Operations Research 2021/2022

Date: 3rd February 2022 **Exam – 1st Call** **Duration: 2 hours**

Note: Present **all the calculations** you make and properly justify your answers.

1. Consider the following (fictional) problem:

*"A certain plant for bottling natural spring water, located in the region of Coimbra, wants help to solve a planning problem. In this factory there are two filling machines, which we will call **A** and **B**, with **A** being designed for **0.75**-liter bottles and **B** for **1.5**-liter bottles. Nevertheless, each of them can be used for both types of bottles, although with some loss of efficiency. The following table shows the data on the performance of the two machines when filling the two sizes of bottles.*

Machine	*0.75-liter bottles*	*1.5-liter bottles*
A	*100/minute*	*40/minute*
B	*60/minute*	*75/minute*

*On the other hand, it is known that these machines can work for **8** hours a day, **5** days a week. It is also known that the profit for filling each **0.75**-liter bottle is **€0.10** and for each **1.5**-liter bottle it is **€0.20**. In addition, the factory has **30,000** liters of water available for bottling every week, and there is an indication that the market cannot absorb more than **25,000** bottles of **0.75** liter and **7,000** bottles of **1.5** liter, per week. Given the above, the factory wants to know the weekly bottling plan that will allow it to maximize profit."*

To help the factory with this issue, **formulate the problem described in terms of a linear programming model**, indicating the meaning of the decision variables and the objective function.

2. Consider the following linear programming problem:

Minimize $z = 4x_1 + x_2$

subject to

$2x_1 + x_2 \geq 4$

$-x_1 + x_2 \leq 1$

$x_1 - x_2 \leq 2$

$x_1 \geq 0, x_2 \geq 0$

a) Solve it by the **Simplex method**, using the **Two-Phase technique**;

b) Formulate the corresponding **dual problem**;

c) **Without solving the dual problem**, present its optimal solution, as well as the optimal value of its objective function.

Quotation: 1 - 3.5 values; 2 - 6.0 values; 3 - 4.5 values; 4 - 6.0 values

3. Now consider the following linear programming problem:

Minimize $z = x_1 + 2x_2$

subject to

$x_1 - 2x_2 \leq 6$

$-x_1 + 3x_2 \leq 3$

$x_1 + x_2 \leq 8$

$x_1 \geq 0$

a) Solve it by the **graphical method**;

b) Indicate which variable replacement should be carried out, so that the above model could be solved by the Simplex method (note that it is not necessary to present the reformulated model). In view of the resolution of **a)**, also indicate what would be the value of these variables in the optimal tableau.

c) In graphical terms, two feasible extreme points are adjacent when they are joined by a single edge in the feasible region. Correspondingly, when are two basic feasible solutions adjacent?

4. An agricultural cooperative of milk producers wants to optimize its process of collecting the milk produced from its members. The cooperative has three large farms as members (**F1**, **F2** and **F3**) whose daily production is **7**, **4** and **6** thousand liters of milk, respectively. The milk must be collected from the farms and delivered to three processing centers (**C1**, **C2** and **C3**) where it will undergo a series of operations in order to transform it into the final product. Centers **C1**, **C2**, and **C3** have the capacity to process **3**, **5** and **9** thousand liters of milk per day, respectively.

Transportation costs, per thousand liters, between farms and processing centers are given in the table below. It should be noted that due to maintenance works, the route between the farm **F2** and the processing center **C2** is closed.

	C1	**C2**	**C3**
F1	8	2	6
F2	5	---	5
F3	2	6	3

Costs in currency units (CU)

a) Determine an initial feasible basic solution by the **northwest corner method**;

b) Starting from the solution determined in **a)**, solve the problem using the **transportation method** in order to minimize the total cost of transportation;

c) According to the optimal solution obtained in **b)**, indicate the origin of the milk that will be daily processed in **C3**.

Quotation: 1 - 3.5 values; 2 - 6.0 values; 3 - 4.5 values; 4 - 6.0 values