Computer Graphics (COMP0027) 2022/23

Rasterization

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Ray-tracing: photo-realistic, but not usually real-time

Goal: Photo-realism at interactive rate for VR/games

Summary

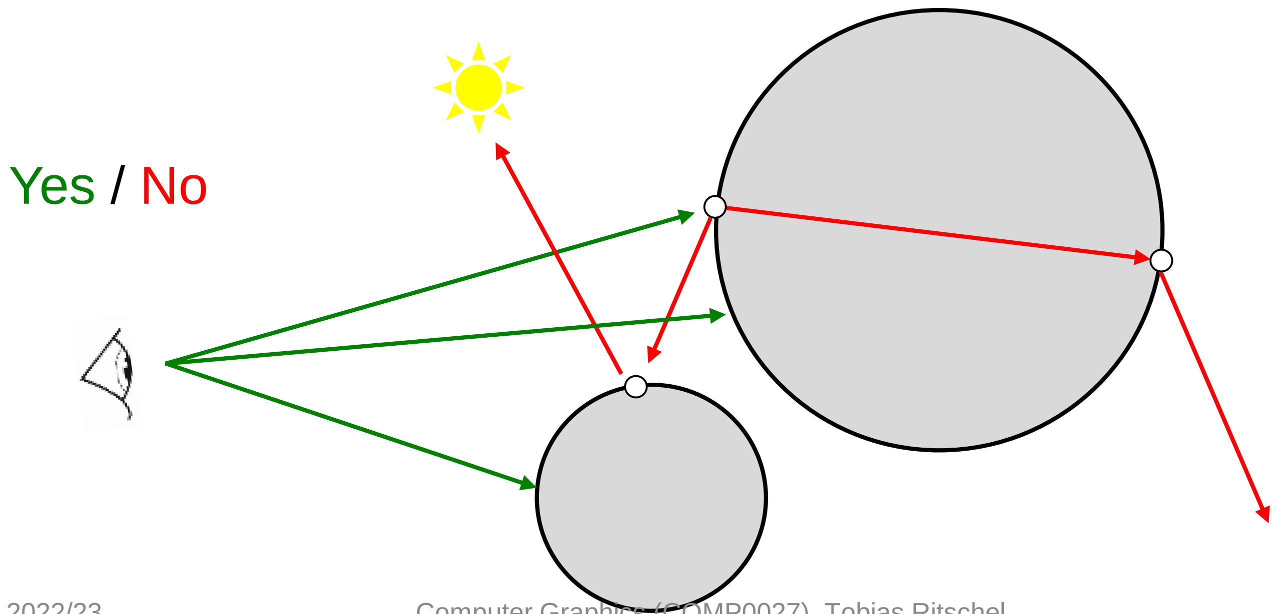
- Ray-tracing is elegant and general but slow
- Now we want to speed this up
- Main idea:
  - Don't **trace** rays
  - **Project** primitives
- Creates many new challenges

Ray-tracing cost

- The process of casting rays is very slow
- Example
  - 10,000 triangles,
  - 1000x1000 pixels
  - Result in 1,000,000 primary rays to cast
  - Each one testing for intersection with the 10,000 triangles (and then reflections, shadow rays, etc…)

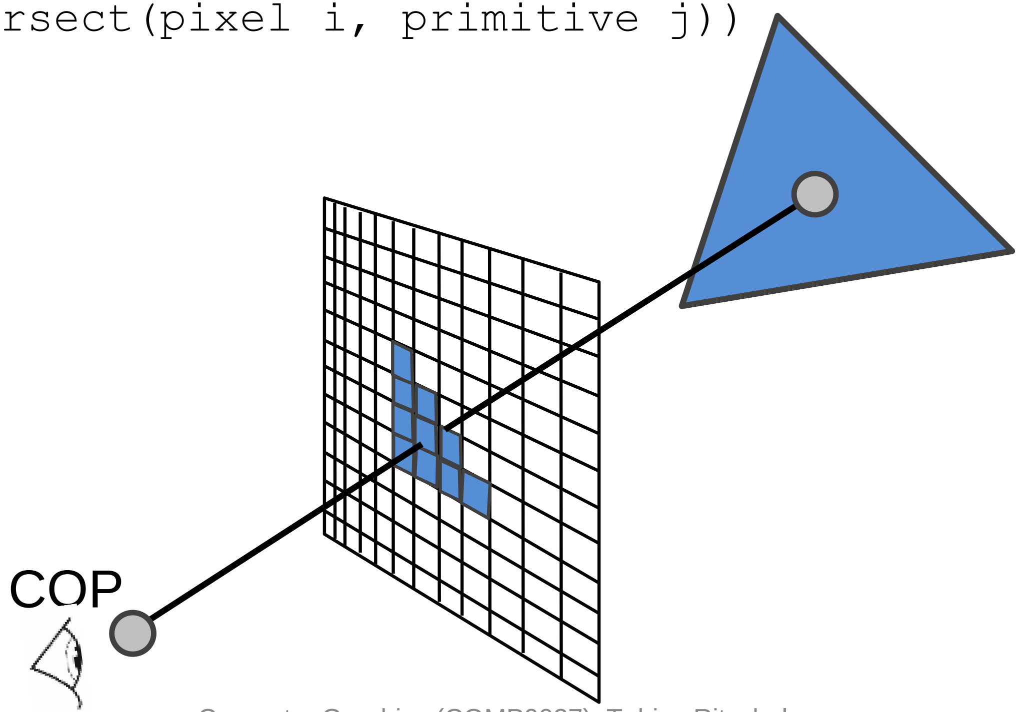
Simplifications from ray-tracing

- Rays only from one or a few specific points
- Only convex polygons
- No global illumination part (i.e. no recursion)



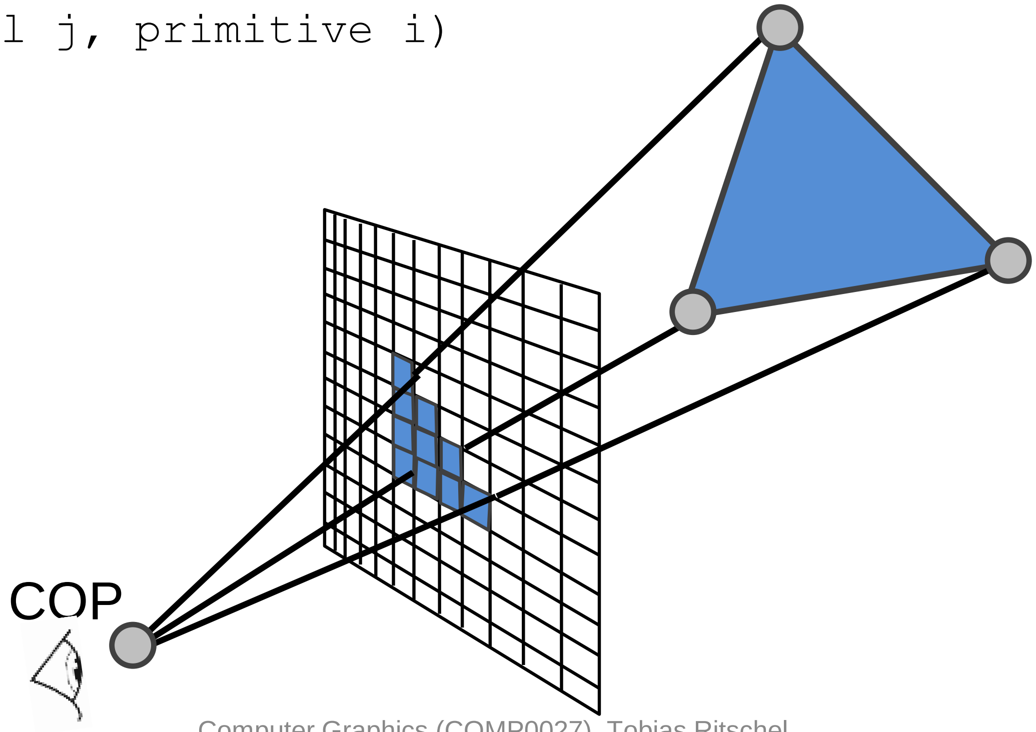
Ray-tracing

```
for all pixel rays i
 for all primitives j
  shade(intersect(pixel i, primitive j))
```

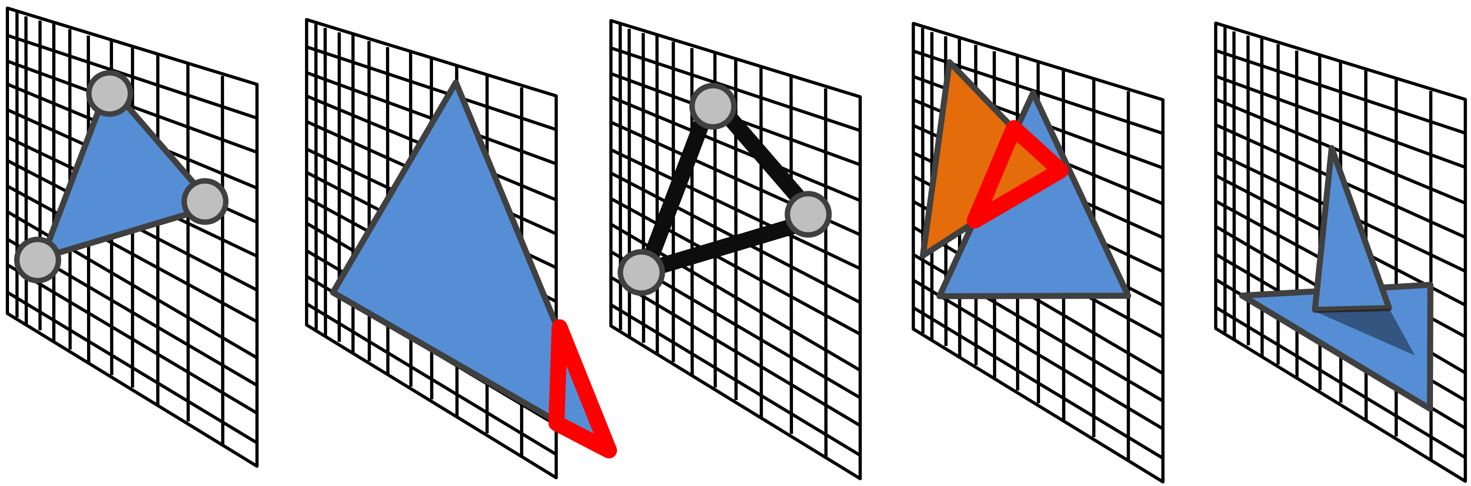


Rasterization

```
for all primitves i
 for all pixels j in projection of primitive i
  shade(pixel j, primitive i)
```

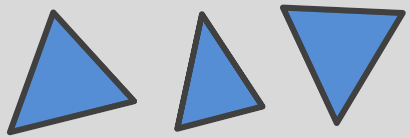


Challenges



Projection Clipping Rasterization Visibility Shading

Pipeline



Primitives

Projection

Clipping

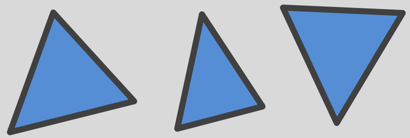
Culling

Rasterisation

z test

Shading

Pipeline



Primitives

Projection

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Culling

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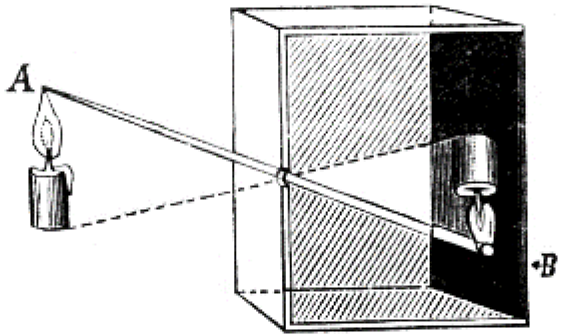
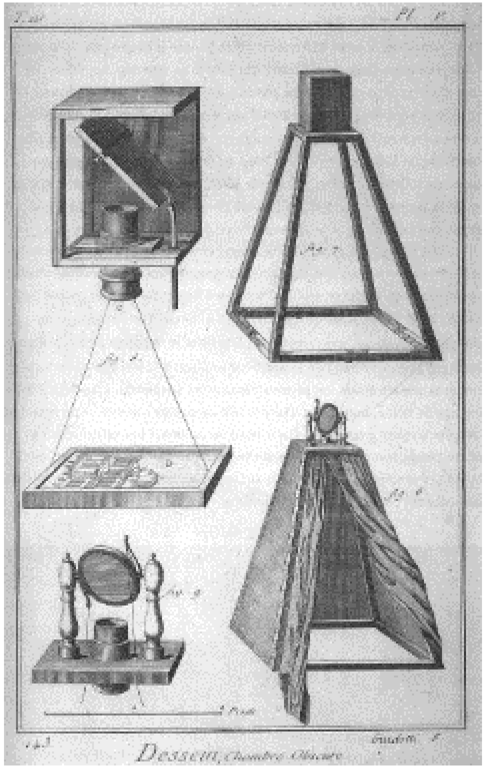
z test

Shading

History of projection

- Camera Obscura
  - Earliest form of projection
  - Mo-Ti (470-390 BC)
  - Aristotle (384-322 BC)





History of Projection

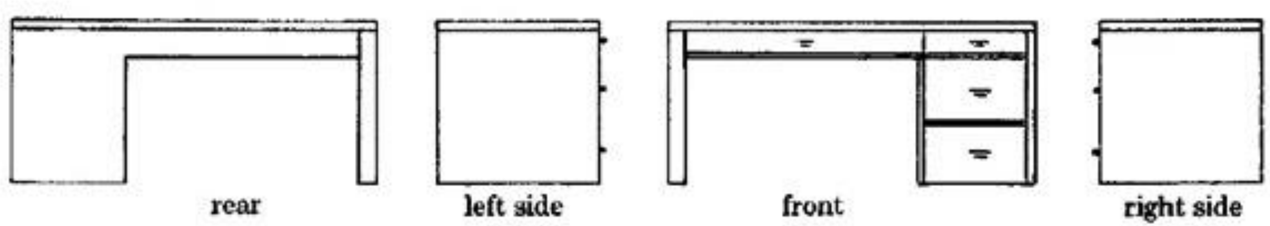
- Ancient Greeks knew the laws of perspective
- Renaissance: perspective is adopted by artists

