Computer Graphics (COMP0027) 2022/23

Spline surfaces

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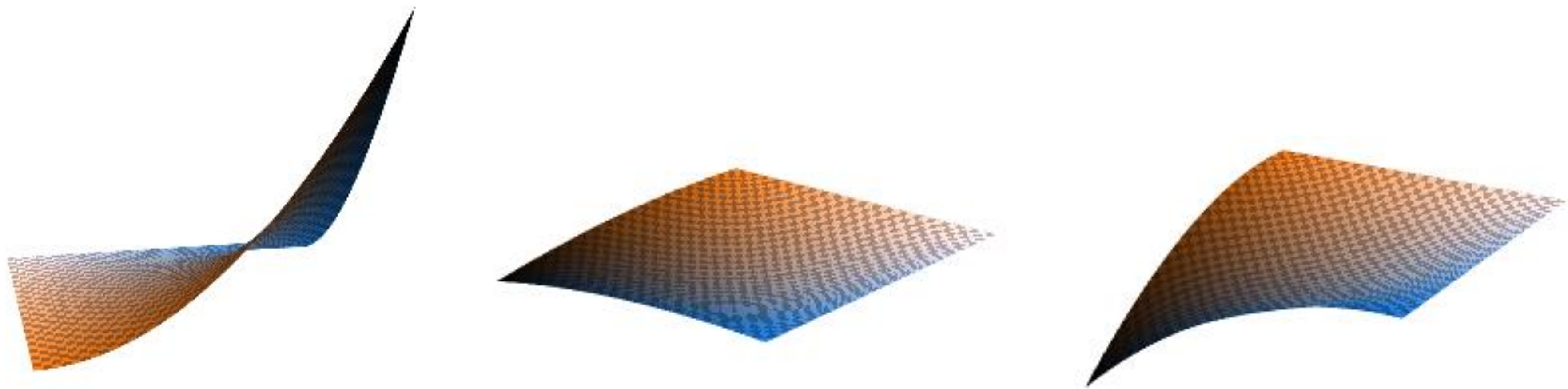
Remember: we want to remove facets

Example `cg.cs.ucl.ac.uk`



Bezier Surfaces Introduction

- Constructing a surface relies very much on the ideas behind constructing curves
- Surfaces can be thought of as ‘Bezier curves in all directions’ across the surface
- Tensor products of Bezier curves
- Teapot most famous example
  - produced entirely by Bezier surfaces

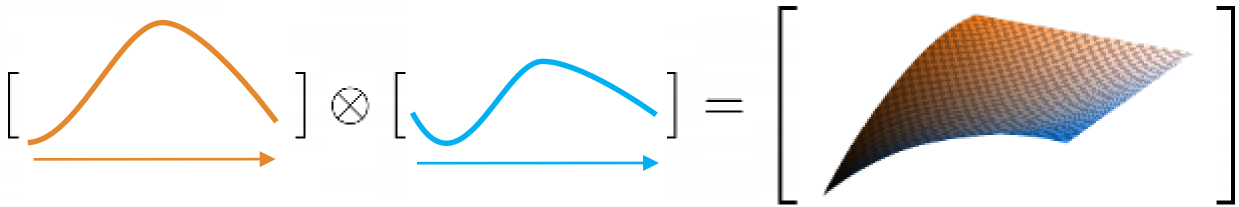
Tensor Product

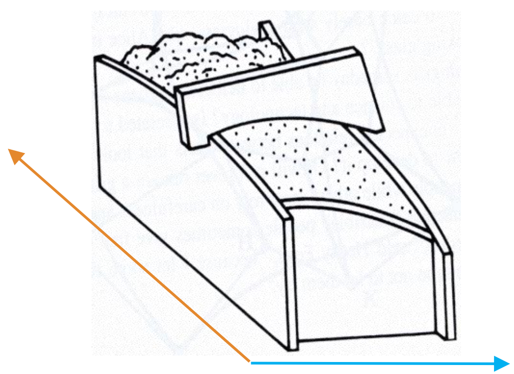
- Tensor product of two **vectors**

$$\mathbf{a} \otimes \mathbf{b} = \begin{bmatrix} a_1 & a_2 & a3 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b3 \end{bmatrix} = \begin{bmatrix} a_1 b_1 & a_2 b_1 & a3 b_1 \\ a_1 b_2 & a_2 b_2 & a3 b_2 \\ a_1 b_3 & a_2 b_3 & a3 b_3 \end{bmatrix}$$

