Computer Graphics (COMP0027) 2022/23

Rendering Equation

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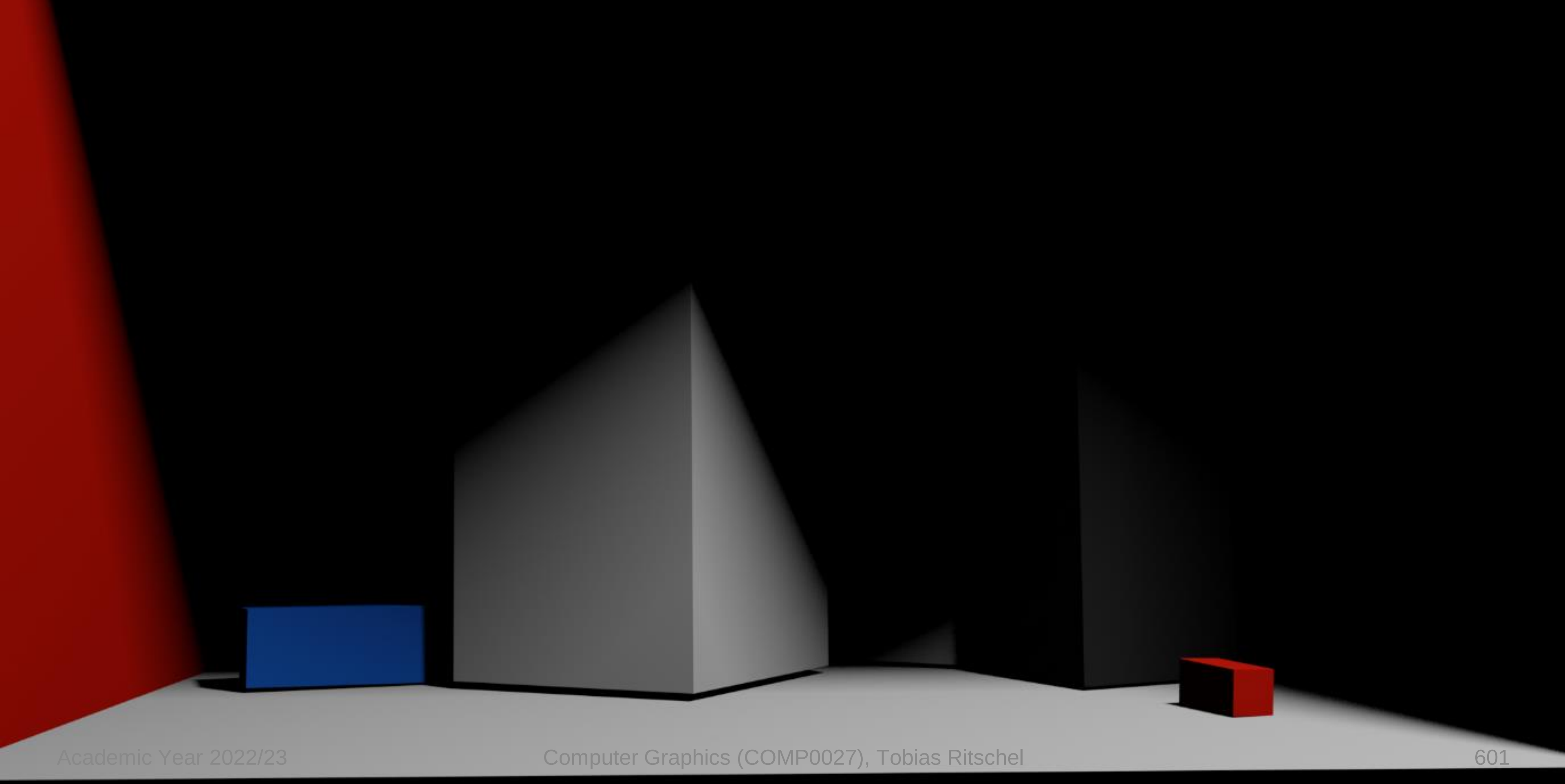
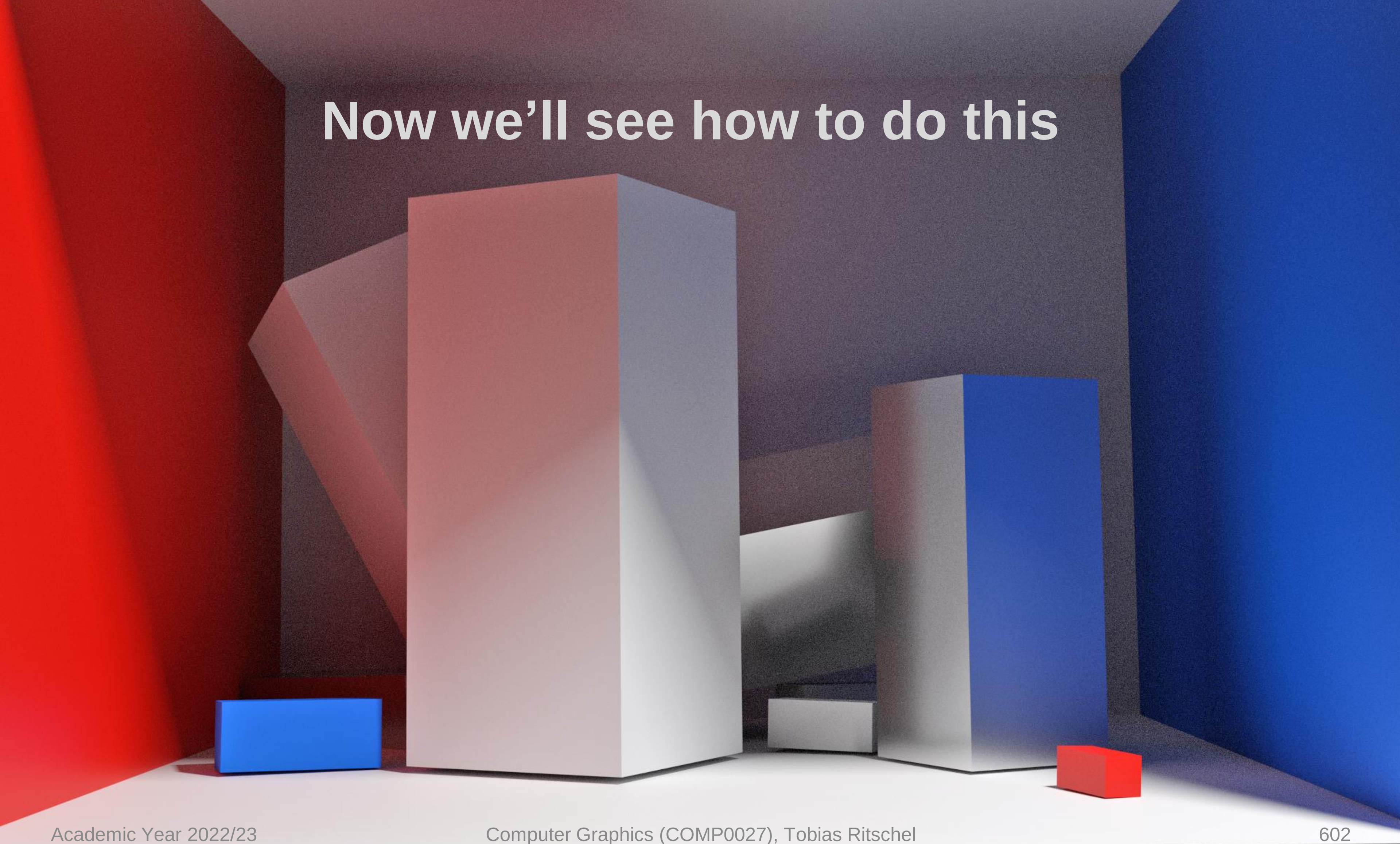
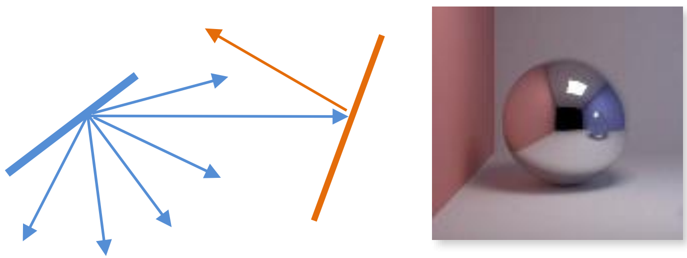
You’ve learned how to do this



