Final Exam is on Tuesday, December 12th, 6pm-8:30pm.

Students are allowed to bring one 3x5 index card with notes on both sides.

Find both the parametric and the vector equation of the line.

1) The line through (0, 1, 0) in the direction of the vector v = (3, 0, -1)

2) The line through (4, -1, 3) that is perpendicular to both u = (2, 0, -1) and the y-axis.

Write the equation for the plane.

3) The plane through the points P(5, -7, 13) , Q(-3, 8, -8) and R(-1, 1, 7).

4) The plane passing through the point $P_0(-3, 1, 7)$ that is perpendicular to the line x = -1 + 6t, y = 2 - 5t, z = 3 + 4t

Find the angle between the planes to the nearest thousandth of a radian.

5) 10x - 9y + 3z = -7 and -6x - 7y + 5z = -10

Find the length of the curve with the given vector equation.

6) $r(t) = (3 + 2t^3)i + (2t^3 - 9)j + (1 - t^3)k, \ -1 \le t \le 4$

7) $r(t) = (2\cos t)i + (2\sin t)j + 7tk, \ 0 \le t \le \pi/2$

The position vector of a particle is r(t). Find the requested vector.

8) The velocity at t = 0 for $r(t) = \cos(4t)i + 8\ln(t - 5)j - \frac{t^3}{10}k$

9) The acceleration at t = 3 for $r(t) = (3t - 3t^4)i + (10 - t)j + (5t^2 - 6t)k$

Solve the problem.

10) Find the equation for the tangent plane to the surface $z = -2x^2 - 7y^2$ at the point (2, 1, -15).

11) Find the equation for the tangent plane to the surface $z = e^{3x^2 + 7y^2}$ at the point (0, 0, 1).

Find the derivative of the function at the given point in the direction of A.

12) $f(x, y) = \ln(-2x + 9y)$, (10, 7), A = 6i + 8j

Provide an appropriate response.

13) Find the direction in which the function is increasing most rapidly at the point P_0.

$f(x, y) = xe^y - \ln(x)$, $P_0(2, 0)$

Solve the problem.

14) Find the maximum rate of change of $f(x, y) = x^2 + xy + y^2$ at the point (7, 8).

15) Find the maxmum rate of change of $f(x, y) = e^{xy}$ at the point (0, 5).

Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.

16) $f(x, y) = x^2 - 18x + y^2 + 20y - 9$

17) $f(x, y) = 8x^2y + 4xy^2$

Use Lagrange Multipliers to find the extreme values of the function subject to the given constraint.

18) $f(x, y) = 7x^2 + 6y^2, \ x^2 + y^2 = 1$

19) $f(x, y) = xy, \ x^2 + y^2 = 128$

Use the given transformation to evaluate the integral.

20) u = -6x + y, v = 7x + y;

$$\iint_R (y - 6x)\,dx\,dy,$$

where R is the parallelogram bounded by the lines y = 6x + 3, y = 6x + 6, y = -7x + 2, y = -7x + 10

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

21) The lines $x = 0$, $y = 6x$, and $y = 9$

22) The parabola $y = x^2$ and the line $y = -2x + 15$

Integrate the function f over the given region.

23) $f(x, y) = \frac{x}{6} + \frac{y}{5}$ over the trapezoidal region bounded by the x-axis, y-axis, line $x = 6$, and line $y = -\frac{2}{3}x + 9$

24) $f(x, y) = xy$ over the triangular region with vertices $(0, 0)$, $(9, 0)$, and $(0, 4)$

Find the volume of the indicated region.

25) the region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $x = 6$, $y = 0$, and $y = 5x$

26) the region that lies under the plane $z = 6x + 2y$ and over the triangle with vertices at $(1, 1)$, $(2, 1)$, and $(1, 2)$

Find the volume of the indicated region.

27) the region enclosed by the cylinder $x^2 + y^2 = 25$ and the planes $z = 0$ and $x + y + z = 10$

Solve the problem.

28) Let D be the region that is bounded below by the cone $\varphi = \frac{\pi}{4}$ and above by the sphere $\varrho = 9$. Set up the triple integral for the volume of D in cylindrical coordinates.

Find the volume of the indicated region.

29) the region bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \sqrt{x^2 + y^2}$

Evaluate the line integral along the curve C.

30) $\int_C \left(\frac{x^2 + y^2}{z^2}\right) ds$, C is the curve $r(t) = (-8 - t)i - j + (-8 - t)k$, $0 \le t \le 1$

31) $\int_C \left(\frac{x^2 + y^2}{z^2}\right) ds$, C is the curve $r(t) = (3 \sin 4t)i + (3 \cos 4t)j + 5tk$, $2 \le t \le 4$

Evaluate the line integral of the given vector field F along the curve C.

