CHAPTER 25
Credit Derivatives

Practice Questions

Problem 25.1.
Explain the difference between a regular credit default swap and a binary credit default swap.

Both provide insurance against a particular company defaulting during a period of time. In a credit default swap the payoff is the notional principal amount multiplied by one minus the recovery rate. In a binary swap the payoff is the notional principal.

Problem 25.2.
A credit default swap requires a semiannual payment at the rate of 60 basis points per year. The principal is $300 million and the credit default swap is settled in cash. A default occurs after four years and two months, and the calculation agent estimates that the price of the cheapest deliverable bond is 40% of its face value shortly after the default. List the cash flows and their timing for the seller of the credit default swap.

The seller receives

$$300,000,000 \times 0.0060 \times 0.5 = \$900,000$$

at times 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, and 4.0 years. The seller also receives a final accrual payment of about $300,000 ($= \$300,000,000 \times 0.060 \times 2/12$) at the time of the default (4 years and two months). The seller pays

$$300,000,000 \times 0.6 = \$180,000,000$$

at the time of the default. (This does not consider day count conventions.)

Problem 25.3.
Explain the two ways a credit default swap can be settled.

Sometimes there is physical settlement and sometimes there is cash settlement. In the event of a default when there is physical settlement the buyer of protection sells bonds issued by the reference entity for their face value. Bonds with a total face value equal to the notional principal can be sold. Cash settlement is now more usual. A calculation agent, often using an auction process, estimates the value of the cheapest-to-deliver bonds issued by the reference entity a specified number of days after the default event. The cash payoff is then based on the excess of the face value of these bonds over the estimated value.

Problem 25.4.
Explain how a cash CDO and a synthetic CDO are created.

A cash CDO is created by forming a portfolio of credit sensitive instruments such as bonds. The returns from the bond portfolio flow to a number of tranches (i.e., different categories of investors). A waterfall defines the way interest and principal payments flow to tranches. The more senior a tranche the more likely it is receive promised payments. In a synthetic CDO n portfolio is created. Instead a portfolio of credit default swaps is sold and the resulting credit risks are allocated to tranches.

Problem 25.5.
Explain what a first-to-default credit default swap is. Does its value increase or decrease as the default correlation between the companies in the basket increases? Explain.

In a first-to-default basket CDS there are a number of reference entities. When the first one defaults there is a payoff (calculated in the usual way for a CDS) and basket CDS terminates. The value of a first-to-default basket CDS decreases as the correlation between the reference entities in the basket increases. This is because the probability of a default is high when the correlation is zero and decreases as the correlation increases. In the limit when the correlation is one there is in effect only one company and the probability of a default is quite low.

Problem 25.6.
Explain the difference between risk-neutral and real-world default probabilities.

Risk-neutral default probabilities are backed out from credit default swaps or bond prices. Real-world default probabilities are calculated from historical data.

Problem 25.7.
Explain why a total return swap can be useful as a financing tool.

Suppose a company wants to buy some assets. If a total return swap is used, a financial institution buys the assets and enters into a swap with the company where it pays the company the return on the assets and receives from the company LIBOR plus a spread. The financial institution has less risk than it would have if it lent the company money and used the assets as collateral. This is because, in the event of a default by the company, it owns the assets.

Problem 25.8.
Suppose that the risk-free zero curve is flat at 7% per annum with continuous compounding and that defaults can occur half way through each year in a new five-year credit default swap. Suppose that the recovery rate is 30% and the hazard rate is 3%. Estimate the credit default swap spread. Assume payments are made annually.

The table corresponding to Tables 25.1, giving unconditional default probabilities, is

Time (years)	*Probability of surviving to year end*	*Default Probability during year*
1	0.9704	0.0296
2	0.9418	0.0287
3	0.9139	0.0278
4	0.8869	0.0270
5	0.8607	0.0262

The table corresponding to Table 25.2, giving the present value of the expected regular payments (payment rate is s per year), is

Time (yrs)	*Probability of survival*	*Expected Payment*	*Discount Factor*	*PV of Expected Payment*
1	0.9704	0.9704s	0.9324	0.9048s
2	0.9418	0.9418s	0.8694	0.8187s
3	0.9139	0.9139s	0.8106	0.7408s
4	0.8869	0.8869s	0.7558	0.6703s
5	0.8607	0.8607s	0.7047	0.6065s
Total				3.7412s

The table corresponding to Table 25.3, giving the present value of the expected payoffs (notional principal =$1), is