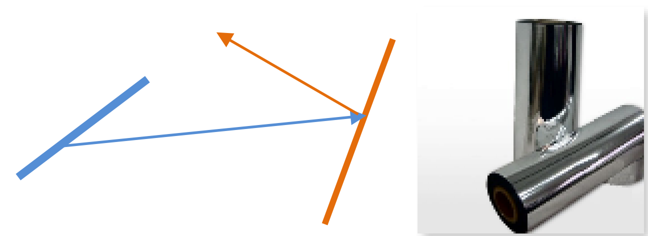
Now we’ll see how to do this



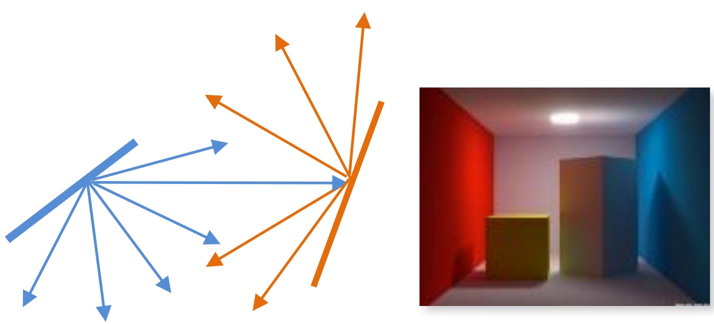
Types of Light Transport



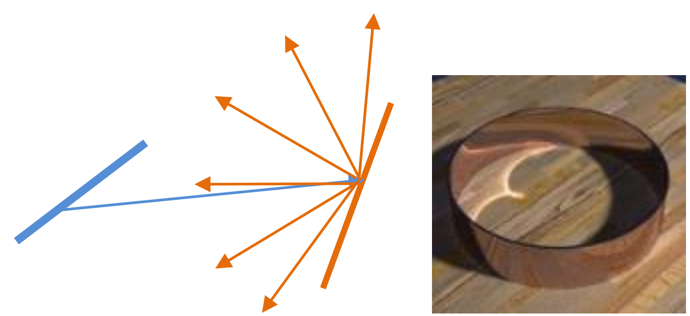
Diffuse-specular



Specular-specular

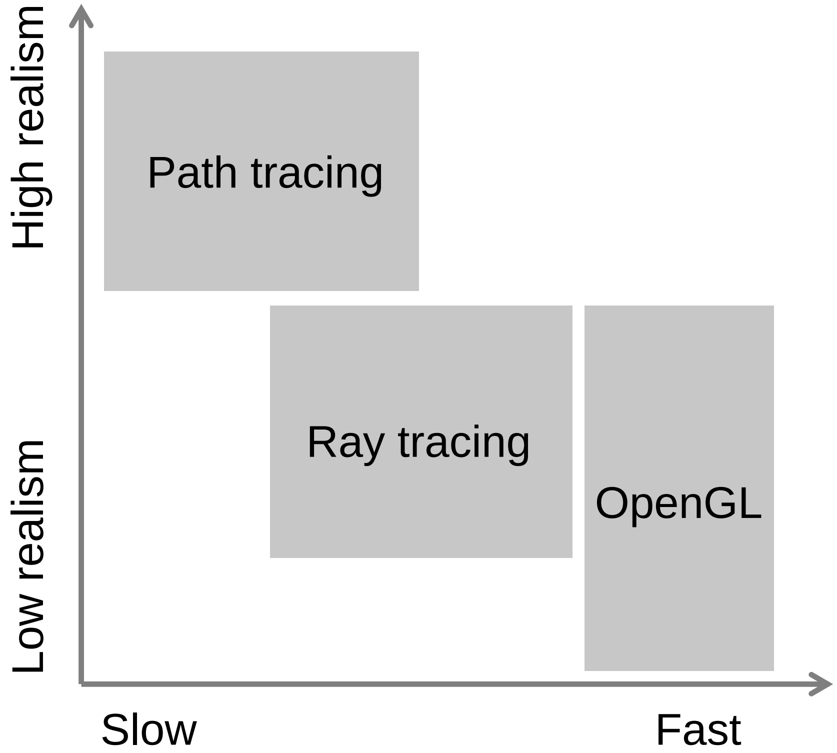


Diffuse-diffuse



Specular-diffuse

Speed/quality Domain



In the next lectures

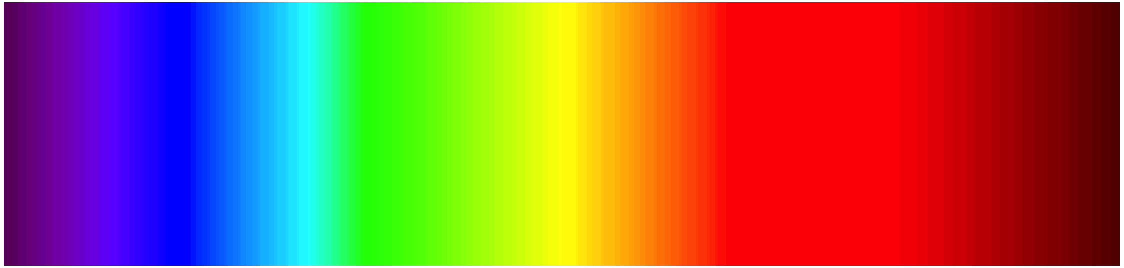
- This lecture (1h): The rendering equation
  - Units
  - Definition
  - Light
  - Reflectance (BRDF)
- Next lectures (2+1+2hs): Methods to solve it
  - Path tracing (2+1hs)
  - Photon mapping (2hs)

Physically-based Rendering

- Simulation of light transport
- Light
  - The nature of light, how it travels in the environment
- Material
  - Anything that interacts with light, how it reflects, refracts or scatters light
  - Bidirectional Reflectance Distribution Function
- Geometry

Light

Visible light is electromagnetic radiation with wavelengths approximately in the range from 400 nm to 700 nm



400 nm 700 nm

What is light?

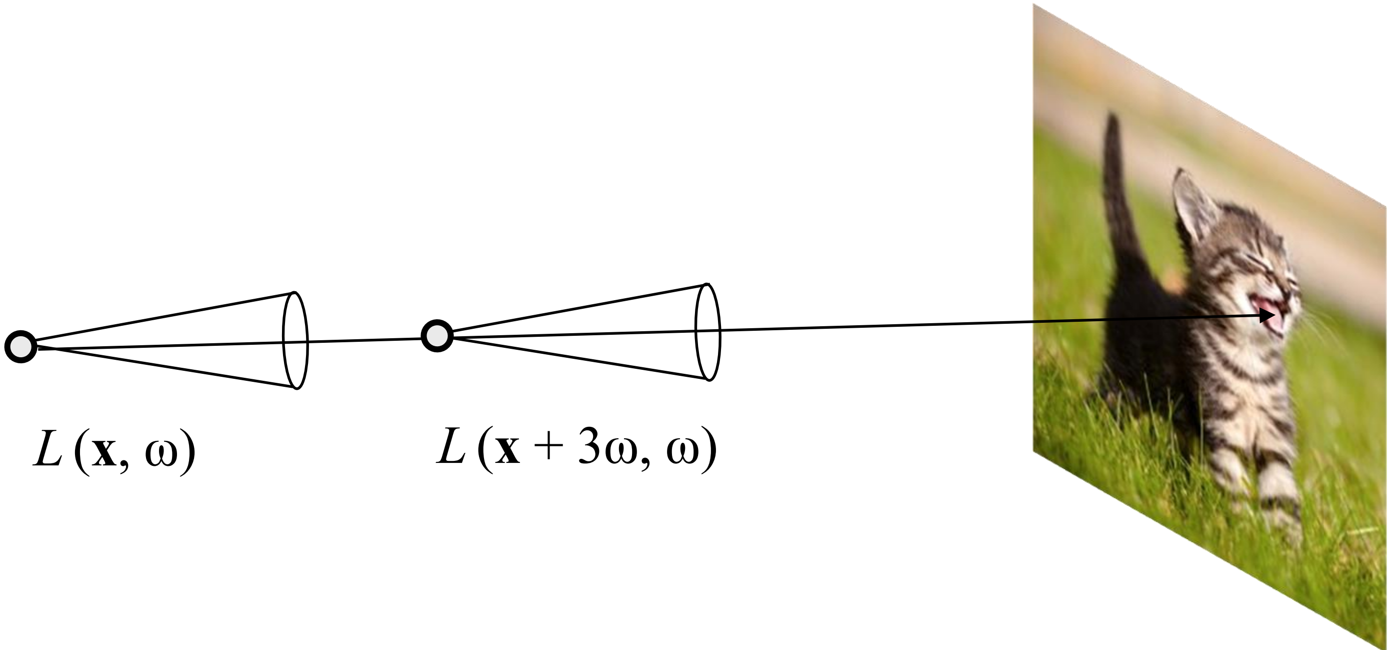
- Light can be viewed as
  - Wave or
  - Particle phenomenon
- Particles are photons
  - Packets of energy which travel in a straight line in vacuum with velocity c (~300,000 km/s)
- For us here:
  Continuous quantity at infinite speed