Tensor Product

- Tensor product of two **functions**

$\mathbf{a} \otimes \mathbf{b} =$ 



Bicubic Bezier Patch

- Let

$$\mathbf{c}(t|\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$$

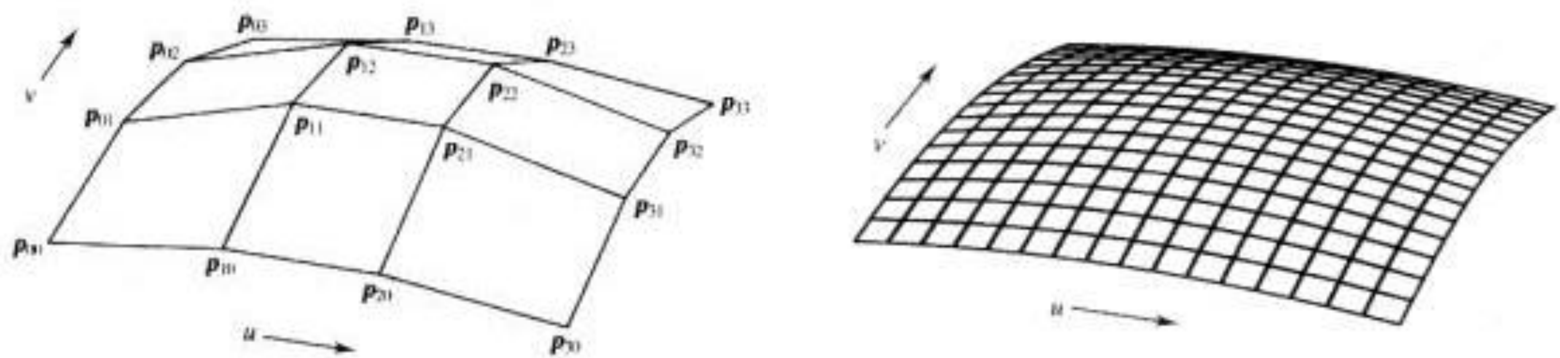
be a 1D spline at t through the control points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$

- Then the surface is

$$\begin{aligned}\mathbf{p}(s,t) = \mathbf{c}(s|&\mathbf{c}(t|\mathbf{p}_{00}, \mathbf{p}_{01}, \mathbf{p}_{02}, \mathbf{p}_{03}),\\ &\mathbf{c}(t|\mathbf{p}_{10}, \mathbf{p}_{11}, \mathbf{p}_{12}, \mathbf{p}_{13}),\\ &\mathbf{c}(t|\mathbf{p}_{20}, \mathbf{p}_{21}, \mathbf{p}_{22}, \mathbf{p}_{23}),\\ &\mathbf{c}(t|\mathbf{p}_{30}, \mathbf{p}_{31}, \mathbf{p}_{32}, \mathbf{p}_{33}))\end{aligned}$$