Time (yrs)	*Probability of default*	*Recovery Rate*	*Expected Payoff*	*Discount Factor*	*PV of Expected Payment*
0.5	0.0296	0.3	0.0207	0.9656	0.0200
1.5	0.0287	0.3	0.0201	0.9003	0.0181
2.5	0.0278	0.3	0.0195	0.8395	0.0164
3.5	0.0270	0.3	0.0189	0.7827	0.0148
4.5	0.0262	0.3	0.0183	0.7298	0.0134
Total					0.0826

The table corresponding to Table 25.4, giving the present value of accrual payments, is

Time (yrs)	*Probability of default*	*Expected Accrual Payment*	*Discount Factor*	*PV of Expected Accrual Payment*
0.5	0.0296	0.0148s	0.9656	0.0143s
1.5	0.0287	0.0143s	0.9003	0.0129s
2.5	0.0278	0.0139s	0.8395	0.0117s
3.5	0.0270	0.0135s	0.7827	0.0106s
4.5	0.0262	0.0131s	0.7298	0.0096s
Total				0.0590s

The credit default swap spread s is given by:

$$3.7412s+0.0590s = 0.0826$$

It is 0.0217 or 217 basis points. This can be verified with DerivaGem.

Problem 25.9.
What is the value of the swap in Problem 25.8 per dollar of notional principal to the protection buyer if the credit default swap spread is 150 basis points?

If the credit default swap spread is 150 basis points, the value of the swap to the buyer of protection is:

$$0.0826 - (3.7412+0.0590)\times 0.0150 = 0.0256$$

per dollar of notional principal.

Problem 25.10.
What is the credit default swap spread in Problem 25.8 if it is a binary CDS?

If the swap is a binary CDS, the present value of expected payoffs is calculated as follows

Time (years)	*Probability of Default*	*Expected Payoff*	*Discount Factor*	*PV of expected Payoff*
0.5	0.0296	0.0296	0.9656	0.0285
1.5	0.0287	0.0287	0.9003	0.0258
2.5	0.0278	0.0278	0.8395	0.0234
3.5	0.0270	0.0270	0.7827	0.0211
4.5	0.0262	0.0262	0.7298	0.0191
				0.1180

The credit default swap spread s is given by:

$$3.7412s+0.0590s = 0.1180$$

$$3.7364s + 0.0598s = 0.1197$$

It is 0.0310 or 310 basis points.

Problem 25.11.
How does a five-year nth-to-default credit default swap work? Consider a basket of 100 reference entities where each reference entity has a probability of defaulting in each year of 1%. As the default correlation between the reference entities increases what would you expect to happen to the value of the swap when a) $n=1$ and b) $n=25$. Explain your answer.

A five-year nth to default credit default swap works in the same way as a regular credit default swap except that there is a basket of companies. The payoff occurs when the nth default from the companies in the basket occurs. After the nth default has occurred the swap ceases to exist. When $n=1$ (so that the swap is a "first to default") an increase in the default correlation lowers the value of the swap. When the default correlation is zero there are 100 independent events that can lead to a payoff. As the correlation increases the probability of a payoff decreases. In the limit when the correlation is perfect there is in effect only one company and therefore only one event that can lead to a payoff.
When $n=25$ (so that the swap is a 25th to default) an increase in the default correlation increases the value of the swap. When the default correlation is zero there is virtually no chance that there will be 25 defaults and the value of the swap is very close to zero. As the correlation increases the probability of multiple defaults increases. In the limit when the correlation is perfect there is in effect only one company and the value of a 25th-to-default credit default swap is the same as the value of a first-to-default swap.

Problem 25.12.
What is the formula relating the payoff on a CDS to the notional principal and the recovery rate?

The payoff is $L(1-R)$ where L is the notional principal and R is the recovery rate.

Problem 25.13.
Show that the spread for a new plain vanilla CDS should be $(1-R)$ *times the spread for a similar new binary CDS where* R *is the recovery rate.*

The payoff from a plain vanilla CDS is $1-R$ times the payoff from a binary CDS with the same principal. The payoff always occurs at the same time on the two instruments. It follows that the regular payments on a new plain vanilla CDS must be $1-R$ times the payments on a new binary CDS. Otherwise there would be an arbitrage opportunity.