Units: Radiance

- There is a large number of radiometric units
- We will simulate in units of **radiance**
- Radiance $L(\mathbf{x}, \omega)$ is the quantity that is high if you look at a bright point $\mathbf{x}$ from angle ω
- How many photons at a wavelength per unit **time**, unit **area** and unit **solid angle**
- Does not change when moving along ω in free space

Radiance

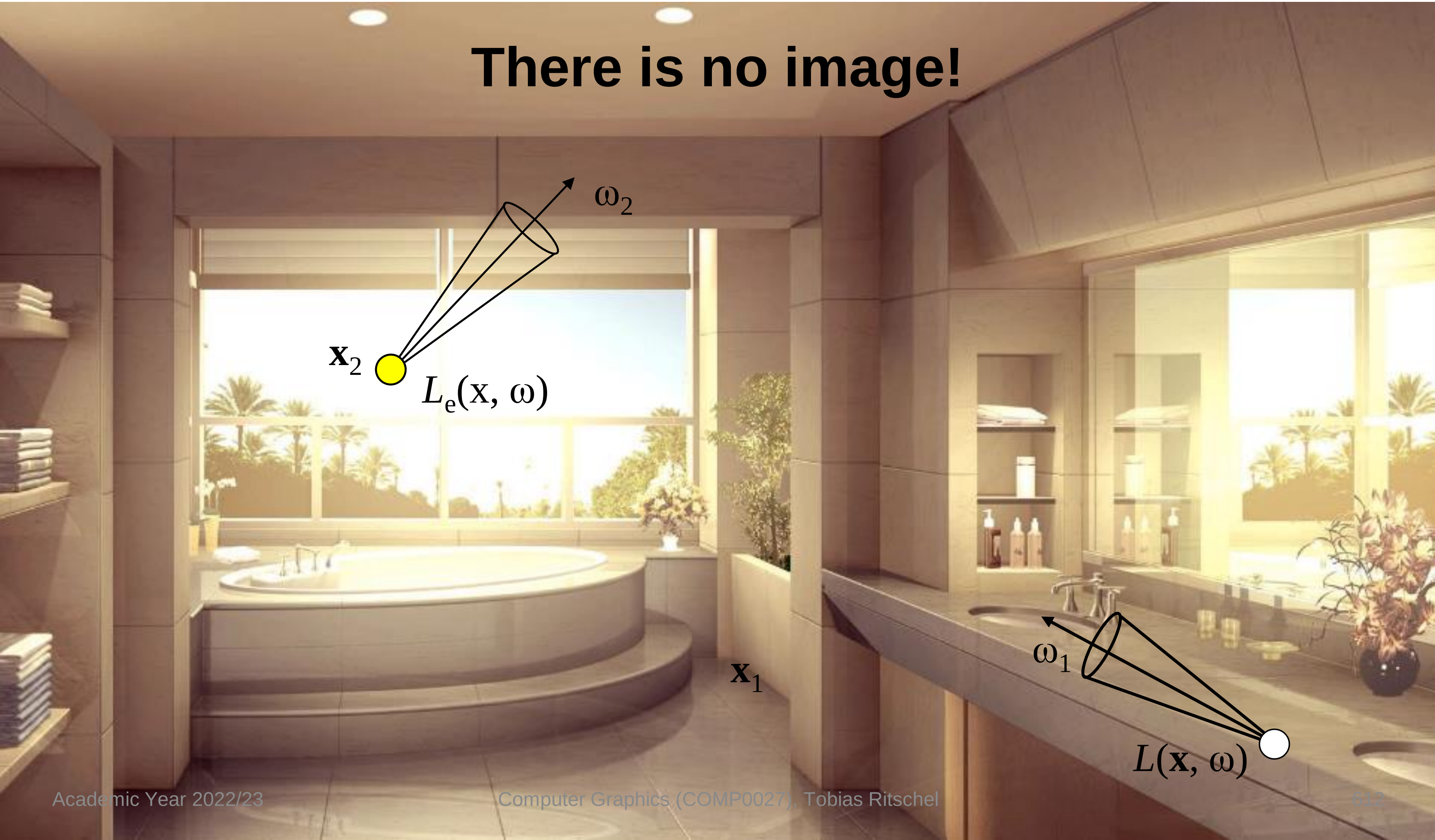
Radiance does not change when moving along ω in free space



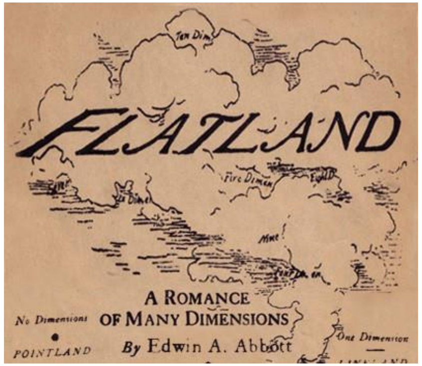
Simplifying Assumptions

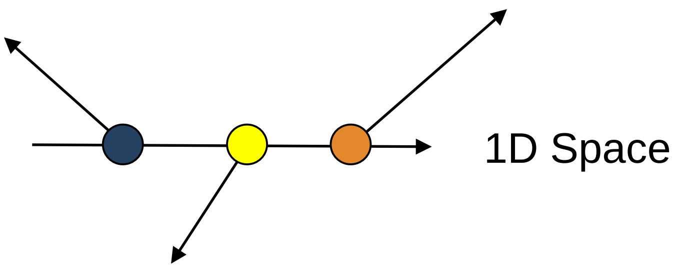
1. Wavelength-independence
   – No interaction between wavelengths
     (no fluorescence)
2. Time-invariance
   – Solution valid over time unless scene changes
     (no phosphorescence)
3. Vacuum
   – Interaction only occurs at the surfaces of objects
     (non-participating medium)

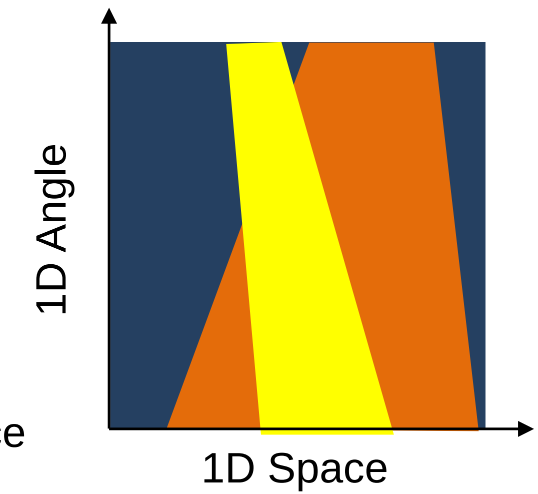
There is no image!



