Departamento de Engenharia Informática e de Sistemas

Operations Research 2018/2019		
Date: 07/01/2019	**Exam – First Call**	**Duration: 2 hours**

Note: Present **all the calculations** and conveniently **justify** you answers.

1. Consider the following problem:

"*Mr. Faustino inherited, from his deceased master, a farm of 100 hectares in the interior of Alentejo and he is studying a way of dividing his property into the following productive activities:*

Land leasing - allocate a certain area to a local agricultural company for planting sunflowers, which will be responsible for this activity and that pays 30,000 €/hectare/year for the lease of the land;

Cattle farming - use other part for raising cattle. The recovery of pastures requires fertilization (100kg/hectare /year) and irrigation (100,000 liters of water/hectare/year). The estimated profit of this activity is 40,000€/ hectare/year;

Soybean plantation - use a third part to plant soybeans, whose consumption in Portugal has been growing in recent years. This plantation requires, on average, 200 kg of fertilizers/hectare/year and 200,000 liters of irrigation water, per hectare, per year. The estimated profit of this activity is 50,000€/hectare/year.

Annually, the availability of fertilizers is around 15,000 kg, and the availability of water is around 12,750,000 liters for consumption in the mentioned activities.

In order to respect the will of the former owner, Mr. Faustino intends that the area to be rented should not be less than half the area to be planted with soy, and that the area destined to cattle farming should not exceed 25% of the area of the other two activities."

Knowing that Mr. Faustino wants to obtain the maximum annual income of his farm, **formulate the problem in terms of a linear programming model**, indicating the meaning of the decision variables and the objective function.

2. Consider the following linear programming model:

Minimize $z = x_1 + x_2$
subject to
$-3x_1 - 3x_2 \leq 6$
$-4x_1 + 4x_2 \leq 2$
$x_1 \geq -1, x_2 \geq 0$

a) **Reformulate it** so that all variables have a **non-negativity constraint**;

b) Solve the reformulated problem by the **dual Simplex method**;

c) Comment the following statement: "*In graphic terms, when solving any problem, the dual Simplex method only moves within the feasible region.*"

Quotations: 1 – 3,5 values 2 – 5,5 values 3 – 5,5 values 4 – 5,5 values

3. Consider now the following linear programming model:

Maximize $z = x_1 + 2x_2$

subject to

$$-x_1 + x_2 \leq 2$$
$$x_1 + 3x_2 \leq 12$$
$$-x_1 + 2x_2 \geq 1$$
$$x_1 \geq 0 \, , \, x_2 \geq 0$$

a) Solve it by the **graphical method**;

b) Solve it by the **Simplex method** with the **"Big M" technique** and indicate, at each iteration, the corresponding basic solution;

c) Formulate the **dual problem** corresponding to the problem presented above.

4. A certain pulp industry has three manufacturing units (**F1**, **F2** and **F3**), where excellent pulp is produced for export. The pulp produced in these units is stored in containers which are transported by truck to three export companies (**E1**, **E2** and **E3**). The latter, located on the coast, send the containers to several countries by sea. It is known that **F1**, **F2** and **F3** are able to produce **100**, **20** and **40** pulp containers monthly, respectively. On the other hand, **E1**, **E2** and **E3** need **80**, **30** and **50** containers of that product, respectively, per month.

The table with transportation costs per container, from each of the origins, to each of the destinations, is as follows:

	E1	**E2**	**E3**
F1	**1**	**9**	**3**
F2	**4**	**7**	**2**
F3	**8**	**5**	**4**

(Values in hundreds of euros)

a) Obtain an initial basic feasible solution to the problem using the **Northwest Corner method**;

b) From the solution obtained in a), solve the problem by the **transportation method**;

c) Explain what happens when in a given basic solution the number of basic variables is less than **m + n-1** (where **m** is the number of rows and **n** is the number of columns in the table).

Quotations: 1 – 3,5 values 2 – 5,5 values 3 – 5,5 values 4 – 5,5 values