Q- What do you mean by Number system? Explain its types.

Ans.– ***A number system of base or radix "r" is a system that uses r distinct symbols to represent any number. Numbers are represented by a string of digit symbols. To determine the quantity that the number represents, it is necessary to multiply each digit by an integer power of r and then form the sum of all weighted digit.***

You realize that you need more than your fingers and toes to keep track of the numbers in your daily routine. Ever since people discovered that it was necessary to count objects, they have been looking for easier ways to count them. The abacus, developed by the Chinese, is one of the earliest known calculators. It is still in use in some parts of the world. Basically, there are two types of number system-

In a non-positional number system, each number in each position does not have to be positional itself. Every system varies by country and it depends on symbols and values set by the people of that country. For example, the Egyptians use Hieroglyphics, and the Greeks use a numeral system.

A positional notation numeral system in which each position is related to the next by a constant multiplier of that numeral system. For each position that the number is in, in that system has a relative symbol or meaning, and in a way relates to the number directly next to it. The total value of a positional number is the total of the resultant values of all positions.

Number System	Base / Radix	Distinct Symbols
Binary	2	0, 1
Octal	8	0 – 7
Decimal	10	0 – 9
Hexadecimal	16	0 – 9, A, B, C, D, E, F

The table below summarizes four positional number systems, their base and the distinct symbols used by these number systems to represent any number:

Decimal number system- A number system which uses digits from 0 to 9 to represent a number with base 10 is the decimal system number. In the **decimal number system**, the numbers are represented with base 10. The way of denoting the decimal numbers with base 10 is also termed as decimal notation. This number system is widely used in computer applications. It is also called the base-10 number system which consists of 10 digits, such as, 0,1,2,3,4,5,6,7,8,9. Each digit in the decimal system has a position and every digit is ten times more significant than the previous digit. Suppose, 25 is a decimal number, then 2 is ten times more than 5. Some examples of decimal numbers are:-

$(12)_{10}$, $(345)_{10}$, $(119)_{10}$, $(200)_{10}$, $(313.9)_{10}$

Each value in this number system has the place value of power 10. It means the digit at the tens place is ten times greater than the digit at the unit place. Let us see some more examples:

$(92)_{10} = 9\times10^1+2\times10^0$

$(200)_{10} = 2\times10^2+0\text{x}10^1+0\text{x}10^0$

The decimal numbers which have digits present on the right side of the decimal (.) denote each digit with decreasing power of 10. Some examples are:

$(30.2)_{10}= 3\times10^1+0\text{x}10^0+2\times10^{-1}$

$(212.367)_{10} = 2\times10^2+1\times10^1+2\times10^0+3\times10^{-1}+6\times10^{-2}+7\times10^{-3}$

Advantages and Disadvantages

The main advantages of Decimal Number System are easy readable, used by humans, and easy to manipulate.

However, there are some disadvantages, like wastage of space and time. Since digital system (e.g., Computers) and hardware is based on binary system (either 0 or 1), so we need to 4 bit space to store each bit of Decimal number, whereas Hexadecimal number is also needed only 4 bit and hexadecimal number has more digits than decimal number which is an advantage of Hexadecimal Number System.

Binary Number System: According to digital electronics and mathematics, a binary number is defined as a number that is expressed in the binary system or base 2 numeral system. It describes numeric values by two separate symbols; 1 (one) and 0 (zero). The base-2 system is the positional notation with 2 as a radix.

The binary system is applied internally by almost all latest computers and computer-based devices because of its direct implementation in electronic circuits using logic gates. Every digit is referred to as a **bit**.

A single binary digit is called a "**Bit".** A binary number consists of several bits. Examples are:

- 10101 is a five-bit binary number
- 101 is a three-bit binary number

In binary system operates in base 2 and the digits 0-1 represent numbers, and the base is known as **radix**.

In the Binary system, we have ones, twos, fours etc…

For example 1011.110

It is shown like this:

$1 \times 8 + 0 \times 4 + 1 \times 2 + 1 + 1 \times ½ + 1 \times ¼ + 0 \times 1/8$

= 11.75 in Decimal

Octal Number system- A number system which has its base as 'eight' is called an Octal number system. It uses numbers from 0 to 7. Let us take an example, to understand the concept. As we said, any number with base 8 is an octal number like 24_8, 109_8, 55_8, etc.

If we solve an octal number, each place is a power of eight.

- $124_8 = 1 \times 8^2 + 2 \times 8^1 + 4 \times 8^0$
- Octal Numbers Chart
- We use only 3 bits to represent Octal Numbers. Each group will have a distinct value between 000 and 111.

Octal Digital Value	***Binary Equivalent***
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111

Applications

The octal Number system is widely used in computer application sectors and also in the aviation sector to use the number in the form of code.

Based on octal number system applications, several computing systems are developed. All the modern generation computing system uses 16-bit, 32-bit or 64-bit word which is further divided into 8-bit words. Similarly, for various programming languages, octal numbers are used to do coding or to write the encrypted language, which is only understood by the computing machine.