Light fields







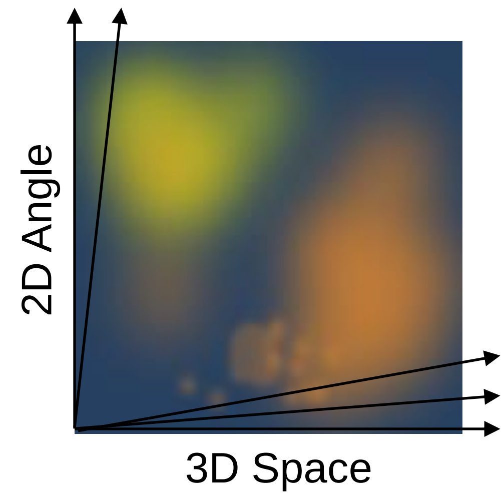
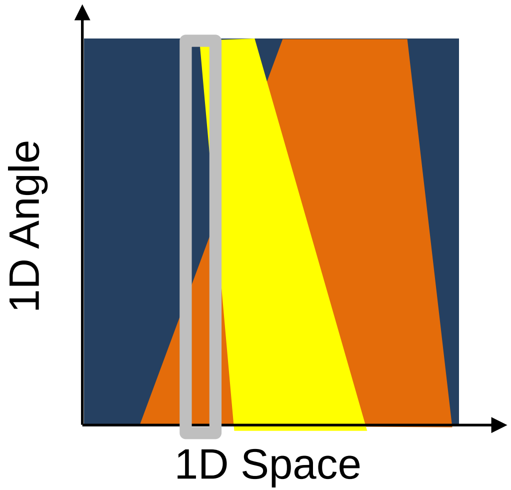
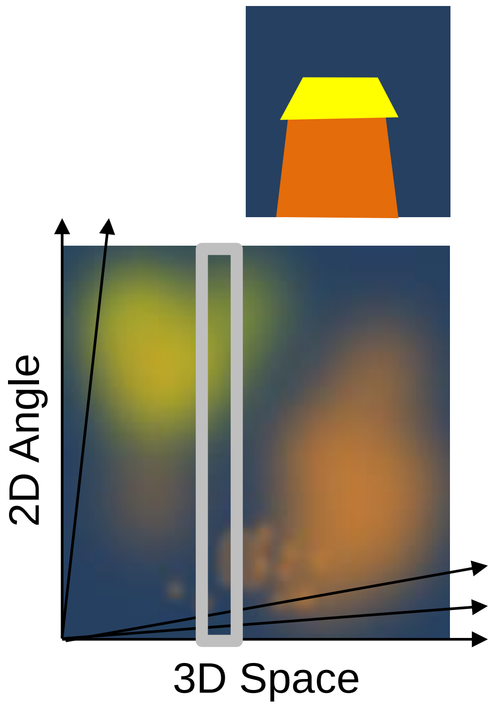


Image is light field slice

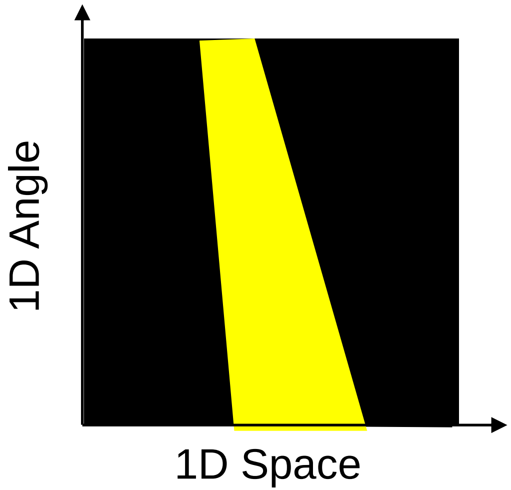
- Making an image is
  - Fixing location
  - Looking at varying angles

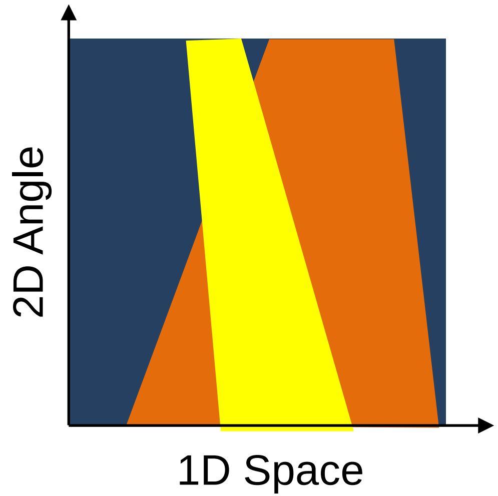




Global illumination, mapping between light fields

- Map from a field of initial radiance
- To a field of reflected radiance

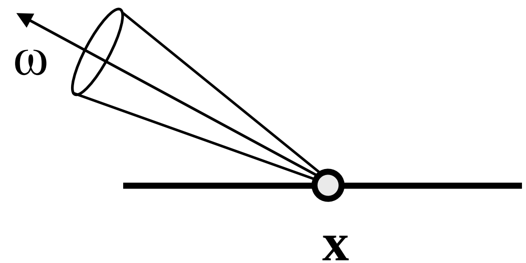




The Rendering Equation

- Rendering Equation [Kajiya 1986]
  - Integral equation
  - Solution is a radiance distribution over space and angle
- A solution of this equation =
  A solution to the whole rendering problem
- Each approach to rendering is a different type of solution to this equation
- Popular approaches:
  - Finite Element
  - Monte Carlo
  - Density Estimation

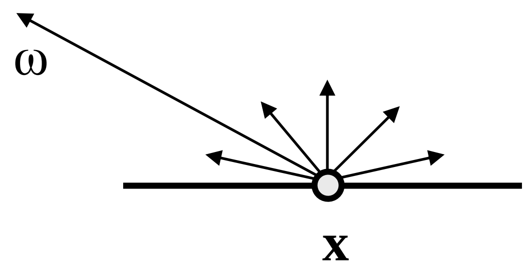
The Rendering Equation



$$L(\mathbf{x}, \omega_\mathrm{o}) = L_\mathrm{e}(\mathbf{x}, \omega_\mathrm{o}) + \int f_\mathrm{r}(\omega_\mathrm{i}, \omega_\mathrm{o}) L(\mathbf{y}, -\omega_\mathrm{i}) \cos\theta_\mathrm{i} \mathrm{d}\omega_\mathrm{i}$$

$L(\mathbf{x}, \omega)$ is the radiance from a point on a surface in a given direction ω

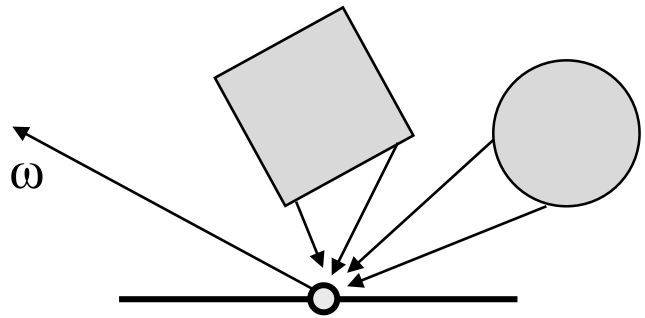
The Rendering Equation



$$L(\mathbf{x}, \omega_\mathrm{o}) = L_\mathrm{e}(\mathbf{x}, \omega_\mathrm{o}) + \int f_\mathrm{r}(\omega_\mathrm{i}, \omega_\mathrm{o}) L(\mathbf{y}, -\omega_\mathrm{i}) \cos\theta_\mathrm{i} \mathrm{d}\omega_\mathrm{i}$$

$L_\mathrm{e}(\mathbf{x},\omega)$ is the emitted radiance from a point: L_e is non-zero only if $\mathbf{x}$ is emissive, i.e., a light source.

