7. Determine the type of Differential Equation then Solve

$$\overset{M(x,y)}{(6x^2-y^2)}dx + \overset{N(x,y)}{(xy-2x^3y^{-1})}dy=0$$

Homogeneous D. E.

Test if Homogeneous D.E.

$$M(tx,ty) = (6t^2x^2 - t^2y^2) = t^2(6x^2-y^2) = \underbrace{t^2 M(x,y)}_{\text{Homogeneous 2nd degree}}$$

same

$$N(tx,ty) = (t^2xy - 2t^{\cancel{3}\,2}x^3/\cancel{t}y) = t^2(xy-2x^3y^{-1}) = \underbrace{t^2 N(x,y)}_{\text{Homogeneous 2nd degree}}$$

Make Substitution to turn into Seperable

Let $x = uy$ Product Rule

$dx = u\,dy + y\,du$

Plug in for x and dx

$$(6u^2y^2 - y^2)(u\,dy + y\,du) + (uy^2 - 2u^3y^2)dy = 0$$

$$(6u^2y^2 - y^2)(u\,dy + y\,du) + (uy^2 - 2u^3y^2)\,dy = 0$$

$$6u^3y^2dy + 6u^2y^3du - uy^2dy - y^3du + uy^2dy - 2u^3y^2dy = 0$$ Foil

$$4u^3y^2dy + 6u^2y^3du - y^3du = 0$$ Add Like Terms

$$y^2dy\,(4u^3) = y^3du\,(1 - 6u^2)$$

$$\frac{1}{y}dy = \frac{1-6u^2}{4u^3}du$$

Seperate variables

$$\int \frac{1}{y}dy = \int \frac{1-6u^2}{4u^3}du$$ Integrate both Sides

$$\ln|y| = -\frac{1}{8u^2} - \frac{3}{2}\ln|u| + C$$

$$\ln|y| = -\frac{1}{8\left(\frac{x}{y}\right)^2} - \frac{3}{2}\ln\left|\frac{x}{y}\right| + C$$

Substitute $\frac{x}{y}$ back for u

19. In 1980 the population of alligators in a particular region was estimated to be 1300. In 2004 the population had grown to an estimated 6000. Estimate the alligator population in this region by the year 2020.

Write Differential Equation

$$\frac{dP}{dt} = KP$$

Write in Standard Form

$$\frac{dP}{dt} - KP = 0$$

Find Integration Factor

$$P(x) = -K \qquad e^{\int P(x)\,dt} = e^{-K\int dt} = e^{-Kt}$$

Multiply everything by Integration Factor

$$e^{-Kt}\left[\frac{dP}{dt} - KP\right] = 0$$

Rewrite as derivative of product

$$\frac{d}{dt}\left[e^{-kt}P\right] = 0$$

Integrate both sides respect to t

$$\int \frac{d}{dt}\left[e^{-kt}P\right] dt = \int 0 \, dt$$

$$e^{-kt}P = C$$

Solve for P

$$P = Ce^{kt}$$

Plug in initial value and solve for C

$$1300 = Ce^{0}$$

Plug in $P(t) = 6000$. Use $C = 1300$ and $t = 2004 - 1980 = 24$

$$6000 = 1300\, e^{-24K}$$

Solve for K

$$\frac{-\ln\left(\frac{6000}{1300}\right)}{24} = K$$

Solve for population in 2020. Use $t = 2020 - 1980 = 40$

$$P(40) = 1300\, e^{\frac{-\ln\left(\frac{6000}{1300}\right)}{24}(40)} = 16632$$