

Appendix

1. Omitted Variables Bias

True model: $Y = X_1\beta_1 + X_2\beta_2 + \varepsilon$

$$E(\varepsilon|X_1, X_2) = 0$$

$$X_2 = X_1\delta_1 + u$$

Omit X_2 : $Y = X_1\tilde{\beta}_1 + \tilde{\varepsilon}$

$$E(\tilde{\beta}_1) = \beta_1 + \delta_1\beta_2$$

\therefore

| | | | |
|-------------|------------------|-----|-------------------|
| No problem: | $\beta_2 = 0$ | or | $\delta_1 = 0$ |
| Problem: | $\beta_2 \neq 0$ | and | $\delta_1 \neq 0$ |

2. Measurement Error

(i) $Y = Y^* + e$ (Y^* : true value, e : ME, Y : observed value)

True model: $Y^* = X\beta + \varepsilon$, $E(\varepsilon|X) = 0$

With ME: $Y = X\tilde{\beta} + \tilde{\varepsilon}$

$$Y = X\beta + \varepsilon + e$$

If $E(e|X) = 0$: ① unbiased and consistent $\hat{\beta}$
 (e 与 Y 相关, 与 Y^* 不相关) ② Inference problem: $\text{Var}(\varepsilon + e) > \text{Var}(\varepsilon)$
 If $E(e|X) \neq 0$: ① biased and inconsistent $\hat{\beta}$
 (e 与 Y^* 相关, 与 Y 不相关) ② ..

(ii) $X = X^* + e$ (X^* : true value, e : ME, X : observed value)

True model: $Y = X^*\beta + \varepsilon$

With ME: $Y = X\tilde{\beta} + \tilde{\varepsilon}$

$$\begin{aligned} Y &= (X - e)\beta + \varepsilon \\ &= X\beta + (\varepsilon - e\beta) \end{aligned}$$

If $E(e|X) = 0$: ① unbiased and consistent $\hat{\beta}$
 (e 与 X 不相关, 与 X^* 相关) ② Inference problem

If $E(e|X) \neq 0$: [Classical Errors-in-Variables (CEV) problem]
 (e 与 X^* 不相关, 与 X 相关) $\hat{\beta} = \frac{\text{cov}(Y, X)}{\text{Var}(X)} = \frac{\text{cov}(X\beta + \varepsilon - e\beta, X)}{\text{Var}(X)} = \beta(1 - \frac{\text{cov}(e, X)}{\text{Var}(X)})$
 $= \beta(1 - \frac{\sigma_e^2}{\sigma_{X^*}^2 + \sigma_e^2})$
 $\therefore |\hat{\beta}| < |\beta|$ attenuation bias

3. Power (in lab / field experiment)

power = 1 - type II error 该拒绝时拒绝的概率.

Data: $\{y_{11}, y_{12}, \dots, y_{1i}, \dots, y_{1n}\}, \{y_{21}, \dots, y_{2i}, \dots, y_{2n}\}, \text{Var}(y_{ji}) = \sigma_y^2$

$$\text{Var}(\Delta \bar{y}) = \text{Var}(\bar{y}_1 - \bar{y}_2) = \text{Var}(\bar{y}_1) + \text{Var}(\bar{y}_2) = \frac{2}{n} \sigma_y^2$$

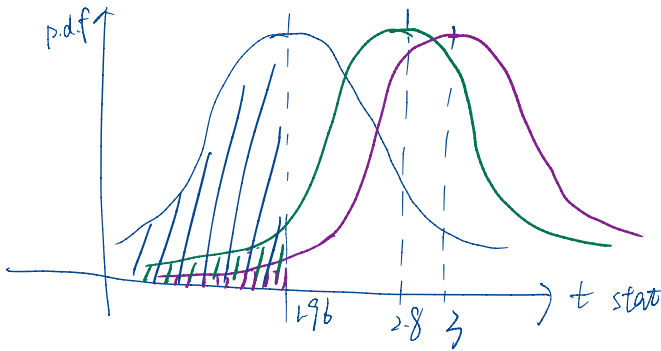
$$\therefore \hat{SD}(\Delta \bar{y}) = \sqrt{\frac{2}{n} \sigma_y^2}$$

$$\therefore E(t \text{ stat}) = E\left(\frac{\Delta \bar{y}}{\hat{SD}(\Delta \bar{y})}\right) = \sqrt{\frac{n}{2}} \cdot \frac{\Delta \bar{y}}{\sigma_y} \xrightarrow{P} \text{Normal distr.}$$

• If $E(t \text{ stat}) = 1.96$, then power = $\Pr(t > 1.96) = 50\%$

• " " 2.8, " " = 80%

• " " 3, " " = 90%



∴ (n, α, β) 已知其二, 可推余下的.

| | | |
|----------------|---------------------|-------|
| ↓ | ↓ | ↓ |
| sample size | confidence level | power |

4. Regression table

. regress price mpg rep78

| Source | SS | df | MS | | Number of obs = 69 |
|----------|-----------|----|------------|--------------------------------|------------------------|
| Model | 144754063 | 2 | 72377031.7 | $\frac{\sum \hat{y}_i^2}{k-1}$ | F(2, 66) = 11.06 |
| Residual | 432042896 | 66 | 6546104.48 | $\frac{\sum e_i^2}{n-(k-1)}$ | Prob > F = 0.0001 |
| Total | 576796959 | 68 | 8482308.22 | $\frac{\sum y_i^2}{n-1}$ | R-squared = 0.2510 |
| | | | | | Adj R-squared = 0.2283 |
| | | | | | Root MSE = 2558.5 |

| price | Coef. | Std. Err. | t | P> t | [95% Conf. Interval] | |
|-------|-----------|-----------|-------|-------|----------------------|-----------|
| mpg | -271.6425 | 57.77115 | -4.70 | 0.000 | -386.9864 | -156.2987 |
| rep78 | 666.9568 | 342.3559 | 1.95 | 0.056 | -16.5789 | 1350.492 |
| _cons | 9657.754 | 1346.54 | 7.17 | 0.000 | 6969.3 | 12346.21 |

$$\approx [\hat{\beta} - 1.96 \times SE(\hat{\beta}), \hat{\beta} + 1.96 \times SE(\hat{\beta})]$$