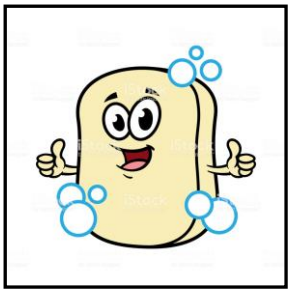
Departamento de Engenharia Informática e de Sistemas

Operations Research 2019/2020

Date: 06/02/2020 **Exam – Second Call** **Duration: 2 hours**

Note: Present **all the calculations performed** and conveniently **justify** your answers.

1. Consider the following problem (fictitious):



"*In recent years there has been a growing concern of the industry in general, with environmental sustainability, trying to create production methods that are environmentally friendly. In this context, some alternative brands of cosmetic products have appeared in Portugal, which pride themselves on using only organic and vegan ingredients, and not testing their products on animals. One of these brands, which is based in Coimbra, is GreenCosmetics, which is preparing a set of workshops for the month of May, with the theme "Learn to make a 100% organic soap". For its dissemination, next April, GreenCosmetics is considering, in addition to advertising on social networks, alternatives such as advertisements in the local newspaper and radio, as well as on outdoors located in various parts of the city. Daily advertising, In the radio corresponds to a 15-second advertisement before each news period; in the newspaper, it corresponds to an eighth page on an inside sheet of that diary; on outdoors, it corresponds to the posting, for 24 hours, of the advertisement in 6 bus stops spread throughout the city. The advertising prices in each of the mentioned media and the expected audience are shown in the following table:*

Daily Advertising:	*Radio*	*Newspaper*	*Outdoors*
Cost (€)	*200*	*60*	*40*
Expected audience	*10,000*	*5,000*	*1,800*

The company that manages the outdoors, requires a minimum of 10 days of advertising per month (do not have to be consecutive). On the other hand, to balance the advertising among the three media types, no more than half should fall in the newspaper and at least 10% of total advertising should occur on the radio. In addition, the company has a monthly budget of € 9,200 for advertising. The GreenCosmetics wants to know how many days of advertising should buy in each media, in order to maximize the total audience in the month of April."

To help in this task, f**ormulate the described problem in terms of a linear programming model, i**ndicating the meaning of the decision variables and objective function.

2. Consider the following linear programming problem:

Minimize $z = 2x_1 + 3x_2 + 7x_3$
subject to
$4x_1 - x_2 + x_3 \geq 14$
$2x_1 - 3x_2 + 2x_3 \geq 12$
$x_1 \geq 0\ x_2 \geq 0,\ x_3 \geq 0$

a) Solve it by the **dual simplex method** and present, in each iteration, the corresponding basic solution;

b) **Formulate** the corresponding **dual problem**;

c) **Without solving the dual problem**, present its optimal solution as well as the optimal value of its objective function;

d) If the dual problem had an unbounded solution, how could this situation be detected in the resolution of a)?

Quotations: 1 - 3.5 points; 2 - 6.5 points; 3 - 4.5 points; 4 - 5.5 points

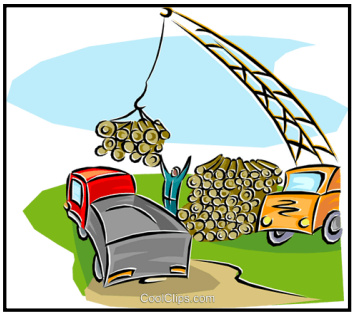
3. Now consider the following linear programming problem:

Minimize $z = x_1 + 2x_2$
subject to
$-2x_1 + x_2 \leq 4$
$5x_1 + 3x_2 \leq 15$
x_1 free, $x_2 \geq -1$

a) Solve it by the **graphical method**;

b) **Reformulate the problem** so that all decision variables have non-negativity constraint.

4. A given furniture industry, urgently needs to transport wood from three forest regions (**R1**, **R2** and **R3**) to its three factories (**F1**, **F2** and **F3**), where it is transformed and used as raw material in the production of furniture of different styles. It is known that in **R1**, **R2** and **R3**, there are currently **20**, **12** and **20** tons of wood, respectively, and that **F1**, **F2** and **F3** need 15, 8 and 10 tons, respectively.



The table with transportation costs (in monetary units - MU), per ton of wood, from each of the forest regions to each of the factories, is as follows:

	F1	F2	F3
R1	9	2	1
R2	3	7	2
R3	6	4	6

a) Get an initial feasible basic solution using **the Vogel's method**;

b) Starting from the solution obtained in a), solve the problem by **the transportation method**;

c) According to the solution obtained in the previous paragraph, indicate if any of the forest regions will have surplus of wood and in what quantity.

Quotations: 1 - 3.5 points; 2 - 6.5 points; 3 - 4.5 points; 4 - 5.5 points