Departamento de Engenharia Informática e de Sistemas

Operations Research 2019/2020		
Date: 20/01/2020	**Exam – First Call**	**Duration: 2 hours**

Note: Present **all the calculations performed** and conveniently **justify** your answers.

1. Consider the following problem (*fictitious*):

"*A team of scientists from a Portuguese university studied how much (and where) Portugal trembled over several hundred years, more precisely between 1300 and 2014. Based on these data, they produced a seismic intensity map from which we can observe that in Coimbra region there are two well-defined zones (A and B) with high seismic risk. As it is impossible to accurately predict when and where such a disaster will occur, it is of vital importance to devise an emergency plan to minimize damages. In this sense, the Intermunicipal Community of the Coimbra Region intends to draw up a plan to transport victims to hospitals in the event of a serious earthquake. For the sake of simplicity, it is assumed that only the two previously mentioned zones, A and B, will be affected, and that the victims will be transported to three hospitals: Coimbra Hospital and University Center (1), Covões Hospital (2) and Figueira da Foz District Hospital (3). In such a situation, it is estimated that there will be* ***300*** *victims in zone A and* ***200*** *in zone B. The travel time from point A to hospitals 1, 2 and 3 is* ***10****,* ***15*** *and* ***35*** *minutes, respectively. In turn, from point B, the transport times are* ***20****,* ***15*** *and* ***15*** *minutes, respectively. The capacity of the three hospitals for these emergencies is* ***250****,* ***170****,* ***150*** *victims, respectively. The plan under preparation should indicate the distribution scheme for victims that will minimize the total time spent transporting them to hospitals.*"

To help in this task, **formulate the described problem in terms of a linear programming model**, indicating the meaning of the decision variables and the objective function.

2. Consider the following linear programming problem:

Maximize $z = 2x_1 - 4x_2 + x_3$

subject to

$x_1 + x_3 \leq 4$

$x_1 + 2x_2 + x_3 \geq 8$

$x_1 \geq 0 , x_2 \geq 0 , x_3 \geq 0$

a) Solve it by the **Simplex method**, using the **Two-Phase technique**;

b) Formulate the corresponding **dual problem**;

c) **Without solving the dual problem**, present its optimal solution, as well as the value of its objective function;

d) Comment the following statement*: "The first phase of the Two-Phase technique aims to obtain a feasible basic solution to the linear programming problem. When this goal is not met, it can be concluded that the problem has no finite optimal solution."*

Quotations: 1 – 3,5 values 2 – 6,5 values 3 – 4,5 values 4 – 5,5 values

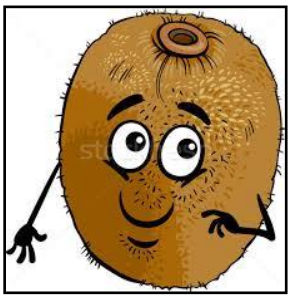
3. Consider this other linear programming problem:

Minimize $z = x_1 - x_2$
subject to
$5x_1 - x_2 \leq 5$
$-x_1 - 3x_2 \leq -3$
$2x_1 + x_2 \leq 4$
$x_1 \geq 0\ ,\ x_2 \geq 0$

a) Solve it by the **graphical method**;

b) In your opinion, could this problem be solved by the **dual Simplex method**? Conveniently justify your answer.

4. The company *JuicyKiwi*, a kiwi fuit producer in northern Portugal, intends to transport the fruit produced in its three plantations (**P1**, **P2**, **P3**) to three companies (**C1**, **C2** and **C3**) responsible for its exportation to several European countries. Plantations **P1**, **P2** and **P3** produce **3**, **9** and **3** tonnes of kiwi fruit, respectively, each year. On the other hand, each year **C1**, **C2** and **C3** order **6**, **4** and **5** tonnes of this fruit, respectively. The transportation costs of each ton of kiwi fruit, from each plantation to each company, are given in the following table:



	C1	**C2**	**C3**
P1	6	4	2
P2	1	3	4
P3	4	2	3

(*Values in Currency Units*)

a) Obtain an initial basic feasible solution to the problem using the **Least Cost method**;

b) From the solution obtained in a), solve the problem by the **transportation method**;

c) Analyse the solution in light of the real problem, presenting it as it will be communicated to *JuicyKiwi*.

Quotations: 1 – 3,5 values 2 – 6,5 values 3 – 4,5 values 4 – 5,5 values