 3 4				1
1 2 4 3			1	
1 3 2 4				1
1 3 4 2		1		
1 4 2 3			1	
1 4 3 2		1		
2 1 3 4				1
2 1 4 3			1	
2 3 1 4				1
2 3 4 1	1			
2 4 1 3			1	
2 4 3 1	1			
3 1 2 4				1
3 1 4 2		1		
3 2 1 4				1
3 2 4 1	1			
3 4 1 2		1		
3 4 2 1	1			
4 1 2 3			1	
4 1 3 2		1		
4 2 1 3			1	
4 2 3 1	1			
4 3 1 2		1		
4 3 2 1	1			
Occurrences	6	6	6	6

We observe that the probabilities of $A[i]$ to get placed in $j = 1$, $j = 2$, $j = 3$ and $j = 4$ are equal and given by $1/4$.

We can thus say that the probability that $A[i]$ will be inserted at the j^{th} location is $1/i$. ■

2.3.1 Part (a)

For every $i = 2..4$, we report the total number of comparisons done to insert $A[i]$ to the j^{th} location and compute the average number of comparisons over all possible permutations. The observations are recorded in the table below.

Permutation	i=2	i=3	i=4	Sum
1 2 3 4	1	1	1	3
1 2 4 3	1	1	2	4
1 3 2 4	1	2	1	4
1 3 4 2	1	1	3	5
1 4 2 3	1	2	2	5
1 4 3 2	1	2	3	6
2 1 3 4	1	1	1	3
2 1 4 3	1	1	2	4

Permutation	i=2	i=3	i=4	Sum
2 3 1 4	1	2	1	4
2 3 4 1	1	1	3	5
2 4 1 3	1	2	2	5
2 4 3 1	1	2	3	6
3 1 2 4	1	2	1	4
3 1 4 2	1	1	3	5
3 2 1 4	1	2	1	4
3 2 4 1	1	1	3	5
3 4 1 2	1	2	3	6
3 4 2 1	1	2	3	6
4 1 2 3	1	2	2	5
4 1 3 2	1	2	3	6
4 2 1 3	1	2	2	5
4 2 3 1	1	2	3	6
4 3 1 2	1	2	3	6
4 3 2 1	1	2	3	6
Sum	24	40	54	118
Average	1.0000	1.6667	2.2500	4.9167

2.3.2 Part (b)

We construct a 2D-table with i in rows and j in columns where each $P[i, j]$ represents the probability that $A[i]$ will be inserted in location j in the i^{th} iteration.

i\j	1	2	3	4
1	24/24	0	0	0
2	12/24	12/24	0	0
3	8/24	8/24	8/24	0
4	6/24	6/24	6/24	6/24

2.3.3 Part (c)

We calculate the average number of comparisons using probabilities values $P[i, j]$ as calculated in part (b) and values of the function $comps(i, j)$ which was defined earlier.

$$
\begin{aligned}
E(X) &= \sum_{i=2}^{n} \sum_{j=1}^{i} comps(i,j) \cdot P[i,j] \\
&= comps(2,1) \cdot P[2,1] + comps(2,2) \cdot P[2,2] + comps(3,1) \cdot P[3,1] \\
&+ comps(3,2) \cdot P[3,2] + comps(3,3) \cdot P[3,3] + comps(4,1) \cdot P[4,1] \\
&+ comps(4,2) \cdot P[4,2] + comps(4,3) \cdot P[4,3] + comps(4,4) \cdot P[4,4] \\
&= 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 3 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} +, 2 \cdot \frac{1}{4} + 1 \cdot \frac{1}{4} \\
&= 4.9167
\end{aligned}
$$

We observe that the expected theoretical value of the average number of comparisons is equal to the actual observed average number of comparisons. ■

2.4 Problem 4

We observe the placement of the elements of a randomly generated input sequence of size 3.

For $i = 1$, all $A[i]$ get placed at location $j = 1$ with probability 1.

For $i = 2$, we have

Permutation	j=1	j=2
3 4 5		1
3 5 4		1

Permutation	j=1	j=2
4 3 5	1	
4 5 3		1
5 3 4	1	
5 4 3	1	
Occurrences	3	3

We observe that the probabilities of $A[i]$ to get placed in $j = 1$, and $j = 2$ are equal and given by $1/2$.

For $i = 3$, we have

Permutation	j=1	j=2	j=3
3 4 5			1
3 5 4		1	
4 3 5			1
4 5 3	1		
5 3 4		1	
5 4 3	1		
Occurrences	2	2	2

We observe that the probabilities of $A[i]$ to get placed in $j = 1$, $j = 2$ and $j = 3$ are equal and given by $1/3$.

We can thus say that the probability that $A[i]$ will be inserted at the j^{th} location is $1/i$. ■

3 Raw Data

permutation	i	j	comparisons
2 3 4 1	2	2	1
2 3 4 1	3	3	1
2 3 4 1	4	1	3
3 2 4 1	2	1	1
3 2 4 1	3	3	1
3 2 4 1	4	1	3
3 4 2 1	2	2	1
3 4 2 1	3	1	2
3 4 2 1	4	1	3
4 3 2 1	2	1	1
4 3 2 1	3	1	2
4 3 2 1	4	1	3
2 4 3 1	2	2	1
2 4 3 1	3	2	2
2 4 3 1	4	1	3
4 2 3 1	2	1	1
4 2 3 1	3	2	2
4 2 3 1	4	1	3
4 3 1 2	2	1	1
4 3 1 2	3	1	2
4 3 1 2	4	2	3
3 4 1 2	2	2	1
3 4 1 2	3	1	2
3 4 1 2	4	2	3
3 1 4 2	2	1	1
3 1 4 2	3	3	1
3 1 4 2	4	2	3
1 3 4 2	2	2	1
1 3 4 2	3	3	1

permutation	i	j	comparisons
1 3 4 2	4	2	3
4 1 3 2	2	1	1
4 1 3 2	3	2	2
4 1 3 2	4	2	3
1 4 3 2	2	2	1
1 4 3 2	3	2	2
1 4 3 2	4	2	3
2 4 1 3	2	2	1
2 4 1 3	3	1	2
2 4 1 3	4	3	2
4 2 1 3	2	1	1
4 2 1 3	3	1	2
4 2 1 3	4	3	2
4 1 2 3	2	1	1
4 1 2 3	3	2	2
4 1 2 3	4	3	2
1 4 2 3	2	2	1
1 4 2 3	3	2	2
1 4 2 3	4	3	2
2 1 4 3	2	1	1
2 1 4 3	3	3	1
2 1 4 3	4	3	2
1 2 4 3	2	2	1
1 2 4 3	3	3	1
1 2 4 3	4	3	2
2 3 1 4	2	2	1
2 3 1 4	3	1	2
2 3 1 4	4	4	1
3 2 1 4	2	1	1
3 2 1 4	3	1	2
3 2 1 4	4	4	1
3 1 2 4	2	1	1
3 1 2 4	3	2	2
3 1 2 4	4	4	1
1 3 2 4	2	2	1
1 3 2 4	3	2	2
1 3 2 4	4	4	1
2 1 3 4	2	1	1
2 1 3 4	3	3	1
2 1 3 4	4	4	1
1 2 3 4	2	2	1
1 2 3 4	3	3	1
1 2 3 4	4	4	1