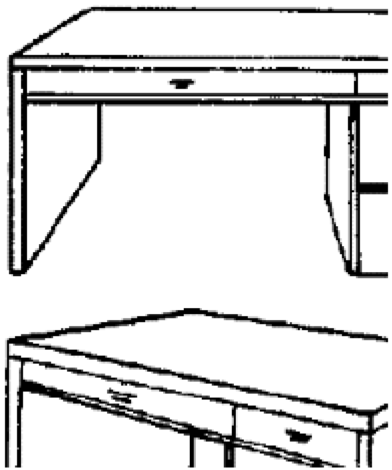
196. From an Apulian calyx-krater. *Würzburg*

History of Projection

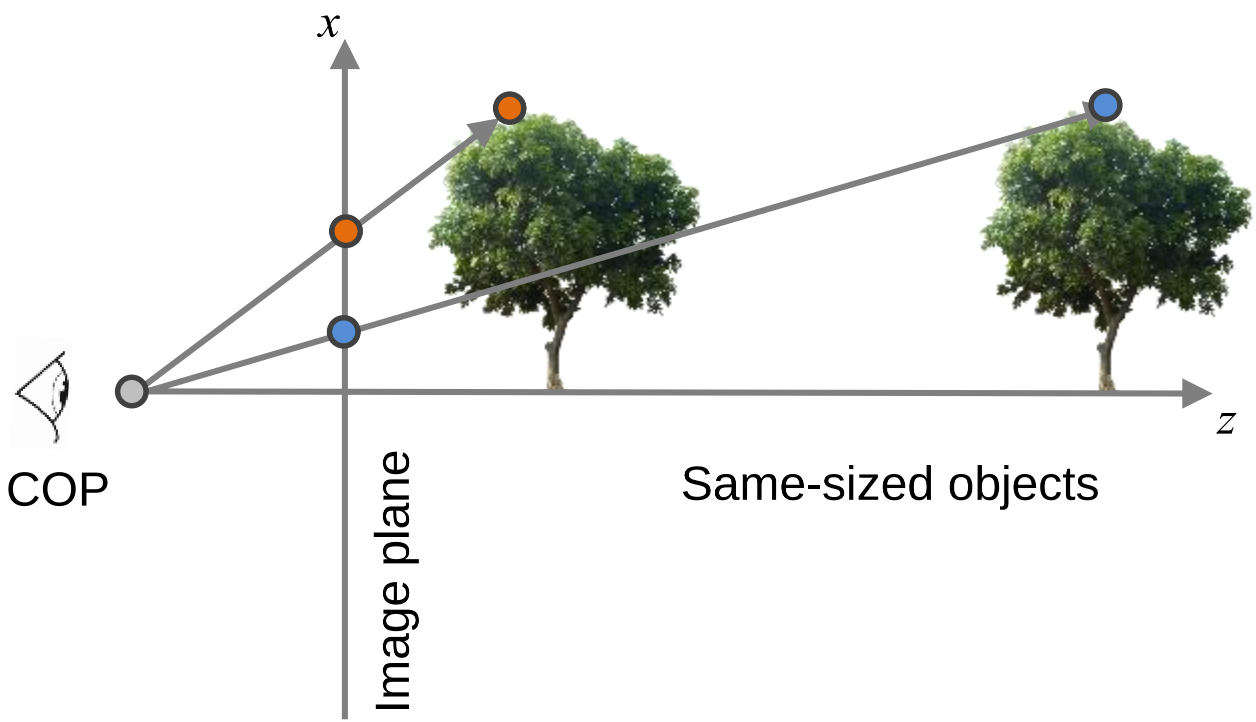
- High Renaissance, 15th century: perspective formalized

Orthographic vs. Perspective

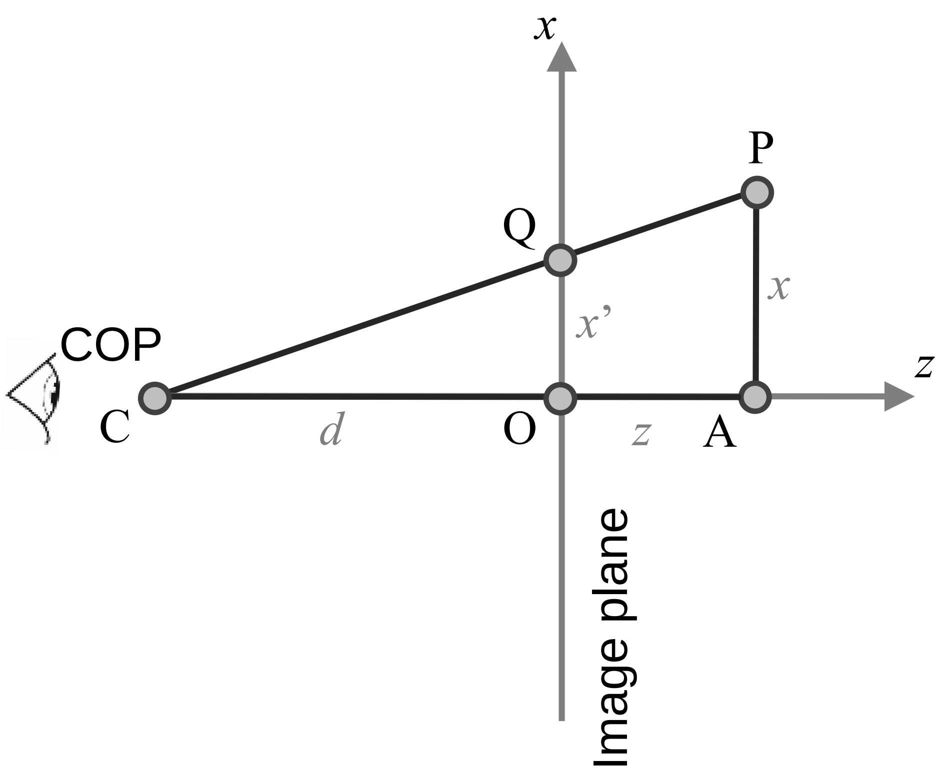




Perspective projection



Perspective projection



We know:
QO/CO = PA/CA

For x, we define
x' = QO
x = PA
d = CO
z = AO

$x'/d = x / (d + z)$ so
$x' = d\,x / (d + z)$

E.g., for $d = 1$:
$x' = x / (z + 1)$

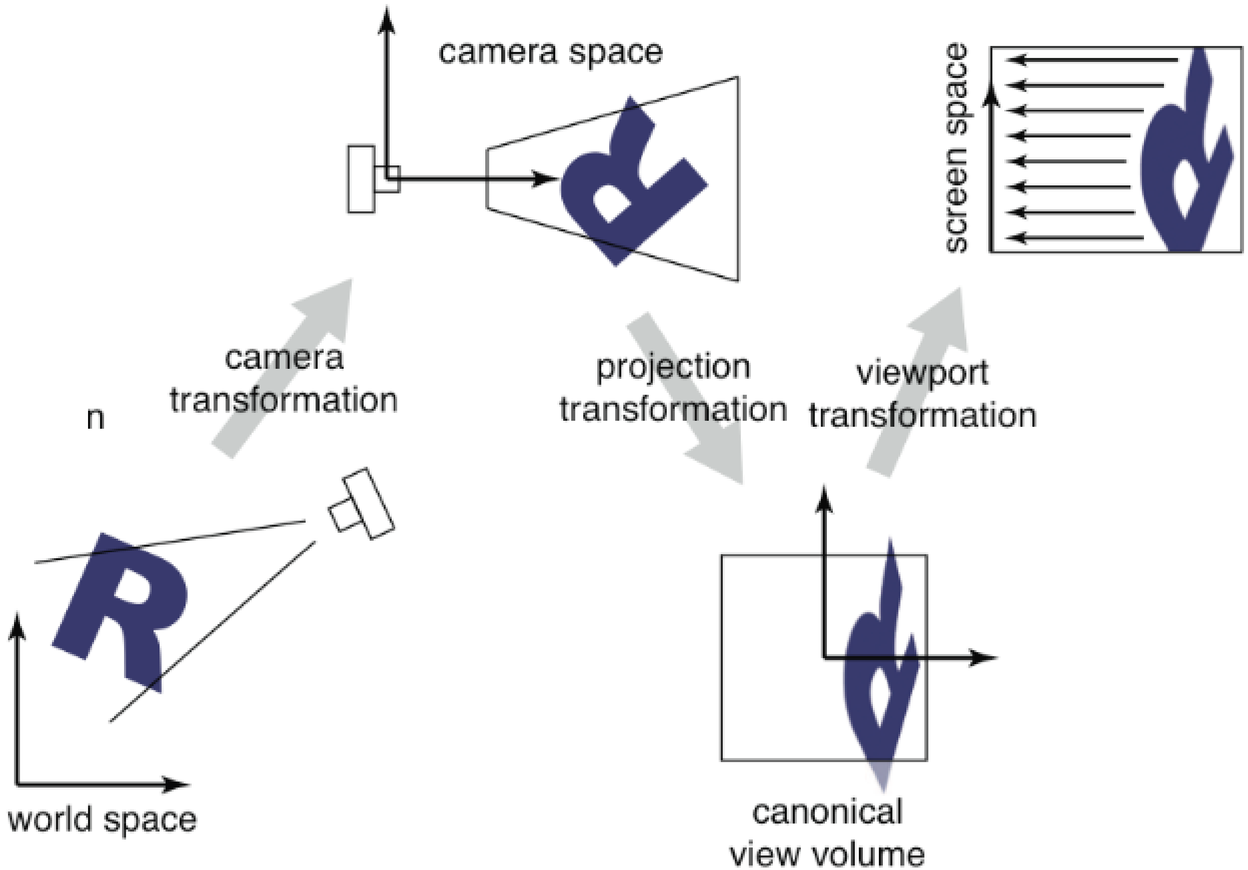
Perspective division

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

- Given a point $(x, y, z, 1)$
- Another good reason for 4x4 matrices
- Its transform is

$$\begin{pmatrix} x \\ y \\ z \\ z+1 \end{pmatrix} = \begin{pmatrix} \frac{x}{z+1} \\ \frac{y}{z+1} \\ \frac{z}{z+1} \\ 1 \end{pmatrix}$$

Projection Pipeline



Camera transform

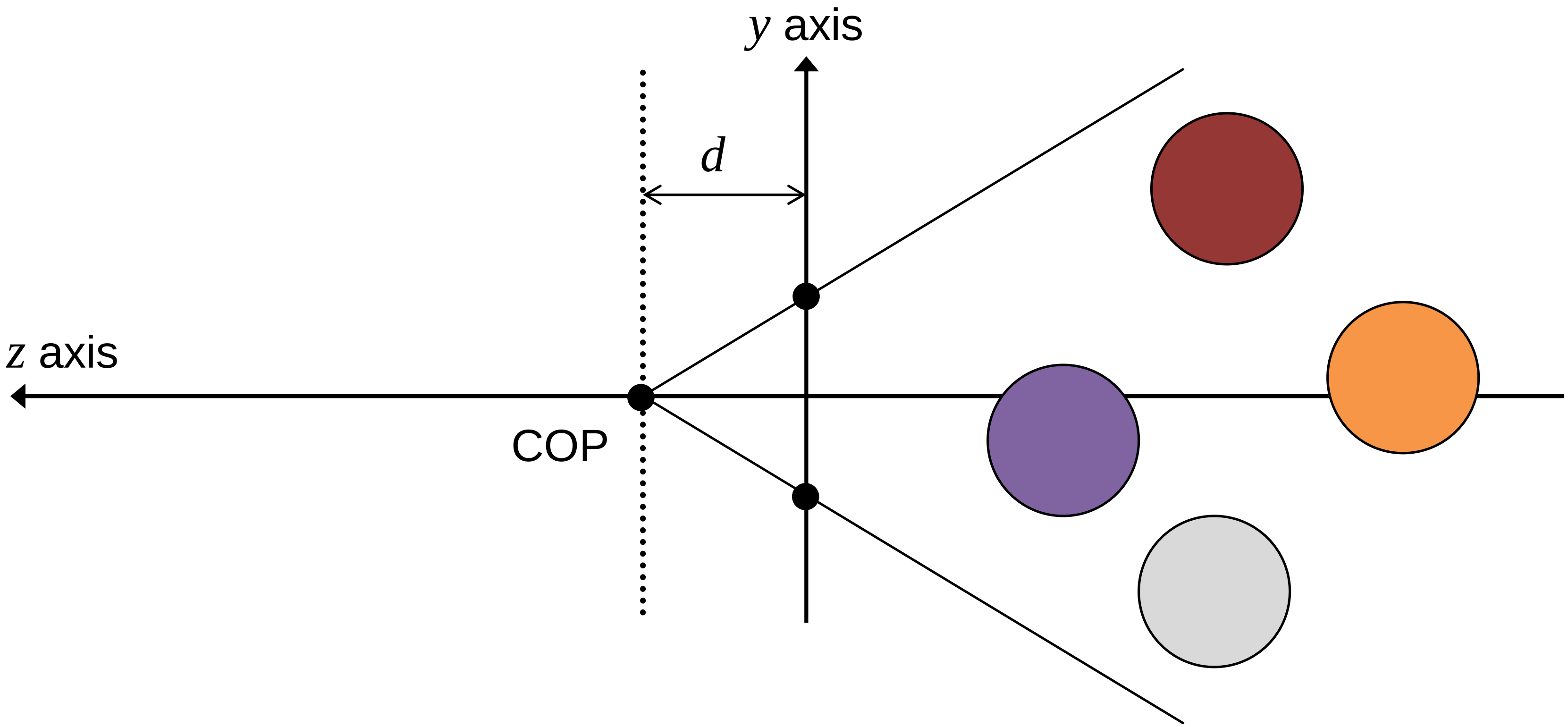
Two simplifications

1. The COP might not fall on the z axis
   - In our model it does
   - Required in professional photography
   - Required in virtual reality (Stereo, HMDs)
2. Might want to re-scale z to fit a range
   - Will get back to this adjustment later