Also in the aviation sector or field or say aviation industry, Transponders used in the aircraft transmits a code which is expressed as four octal digit number. These codes are interrogated by ground radar.

Hexadecimal Number System

In the **hexadecimal number system**, the numbers are represented with base 16. It is also pronounced sometimes as **'hex'**. Just like the **binary number, octal number and decimal number** whose base representation are 2, 8 and 10, respectively, the hexadecimal conversion is also possible which can be represented in a table. This concept is widely explained in the syllabus of Class 9. The **list of 16 hexadecimal digits** with their decimal, octal and binary representation is provided here in the form of a table, which will help in number system conversion. This list can be used as a translator also.

Decimal Numbers	4-bit Binary Number	Hexadecimal Number
0	0	0
1	1	1
2	10	2
3	11	3
4	100	4
5	101	5
6	110	6
7	111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Decimal Number System to Other Bases

Decimal to Binary

To convert a decimal number into an equivalent binary number we have to divide the original number system by 2 until the quotient is 0, when no more division is possible. The remainder so obtained is counted for the required number in the order of LSB (Least significant bit) to MSB (most significant bit). Let us go through the example.

Steps

- **Step 1** – Divide the decimal number to be converted by the value of the new base.
- **Step 2** – Get the remainder from Step 1 as the rightmost digit (least significant digit) of new base number.

- **Step 3** – Divide the quotient of the previous divide by the new base.
- **Step 4** – Record the remainder from Step 3 as the next digit (to the left) of the new base number.

Repeat Steps 3 and 4, getting remainders from right to left, until the quotient becomes zero in Step 3.

The last remainder thus obtained will be the Most Significant Digit (MSD) of the new base number.

Example: Convert 26_{10} into a binary number.

Solution: Given 26_{10} is a decimal number.

Divide 26 by 2

26/2 = 13 Remainder →0 (MSB)

13/2 = 6 Remainder →1

6/2 = 3 Remainder →0

3/2 = 1 Remainder →1

½ = 0 Remainder →1 (LSB)

Hence, the equivalent binary number is $(11010)_2$

$(125.125)_{10}$ = (? $)_2$

Decimal number = 125			
Division expression	Quotient	Remainder	Direction
125 / 2	62	1	
62 /2	31	0	
31 /2	15	1	
15 /2	7	1	
7 /2	3	1	
3 /2	1	1	
1 /2	0	1	
Binary number = $(1111101)_2$			

Hence $(125)_2$ = (1111101 $)_2$

Fractional part =(.125)

Direction	Binary bit	.125 * 2
	0	.250 * 2
	0	.500 * 2
	1	.00
binary number = .001		

Hence $(125.125)_{10}$ = $(1111101.001)_2$

Decimal to Octal

Here the decimal number is required to be divided by 8 until the quotient is 0. Then, in the same way, we count the remainder from LSB to MSB to get the equivalent octal number.

Example: Convert 65_{10} into an octal number.

Solution: Given 65_{10} is a decimal number.

Divide by 8

65/8 = 8 Remainder →1 (MSB)

8/8 = 1 Remainder →0

1/8 = 0 Remainder →1 (LSB)

Hence, the equivalent octal number is $(101)_8$

$(654)_{10}$ = ( )$_8$

Decimal number = 654			
Division expression	Quotient	Remainder	Direction
654 / 8	81	6	↑
81 / 8	10	1	
10 / 8	1	2	
1 / 8	0	1	
Octal number = $(1216)_8$			

Hence, $(654)_{10}$ = $(1216\ )_8$

Decimal to Hexadecimal

The given decimal number here is divided by 16 to get the equivalent hex. The division of the number continues until we get the quotient 0. Follow the below steps:

- Firstly divide the number by 16
- Take the quotient and divide again by 16
- The remainder left will produce the hex value
- Repeats the steps until the quotient has become 0

Example: Convert $(242)_{10}$ into hexadecimal.

Solution: Divide 242 by 16 and repeat the steps, till the quotient is left as 0.

Therefore, $(242)_{10}$ = $(F2)_{16}$

$$\begin{array}{r|l} 16 & 242 \\ \hline 16 & 15 \quad 2 \rightarrow 2 \\ \hline & 0 \quad 15 \rightarrow F \end{array}$$

Example: Convert 127_{10} to a hexadecimal number.
Solution: Given 127_{10} is a decimal number.
Divide by 16
127/16 = 7 Remainder →15
7/16 = 0 Remainder → 7
In the hexadecimal number system, alphabet F is considered as 15.
Hence, 127_{10} is equivalent to $7F_{16}$

$(43.52)_{10}$ in hexadecimal form-

Integer number = 43			
Division expression	Quotient	Remainder	Direction
43 / 16	2	11=B	↑
2 /16	0	2	
Hexa number = $(2B)_{16}$			

Fractional part = .52

Direction	Hexa digit	.52 * 16
↓	8	.32 * 16
	5	.12 * 16
	1	.92 * 16
	14 = E	.72 * 16
	11 = B	.52
Hexa number = .851EB….. recurring		

Hence $(43.52)_{10}$ = $(2B.851EB\ldots)_{16}$

Other Base System to Decimal System

Steps

- **Step 1** – Determine the column (positional) value of each digit (this depends on the position of the digit and the base of the number system).
- **Step 2** – Multiply the obtained column values (in Step 1) by the digits in the corresponding columns.
- **Step 3** – Sum the products calculated in Step 2. The total is the equivalent value in decimal.

