Algebraic number theory - Fermat last theorem part3

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Abstract

in this paper we will provide a simple proof the Fermat conjecture using a very elementary proof on a special case.

1 4-proving the last Fermat conjecture on the case that n is a prime

let x,y,z three positive integers and p is an odd prime and (p,z) = 1, suppose that

$$x^p + y^p = z^p \tag{1}$$

Lemma 1 (Fermat little theorem)**.** $(\forall p \in \mathbb{P}) : \forall n \in \mathbb{N} : n^p \equiv n[p]$

Lemma 2. $(\forall p \in \mathbb{P})(\forall k \in \mathbb{N}) : 0 < k < p : p|\mathrm{C}_p^k$

so we have that(using **lemma 1**) $x^p + y^p + z^p \equiv x + y + z[p]$
$2z^p \equiv x + y + z[p]$ then $2^p z^{p^2} \equiv (x + y + z)^p[p]$

$\therefore$

$$2^p z \equiv \sum_{k=0}^{p} \mathrm{C}_p^k (x+y)^{p-k} z^k [p]$$

as a consequence of **lemma 2**, we have that :
$2^p z \equiv (x+y)^p + z^p \Leftrightarrow 2^p z \equiv (x+y)^p + z$
$\Leftrightarrow z(2^p - 1) \equiv \sum_{k=0}^{p} \mathrm{C}_p^k x^{p-k} y^k [p]$
again using **lemma 2** : we will find that
$z(2^p - 1) \equiv x^p + y^p [p]$
$\therefore z(2^p - 1) \equiv z[p]$ using gauss theorem
$\because$ (z,p) = 1 we could subtract z , therefore :
$2^p \equiv 1[p]$, and while that $p > 2$ we have (p,2) = 1 so using lemma 1 : we will have that : $2^p \equiv 2^{p-1}$ therefore : p = 1
hence! contradiction.
the equation (1) Have no solution .
QED.

2 References

[1]-Ore, Oystein (1988) [1948], Number Theory and Its History, Dover, ISBN 978-0-486-65620-5.