ECE-GY 6123 Image and Video Processing, Spring 2023
Computer Assignment 3: Pyramid Transforms
Due 3/23/2023 at 11:59 PM

Please submit your solutions, including Python code and outputs (when relevant), **as a single PDF file to GradeScope**.

In this assignment, you will implement 2D Gaussian, Laplacian pyramids and a Wavelet transform in Python, and look at their reconstruction qualities and representation efficiency.

You'll need to have python packages for `OpenCV` and `PyWavelets` installed. Throughout the following problems, keep your input images on the `uint8` scale of [0,255].

Problem 1 (Gaussian and Laplacian pyramids).

(a) Write functions `gaussian_pyramid` and `laplacian_pyramid` that decompose an input grayscale image into a J-level Gaussian and Laplacian pyramid, respectively, where J is an input to each function.

You can use `cv2.resize` for downsampling and upsampling with `INTER_LINEAR` and `INTER_CUBIC` filters, respectively.

(b) Write a function `reconstruct_laplacian` that reconstructs the original image from a J-level Laplacian pyramid. Verify it works correctly on a test image. Display the Gaussian and Laplacian pyramid images for $J = 3$.

(c) Write a function `quantize_pyramid` that takes in a Laplacian pyramid and quantizes the coefficients c with quantization step-size q as follows,

$$Q(c, q) = q \left\lfloor \frac{c - \mu}{q} + \frac{1}{2} \right\rfloor + \mu$$

where μ is the mean of the coefficient map, assumed to be $\mu = 0$ for residual (Laplacian) images and $\mu = 128$ otherwise (Gaussian images).

(d) For pyramid levels $J = 0, 1, 2, 3$ (where $J = 0$ is simply the original image) plot the reconstruction PSNR,

$$\text{PSNR} = 10 \log_{10} \left(\frac{255^2}{\text{MSE}} \right),$$

between the original and reconstructed image vs. the number of non-zeros in the representation for pyramids quantized at steps $q = 2^n$, $n = 0, 1, \ldots, 8$. Plot PSNR on the y-axis and NNZ on the x-axis.

What relationship do you observe between pyramid depth and representation efficiency for a desired reconstruction PSNR? Is this expected?

Such a curve helps us evaluate the representation efficiency of the Laplacian pyramid. Note that we're using the number of non-zeros (NNZ) as a surrogate for the number of bits needed to describe the image.

(e) For $J = 3$, determine qualitatively at what point the quantization level is unnoticeable. How do the number of non-zeros compare to the original image?

Help!: If your pyramid functions from problem 1 aren't working, you can use the skimage packages's functions.

Problem 2 (Wavelet Transform)**.**

(a) Use PyWavelet's `dwt2` and `idwt2` to write J-level wavelet transform (`wavelet_transform`) and reconstruction (`reconstruct_wavelet`) functions, for a grayscale input image. Use symmetric boundary conditions.

(b) Repeat parts (b,c,d,e) from problem 1, now with your wavelet transform functions and wavelet subimages.

(c) Based on your results, compare the representation efficiency of the Laplacian pyramid and wavelet transform in terms of the NNZ required to represent an image to the same quality (PSNR).

Help!: If your wavelet functions from problem 2 aren't working, you can use the PyWavelet's `wavedec2`, `waverec2` functions.