Problem 25.14.
Verify that if the CDS spread for the example in Tables 25.1 to 25.4 is 100 basis points, the hazard rate must be 1.63% per year. How does the hazard rate change when the recovery rate is 20% instead of 40%? Verify that your answer is consistent with the implied probability of default being approximately proportional to $1/(1-R)$ *where* R *is the recovery rate.*

The 1.63% hazard rate can be calculated by setting up a worksheet in Excel and using Solver. To verify that 1.63% is correct we note that, with a hazard rate of 1.63%

Time (years)	*Probability of surviving to year end*	*Default Probability during year*
1	0.9838	0.0162
2	0.9679	0.0159
3	0.9523	0.0156
4	0.9369	0.0154
5	0.9217	0.0151

The present value of the regular payments becomes 4.1162*s*, the present value of the expected payoffs becomes 0.0416, and the present value of the expected accrual payments becomes 0.0347*s*.When $s=0.01$ the present value of the expected payments equals the present value of the expected payoffs.
When the recovery rate is 20% the implied hazard rate(calculated using Solver) is 1.22% per year. Note that 1.22/1.63 is approximately equal to $(1-0.4)/(1-0.2)$ showing that the implied hazard is approximately proportional to $1/(1-R)$.
In passing we note that if the CDS spread is used to imply an unconditional default probability (assumed to be the same each year) then this implied unconditional default probability is exactly proportional to $1/(1-R)$. When we use the CDS spread to imply a hazard rate (assumed to be the same each year) it is only approximately proportional to $1/(1-R)$.

Problem 25.15.
A company enters into a total return swap where it receives the return on a corporate bond paying a coupon of 5% and pays LIBOR. Explain the difference between this and a regular swap where 5% is exchanged for LIBOR.

In the case of a total return swap a company receives (pays) the increase (decrease) in the value of the bond. In the regular swap this does not happen.

Problem 25.16.
Explain how forward contracts and options on credit default swaps are structured.

When a company enters into a long (short) forward contract it is obligated to buy (sell) the protection given by a specified credit default swap with a specified spread at a specified future time. When a company buys a call (put) option contract it has the option to buy (sell) the protection given by a specified credit default swap with a specified spread at a specified future time. Both contracts are normally structured so that they cease to exist if a default occurs during the life of the contract.

Problem 25.17.
"The position of a buyer of a credit default swap is similar to the position of someone who is long a risk-free bond and short a corporate bond." Explain this statement.

A credit default swap insures a corporate bond issued by the reference entity against default. Its approximate effect is to convert the corporate bond into a risk-free bond. The buyer of a credit default swap has therefore chosen to exchange a corporate bond for a risk-free bond. This means that the buyer is long a risk-free bond and short a similar corporate bond.

Problem 25.18.
Why is there a potential asymmetric information problem in credit default swaps?

Payoffs from credit default swaps depend on whether a particular company defaults. Arguably some market participants have more information about this than other market participants. (See Business Snapshot 25.3.)

Problem 25.19.
Does valuing a CDS using real-world default probabilities rather than risk-neutral default probabilities overstate or understate its value? Explain your answer.

Real world default probabilities are less than risk-neutral default probabilities. It follows that the use of real world (historical) default probabilities will tend to understate the value of a CDS.

Problem 25.20.
What is the difference between a total return swap and an asset swap?

In an asset swap the bond's promised payments are swapped for LIBOR plus a spread. In a total return swap the bond's actual payments are swapped for LIBOR plus a spread.

Problem 25.21.
Suppose that in a one-factor Gaussian copula model the five-year probability of default for each of 125 names is 3% and the pair wise copula correlation is 0.2. Calculate, for factor values of –2, –1, 0, 1, and 2, a) the default probability conditional on the factor value and b) the probability of more than 10 defaults conditional on the factor value.

Using equation (25.5) the probability of default conditional on a factor value of F is

$$N\left(\frac{N^{-1}(0.03)-\sqrt{0.2}F}{\sqrt{1-0.2}}\right)$$

For F equal to –2, –1, 0, 1, and 2 the probabilities of default are 0.135, 0.054, 0.018, 0.005, and 0.001 respectively. To six decimal places the probability of more than 10 defaults for these values of F can be calculated using the BINOMDIST function in Excel. They are 0.959284, 0.079851, 0.000016, 0, and 0, respectively.

Problem 25.22.
Explain the difference between base correlation and compound correlation

Compound correlation for a tranche is the correlation which when substituted into the one-factor Gaussian copula model produces the market quote for the tranche. Base correlation is the correlation which is consistent with the one-factor Gaussian copula and market quotes for the 0 to X% tranche where X% is a detachment point. It ensures that the expected loss on the 0 to X% tranche equals the sum of the expected losses on the underlying traded tranches.

Problem 25.23.
In Example 25.2, what is the tranche spread for the 9% to 12% tranche assuming a tranche correlation of 0.15?