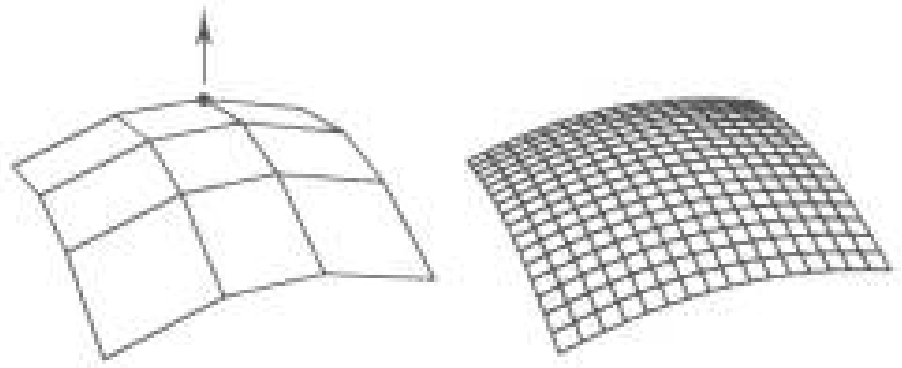
Bicubic Bezier Patch

$$\mathbf{p}(s,t) = \mathbf{c}(s|\mathbf{c}(t|\mathbf{p}_{00},\mathbf{p}_{01},\mathbf{p}_{02},\mathbf{p}_{03}),\\ \mathbf{c}(t|\mathbf{p}_{10},\mathbf{p}_{11},\mathbf{p}_{12},\mathbf{p}_{13}),\\ \mathbf{c}(t|\mathbf{p}_{20},\mathbf{p}_{21},\mathbf{p}_{22},\mathbf{p}_{23}),\\ \mathbf{c}(t|\mathbf{p}_{30},\mathbf{p}_{31},\mathbf{p}_{32},\mathbf{p}_{33}))$$

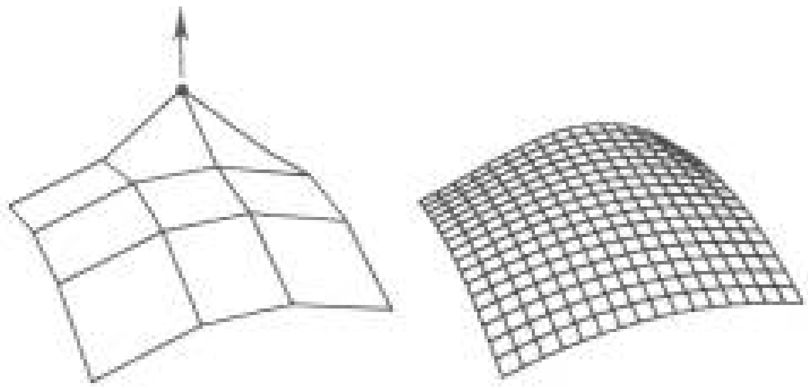


(a) (b)

