Departamento de Engenharia Informática e de Sistemas

Operations Research 2017/2018		
Date: 29/01/2018	**Exam – 1st Call**	**Duration: 2 hours**

Note: Present **all the calculations** that you perform and appropriately **justify** your answers.

1. Consider the following problem:

"*Anaemia is a disease caused by low levels of haemoglobin in the blood, which is the protein responsible for transporting oxygen. To prevent it, the diet should be rich in iron, vitamin A, vitamin B12 and folic acid, nutrients that can be found in several foods, such as spinach, carrots and eggs.*

Given the high prevalence of this disease in Portugal, the MoreHealth clinic decided to bet on prevention, recommending to its patients a diet that, at the lowest possible cost, includes in the main meals of the day, food that allow to satisfy 100% of the daily requirements of each one of the nutrients. The table below shows the values of these requirements, as well as other important data. To make it easier, food is quantified in portions, with one portion being 100g.

		Nutrients in each portion of food				
		Iron (mg)	**Vitamin A (UI)**	**Vitamin B12 (μg)**	**Folic Acid (mg)**	**Price *per* portion (€)**
Food	**Spinach**	3	7400	0	0.4	0.53
	Carrots	1	14500	0.1	0.005	0.35
	Eggs	0.9	3215	1	0.05	0.72
	Daily Requirements	**8**	**4500**	**2**	**0.4**	

In addition, there is a recommendation that the total amount of the mentioned food to be daily taken should not exceed 1.5 kg."

To help clinicians at MoreHealth in establishing the diet, formulate the problem in terms of a linear programming model, indicating the meaning of the decision variables and the objective function.

2. Consider the following linear programming model:

Minimize $z = -x_1 + 2x_2$

subject to

$x_1 + 3x_2 \geq 6$

$x_1 - x_2 \leq 1$

$x_1 \geq 0,\ x_2 \geq 0$

a) Solve it by the Simplex method using the "Big M" technique and identify, in each iteration, the corresponding basic solution;

b) Explain the goal of using the "Big M" technique and how this technique can ensure that this goal is achieved.

Scores: 1 – 3,5 points 2 – 5,5 points 3 – 5,5 points 4 – 5,5 points

3. Consider the following linear programming model:

Maximize $z = -x_1 + 4x_2$
subject to
$3x_1 + 2x_2 \geq 12$
$-3x_1 + 4x_2 \leq 12$
$x_1 \leq 5$
$x_1 \geq 0, x_2 \geq 0$

a) Solve it by the graphical method;

b) Formulate the dual problem corresponding to the problem presented above;

c) Refer if any of the following solutions could be an optimal solution for the dual problem:

A = $(u_1,u_2,u_3) = (2,-1, 2)$; **B** = $(u_1,u_2,u_3) = (-1, 2, 3)$; **C** = $(u_1,u_2,u_3) = (-1, 2, 2)$

4. A certain pharmaceutical company has three laboratories, **L1**, **L2** and **L3**, which produce a known medication that is subsequently transported to two distribution centres, **C1** and **C2**, from which it is distributed to pharmacies throughout the country. It is known that laboratories daily produce **6**, **6** and **4** batches of the medication, respectively. The distribution centres need to receive, per day, **7** and **7** batches of the medication, respectively. The following table shows the transportation costs of each batch, expressed in monetary units (MU):

	C1	C2
L1	2	6
L2	3	5
L3	1	4

a) Obtain an initial basic feasible solution for the problem by the Penalties method;

b) Starting from the solution obtained in a) solve the problem by the transportation method;

c) Interpret the obtained optimal solution, indicating how many batches of medication will be transported from each laboratory to each distribution centre and whether there will be any excess in any of the laboratories.

Scores: 1 – 3,5 points 2 – 5,5 points 3 – 5,5 points 4 – 5,5 points