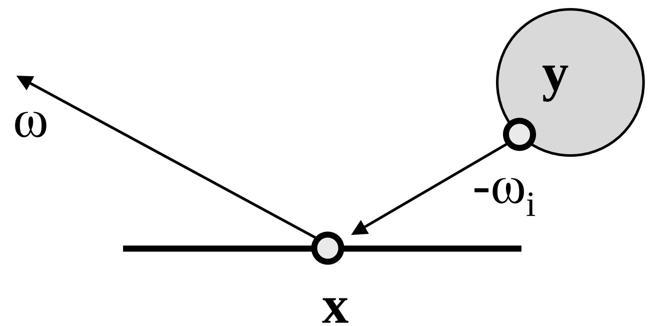
The Rendering Equation



$$\underbrace{L(\mathbf{x},\omega_\mathrm{o}) = L_\mathrm{e}(\mathbf{x},\omega_\mathrm{o}) + \int f_\mathrm{r}(\omega_\mathrm{i},\omega_\mathrm{o})L(\mathbf{y},-\omega_\mathrm{i})\cos\theta_\mathrm{i}\mathrm{d}\omega_\mathrm{i}}$$

Reflected light. Summed contribution from all other surfaces in the scene

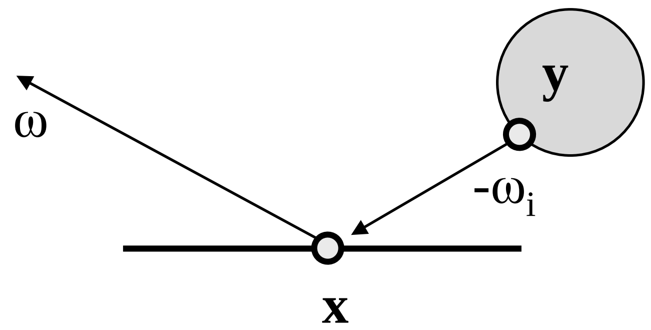
The Rendering Equation



$$L(\mathbf{x}, \omega_\mathrm{o}) = L_\mathrm{e}(\mathbf{x}, \omega_\mathrm{o}) + \int f_\mathrm{r}(\omega_\mathrm{i}, \omega_\mathrm{o}) L(\mathbf{y}, -\omega_\mathrm{i}) \cos\theta_\mathrm{i} \mathrm{d}\omega_\mathrm{i}$$

For each ω_i, compute $L(\mathbf{y}, -\omega_i)$: the radiance at point $\mathbf{y}$ in the direction $-\omega_i$ (i.e., radiance arriving at $\mathbf{x}$)

The Rendering Equation



$$L(\mathbf{x},\omega_\mathrm{o}) = L_\mathrm{e}(\mathbf{x},\omega_\mathrm{o}) + \int f_\mathrm{r}(\omega_\mathrm{i},\omega_\mathrm{o}) L(\mathbf{y},-\omega_\mathrm{i}) \cos\theta_\mathrm{i} \mathrm{d}\omega_\mathrm{i}$$

Scale the contribution by $f_r\,(\mathbf{x}, \omega_i, \omega)$, the reflectivity (BRDF) of the surface at $\mathbf{x}$,

Recap

- What are the players?
  1. Emission, i.e., light sources
  2. Spherical integration
  3. Visibility, i.e., finding $\mathbf{y}$
  4. Reflectivity, i.e., BRDF aka. material
- Will see all of them in detail next

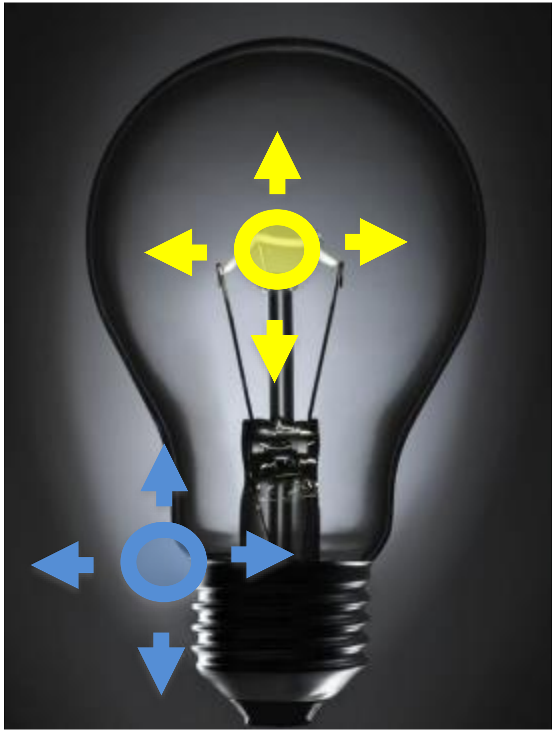
Light sources

Light sources

- Forget about points lights
- From now on, every location $\mathbf{x}$ can send light into every direction ω
- Emission function $L_\mathrm{e}(\mathbf{x}, \omega)$

Example light

Emission is zero, except at the center, where it is L_e for all directions

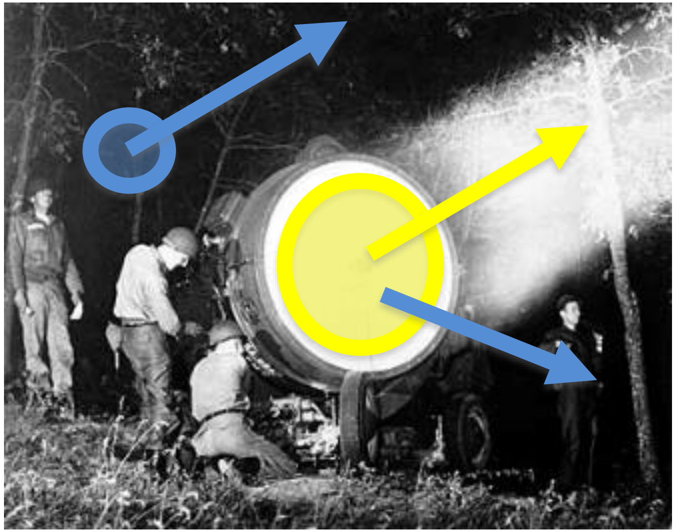


Example light

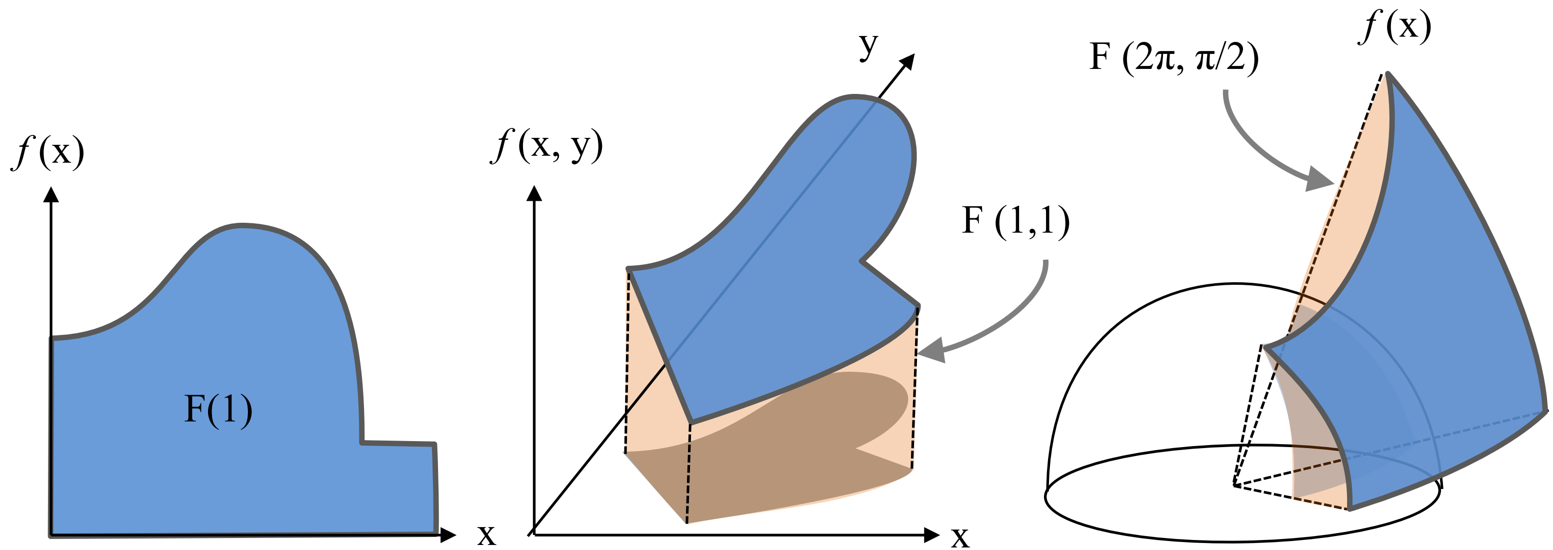
Emission is zero except at all points on the TV in all direction, where it is L_e

Example light

Emission is zero except at all points on the surface in direction of the search light, where it is L_{e}

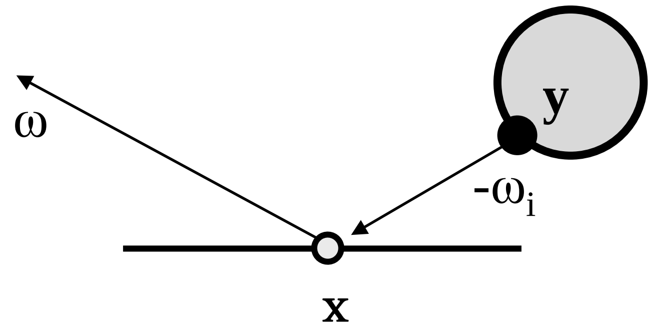


(Hemi)-spherical integration



1D Integration | 2D Integration | Spherical Integration

What is $\mathbf{y}$?