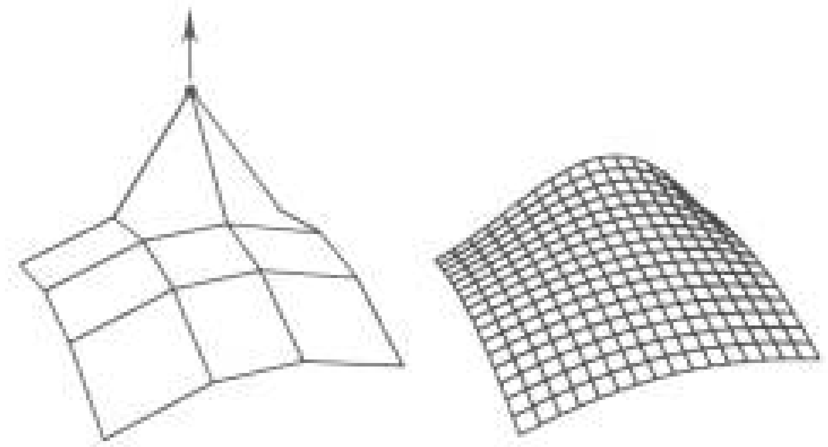
Editing Bicubic Bezier Patches



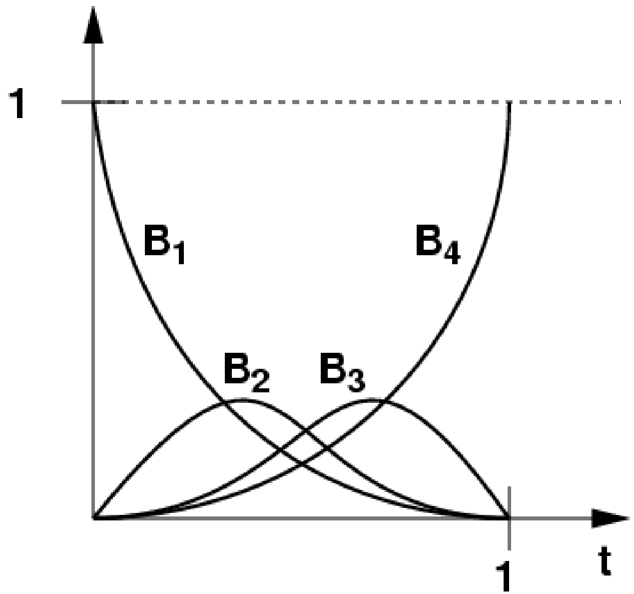
Editing Bicubic Bezier Patches



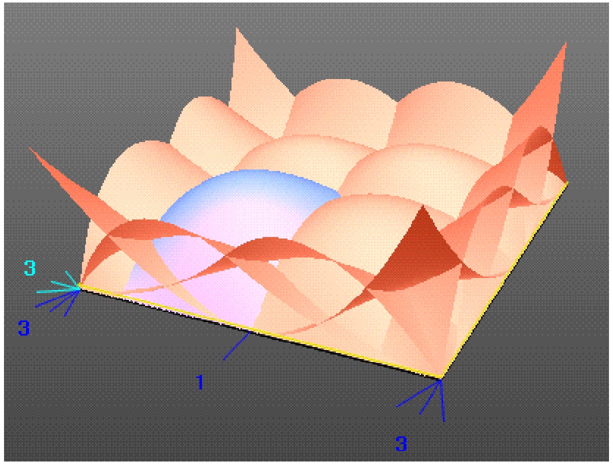
Editing Bicubic Bezier Patches



Editing Bicubic Bezier Patches



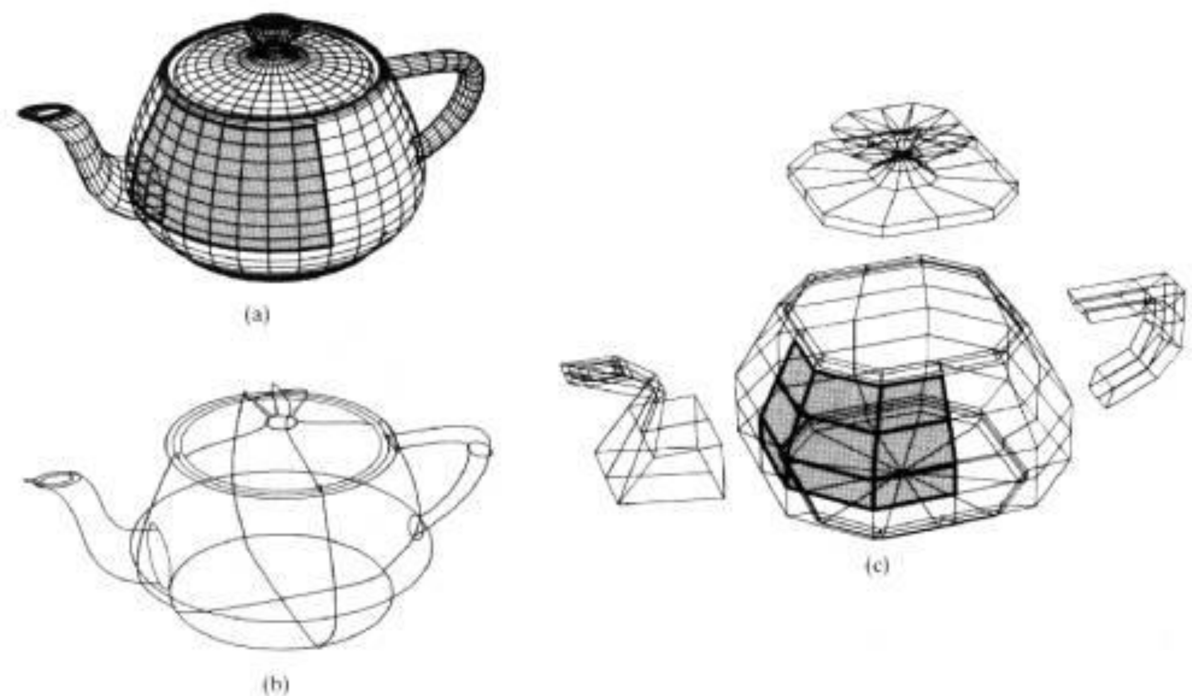
1D Basis Functions



2D Basis Functions

Patch Modelling

- Original Teapot specified with Bezier Patches



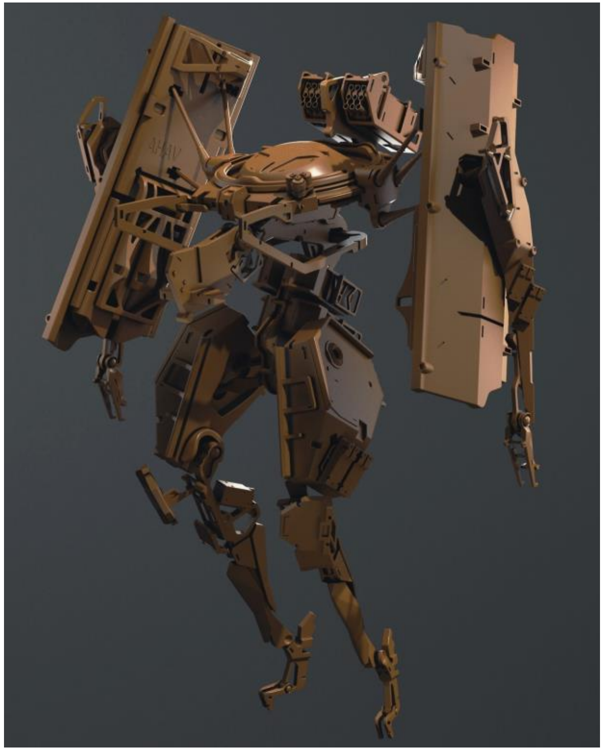
Alternative Splines Surfaces

- You can make surfaces from B-Splines in a similar way