Overview

- Simple camera is limiting: fixed in position and orientation
- We need a camera that can be moved and rotated
- We will define parameters for a camera in terms of where it “is”, the direction it points and the direction it considers to be “up” on the image

Simple Camera (Cross section)

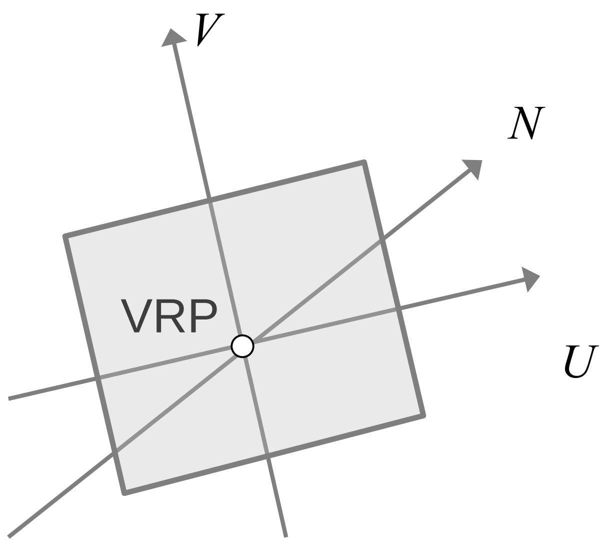


General Camera

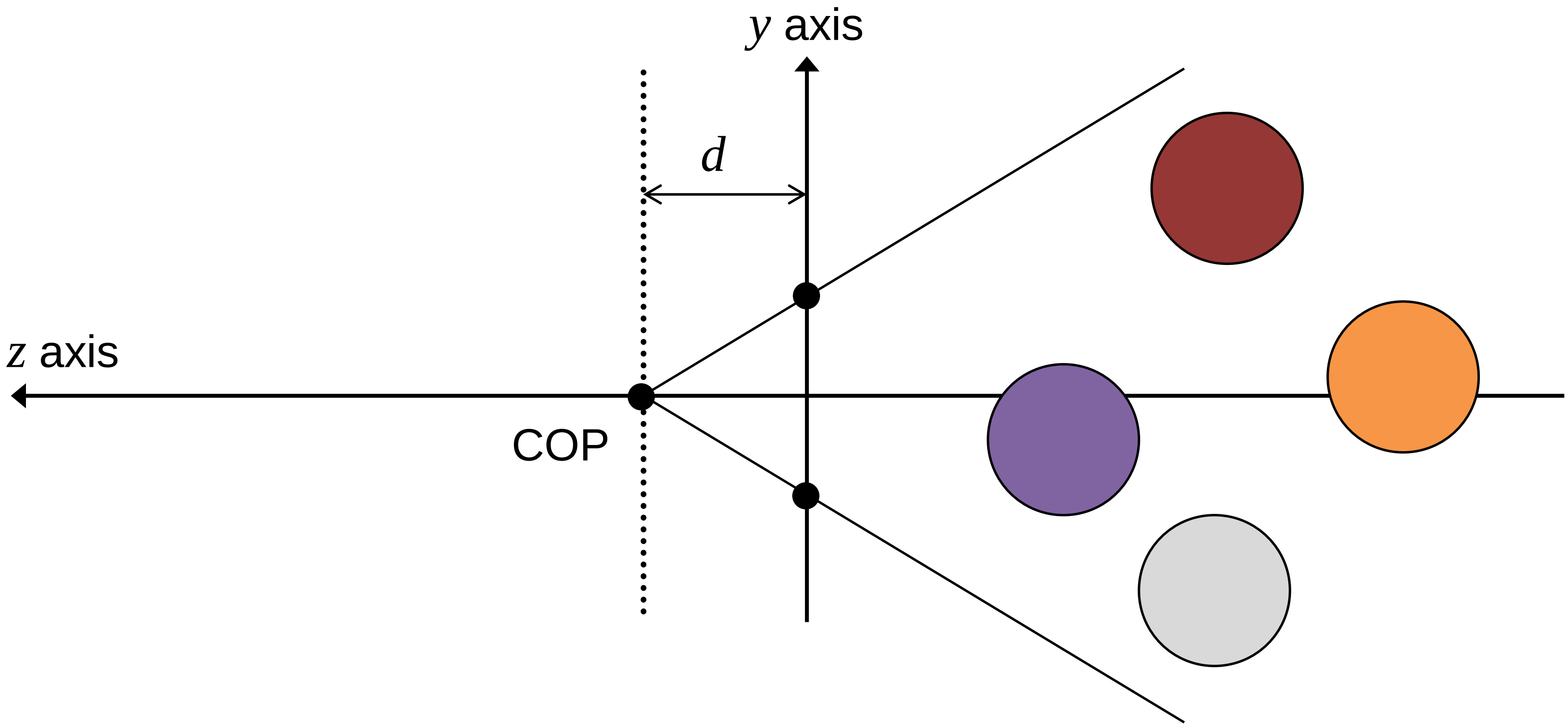
- View reference point (VRP)
  Where the camera is
- View Plane Normal (VPN)
  Where the camera points
- View up Vector
  Which way is up in the camera
- X (or U axis) forms LHS

UVN Coordinates

- View reference point (**VRP**)
  Origin of VC system
- View Plane Normal (**VPN**)
  Z *(or* N*-axis) of VC system*
- View up Vector (**VUV**)
  Determines Y (or V-axis) of VCS
- X (or U axis) forms LHS

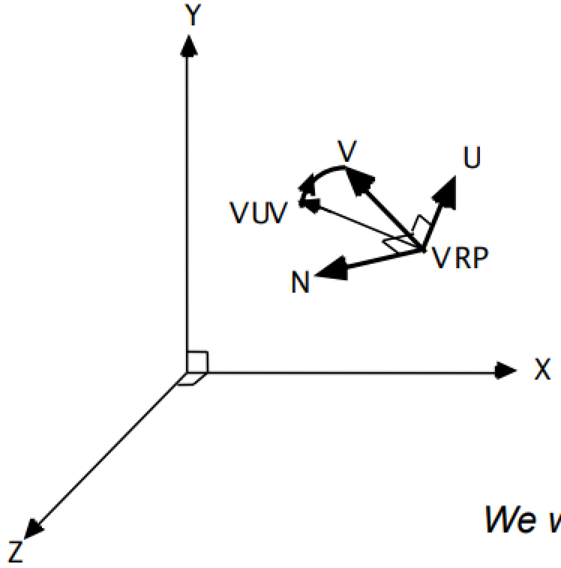


Simple Camera (Cross section)



World Coords and View Coords

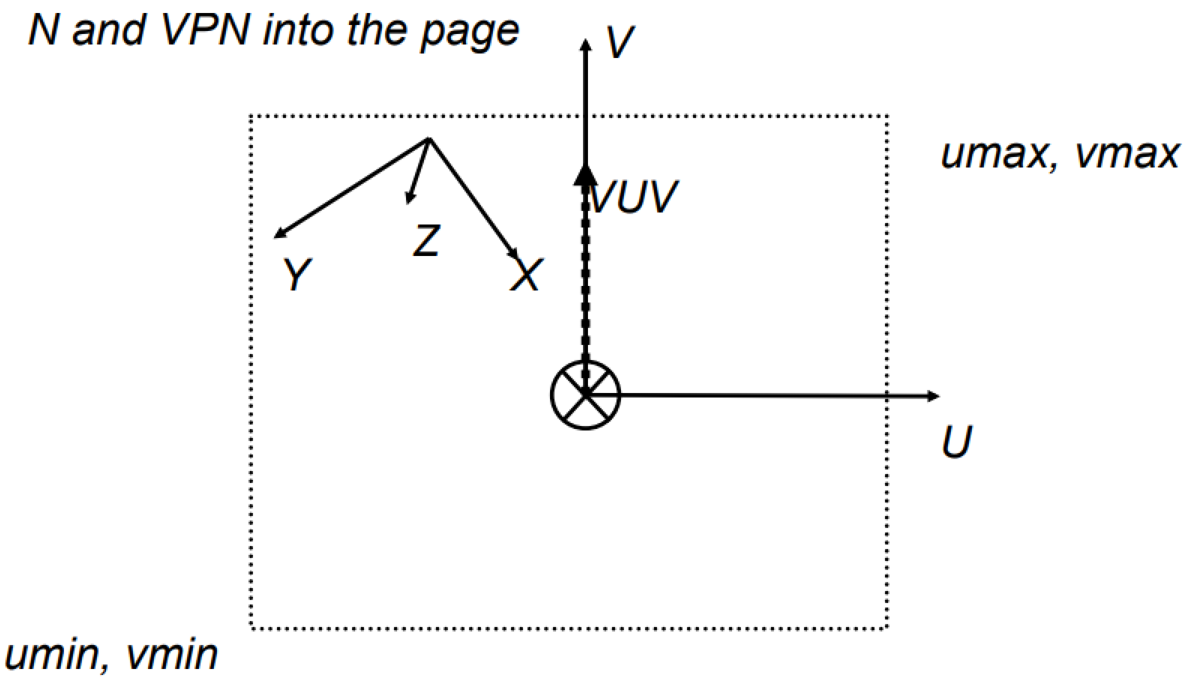
World Co-ordinates: XYZ
Viewing Co-ordinates: UVN



$$M = \begin{pmatrix} R_1 & R_2 & R_3 & 0 \\ R_4 & R_5 & R_6 & 0 \\ R_7 & R_8 & R_9 & 0 \\ T_1 & T_2 & T_3 & 1 \end{pmatrix}$$

We want to find a general transform of the above form that will map WC to VC

View from the Camera



Prior knowledge

We control the placement of the camera in the scene, so we know the following (w.r.t. world co-ordinate system):

- View Reference Point (VRP) - where the camera is
- View Plane Normal (VPN) - where the camera points
- View Up Vector (VUV) - which way is up to the camera

Finding the basis vectors

- Step 1 - find n

$$n = \frac{VPN}{|VPN|}$$

- Step 2 - find u

$$u = \frac{n \times VUV}{|n \times VUV|}$$

- Step 3 - find v

$$v = u \times n$$

Finding the Mapping (Rotation)

- u,v,n must rotate under R to i,j,k of viewing space

$$\begin{pmatrix} u \\ v \\ n \end{pmatrix} \begin{pmatrix} & & \\ & R & \\ & & \end{pmatrix} = \begin{pmatrix} & & \\ & I & \\ & & \end{pmatrix}$$

- In other words:

$uR = \mathrm{i} = [1\ \ 0\ \ 0]$

$vR = \mathrm{j} = [0\ \ 1\ \ 0]$

$nR = \mathrm{k} = [0\ \ 0\ \ 1]$

Finding the Mapping (Rotation)

u, v and n are *orthonormal* vectors (i.e. they are unit vectors, and are all orthogonal to each other)

$\Rightarrow$their dot products u.v, v.n, n.u are all zero

so:

$$u.v = u_1v_1 + u_2v_2 + u_3v_3 = 0$$

$$v.n = v_1n_1 + v_2n_2 + v_3n_3 = 0$$

$$n.u = n_1u_1 + n_2u_2 + n_3u_3 = 0$$

Finding the Mapping (Rotation)

- Also **u**, **v** and **n** are unit vectors so their magnitude is 1 thus:

  $u_1^2 + u_2^2 + u_3^2 = 1$

  $v_1^2 + v_2^2 + v_3^2 = 1$

  $n_1^2 + n_2^2 + n_3^2 = 1$

- We can exploit all this by setting $R = (u^T, v^T, n^T)$

$$R = \begin{pmatrix} u_1 & v_1 & n_1 \\ u_2 & v_2 & n_2 \\ u_3 & v_3 & n_3 \end{pmatrix}$$

- So $R^{-1} = R^T$

Finding the Mapping (Translation)

- For our equation, we call the view reference point *q*
- In uvn system *q* is (0, 0, 0, 1)

=> We want our mapping such that:

$$(q_1, \quad q_2, \quad q_3, \quad 1) \begin{vmatrix} u_1 & v_1 & n_1 & 0 \\ u_2 & v_2 & n_2 & 0 \\ u_3 & v_3 & n_3 & 0 \\ t_1 & t_2 & t_3 & 1 \end{vmatrix} = (\ 0, \quad 0, \quad 0, \quad 1)$$

Finding the Mapping (Translation)

So,

$$\sum_{i=1}^{3} q_i u_i + t_1 = 0 \qquad \sum_{i=1}^{3} q_i v_i + t_2 = 0 \qquad \sum_{i=1}^{3} q_i n_i + t_3 = 0$$

$$\Rightarrow \quad (t_1 \quad t_2 \quad t_3) = -\left(\sum_{i=1}^{3} q_i u_i \quad \sum_{i=1}^{3} q_i v_i \quad \sum_{i=1}^{3} q_i n_i \right)$$

Complete mapping

- Complete matrix

$$M = \begin{pmatrix} u_1 & v_1 & n_1 & 0 \\ u_2 & v_2 & n_2 & 0 \\ u_3 & v_3 & n_3 & 0 \\ -\sum_{i=1}^{3} q_i u_i & -\sum_{i=1}^{3} q_i v_i & -\sum_{i=1}^{3} q_i n_i & 1 \end{pmatrix}$$

What this is not

- Just put three basis into upper 3-times-3 block
- Just put translation into some row or column

Using this for ray-casting

- Use similar camera config (COP is usually, but not always on $-\boldsymbol{n}$)
- To trace an objects one must either
  - Transform object into VC
  - Transform rays into WC

Trade-off

- If more rays than spheres do the former
  - Transform spheres into VC
- For more complex scenes e.g. with polygons
  - Transform rays into WC