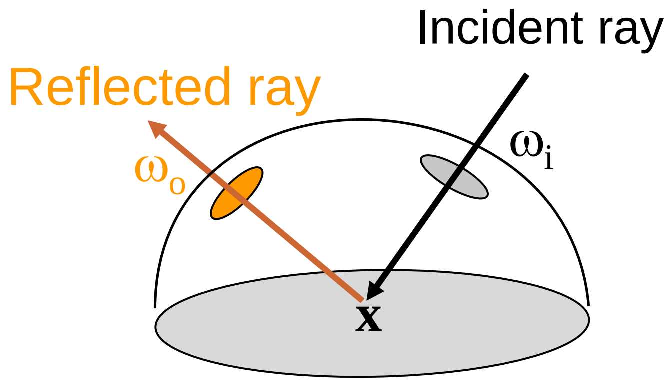


- $\mathbf{y}$ is the first point along ω
- Easy to say but hard to compute: Ray-tracing
- The source of infinite frequencies

BRDF

- Bi-directional Reflectance Distribution Function

- Radiance reflected at direction ω_o from irradiance at direction ω_i
- Symbol $f_r(\omega_o, \omega_i)$



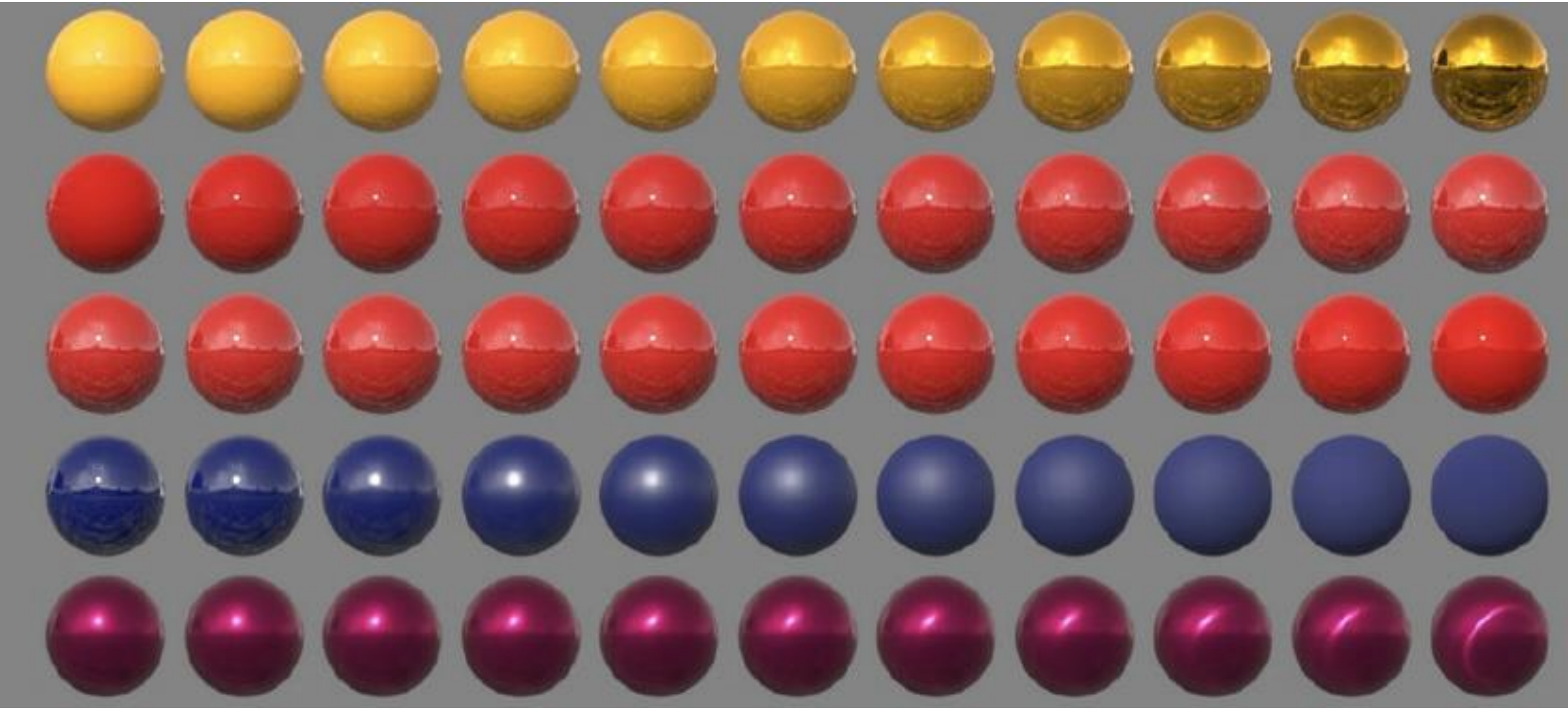
Properties of BRDFs

1. Non-negativity
$$f_{\mathrm{r}}(\omega_{\mathrm{i}}, \omega_{\mathrm{o}}) \geq 0$$

2. Energy conservation
$$\int f_{\mathrm{r}}(\omega_{\mathrm{i}}, \omega_{\mathrm{o}}) \mathrm{d}\omega_{\mathrm{o}}$$

3. Reciprocity
$$f_{\mathrm{r}}(\omega_{\mathrm{i}}, \omega_{\mathrm{o}}) = f_{\mathrm{r}}(\omega_{\mathrm{o}}, \omega_{\mathrm{i}})$$

BRDF examples



Different types of materials

- Matte materials
  - Flour
  - Rubber
  - Matte wall paint

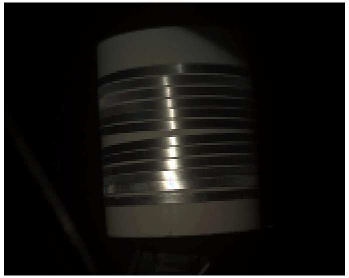
Different types of materials

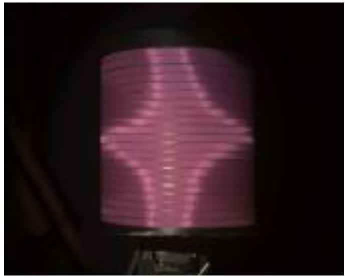
- Specular materials
  - Metals
  - Plastic
  - Glass

