

# Probability & Random Processes

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# CHAPTER 1

## PROBABILITY THEORY

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1.1 INTRODUCTION

1.2 DISCRETE RANDOM VARIABLES

1.3 CONTINUOUS RANDOM VARIABLES

1.4 ORDER STATISTICS

1.5 TRANSFORMS AND SUMS

1.6 CONCENTRATION INEQUALITIES AND CONVERGENCES

# CHAPTER 2

## INFORMATION THEORY

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2.1 INFORMATION MEASURES AND ENTROPY

2.2 SOURCE CODING THEOREM

2.3 HUFFMAN CODING

2.4 CHANNEL MODELS

2.5 CHANNEL CODING THEOREM

# CHAPTER 3

## STOCHASTIC PROCESSES

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3.1 DISCRETE TIME MARKOV CHAINS

3.2 POISSON PROCESSES

3.3 CONTINUOUS TIME MARKOV CHAINS

# CHAPTER 4

## RANDOM GRAPHS

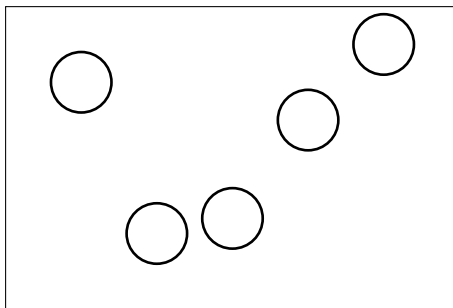
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### 4.1 ERDOS-REYNI MODEL

**Definition 4.1** (Erdos-Reyni random graph).

Given  $n \in \mathbb{Z}^+$  and a probability  $p \in [0, 1]$ , the (E-R) random graph is denoted as  $\mathcal{G}(n, p)$ .

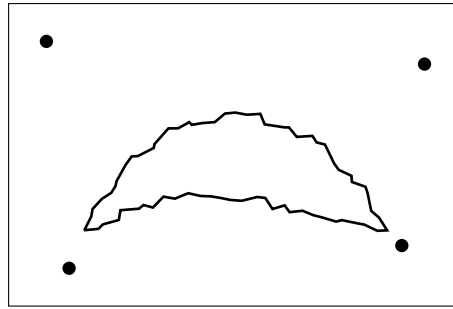
- The (RG)  $\mathcal{G}(n, p)$  is undirected on all  $n$  vertices
- Each of the  $\binom{n}{2}$  edges is formed independently from each other with probability  $p$
- E-R stated a number of result that are based on "thresholds" of  $p$  which are needed to show the structural properties of the graph:
  1.  $p$  is a function of  $n$ ,  $p := p(n)$
  2.  $\frac{1}{n^2}$  is the threshold for the first edges to appear in a (RG)
  3.  $\frac{1}{n^{3/2}}$  is the threshold for the first trees to emerge
  4.  $\frac{1}{n}$  is the threshold for the first cycles to appear in the graph
- If  $p_n = 1/n$  then the "Giant component" emerges. Let  $p_n = (1 - \varepsilon)/n$  for  $0 < \varepsilon \ll 1$ , then each "puddle" represents a region in the graph where all nodes are connected.



Subcritical:  $p = \frac{1-\varepsilon}{n}$

Note that  $p_n$  is to the left of  $\varepsilon$  and that the size of the largest puddles are "small",  $\rightarrow O(\log n)$  w.h.p.

- For  $p_n = (1 + \varepsilon)/n$ , we go to the right of  $\varepsilon \rightarrow 1 + \varepsilon$ .

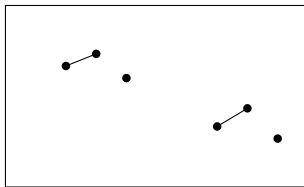


Supercritical:  $p > \frac{1+\varepsilon}{n}$

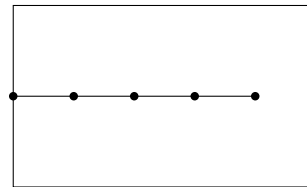
Here we have one large "island" where it's size is relatively,  $O(n)$  w.h.p.

## 4.2 THRESHOLDS AND CONNECTIVITY

When  $p_n$  is below  $\log_e n \implies$  no connectivity,  $P(\text{connectivity}) \longrightarrow 0$ . Otherwise, if  $p_n$  is above  $\log_e n \implies$  ensures connectivity,  $P(\text{connectivity}) \longrightarrow 1$ .



Disconnected:  $p < \frac{\log_e n}{n}$



connected ( $p > (1 + \varepsilon) \frac{\log n}{n}$ )

# CHAPTER 5

## STATISTICAL INFERENCE

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