Alternative Forms of the Camera

- Simple “Look At”
  - Give a VRP and a target (TP)
  - VPN = TP-VRP
  - VUV = (0 1 0) (i.e. “up” in WC)
- Field of View
  - Give horizontal and vertical FOV or one or the other and an aspect ratio
  - Calculate viewport and proceed as before

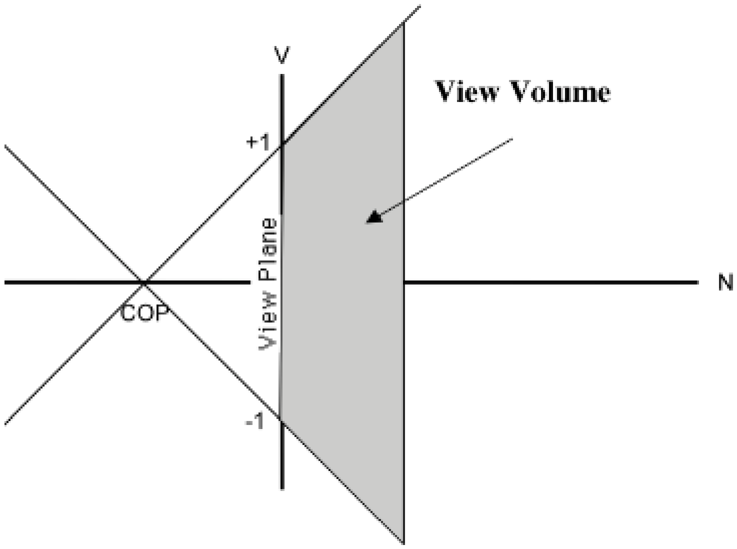
General Camera Recap

- We created a more general camera which we can use to create views of our scenes from arbitrary positions
- Formulation of mapping from WC to VC (and back)

Projection transform

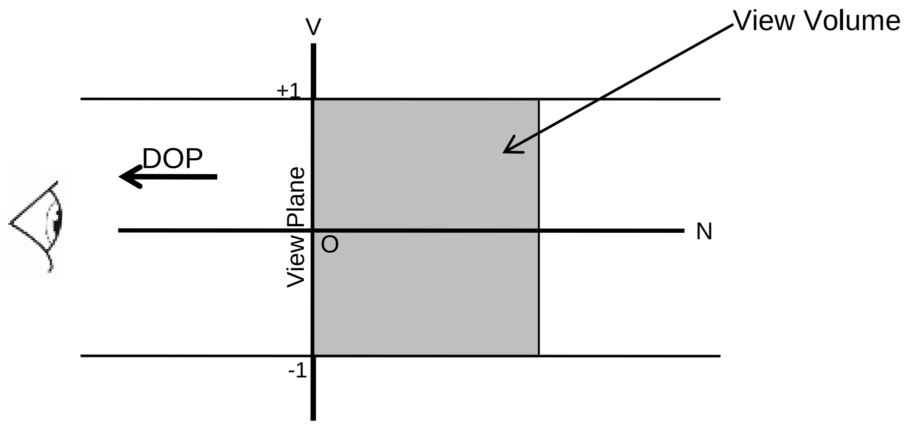
Canonical Frames

- We use **canonical frames** as intermediate stages from which we know how to proceed
- Canonical Frame for Perspective Projection:
  - COP at (0, 0, -1)
  - View plane coincident with UV plane
  - Viewplane window bounded by –1 to +1



Canonical Frame for Parallel Projection

- Orthographic parallel projection
- Direction of projection (DOP) is $(0,0,-1)$
- View volume bounded by -1 and $+1$ on U and V
- And by 0 and 1 on the N axis
- $p' = (x, y, 0)$

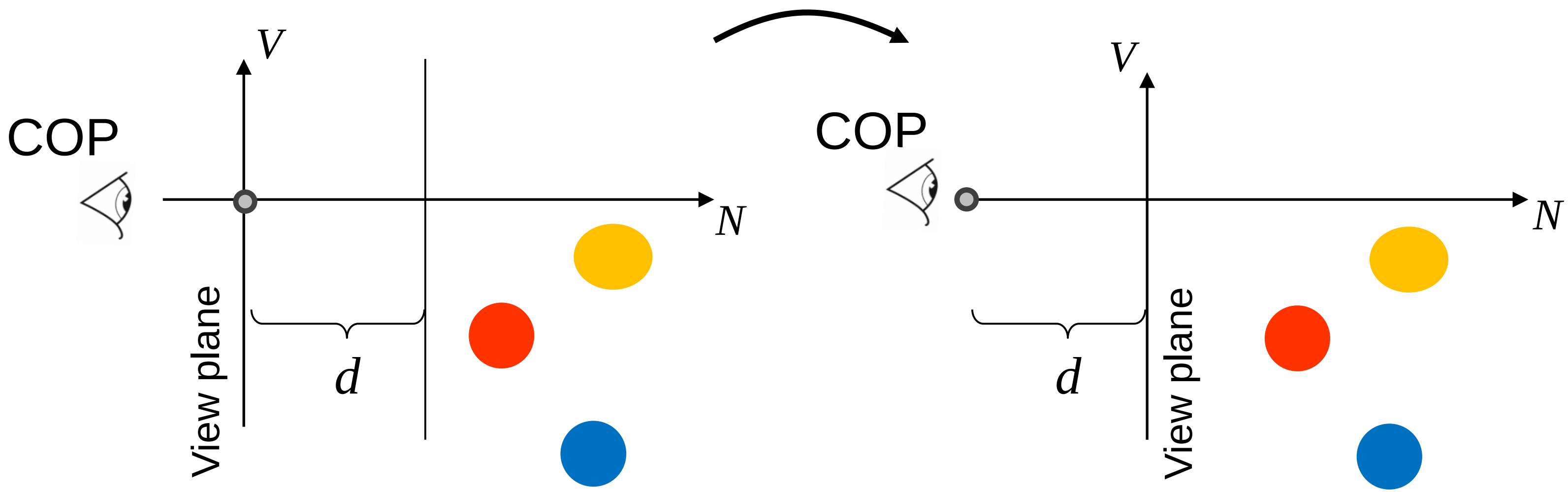


General Persp. to Canonical Persp.

- We will apply **four** transformation matrices
- Each “corrects” one aspect of the projection
- Finally, we multiply them all together

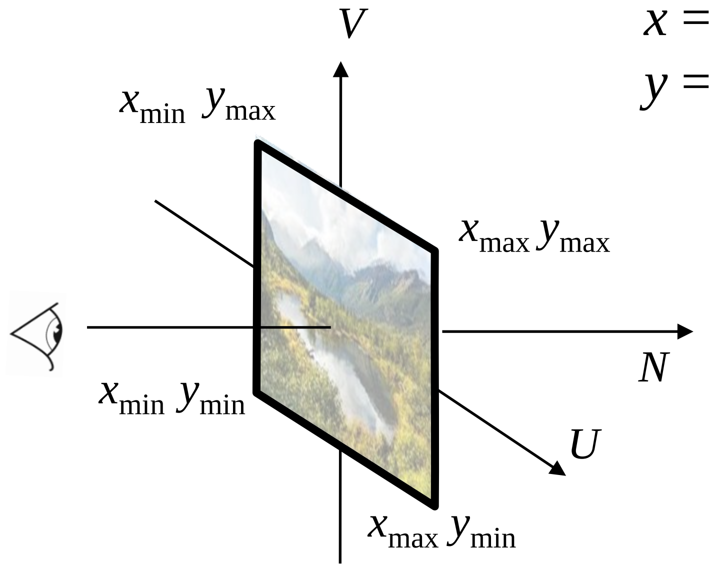
Step 1: Move to UV plane

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Recall the window

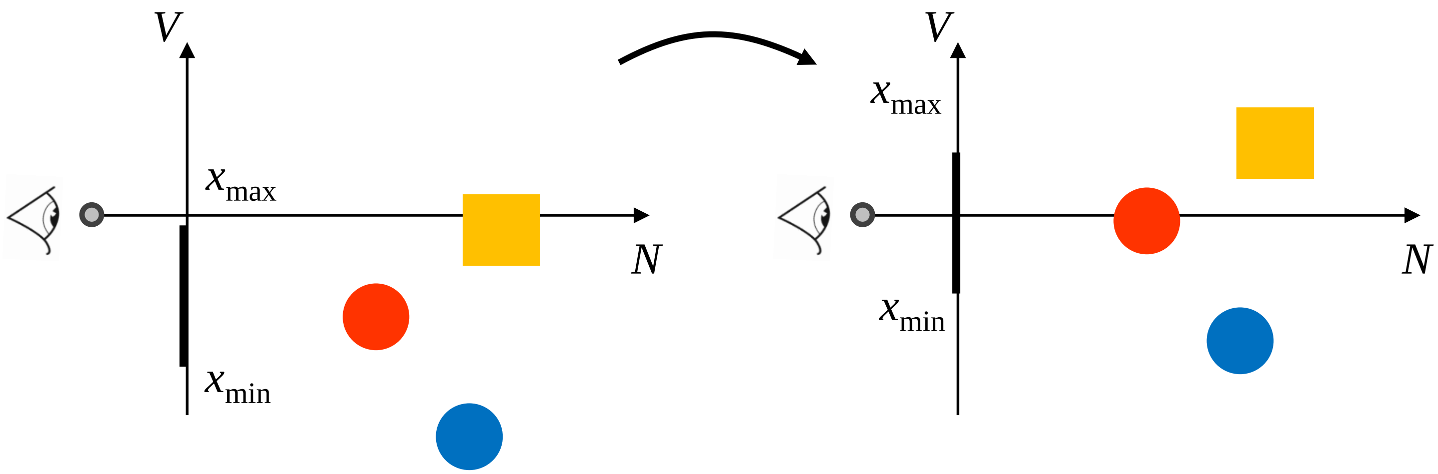
$$w = x_{\text{max}} - x_{\text{min}}$$
$$h = y_{\text{max}} - y_{\text{min}}$$
$$x = x_{\text{max}} + x_{\text{min}}$$
$$y = y_{\text{max}} + y_{\text{min}}$$



Step 2: Regular pyramid

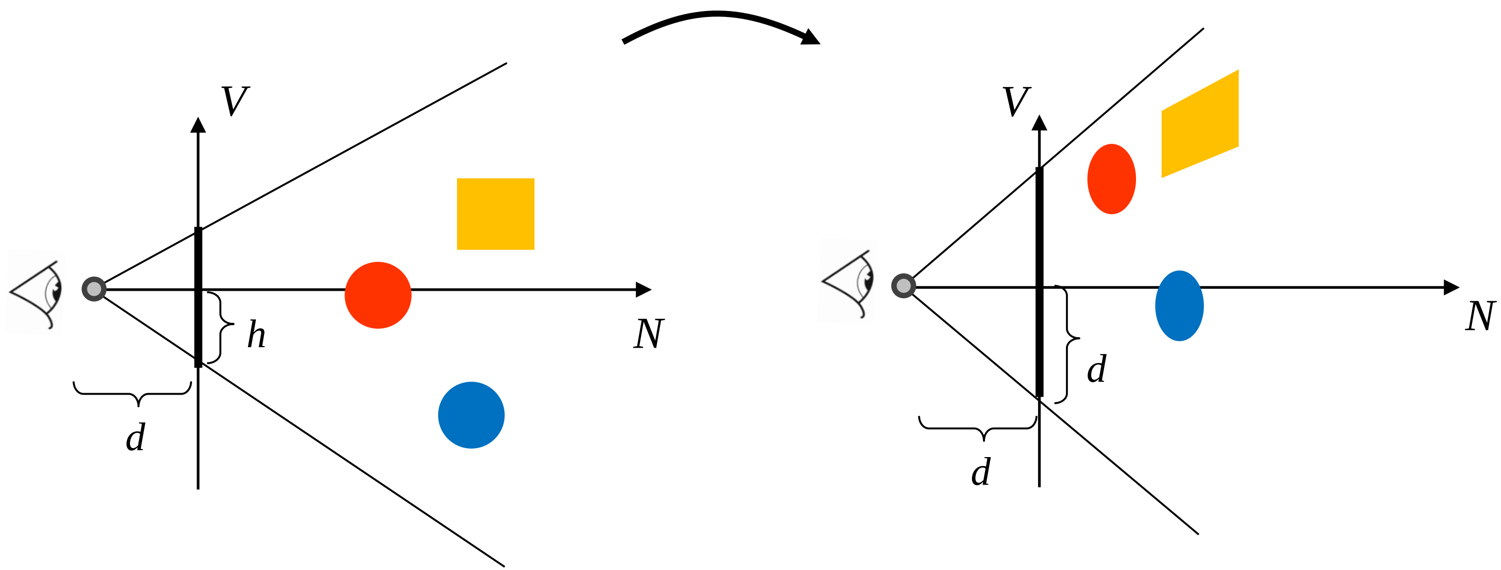
$$\begin{pmatrix} 1 & 0 & 0 & -x \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$w = x_{\max} - x_{\min}$$
$$h = y_{\max} - y_{\min}$$
$$x = x_{\max} + x_{\min}$$
$$y = y_{\max} + y_{\min}$$



Step 3: Regular pyramid

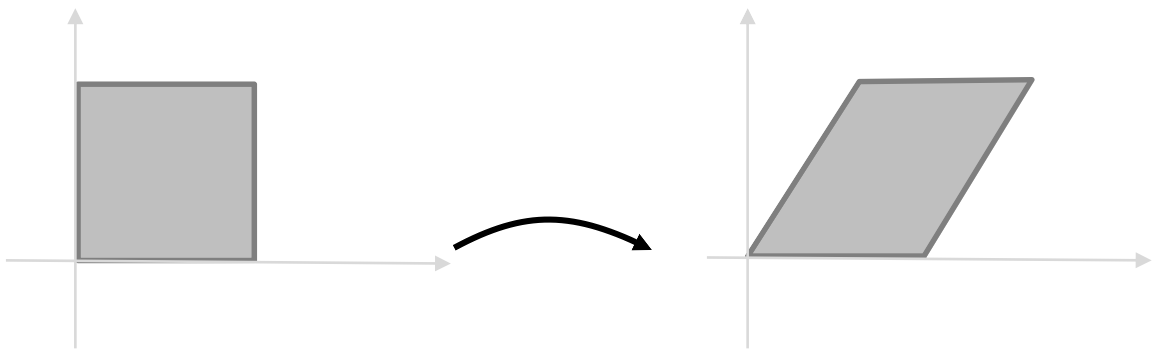
This is not uniform scale: z did not change!



