Singularities

A `singularity` occurs when a robot loses the ability to move the end effector in a certain direction or when the end effector cannot rotate about a certain axis. A singularity occurs when the jacobian matrix becomes non invertible.

Jacobian Matrix

The Jacobian matrix is a representation of the linear/rotational velocities in matrix form. For example:

$$^{n}\omega_e = \begin{bmatrix} A\dot{\theta}_1 + D\dot{\theta}_2 \\ B\dot{\theta}_2 \\ C\dot{\theta}_3 \end{bmatrix} = J_\omega \cdot \dot{\theta} = \begin{bmatrix} A & D & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

similarly for velocity:

$$^{n}v_e = \begin{bmatrix} A\dot{d}_1 + D\dot{d}_2 \\ B\dot{d}_2 \\ C\dot{d}_3 \end{bmatrix} = J_v \cdot \dot{d} = \begin{bmatrix} A & D & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \cdot \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

For linear velocity, the position of the end effector (or any frame) can be determined geometrically pretty easily. This can be expressed as a vector, which can then be differentiated to give the jacobian matrix.

This can be done by finding the determinant of the jacobian matrix (ignoring any 0 rows).

Static Forces

The thing we want to determine is the joint torques required to keep the robot in static equilibrium. This robot could be holding something, or pushing an object.

$$\tau = J_v^T \cdot F$$

Where:

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \dots \\ \tau_n \end{bmatrix} F = \begin{bmatrix} F_z \\ F_x \\ F_y \end{bmatrix}$$

F is the desired force and τ represents the motor torques.

It is easiest for these to be represented with respect to the zero frame.

$$ {}^{0}\tau = {}^{0}J_v^T \cdot {}^{0}F $$

There is a torque for each joint in the robot. Positive torques act in the anti clockwise direction.

Static forces at singularities

At a singularity, no torque is required to withstand large forces.