32) $F(x, y) = (x^4 + y^4)i + xyj$; C is the curve $x = t^2$, $y = t$, $0 \le t \le 1$

33) $F = xyi + 7j + 4xk$; C: $x = \cos 2t$, $y = \sin 2t$, $z = t$, $0 \le t \le \frac{\pi}{4}$

Test the vector field F to determine if it is conservative.

34) $F = xyi + yj + zk$

35) $F = -\cos x \cos y i + \sin x \sin y j - \sec^2 z k$

Find f so that $F = \nabla f$.

36) $F(x,y) = (x - 3y + 7)i + (-3x + 3y + 8)j$

Answer each question.

37) (a) Show that $F(x,y) = (3x^2y - y^2)i + (x^3 - 2xy)j$ is a conserviative vector field.
(b) Use the Fundamental Theorem for Line Integrals to evalue $\int_C F \cdot dr$, where C is the portion of a curve stretching from $(-2,3)$ to $(4,2)$.

Apply Green's Theorem to evaluate the integral.

38) $\oint_C (x + 2y)dx + 4xy\, dy$; C the the triangle with vertices $(0, 0)$, $(3, 0)$ and $(0, 3)$.

39) $\oint_C -3\, dx + 8x\, dy$; C the circle with center $(0, 0)$ and radius 2.

Find the divergence of the field F.

40) $F = -3x^3i + 7y^3j + 6z^3k$

Evaluate the surface integral of G over the surface S.

41) S is the plane $x + y + z = 2$ above the rectangle $0 \leq x \leq 3$ and $0 \leq y \leq 3$; $G(x,y,z) = 3z$

Find the flux of the vector field F across the surface S in the indicated direction.

42) $F = 9x\mathbf{i} + 9y\mathbf{j} + z\mathbf{k}$; S is portion of the plane $x + y + z = 8$ for which $0 \leq x \leq 3$ and $0 \leq y \leq 3$; direction is outward (away from origin)

43) $F = 7x\mathbf{i} + 7y\mathbf{j} + 2\mathbf{k}$; S is "nose" of the paraboloid $z = 7x^2 + 7y^2$ cut by the plane $z = 2$; direction is outward

Find the flux of the curl of field F through the shell S.

44) $F = e^x\mathbf{i} + e^y\mathbf{j} + 3xy\mathbf{k}$; S is the portion of the paraboloid $1 - x^2 - y^2 = z$ that lies above the xy-plane

45) $F = -7z\mathbf{i} + 3x\mathbf{j} + 9y\mathbf{k}$; S is the portion of the cone $z = 6\sqrt{x^2 + y^2}$ below the plane $z = 5$

Answer Key
Testname: MAT-1C FINAL EXAM REVIEW TTH CLASS

1) $x = 3t,\ y = 1,\ z = -t;\ r = (0, 1, 0) + t(3, 0, -1)$
2) $x = 4 - t,\ y = -1,\ z = 3 + 2t;\ r = (4, -1, 3) + t(-1, 0, 2)$
3) $3x + 3y + z = 7$
4) $6x - 5y + 4z = 5$
5) 1.446 rad
6) 195
7) $\frac{\sqrt{53}t}{2}\pi$
8) $v(0) = -\frac{8}{5}j$
9) $a(3) = -324i + 10k$
10) $-8x - 14y - z = -15$
11) $z = 1$
12) $\frac{6}{43}$
13) $\left(\frac{1}{\sqrt{17}}\right)i + \left(\frac{4}{\sqrt{17}}\right)j$
14) $\sqrt{1013}$
15) 5
16) f(, -10) = -190, local minimum
17) f(0, 0) = 0, saddle point
18) Minimum: 6 at (0, ±1); maximum: 7 at (±1, 0)
19) Maximum: 64 at (8, 8) and (-8, -8); minimum: -64 at (8, -8) and (-8, 8)
20) $\frac{108}{13}$
21) $\int_0^9 \int_0^{y/6} dx\, dy$
22) $\int_{-5}^3 \int_{x^2}^{-2x + 15} dy\, dx$
23) $\frac{246}{5}$
24) 54
25) 15,120
26) $\frac{16}{3}$
27) 250π
28) $\int_0^{2\pi} \int_0^{9/\sqrt{2}} \int_r^{\sqrt{81 - r^2}} r\, dz\, dr\, d\theta$
29) $\frac{8}{3}\pi(2 - \sqrt{2})$
30) $\frac{73}{72}\sqrt{2}$
31) $\frac{117}{100}$
32) $\frac{47}{60}$
33) $\frac{26}{3}$
34) Not conservative
35) Conservative
36) $f(x, y) = \frac{x^2}{2} - 3xy + 7x + \frac{3y^2}{2} + 8y + C$
37) 118
38) 9
39) 32π
40) $-9x^2 + 21y^2 + 18z^2$
41) $-27\sqrt{3}$
42) 288
43) $\frac{24}{7}\pi$
44) 0
45) $-\frac{25}{12}\pi$