Shear

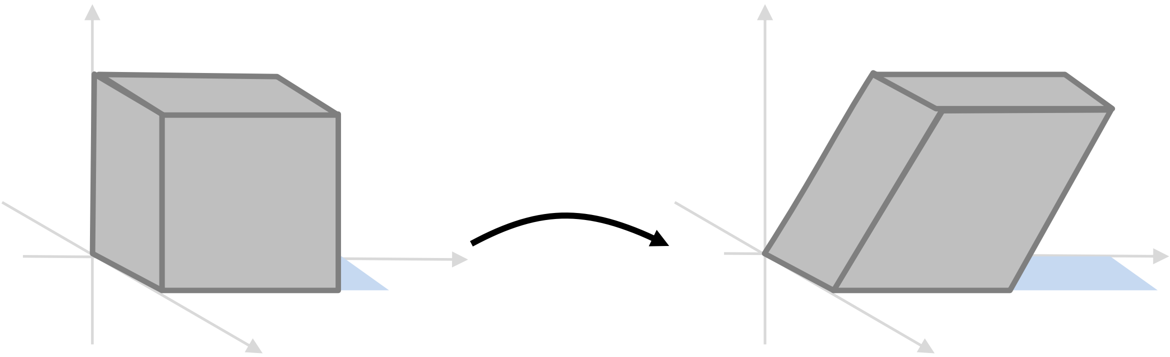
- This needs to be a **shear**
- If you want a regular pyramid to become a box ..
- A box has to become some other regular pyramid
- What is shear?

Interlude: Shear 2D



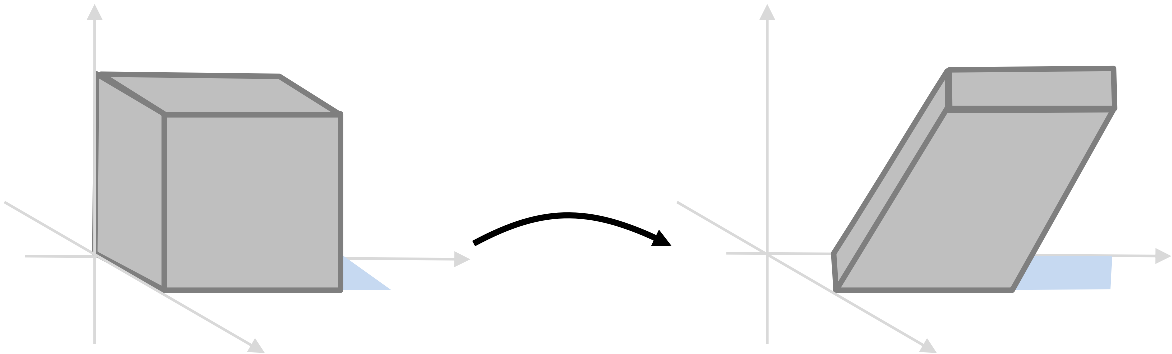
$$\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Interlude: Shear 3D



$$\begin{pmatrix} 1 & a & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Interlude: Shear 3D

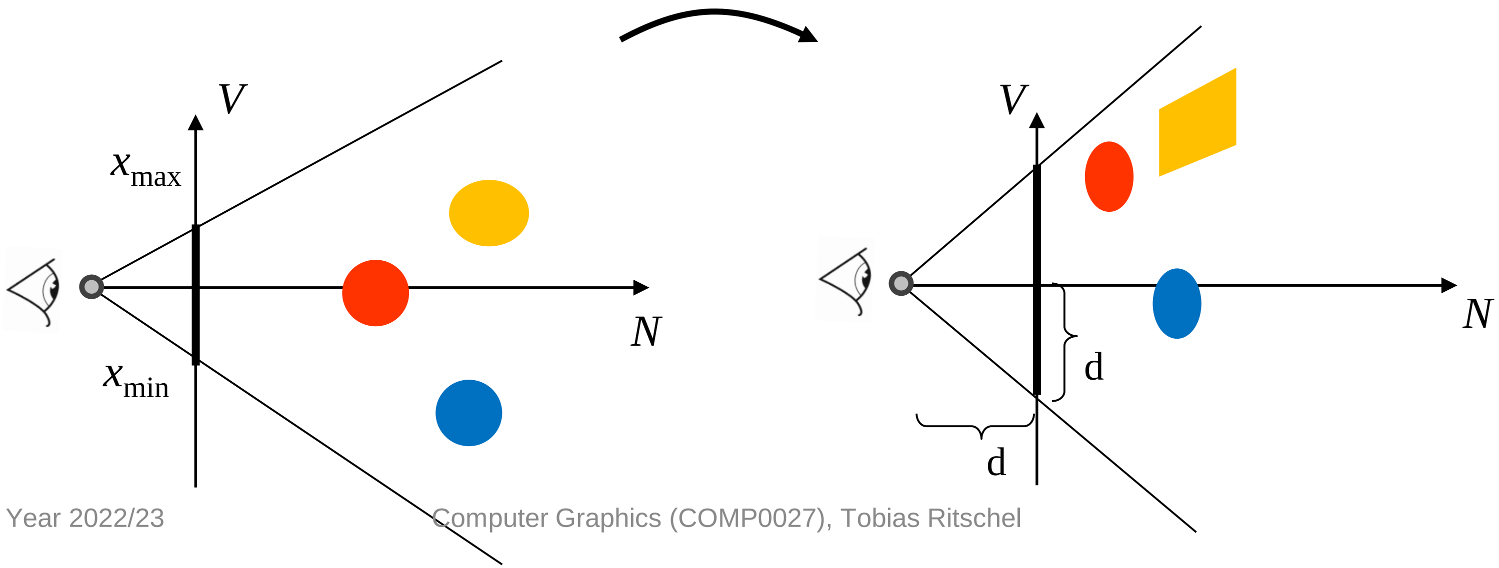


$$\begin{pmatrix} 1 & a & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Step 3: Regular pyramid

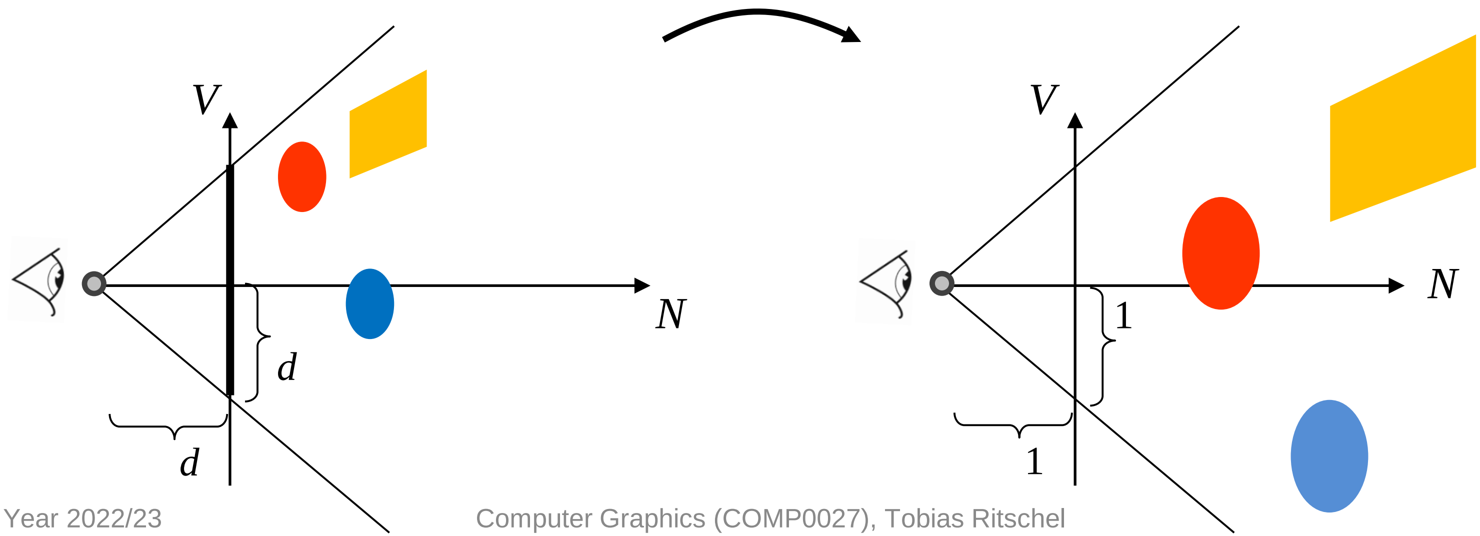
$$\begin{pmatrix} d/w & 0/w & -x/w & 0/w \\ 0/h & d/h & -y/h & 0/h \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$w = x_{\max} - x_{\min}$$
$$h = y_{\max} - y_{\min}$$
$$x = x_{\max} + x_{\min}$$
$$y = y_{\max} + y_{\min}$$



Step 4: Scale by $1/d$

$$\begin{pmatrix} 1/d & 0 & 0 & 0 \\ 0 & 1/d & 0 & 0 \\ 0 & 0 & 1/d & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

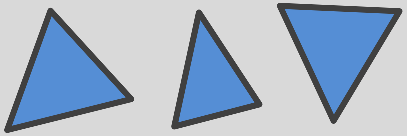


Viewport transform

Viewport transform

- Discrete pixel coordinates
- Quite trivial
- After last step we in $(-1,1) \times (-1,1)$
- Pixel coordinates are $N \times M$
- We did this in the second lecture
  - Add $1, 1$ to make position
  - Divide by $2, 2$ to make 0-to-1
  - Multiply by N and M

Pipeline



Primitives

Projection

Clipping

Culling

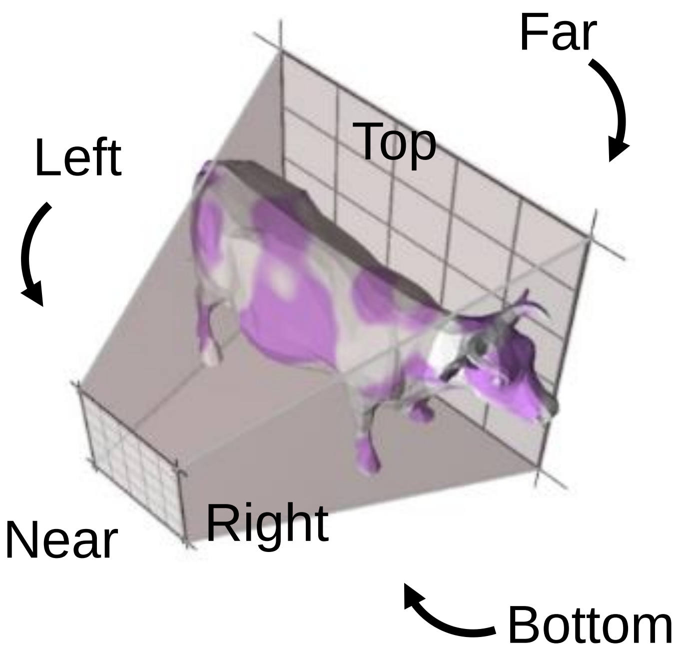
Rasterisation

z test

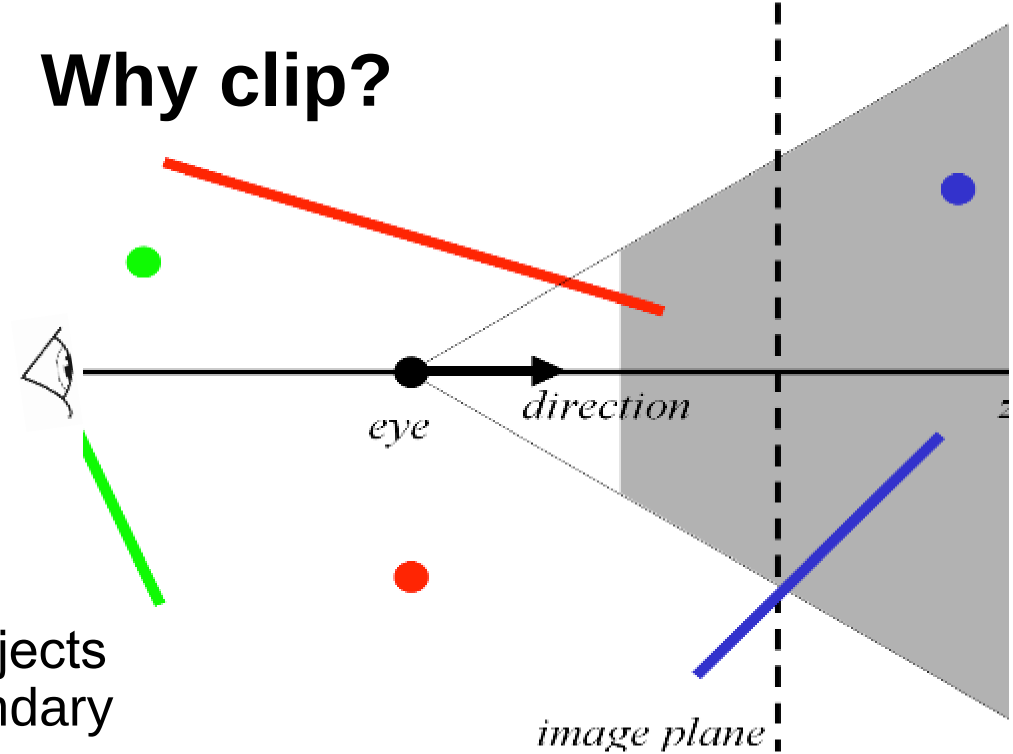
Shading

Clipping

- Eliminate portions of objects outside the viewing frustum
- View frustum
  - Boundaries of the image plane projected in 3D
  - A near & far clipping plane
- User may define additional clipping planes



Why clip?



