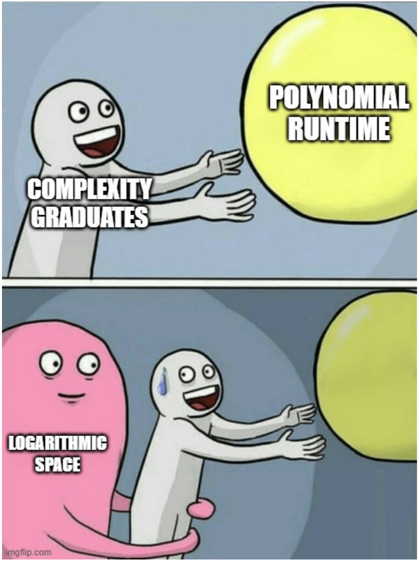
Lecture 13: Log space



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- Show that $\boldsymbol{L} \subseteq P$,
- And also that $\boldsymbol{NL} \subseteq P$.

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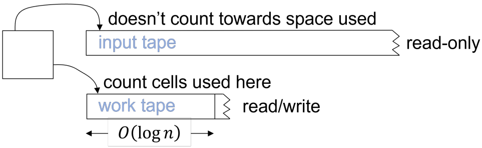
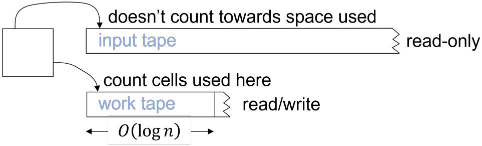
Model: a 2-tape TM with a read-only tape for the input and a RW-tape for computation.



Credit: Sipser

Log-space



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Definition

$\boldsymbol{L} = SPACE(\log n)$

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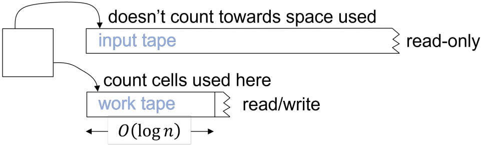

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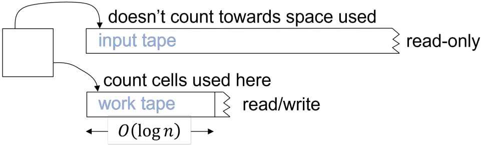

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For example, we can store $O(1)$ pointers.

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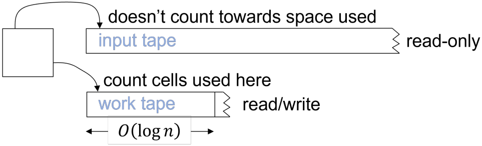

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Another example: $\{ww^R | w \in \Sigma^*\} \in \mathbf{L}$.

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Seems easy enough.
But how is this implemented?

Our input size is $n = O(qw)$.
We are not allowed to keep w bits (for M) in memory!

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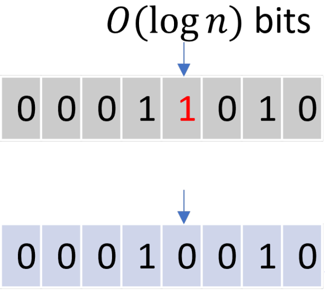

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Given a directed graph $G = (V, E)$ and two vertices s, t, the STCON problem decides whether there exists a path from s to t.

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Algorithm 1 STCON

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Set v ← s.
for ℓ = 0 to |V| − 1 do
    if v = t then
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Notice that ℓ and v both require $O(\log n)$ bits and thus this can be implemented on an ***L*** machine. $\square$

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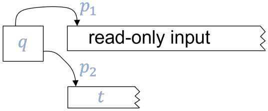
For L machines, we can refine the definition of a configuration. Since the input tape is read-only, we denote a configuration by (q, p_1, p_2, t), where:



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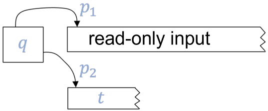

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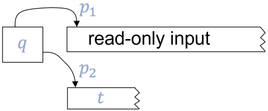

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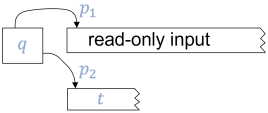

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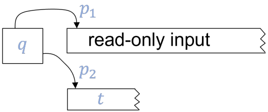

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- q is the current state.
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- $t \in \Gamma^{O(\log n)}$ is the contents of the RW tape.

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$|Q| \times n \times \log n \times |\Gamma|^{O(\log n)} = n^{O(1)}$. □

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Immediate from Savitch's theorem. $\square$

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To simulate a general $SPACE(\log^2 n)$ machine we need $2^{\Omega(\log^2 n)} = n^{\Omega(\log n)}$ time, which is not polynomial.

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Consider the set of configurations

$$C = \{(q, p_1, p_2, t) \in Q \times \{0, \dots, n-1\} \times \{0, \dots, O(\log n)\} \times \Gamma^{O(\log n)}\}.$$

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Construct a graph with C as vertices and connect each pair $(c_i, c_j) \in C^2$ if c_j follows from c_i in one step.

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We then run a BFS to check if c_Y is reachable from c_0 and terminate accordingly. $\square$

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- It was shown that undirected STCON is in $\boldsymbol{L}$.
- There are also P-complete problems (using $\boldsymbol{L}$-reductions) such as max-flow.

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Answer on Mentimeter: .

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- An $\boldsymbol{L}$ machine must be looping after $\log^2 n$ steps. No, it can make $n^{O(1)}$ steps without looping.
- $STCON \in \boldsymbol{L}$. Probably not; all we know is that $STCON \in \boldsymbol{NL}$.

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Next time: hierarchy theorems and EXPTIME-completeness.