

Algebraic Number theory : Fermat last theorem

Nasr-allah Hitar

October 2022

1 1-Introduction :

In number theory, Fermat's Last Theorem (sometimes called Fermat's conjecture, especially in older texts) states that no three positive integers a , b , and c satisfy the equation

$$a^n + b^n = c^n \tag{1}$$

for any integer value of n greater than 2. The cases $n = 1$ and $n = 2$ have been known since antiquity to have infinitely many solutions , The proposition was first stated as a theorem by Pierre de Fermat around 1637 in the margin of a copy of *Arithmetica*. After 358 years of effort by mathematicians, the first successful proof was released in 1994 by Andrew Wiles and formally published in 1995(using Taniyama Shimura conjecture).

2 2-abc conjecture and applications :

Theorem : (Fermat Last Theorem) No three positive integers a , b , and c satisfy the equation

$$a^n + b^n = c^n \quad (2)$$

Before proving it We have to talk about abc conjecture

Conjecture : (abc conjecture) For all triples (a, b, c) of coprime positive integers with $a + b = c$, $\text{rad}(abc)$ is at least $c^{1-o(1)}$

An equivalent formulation is: For every positive real number ϵ , there exists a constant K_ϵ such that for all triples (a, b, c) of coprime positive integers, with $a + b = c$

We have that

$$: c < K_\epsilon (\text{rad}(abc))^{1+\epsilon} \quad (3)$$

The conjecture was proved (not officially) by **Shinichi Mochizuki** he proved it using a new theory he called **inter-universal Teichmüller theory (IUTT)**.

3 3-abc application to fermat last theorem

let x,y,z three integers such that

$$x^n + y^n = z^n \quad (4)$$

(obviously that x,y,z are co-primes because we could subtract the gcd(x,y,z))

let

$$a = x^n, b = y^n, c = z^n \quad (5)$$

so a+b=c and a,b,c are coprimes .

so for every

$$\epsilon > 0 : c < k\epsilon(rad(abc)^{1+\epsilon}) \quad (6)$$

$$\Rightarrow c < \lim_{\epsilon \rightarrow 0} k\epsilon.rad(abc)^{1+\epsilon}$$

(7)

therefore

$$c < k_{0+} rad(abc) \Rightarrow z^n < (k_{0+} rad((xyz)^n)) = k_{0+} rad((xyz)) < k_{0+} \quad (8)$$

(because

$$rad(h^n) = rad(h) \quad (9)$$

and

$$rad(h) < h \quad (10)$$

for every integer h)

then

$$z^n < k_{0+}xyz \Rightarrow z^n < k_{0+}z^3 \quad (11)$$

(because $x < z$ and $y < z$)

therefore

$$\mathbf{n} < \log_z(k_{0+}) + 3(12)$$

4 conclusion

as we see in this paper abc is a powerful conjecture

.