- A particular types of B-Spline generalisation, Non-uniform rational Basis spline (NURBS) surfaces are particularly common

Geri's game by Pixar

Spline domain

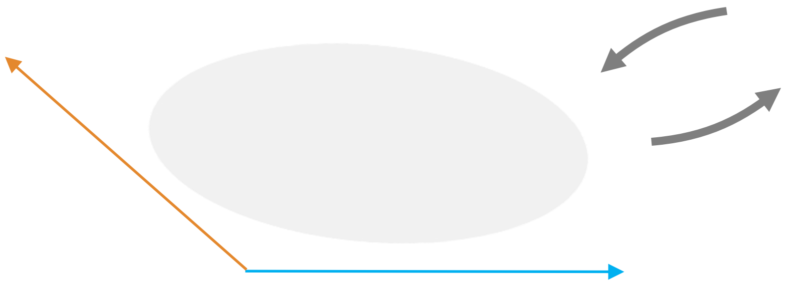
- Splines are defined to map from (0,1)^2 to 3D
- Not every 3D shape can be represented like this
  - Homeomorphism
  - You cannot take every shape and map it to (0,1)^2
  - Counter example seen right

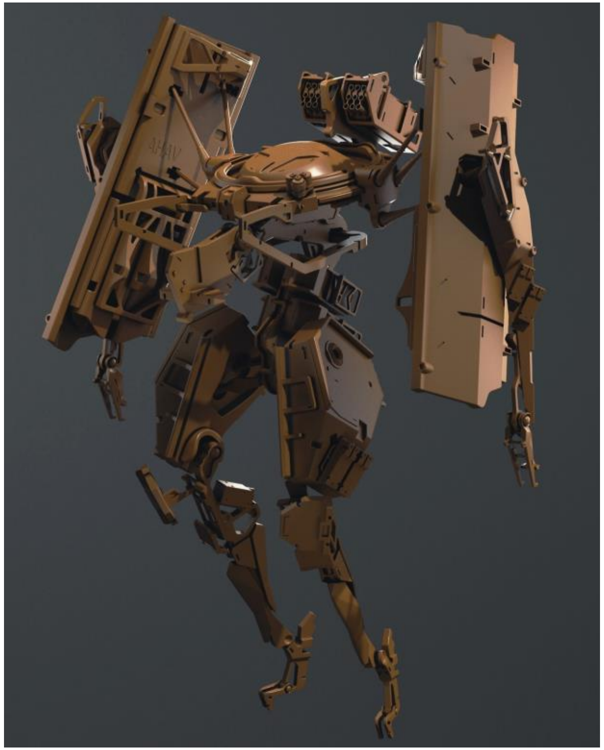


Splines on non-disk topology

- Non-disk topology

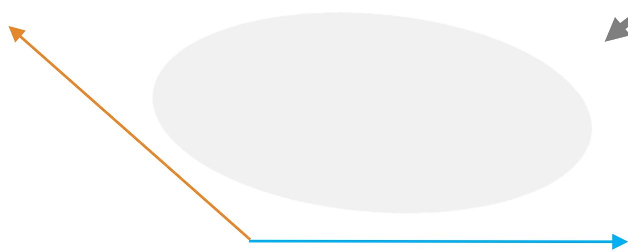
No, sorry!