In this case $a_L = 0.09$ and $a_H = 0.12$. Proceeding similarly in Example 25.2 the tranche spread is calculated as 30 basis points.

Further Questions

Problem 25.24.
Suppose that the risk-free zero curve is flat at 6% per annum with continuous compounding and that defaults can occur at times 0.25 years, 0.75 years, 1.25 years, and 1.75 years in a two-year plain vanilla credit default swap with semiannual payments. Suppose that the recovery rate is 20% and the unconditional probabilities of default (as seen at time zero) are 1% at times 0.25 years and 0.75 years, and 1.5% at times 1.25 years and1.75 years. What is the credit default swap spread? What would the credit default spread be if the instrument were a binary credit default swap?

The table corresponding to Table 25.2, giving the present value of the expected regular payments (payment rate is s per year), is

Time (yrs)	*Probability of survival*	*Expected Payment*	*Discount Factor*	*PV of Expected Payment*
0.5	0.990	0.4950s	0.9704	0.4804s
1.0	0.980	0.4900s	0.9418	0.4615s
1.5	0.965	0.4825s	0.9139	0.4410s
2.0	0.950	0.4750s	0.8869	0.4213s
Total				1.8041s

The table corresponding to Table 25.3, giving the present value of the expected payoffs (notional principal =$1), is

Time (yrs)	*Probability of default*	*Recovery Rate*	*Expected Payoff*	*Discount Factor*	*PV of Expected Payment*
0.25	0.010	0.2	0.008	0.9851	0.0079
0.75	0.010	0.2	0.008	0.9560	0.0076
1.25	0.015	0.2	0.012	0.9277	0.0111
1.75	0.015	0.2	0.012	0.9003	0.0108
Total					0.0375

The table corresponding to Table 25.4, giving the present value of accrual payments, is

Time (yrs)	*Probability of default*	*Expected Accrual Payment*	*Discount Factor*	*PV of Expected Accrual Payment*
0.25	0.010	0.0025s	0.9851	0.0025s
0.75	0.010	0.0025s	0.9560	0.0024s
1.25	0.015	0.00375s	0.9277	0.0035s
1.75	0.015	0.00375s	0.9003	0.0034s
Total				0.0117s

The credit default swap spread s is given by:

$$1.804s + 0.0117s = 0.0375$$

It is 0.0206 or 206 basis points. For a binary credit default swap we set the recovery rate equal to zero in the second table to get the present value of expected payoffs equal to 0.0468 so that

$$1.804s + 0.0117s = 0.0468$$

and the spread is 0.0258 or 258 basis points.

Problem 25.25.

Assume that the hazard rate for a company is λ and the recovery rate is R. The risk-free interest rate is 5% per annum. Default always occurs half way through a year. The spread for a five-year plain vanilla CDS where payments are made annually is 120 basis points and the spread for a five-year binary CDS where payments are made annually is 160 basis points. Estimate R and λ.

The spread for a binary credit default swap is equal to the spread for a regular credit default swap divided by $1-R$ where R is the recovery rate. This means that $1-R$ equals 0.75 so that the recovery rate is 25%. To find λ we search for the conditional annual default rate that leads to the present value of payments being equal to the present value of payoffs. The answer is $\lambda = 0.0156$. The present value of payoffs (per dollar of principal) is then 0.0499. The present value of regular payments is 4.1245. The present value of accrual payments is 0.0332.

Problem 25.26.
Explain how you would expect the yields offered on the various tranches in a CDO to change when the correlation between the bonds in the portfolio increases.

As the correlation increases the yield on the equity tranche decreases and the yield on the senior tranches increases. To understand this, consider what happens as the correlation increases from zero to one. Initially the equity tranche is much more risky than the senior tranche. But as the correlation approaches one the companies become essentially the same. We are then in the position where either all companies default or no companies default and the tranches have similar risk.

Problem 25.27.
Suppose that

(a) *The yield on a five-year risk-free bond is 7%*
(b) *The yield on a five-year corporate bond issued by company X is 9.5%*
(c) *A five-year credit default swap providing insurance against company X defaulting costs 150 basis points per year.*

What arbitrage opportunity is there in this situation? What arbitrage opportunity would there be if the credit default spread were 300 basis points instead of 150 basis points?

