Probability & Random Processes

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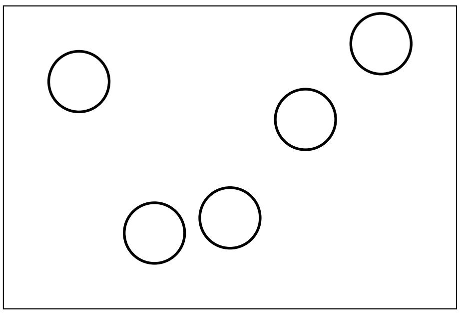
4.1 Erdos-Reyni Model

Definition 4.1 (Erdos-Reyni random graph).

Given $n \in \mathbb{Z}^+$ and a probability $p \in [0, 1]$, the (E-R) random graph is denoted as $\mathcal{G}(n, p)$.

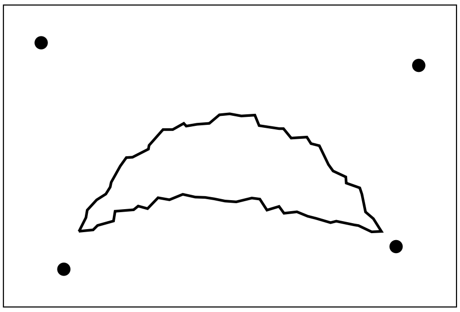
- The (RG) $\mathcal{G}(n, p)$ is undirected on all n vertices
- Each of the $\binom{n}{2}$ edges is formed independently from each other with probability p
- E-R stated a number of result that are based on "thresholds" of p which are needed to show the structural properties of the graph:
  1. p is a function of n, $p :- p(n)$
  2. $\frac{1}{n^2}$ is the threshold for the first edges to appear in a (RG)
  3. $\frac{1}{n^{3/2}}$ is the threshold for the first trees to emerge
  4. $\frac{1}{n}$ is the threshold for the first cycles to appear in the graph
- If $p_n = 1/n$ then the "Giant component" emerges. Let $p_n = (1 - \varepsilon)/n$ for $0 < \varepsilon \ll 1$), then each "puddle" represents a region in the graph where all nodes are connected.



Subcritical: $p = \frac{1-\varepsilon}{n}$

  Note that p_n is to the left of ε and that the size of the largest puddles are "small", $\rightarrow O(\log n)$ w.h.p.

- For $p_n = (1 + \varepsilon)/n$, we go to the right of $\varepsilon \rightarrow 1 + \varepsilon$.

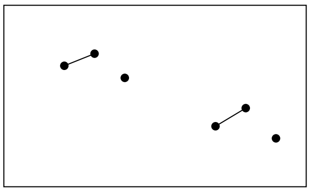


Supercritical: $p > \frac{1+\varepsilon}{n}$

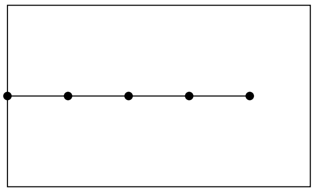
Here we have one large "island" where it's size is relatively, $O(n)$ w.h.p.

4.2 Thresholds and Connectivity

When p_n is below $\log_e n \implies$ no connectivity, $P(\text{connectivity}) \longrightarrow 0$. Otherwise, if p_n is above $\log_e n \implies$ ensures connectivity, $P(\text{connectivity}) \longrightarrow 0$.



Disconnected: $p < \frac{\log_e n}{n}$



connected $(p > (1+\varepsilon)\frac{\log n}{n})$

Chapter 5

Statistical Inference