a new approach to Goldbach conjecture and goldbach numbers density

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Abstract

in this paper we will provide a simple prove of the existence of infinitely many Goldbach numbers.

1 Introduction

The Goldbach Conjecture is a famous unsolved problem in number theory that states that every even integer greater than 2 can be expressed as the sum of two prime numbers. It was first proposed by the German mathematician Christian Goldbach in a letter to the Swiss mathematician Leonhard Euler in 1742. Despite much effort, a proof or counterexample has yet to be found. It is one of the oldest unsolved problems in number theory and in all of mathematics.

2 Demonstration principle

Let

$$P_G(n) : (\exists p \in \mathbb{P} : 2n - p \in \mathbb{P})$$

$$G := \{n \in \mathbb{N} | P_G(n)\}$$

let's suppose that G is finite.
We have that $2 \in G$.

$\because 2 \cdot 2 - 2 \in \mathbb{P}$
and we have that $G \subset \mathbb{N}$, and (G is finite).
Peano axiome : G has an element maximal , let's note it as , $\max(G) := g$.

let $p \in \mathbb{P} : p > 2, \forall m \in \mathbb{N} : m > g : 2m = p + q$ such that $q \notin \mathbb{P}$ and $q \notin 2\mathbb{N}$($\because p$ is an odd prime.)

let $k \in \mathbb{N} : 2m + 2k = p + q + 2k$
$\Longrightarrow 2(m + k) = p + (q + 2k)$,and we have that :
$2(m + k) \geq 2m > 2g \Longrightarrow 2(m + k) \notin G$

$$\therefore q + 2k \notin \mathbb{P} \tag{1}$$

(because if we suppose that :
$q + 2k \in \mathbb{P} \Longrightarrow 2(m + k) - p \in \mathbb{P} \therefore 2(m + k) \in G$
and that's impossible. because g is the maximal element of G. and $m > g$.)

let
$f_q : \mathbb{N} \longrightarrow 2\,[|q, +\infty[+ 1$
$q \longrightarrow q + 2k$
we have that $q + 2k = q + 2k' \Longrightarrow k = k'$
so f_q : is a injective application.

let : $\lambda \in 2\,[|q, +\infty[+ 1$ so $\exists k' \in [|q, +\infty[: \lambda = 2k' + 1$ to prove that f_q is a surjective application , we have to prove the existence of k $\in \mathbb{N}$ such that :
$f_q(k) = 2k' + 1 \Longrightarrow q + 2k = 2k' + 1$

$\Longrightarrow k = \frac{2k'+1-q}{2} \in \mathbb{N}(\because q$ is an odd integer .)
the application f_q : is a bijective application in $2\,[|q, +\infty[+ 1$. using equation (1) we have that

$$\forall k \in \mathbb{N} : q + 2k \notin \mathbb{P}$$

in other way :
$\forall k \in \mathbb{N} : f_q(k) \notin \mathbb{N} \implies \forall \lambda \in 2\,[|q, +\infty[+ 1 : \lambda \notin \mathbb{P}$
hence ! contradication . because there exist infinitely many primes.
the set G is infinite set . QED.