Probability & Random Processes

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4.1 Erdos-Reyni Model

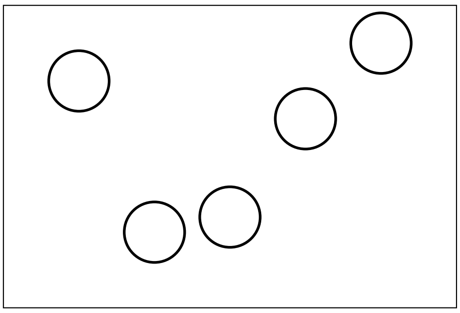
Definition 4.1 (Erdos-Reyni random graph).

Given $n \in \mathbb{Z}^+$ and a probability $p \in [0, 1]$, the (E-R) random graph is denoted as $\mathcal{G}(n, p)$.

- The (RG) $\mathcal{G}(n, p)$ is undirected on all n vertices
- Each of the $\binom{n}{2}$ edges is formed independently from each other with probability p
- E-R stated a number of result that are based on "thresholds" of p which are needed to show the structural properties of the graph:

  1. p is a function of n, $p :- p(n)$
  2. $\frac{1}{n^2}$ is the threshold for the first edges to appear in a (RG)
  3. $\frac{1}{n^{3/2}}$ is the threshold for the first trees to emerge
  4. $\frac{1}{n}$ is the threshold for the first cycles to appear in the graph

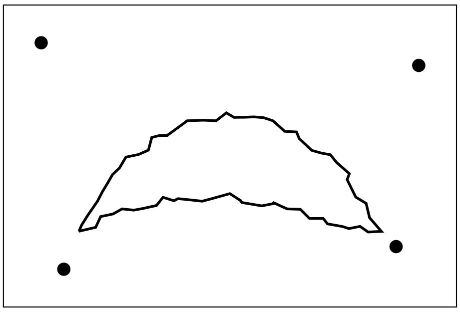
- If $p_n = 1/n$ then the "Giant component" emerges. Let $p_n = (1 - \varepsilon)/n$ for $0 < \varepsilon \ll 1$), then each "puddle" represents a region in the graph where all nodes are connected.



Subcritical: $p = \frac{1-\varepsilon}{n}$

  Note that p_n is to the left of ε and that the size of the largest puddles are "small", $\to O(\log n)$ w.h.p.

- For $p_n = (1 + \varepsilon)/n$, we go to the right of $\varepsilon \to 1 + \varepsilon$.

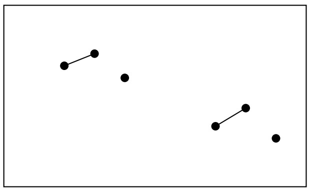


Supercritical: $p > \frac{1+\varepsilon}{n}$

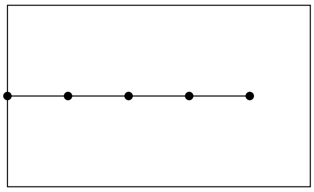
Here we have one large "island" where it's size is relatively, $O(n)$ w.h.p.

4.2 Thresholds and Connectivity

When p_n is below $\log_e n \implies$ no connectivity, $P(\text{connectivity}) \longrightarrow 0$. Otherwise, if p_n is above $\log_e n \implies$ ensures connectivity, $P(\text{connectivity}) \longrightarrow 0$.



Disconnected: $p < \frac{\log_e n}{n}$



connected ($p > (1+\varepsilon)\frac{\log n}{n}$)

Chapter 5

Statistical Inference

5.1 Point Estimation

- **Probability vs Statistics**: There exists two perspectives that surround the field of statistical inference, Bayesian vs Frequentist.

  1. From the Bayesian perspective, unknown quantities are treated as RVs with known or assumed distributions and priors.

  2. While under the Frequentist's point of view, unknowns are deterministic parameters that need to be estimated.

  In essence, a person following the Bayesian perspective starts with a probabilistic model and assumes its distribution and shows through rigorous probabilistic analysis that measurements get smaller.

  On the other hand, a Frequentist would make hidden assumptions about parameters present in the model for which we know knowing about.

- **Bayesian Inference:**