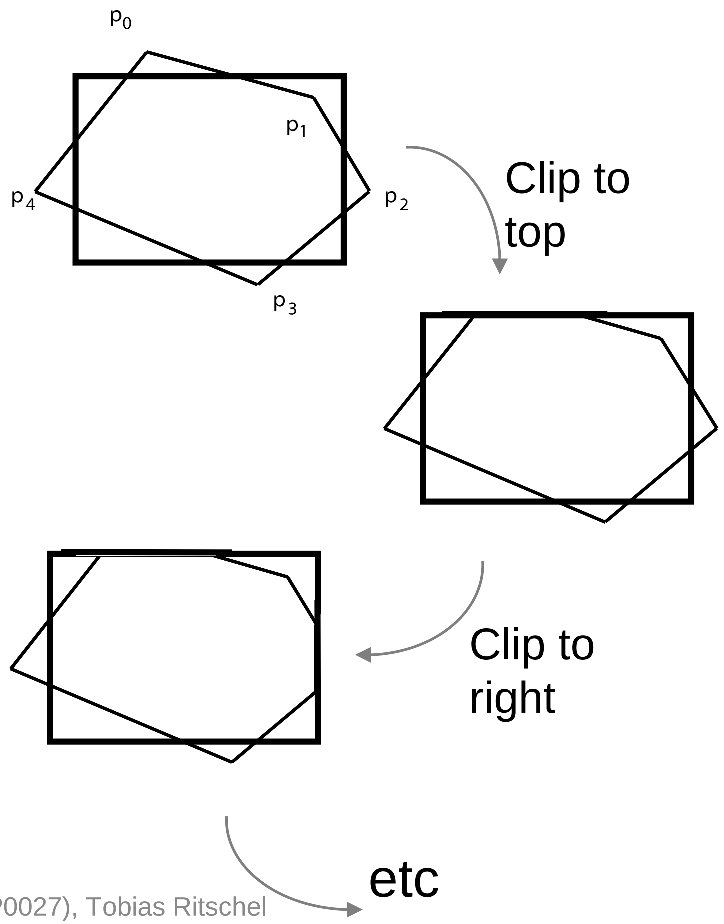
- Avoid degeneracies
  - Don't draw stuff behind the eye
  - Avoid division by 0 and overflow
- Efficiency
  - Don't waste time on objects outside the image boundary
- Other graphics applications (often non-convex)
  - Hidden-surface removal, Shadows, Picking, Binning, CSG (Boolean) operations (2D & 3D)

Clipping Summary

- It’s the process of finding the exact part of a polygon lying inside the view volume
- To maintain consistency, clipping of a polygon should result in a polygon, not a sequence of partially unconnected lines
- We will first look at two different 2D solutions and then extend one to 3D

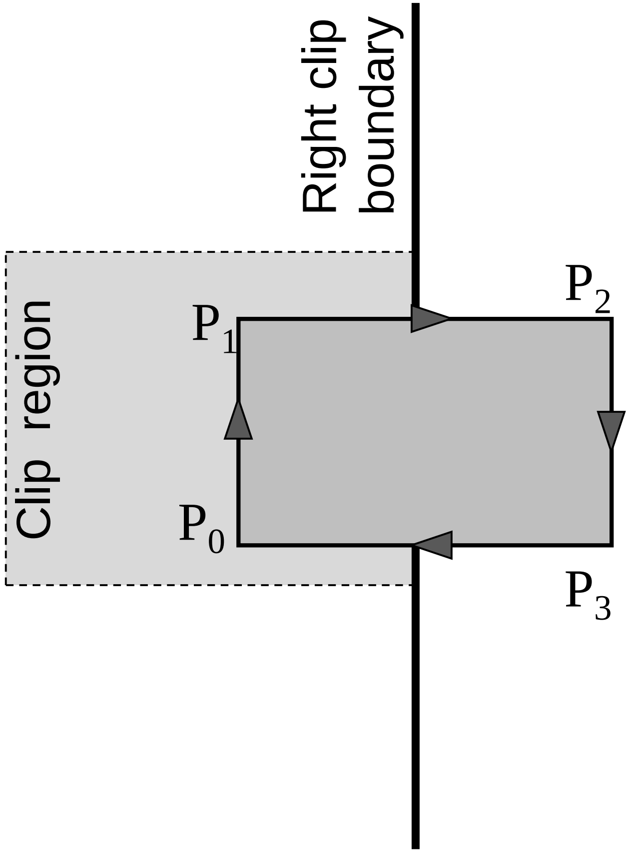
Sutherland-Hodgman Algorithm

- Clip the polygon against each boundary of the clip region successively
- Result is possibly NULL if polygon is outside
- Can be generalised to work for any polygonal clip region, not just rectangular



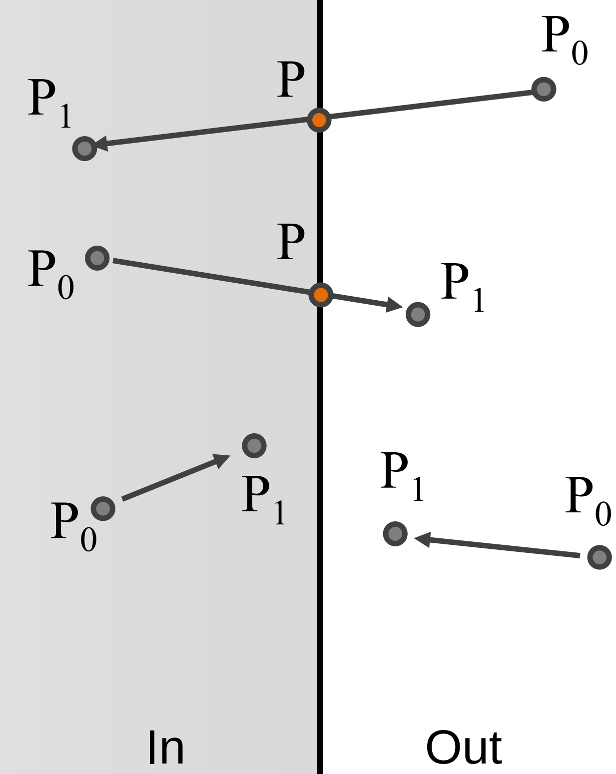
Clipping To A Region

- To find the new polygon
  - iterate through each of the polygon edges and construct a new sequence of points
  - starting with an empty sequence
  - for each edge there are 4 possible cases to consider



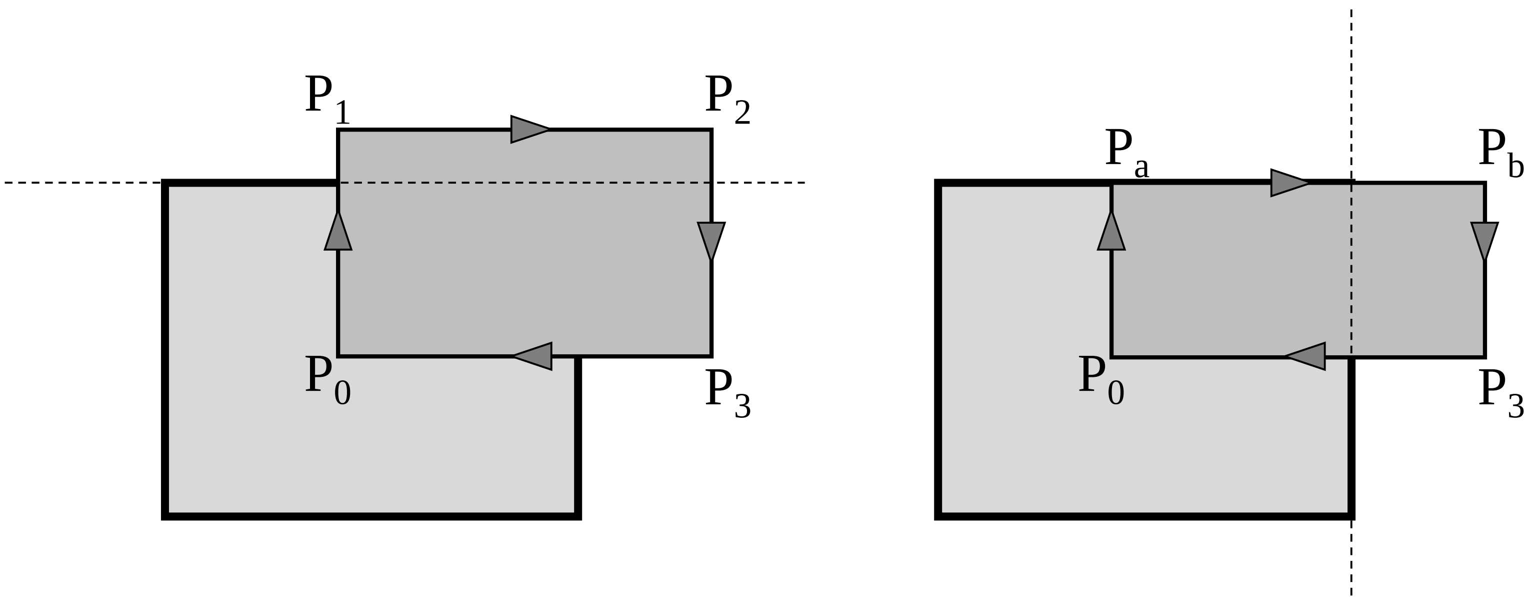
Clipping polygon edge against boundary

- Given an edge P_0, P_1 we have 4 case. Can …
  - enter the clip region, add P and P_1
  - leave the region, add only P
  - be entirely outside, do nothing
  - be entirely inside, add only P_1
- Where P is the point of intersection



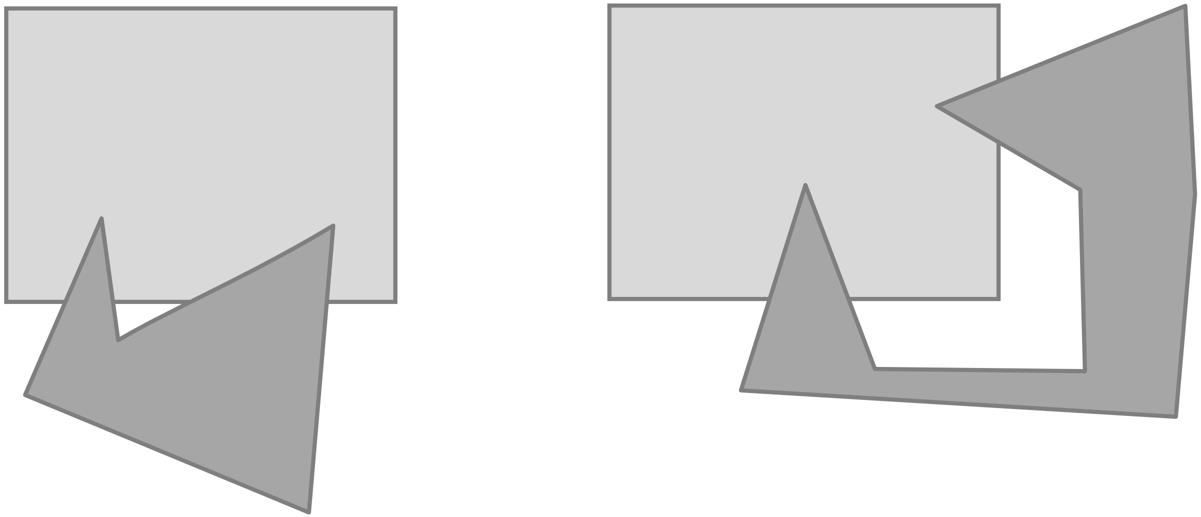
Still the Sutherland-Hodgman

- We can determine which of the 4 cases and also the point of intersection with just if statements
- Example:



Sutherland-Hodgman Problem

- Clipping is an example where convex polys make our lives easier
- Concave: Weiler-Atherton algorithm



Clipping Polygons in 3D

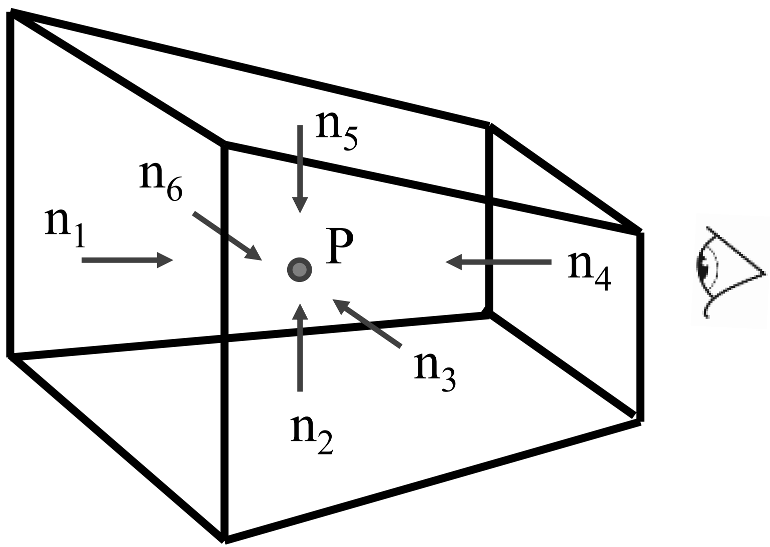
- The Sutherland-Hodgman can easily be extended to 3D
  - Clipping boundaries are 6 planes instead of 4 lines
  - Intersection calculation is done by comparing an edge to a plane instead of edge to edge

When to clip?

