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4-Using Professor S-Mochizuki result

in 2012 Professor **Shinichi Mochizuki** in kyoto university proved that for any three integers a,b,c such that a coprime with b and a+b=c we have

$$c < rad(abc)^2 \tag{1}$$

$$(\forall n \in \mathbb{Z}) : rad(n) \overset{def}{=} \prod_{p|n} p \tag{2}$$

(such that p is a prime)

let x , y , z , n three positive integers > 0 such that n>2 : and

$$x^n + y^n = z^n \tag{3}$$

we could suppose that x,y are coprimes because it is enough ,

okay let's suppose that x,y are not coprimes so

$\exists p \in \mathbb{P}$:

p —x $\wedge p|y \rightarrow \exists (q, q') \in \mathbb{N}^{*2}$:

$x = p.q \wedge y = p.q'$

$\rightarrow x^n + y^n = p^n(q^n + q'^n) = z^n \rightarrow p|z^n \rightarrow p|z \rightarrow \exists d \in \mathbb{N}$:

$z = p.d$

$$\rightarrow z^n = p^n d^n \rightarrow p^n(q^n + q'^n) = p^n d^n \rightarrow q^n + q'^n = d^n \tag{4}$$

and so one until we subtract all coprimes

so after **mochizuki** we have

$z^n < rad((xyz)^n)^2 = rad(xyz)^2 \rightarrow z^n < rad(z^3)^2$

$\rightarrow z^n \leq z^6$

$\rightarrow n \leq 6 \rightarrow n \in \{3, 4, 5, 6\}$

-The case n = 3 was proven by Euler in 1770.

-The case n = 5 was proved by Dirichlet and Legendre around 1825.

-Alternative proofs of the case n = 4 were developed later by Frénicle de Bessy,

-Proofs for n = 6 have been published by Kausler,