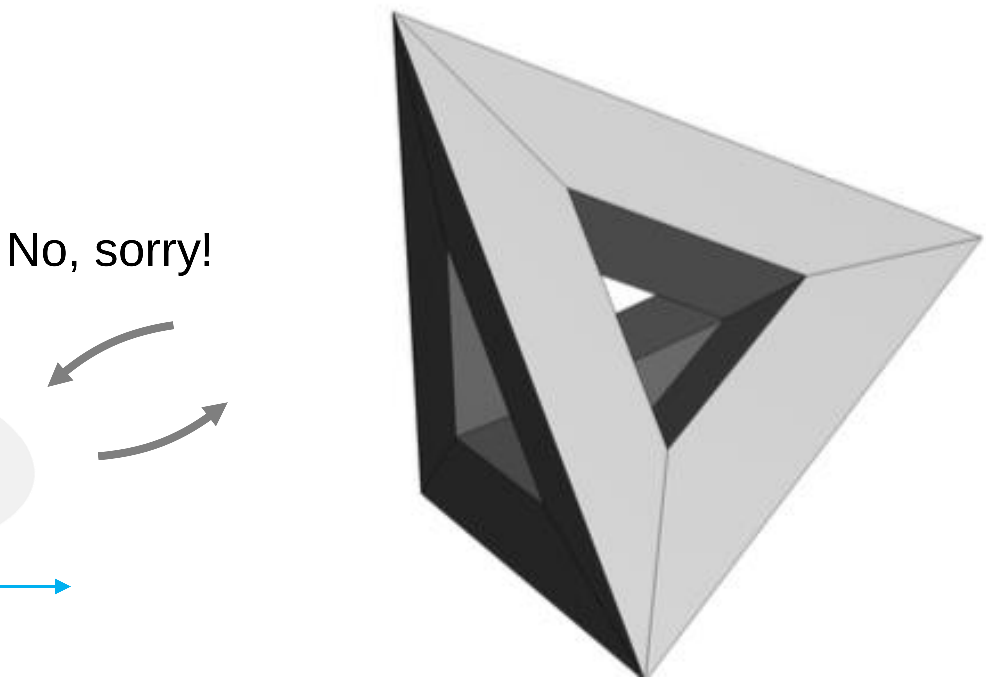




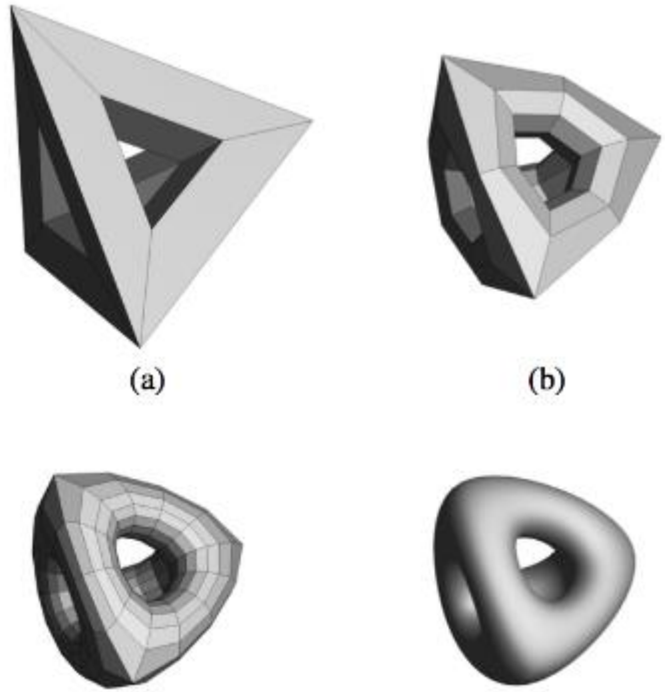
Splines on non-disk topology

- Lets consider a simple case with high genus
- Non-disk topology





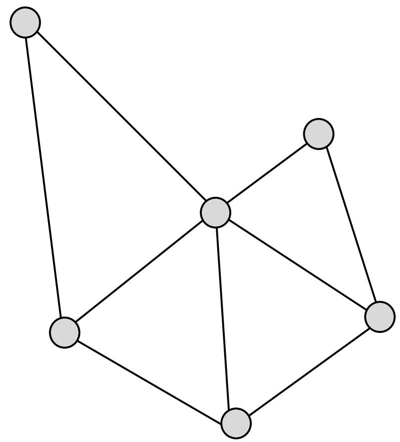
Iterative splitting



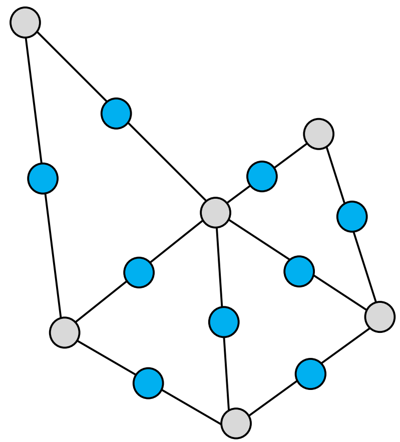
(a) (b)

(c) (d)

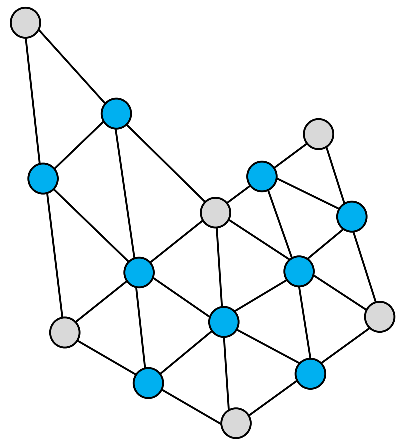
Best to think 2D again



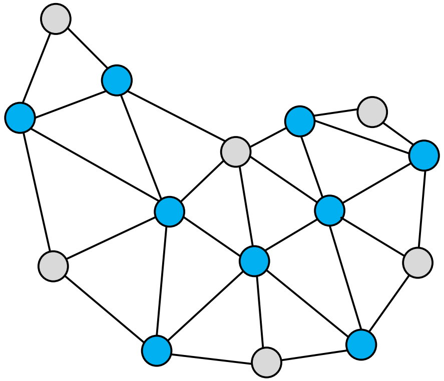
Step 1: Split edges



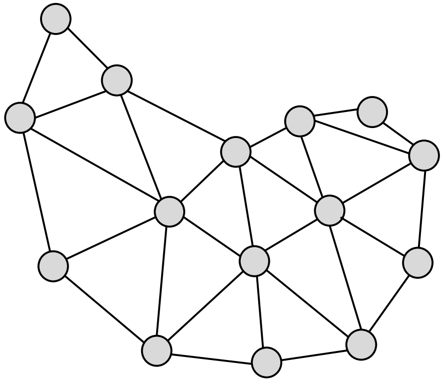
Step 2: Re-topologize



Step 3: Relax



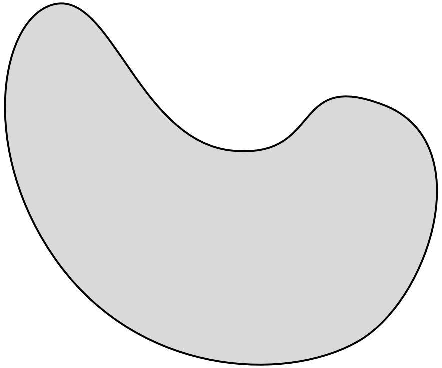
Done



Does not look like much, but ..

… repeat forever …

Does not look like much, but ..



.. Is the key to high-quality 3D geometry

Conclusions

- Surfaces are a simple extension to curves
- Really just a tensor-product between two curves
  - One curve gets extruded along the other
- Subdivision surfaces are another way of generating curves
  - Particularly amenable to GPU implementation!