When the credit default swap spread is 150 basis points, an arbitrageur can earn more than the risk-free rate by buying the corporate bond and buying protection. If the arbitrageur can finance trades at the risk-free rate (by shorting the riskless bond) it is possible to lock in an almost certain profit of 100 basis points. When the credit spread is 300 basis points the arbitrageur can short the corporate bond, sell protection and buy a risk free bond. This will lock in an almost certain profit of 50 basis points. The arbitrage is not perfect for a number of reasons:

(a) It assumes that both the corporate bond and the riskless bond are par yield bonds and that interest rates are constant. In practice the riskless bond may be worth more or less than par at the time of a default so that a credit default swap under protects or overprotects the bond holder relative to the position he or she would be in with a riskless bond.
(b) There is uncertainty created by the cheapest-to-delivery bond option.
(c) To be a perfect hedge the credit default swap would have to give the buyer of protection the right to sell the bond for face value plus accrued interest, not just face value.

The arbitrage opportunities assume that market participants can short corporate bonds and borrow at the risk-free rate. The definition of the credit event in the ISDA agreement is also occasionally a problem. It can occasionally happen that there is a "credit event" but promised payments on the bond are made.

Problem 25.28.
In Example 25.3, what is the spread for a) a first-to-default CDS and b) a second-to-default CDS?

(a) In this case the answer to Example 25.3 gets modified as follows. When $F = -1.0104$ the cumulative probabilities of one or more defaults in 1, 2, 3, 4, and 5 years are 0.3075, 0.5397, 0.6959, 0.7994, and 0.8676. The conditional probability that the first default occurs in years 1, 2, 3, 4, and 5 are 0.3075, 0.2322, 0.1563, 0.1035, and 0.0682, respectively. The present values of payoffs, regular payments, and accrual payments conditional on $F = -1.0104$ are 0.4767, 1.6044*s*, and 0.3973*s*. Similar calculations are carried out for the other factor values. The unconditional expected present values of payoffs, regular payments, and accrual payments are 0.2602, 2.9325*s*, and 0.2168*s*. The breakeven spread is therefore

$$0.2602/(2.9325+0.2168) = 0.0826$$

or 826 basis points.

(b) In this case the answer to Example 25.3 gets modified as follows. When $F = -1.0104$ the cumulative probabilities of two or more defaults in 1, 2, 3, 4, and 5 years are 0.0483, 0.1683, 0.3115, 0.4498, and 0.5709. The conditional probability that the second default occurs in years 1, 2, 3, 4, and 5 are 0.0483, 0.1200, 0.1432, 0.1383, and 0.1211, respectively. The present values of payoffs, regular payments, and accrual payments conditional on $F = -1.0104$ are 0.2986, 3.0351*s*, and 0.2488*s*. Similar calculations are carried out for the other factor values. The unconditional expected present values of payoffs, regular payments, and accrual payments are 0.1264, 3.7428*s*, and 0.1053*s*. The breakeven spread is therefore

$$0.1264/(3.7428+0.1053) = 0.0328$$

or 328 basis points.

Problem 25.29.
In Example 25.2, what is the tranche spread for the 6% to 9% tranche assuming a tranche correlation of 0.15?

(In this case $a_L = 0.06$ and $a_H = 0.09$. Proceeding similarly in Example 25.2 the tranche spread is calculated as 98 basis points assuming a tranche correlation of 0.15.

Problem 25.30. (Excel file)
The 1-, 2-, 3-, 4-, and 5-year CDS spreads are 100, 120, 135, 145, and 152 basis points, respectively. The risk-free rate is 3% for all maturities , the recovery rate is 35%, and payments are quarterly. Use DerivaGem to calculate the hazard rate each year. What is the probability of default in year 1? What is the probability of default in year 2?

The hazard rates in years 1, 2, 3, 4, and 5 are 1.53%, 2.16%, 2.57%, 2.74%, and 2.82%, respectively. The probability of default in year 1 is
$1 - e^{-0.0153\times 1} = 1.5210\%$
The probability of default in year 2 is
$e^{-0.0153\times 1} - e^{-0.0216\times 2} = 2.7090\%$

Problem 25.31. (Excel file)
Table 25.6 shows the five-year iTraxx index was 77 basis points on January 31, 2008. Assume that the risk-free rate is 5% for all maturities, the recovery rate is 40%, and payments are quarterly. Assume also that the spread of 77 basis points applies to all maturities. Use the DerivaGem CDS worksheet to calculate the hazard rate consistent with the spread. Use this in the CDO worksheet with 10 integration points to imply base correlations for each tranche from the quotes for January 31, 2008.

The hazard rate consistent with the data is 1.28%. The compound (tranche) correlations are

0.4017, 0.8425, 0.1136, 0.2198, and 0.3342. The base correlations are 0.4017, 0.5214, 0.5825, 0.6046, and 0.7313. Note that in January 2009 spreads were so high that correlations could not be implied and many dealers had to change their models.