COMP0174 Practical Program Analysis
Available Expressions Analysis

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Four Classic Analyses

	Forward	Backward
Must	**Available Expressions**	Very Busy Expressions
May	Reaching Definitions	Live Variables

Available Expressions

Definition. For each program point, which expressions must have already been computed, and not later modified, on all paths to the program point.

It is a *forward must* analysis.

Application: optimization (don't recompute expressions that are still available).

Example

$[x := a + b]^1;$
$[y := a * b]^2;$
$while\ [y > a + b]^3 do$
$\quad [a := a + 1]^4;$
$\quad [x := a + b]^5;$

The expression $a + b$ is available every time execution reaches the condition 3, therefore the expression need not be recomputed.

Killed Expression

An expression is killed in a block if any of the variables used in the expression are modified in the block:

$$kill_{AE}\left([x := a]^l\right) = \{a' \in AExp_* \mid x \in Vars(a')\}$$
$$kill_{AE}\left([skip]^l\right) = \emptyset$$
$$kill_{AE}\left([b]^l\right) = \emptyset$$

where $AExp_*$ are all expressions in the program

Killed Expression

An expression is killed in a block if any of the variables used in the expression are modified in the block:

$$\boxed{kill_{AE}([x := a]^l) = \{a' \in AExp_* \mid x \in Vars(a')\}}$$

$$kill_{AE}([skip]^l) = \emptyset$$

$$kill_{AE}([b]^l) = \emptyset$$

Assignment ***statement***: kills all expressions that use variable x assigned in the block because they have to be recomputed again

where $AExp_*$ are all expressions in the program

Generated Expression

A generated expression is an expression that is evaluated in the block and where none of the variables used in the expression are later modified in the block:

$$gen_{AE}\left([x \coloneqq a]^l\right) = \{a' \in AExp(a) \mid x \notin Vars(a')\}$$
$$get_{AE}\left([skip]^l\right) = \emptyset$$
$$gen_{AE}\left([b]^l\right) = AExp(b)$$

Analysis

The goal of the analysis is to compute the largest set satisfying the equation for AE_{entry}:

$$AE_{entry}(l) = \begin{cases} \emptyset & if\ l = init(program) \\ \cap \{AE_{exit}(l') \mid (l', l) \in flow(program)\} & otherwise \end{cases}$$

$$AE_{exit}(l) = \left(AE_{entry}(l) \setminus kill(B^l)\right) \cup gen_{AE}(B^l)$$

where $B^l \in blocks(program)$

Example

$$
\begin{aligned}
&[x := a + b]^1;\\
&[y := a * b]^2;\\
&while\ [y > a + b]^3\ do\\
&\qquad [a := a + 1]^4;\\
&\qquad [x := a + b]^5;
\end{aligned}
$$

$\boldsymbol{l}$	$\boldsymbol{kill_{AE}(l)}$	$\boldsymbol{gen_{AE}(l)}$
1	$\emptyset$	$\{a + b\}$
2	$\emptyset$	$\{a * b\}$
3	$\emptyset$	$\{a + b\}$
4	$\{a + b, a * b, a + 1\}$	$\emptyset$
5	$\emptyset$	$\{a + b\}$

Example

$$AE_{entry}(1) = \emptyset$$
$$AE_{entry}(2) = AE_{exit}(1)$$
$$AE_{entry}(3) = AE_{exit}(2) \cap AE_{exit}(5)$$
$$AE_{entry}(4) = AE_{exit}(3)$$
$$AE_{entry}(5) = AE_{exit}(4)$$

$$AE_{exit}(1) = AE_{entry}(1) \cup \{a + b\}$$
$$AE_{exit}(2) = AE_{entry}(2) \cup \{a * b\}$$
$$AE_{exit}(3) = AE_{entry}(3) \cup \{a + b\}$$
$$AE_{exit}(4) = AE_{entry}(4) \setminus \{a + b, a * b, a + 1\}$$
$$AE_{exit}(5) = AE_{entry}(5) \cup \{a + b\}$$

Example

Equations for entry and exit functions:

$$AE_{entry}(1) = \emptyset$$
$$AE_{entry}(2) = AE_{exit}(1)$$
$$AE_{entry}(3) = AE_{exit}(2) \cap AE_{exit}(5)$$
$$AE_{entry}(4) = AE_{exit}(3)$$
$$AE_{entry}(5) = AE_{exit}(4)$$

$$AE_{exit}(1) = AE_{entry}(1) \cup \{a + b\}$$
$$AE_{exit}(2) = AE_{entry}(2) \cup \{a * b\}$$
$$AE_{exit}(3) = AE_{entry}(3) \cup \{a + b\}$$
$$AE_{exit}(4) = AE_{entry}(4) \setminus \{a + b, a * b, a + 1\}$$
$$AE_{exit}(5) = AE_{entry}(5) \cup \{a + b\}$$

AE at the entry of Block 3 = AE available at the exit of Block 2 (when entering the loop for the first time) and of Block 5 (when coming back from the exit of the loop)

Example

$[x \coloneqq a + b]^1;$
$[y \coloneqq a * b]^2;$
$while\ [y > a + b]^3\ do$
$\quad [a \coloneqq a + 1]^4;$
$\quad [x \coloneqq a + b]^5;$

$\boldsymbol{l}$	$\boldsymbol{AE_{entry}(l)}$	$\boldsymbol{AE_{exit}(l)}$
1	$\emptyset$	$\{a + b\}$
2	$\{a + b\}$	$\{a + b, a * b\}$
3	$\{a + b\}$	$\{a + b\}$
4	$\{a + b\}$	$\emptyset$
5	$\emptyset$	$\{a + b\}$