- Before perspective transform in 3D space
  - Use the equation of 6 planes
  - Natural, not too degenerate
- In homogeneous coordinates after perspective transform (Clip space)
  - **Before** perspective divide (4D space, weird w values)
  - Canonical, independent of camera
  - The simplest to implement in fact
- In the transformed 3D screen space after perspective division (canonical par.)
  - Problem: objects in the plane of the camera

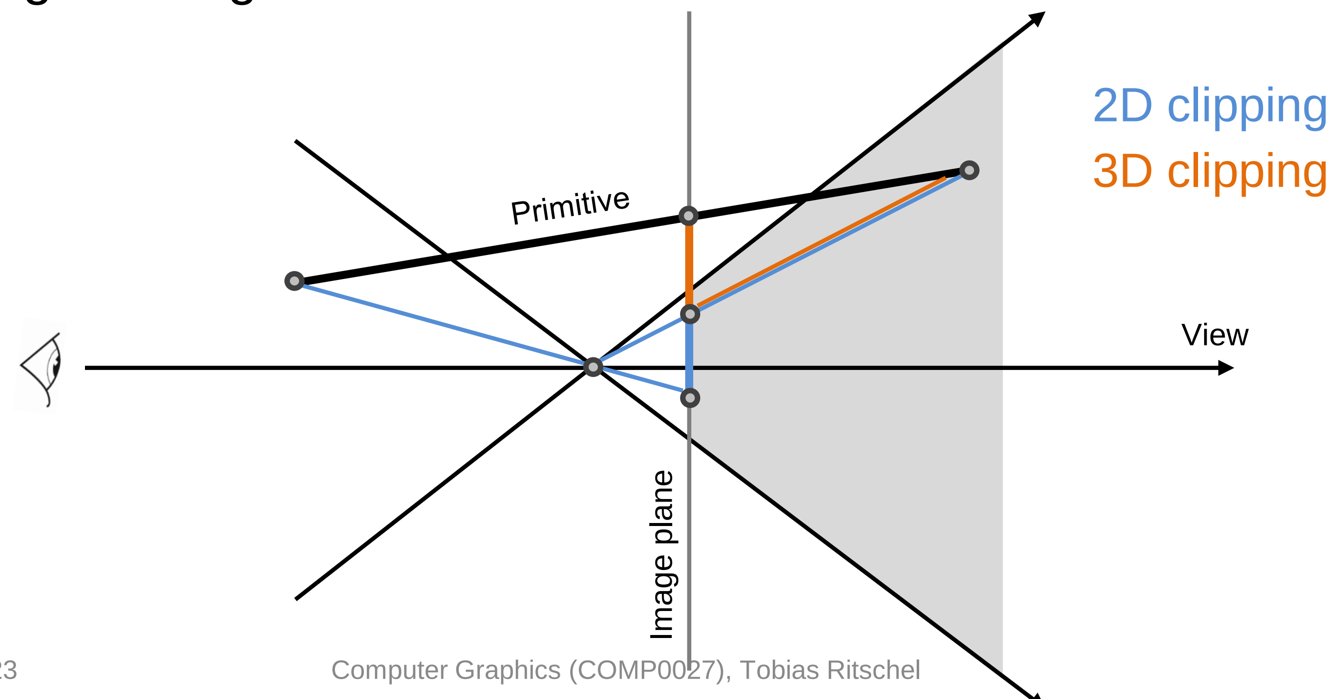
Clipping with respect to View Frustum

- Test against each of the 6 planes
  - Normals oriented towards the interior
- Clip / cull / reject point P if any $<\mathbf{n}_i, \mathbf{p}> < d_\mathrm{i}$



Clipping After Projecting

- When we have an edge that extends from the front to behind the COP, then if we clip after projection (which in effect is what e.g. the PS 1 did) we might get wrong results



A note on implementation

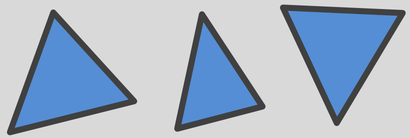
- Vanilla implementation uses dynamic data structure

```
iterate windowVertices edges {
 iterate primitiveVertices
  if(…)
   primitiveVertcies.add(new vertex)
 if(…)
   primitiveVertcies.remove(new vertex)
}}
```

- Better, without

```
for MAX_EDGES edges {
 for MAX_CLIPPED_VERTICES {
  if(…)
   primitiveVertcies[vertexCount++](new vertex)
 if(…)
   vertexCount++
}}
```

Pipeline



Primitives

Projection

Clipping

Culling

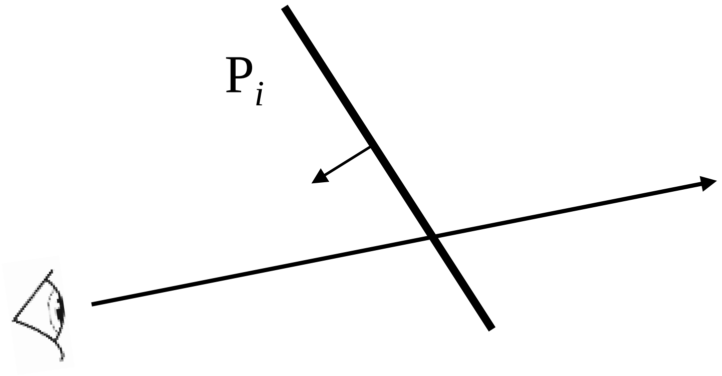
Rasterisation

z test

Shading

Idea

- Optional speed-up
- Remove primitives P_i not facing the camera
- Polygons whose normal does not face the viewpoint, are not rendered



Back Face Culling

- Thus if we have the plane equation

$$l(x, y, z) = ax + by + cz - d = 0$$

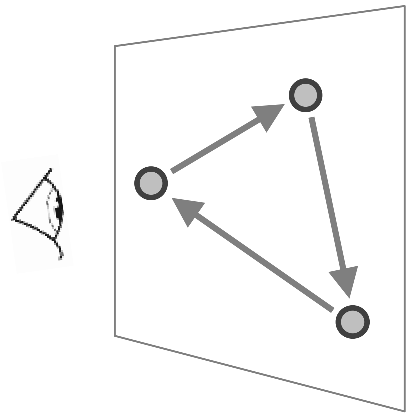
- and the COP (c_x, c_y, c_z), then

$$l(c_x, c_y, c_z) > 0$$

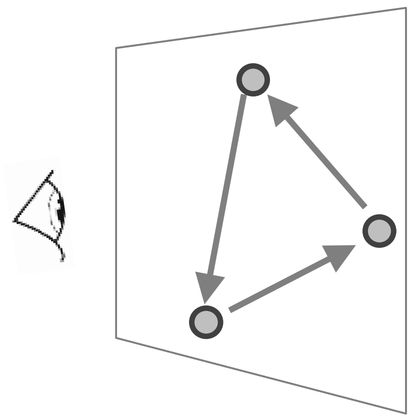
- if the COP is in front of the polygon
- Otherwise: cull!

Clockwise/counter-clockwise

Typically done without normals: All polygons that are not counter-clockwise after projection are culled



Clockwise: Culled



Counter clockwise: Not culled

Limitation

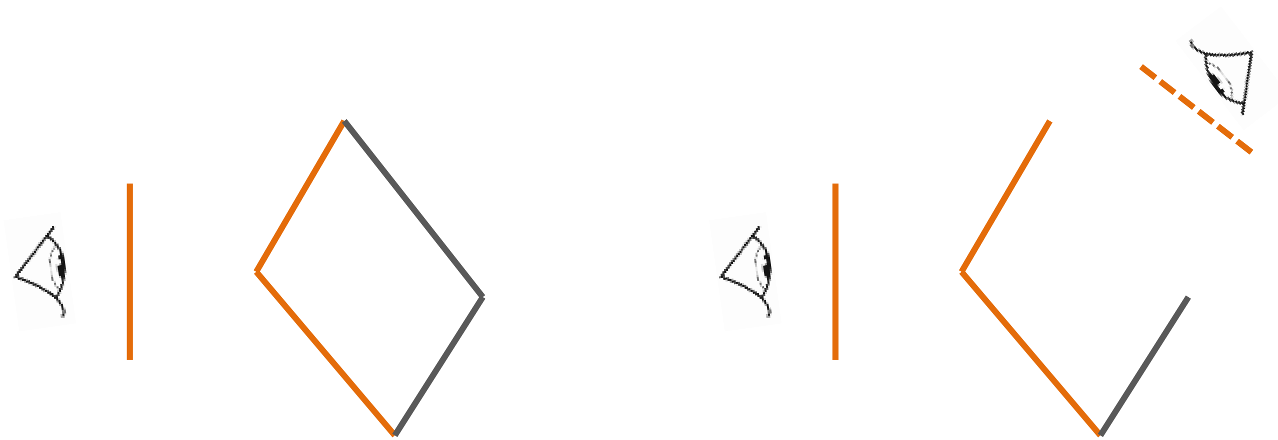
Does not ensure correct visible surfaces determination, unless there is only a single convex object

Convex: Works

Concave: Does not work

Limitation

Only reliable if backsides are never required. Therefore, does not work for non-closed polytopes.



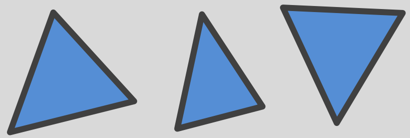
Closed polytop: Works

Non-closed polytop: Does not work

Culling recap

- Culling can easily & quickly remove (some!) invisible polygons from the pipeline
- Still some (partially) invisible ones remain

Pipeline



Primitives

Projection

Clipping

Culling

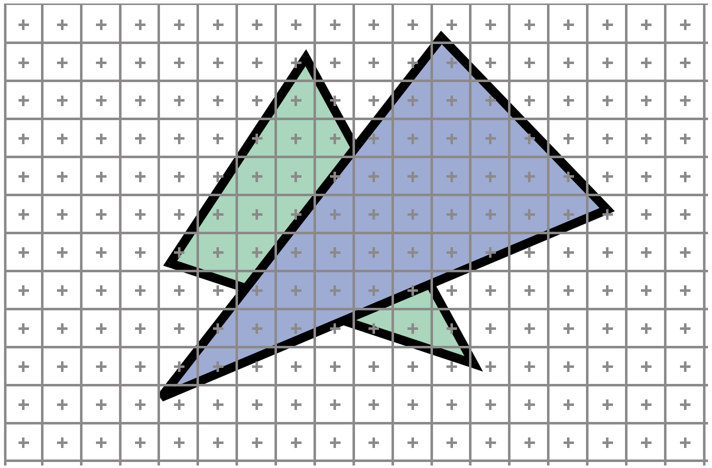
Rasterisation

z test

Shading

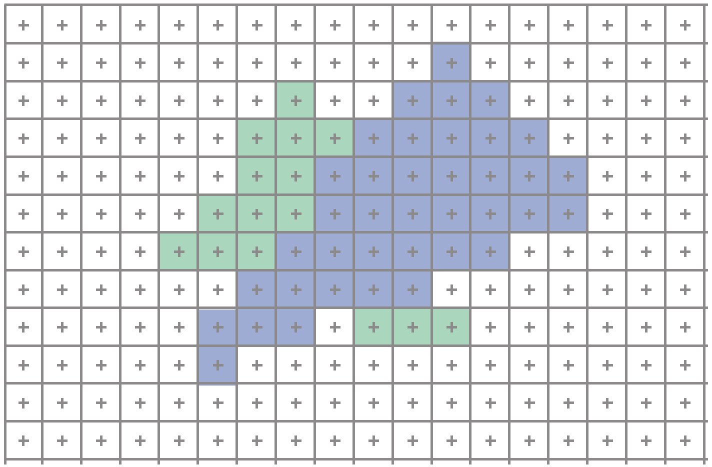
2D Scan Conversion

- Primitives are continuous; screen is discrete
  - Triangles defined by discrete set of vertices
  - But they define a continuous area on screen



2D Scan Conversion

- Solution: compute discrete approximation
- Scan Conversion (Rasterization):
  algorithms for efficient generation of the samples comprising this approximation



Naïve Filling Algorithm

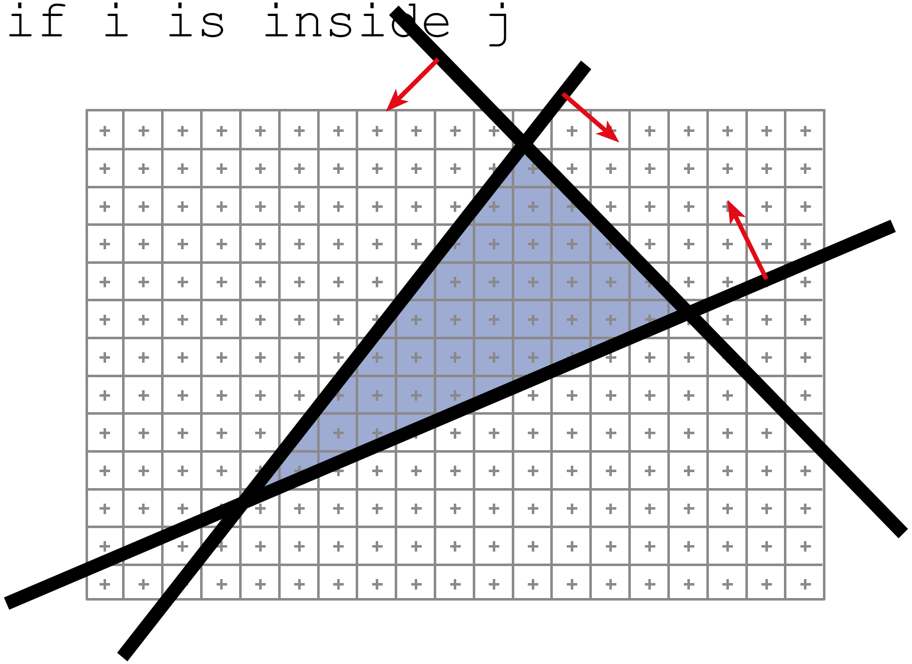
- Find a point inside the polygon
- Do a **flood fill:**
  - Keep a stack of points to be tested
  - When the stack is not empty
    - Pop the top point (Q)
    - Test if Q is inside or outside
      - If Inside, colour Q, push neighbours of Q if not already tested
      - If outside discard
    - Mark Q as tested

Critique

- Horribly slow
  - Explicit in/out test at every point
  - But still very common in paint packages!
  - Only solid color
- Stack might be very deep
- Want to do this
  - Without much memory
  - In parallel