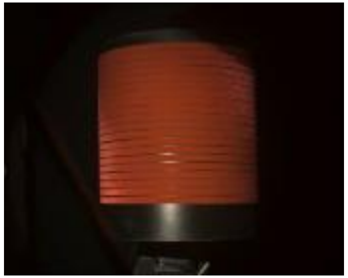



Different types of materials

- Anisotropic Materials
  - Velvet, Brushed metals







Different types of materials

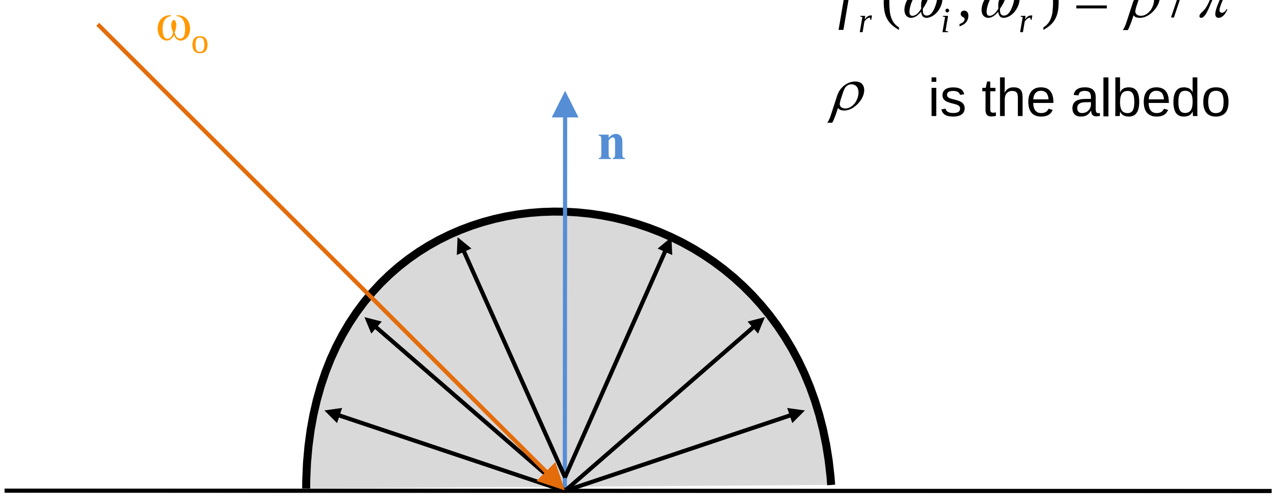
- Translucent materials
  - Skin
  - Wax
  - Marble
  - Paper

Describing the Reflectance

- The full BRDF is a 4D function
- Can sample and store
- Can find more compact BRDF **models**
  - Phong
  - Ward
  - Lafortune
  - etc.

Perfectly diffuse

Radiance reflected equally in every direction independently of the incoming direction

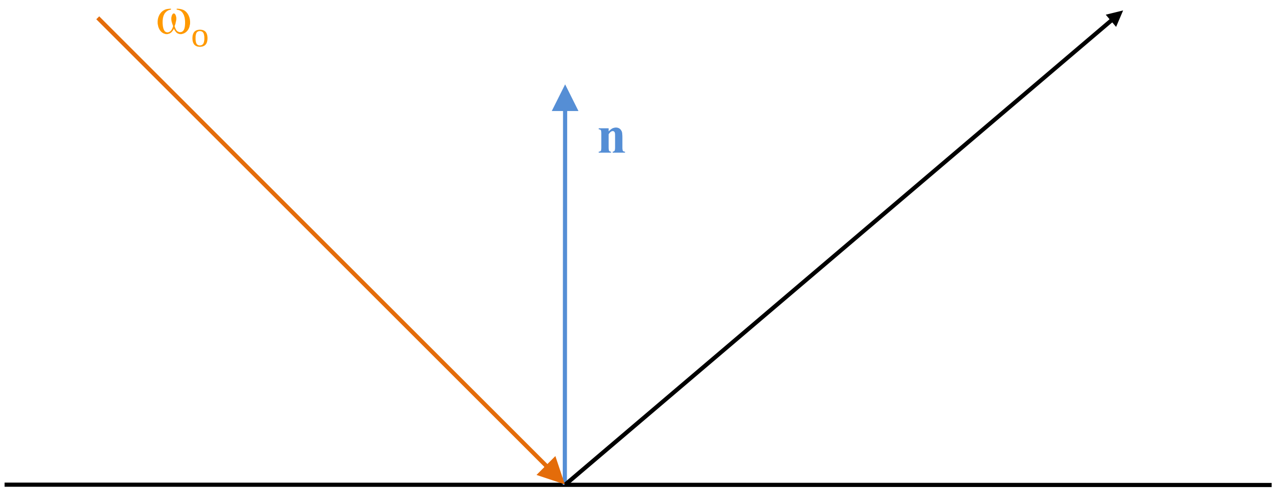


$$f_r(\omega_i, \omega_r) = \rho / \pi$$

ρ is the albedo

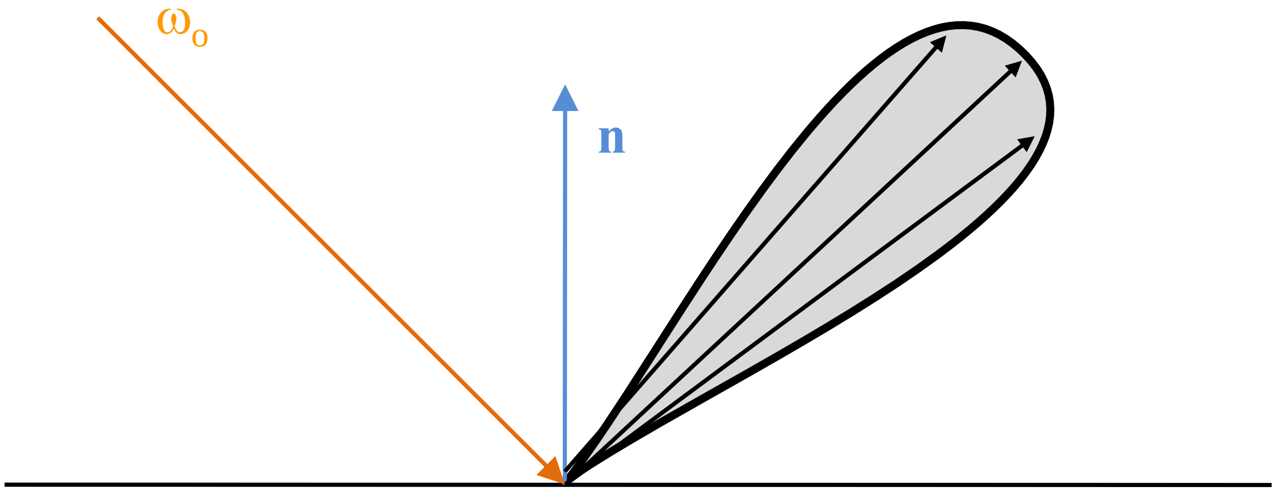
Perfectly specular

- Reflected dependently of the incident light
- What's its BRDF? Dirac.
- Not physically possible