Example

Binary Number – 11101_2

Calculating Decimal Equivalent –

Step	Binary Number	Decimal Number
Step 1	11101_2	$((1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	11101_2	$(16 + 8 + 4 + 0 + 1)_{10}$
Step 3	11101_2	29_{10}

Binary Number – 11101_2 = Decimal Number – 29_{10}

$(1101.1001)_2 = (\ \ )_{10}$

Number	1	1	0	1	.	1	0	0	1
ON/OFF	ON	ON	OFF	ON		ON	OFF	OFF	ON
Place value	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}
Exponential Expression	$1*2^3$	$1*2^2$	$0*2^1$	$1*2^0$		$1*2^{-1}$	$0*2^{-2}$	$0*2^{-3}$	$1*2^{-4}$
Solved Multiplication	8	4	0	1		.5	0	0	.0625
Add to calculate decimal value	8+4+1+0.5+0.0625=13.5625								

$(624)_8 = (\ )_{10}$

Number	6	2	4
ON/OFF	ON	ON	ON
Place value	8^2	8^1	8^0
Exponential Expression	$6 * 8^2$	$2 * 8^1$	$4 * 8^0$
Solved Multiplication	384	16	4
Add to calculate decimal value	384+16+4 = 404		

Hence, $(624)_8 = (404)_{10}$

Other Base System to Non-Decimal System

Steps

- **Step 1** – Convert the original number to a decimal number (base 10).
- **Step 2** – Convert the decimal number so obtained to the new base number.

Example

Octal Number – 25_8

Calculating Binary Equivalent –

Step 1 – Convert to Decimal

Step	Octal Number	Decimal Number
Step 1	25_8	$((2 \times 8^1) + (5 \times 8^0))_{10}$
Step 2	25_8	$(16 + 5)_{10}$
Step 3	25_8	21_{10}

Octal Number – 25_8 = Decimal Number – 21_{10}

Step 2 – Convert Decimal to Binary

Step	Operation	Result	Remainder
Step 1	21 / 2	10	1
Step 2	10 / 2	5	0
Step 3	5 / 2	2	1
Step 4	2 / 2	1	0
Step 5	1 / 2	0	1

Decimal Number – 21_{10} = Binary Number – 10101_2

Octal Number – 25_8 = Binary Number – 10101_2

Shortcut method - Binary to Octal

Steps

- **Step 1** – Divide the binary digits into groups of three (starting from the right).
- **Step 2** – Convert each group of three binary digits to one octal digit.

Example

Binary Number – 10101_2

Calculating Octal Equivalent –

Step	Binary Number	Octal Number
Step 1	10101_2	010 101
Step 2	10101_2	2_8 5_8
Step 3	10101_2	25_8

Binary Number – 10101_2 = Octal Number – 25_8

Shortcut method - Octal to Binary

Steps

- **Step 1** – Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion).
- **Step 2** – Combine all the resulting binary groups (of 3 digits each) into a single binary number.

Example

Octal Number – 25_8

Calculating Binary Equivalent –

Step	Octal Number	Binary Number
Step 1	25_8	2_{10} 5_{10}
Step 2	25_8	010_2 101_2
Step 3	25_8	010101_2

Octal Number – 25_8 = Binary Number – 10101_2

Shortcut method - Binary to Hexadecimal

Steps

- **Step 1** – Divide the binary digits into groups of four (starting from the right).
- **Step 2** – Convert each group of four binary digits to one hexadecimal symbol.

Example

Binary Number – 10101_2

Calculating hexadecimal Equivalent –

Step	Binary Number	Hexadecimal Number
Step 1	10101_2	0001 0101
Step 2	10101_2	1_{10} 5_{10}
Step 3	10101_2	15_{16}

Binary Number – 10101_2 = Hexadecimal Number – 15_{16}

Shortcut method - Hexadecimal to Binary

Steps

- **Step 1** – Convert each hexadecimal digit to a 4 digit binary number (the hexadecimal digits may be treated as decimal for this conversion).
- **Step 2** – Combine all the resulting binary groups (of 4 digits each) into a single binary number.

Example

Hexadecimal Number – 15_{16}

Calculating Binary Equivalent –

Step	Hexadecimal Number	Binary Number
Step 1	15_{16}	1_{10} 5_{10}
Step 2	15_{16}	0001_2 0101_2
Step 3	15_{16}	00010101_2

Hexadecimal Number – 15_{16} = Binary Number – 10101_2

$(6EA)_{16} = (\ )_2$

Hexadecimal number	6	E	A
Binary equivalents to Hexadecimal digit	0110	1110	1010
Binary number	011011101010		

Hence, $(6EA)_{16} = (011011101010)_2$