Brute Force Solution for Triangles

```
For all pixels i
 For all edges j
  Test if i is inside j
```



Half-Space Test Reminder

- For each edge compute line equation (analogue to plane equation):

$$L_i(x, y) = a_i x + b_i y + c_i$$

- If $L_i(x, y) > 0$ point in **positive** half-space
- If $L_i(x, y) < 0$ point in **negative** half-space
- If all $L_{1,2,3}(x, y) \geq 0$ point inside triangle

Half-Space Test Reminder

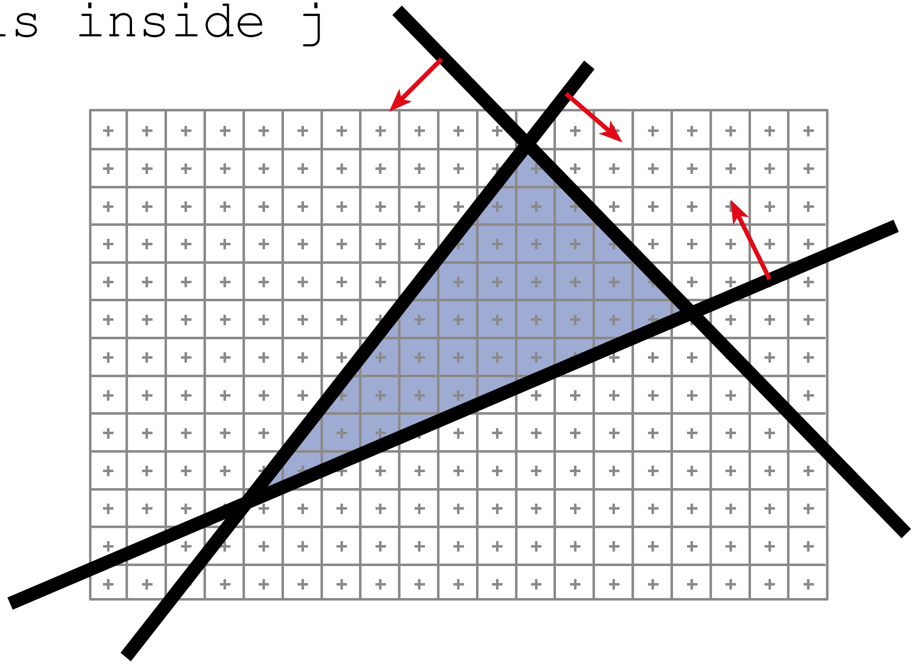
- For each edge compute line equation (analogue to plane equation):

$$L_i(x, y) = a_i x + b_i y + c_i$$

- Example:
  - Assuming $(2, 2)$ to $(4, 7)$,
  - The slope is $\mathrm{d}y/\mathrm{d}x = 5/2 = 2.5$
  - Hence $y - 7 = 2.5(x - 4)$,
  - So $L_1(x, y) = 2.5x - y - 3.$

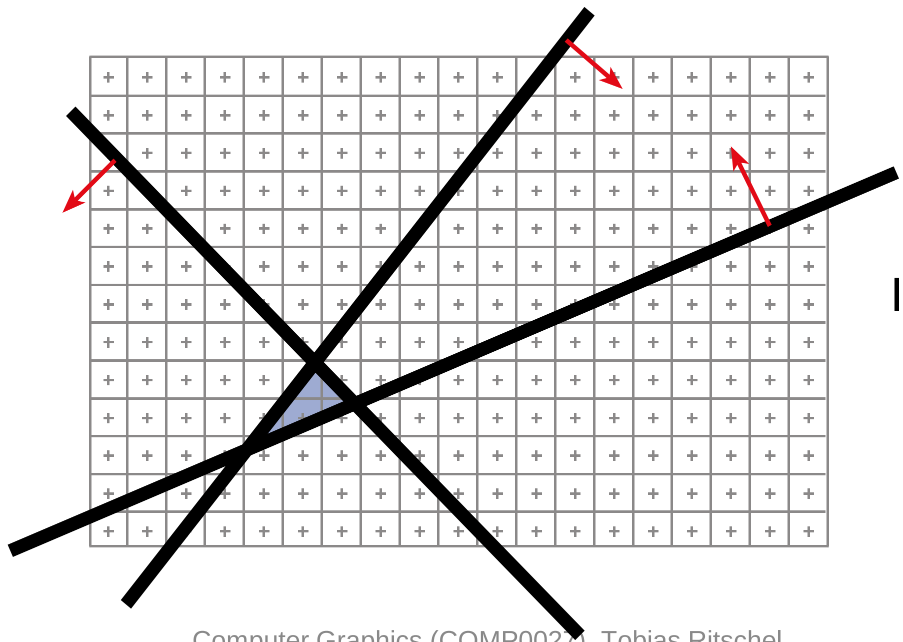
Brute Force Solution for Triangles

```
For all pixels i
 For all edges j
  Test if i is inside j
```



Brute Force Solution for Triangles

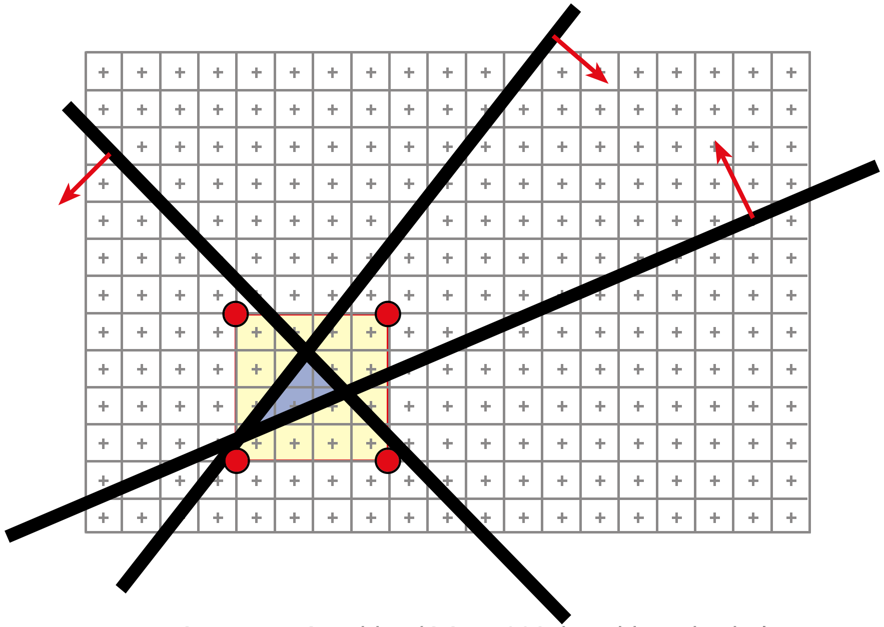
```
For all pixels i
 For all edges j
  Test if i is inside j
```



Problem?
If the triangle is small, a lot of useless computation!

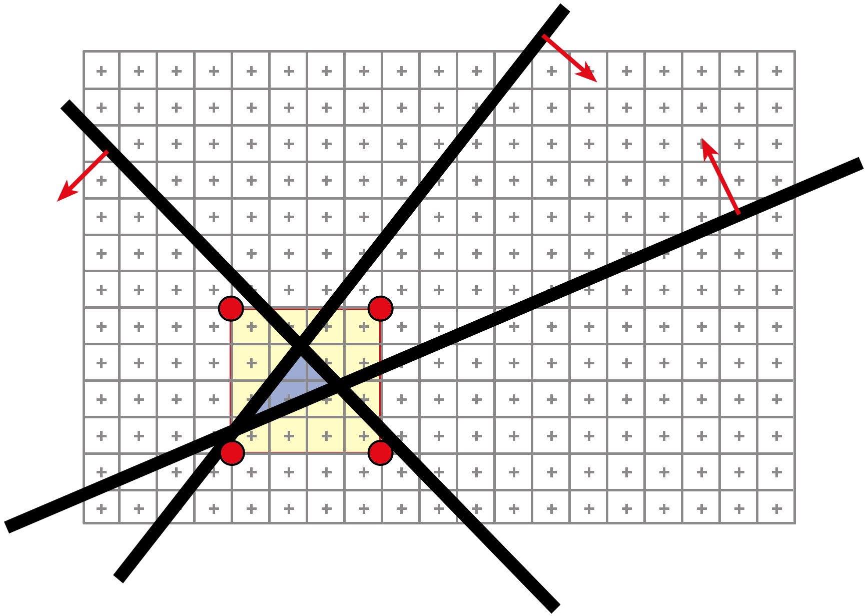
Brute Force Solution for Triangles

- Improvement: Compute only for the *screen bounding box* of the triangle
- How do we get such a bounding box?
  - x_{min}, x_{max}, y_{min}, y_{max} of the triangle vertices



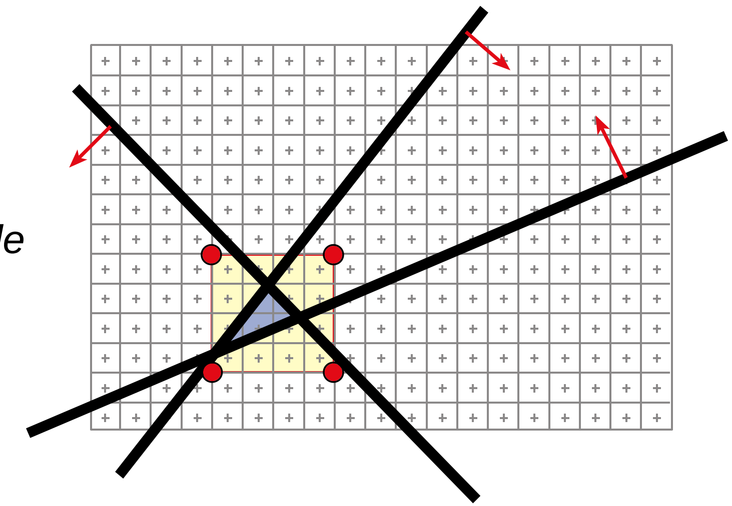
Rasterisation on Graphics Cards

- Triangles are usually very small
  - Setup cost are becoming more troublesome
- Brute force is tractable



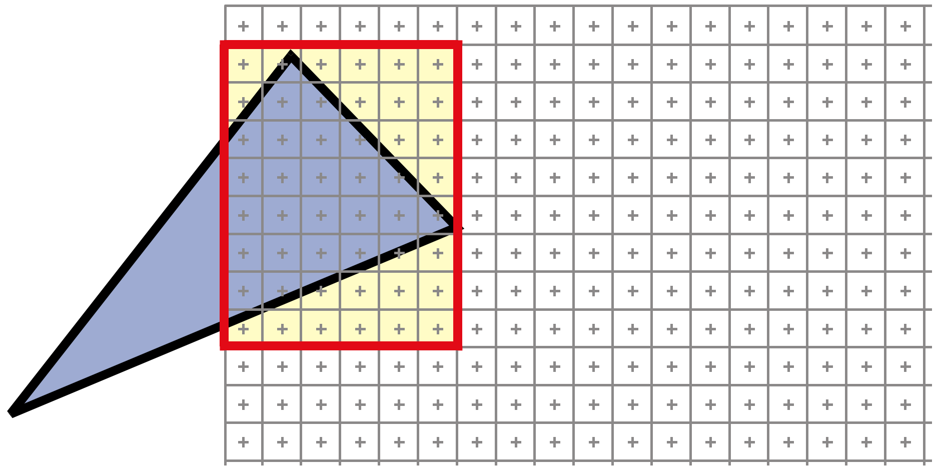
Rasterisation on Graphics Cards

```
For every triangle
 ComputeProjection
 Compute bbox, clip bbox to screen limits
 Compute line equations
 For all pixels in bbox
  If all line equations > 0 // Pixel [x,y] in triangle
   framebuffer[x,y] = color;
```



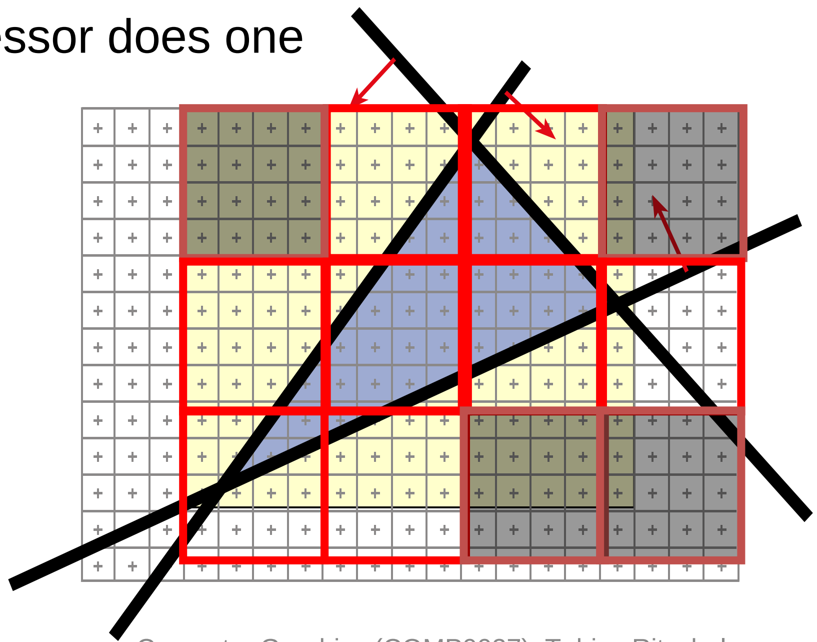
Rasterisation on Graphics Cards

Note that Bbox clipping is trivial, unlike triangle clipping



Tiling

- Subdivide BBox into smaller tiles
  - Early rejection of tiles
  - Memory access coherence
  - Each micrprocessor does one



Recap

- Moving away from ray-tracing to projection
- How to
  - Project a point
  - Clip a polygon
  - Cull a polygon
  - Fill the interior (rasterise)
- We’ll spend next lectures tidying up the remaining problems!