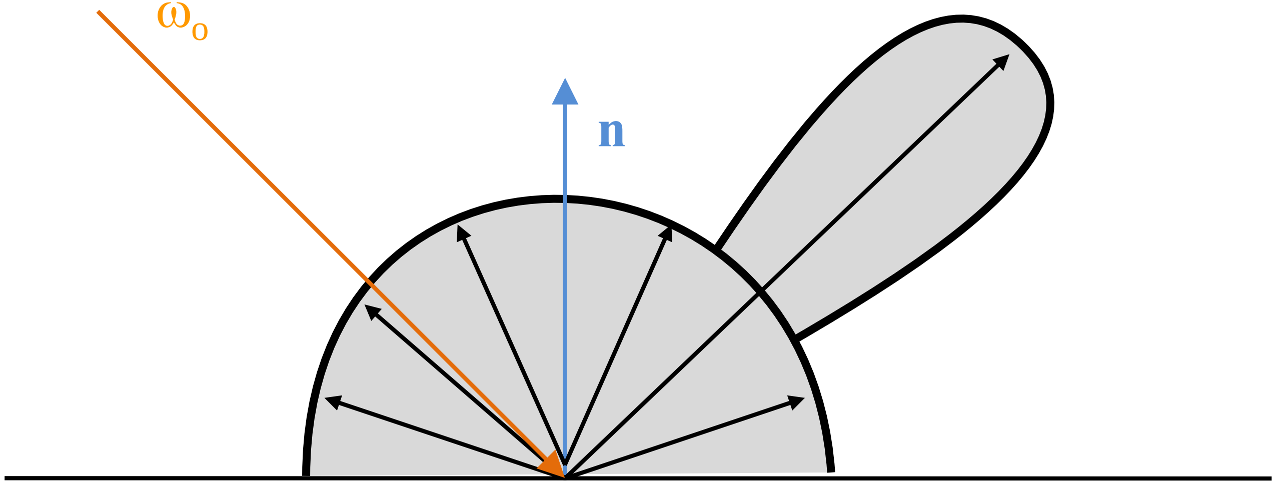
Glossy BRDF

Glossy is a blurry mirror



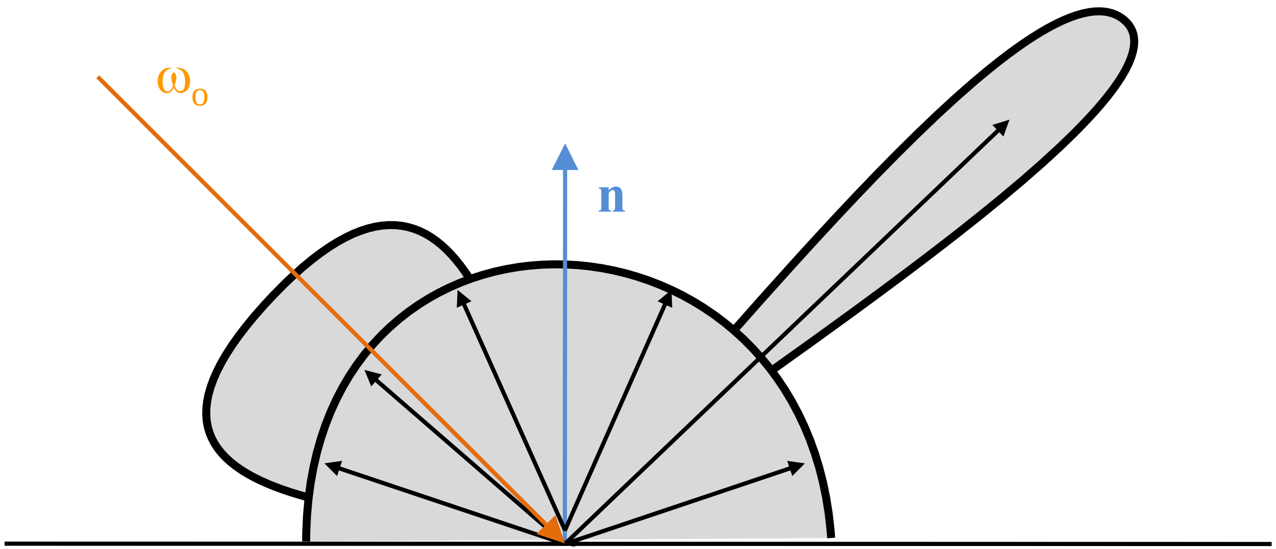
Diffuse and glossy BRDF

Diffuse and glossy



Multiple specular peaks

Multiple specular peaks, e.g. retroreflective

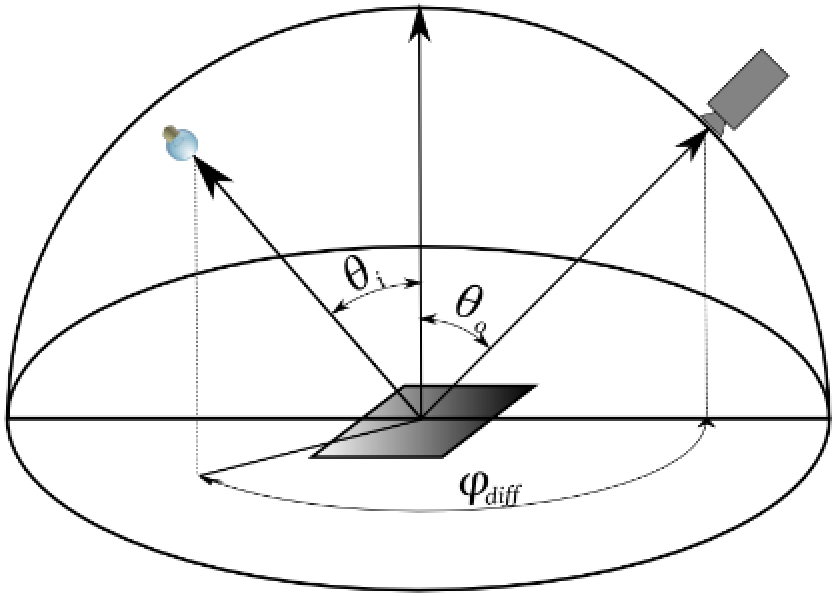


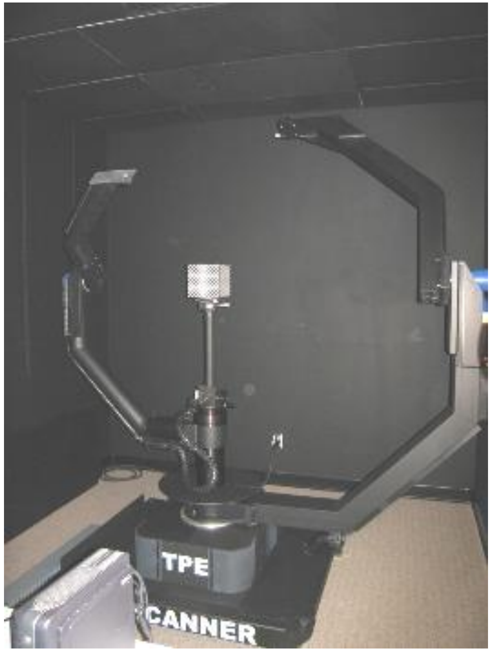
How to define a BRDF

- Three main options
  - Choose model and select parameters
  - Measure
  - Estimate from photographs (inverse illumination)

BRDF Measurment

- There are numerous devices for measuring reflectance
- Gonioreflectometer



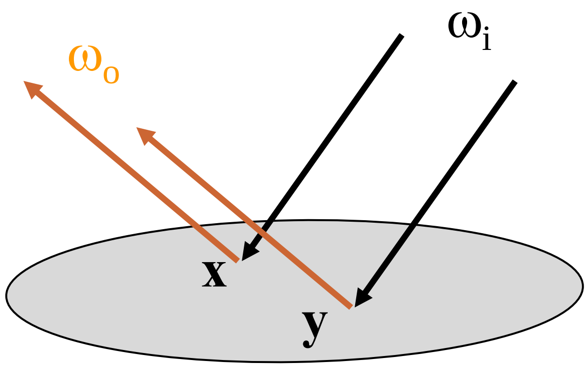


svBRDF

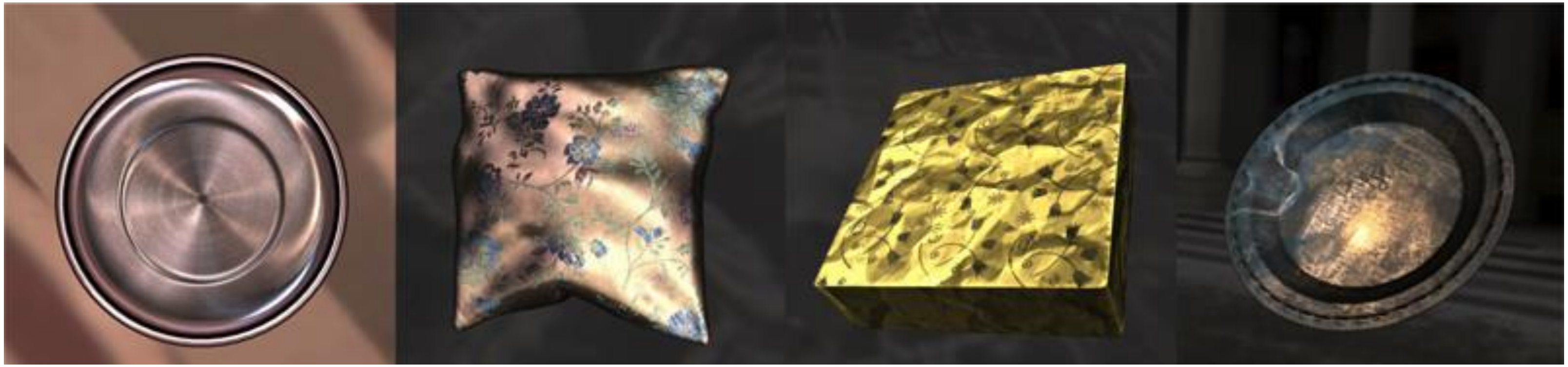
- Spatially-varying BRDF

$$f_r(\mathbf{x}, \omega_i, \omega_r)$$

- The reflection might change from location to location



svBRDF Examples



Recap

- Physically based lighting relies on solving the rendering equation
- Complete solutions to this equation are not tractable, so simple assumptions are made
- Need to be able to describe the reflectance properties of materials with a BRDF