COMP0174 Practical Program Analysis
Live Variables Analysis

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Four Classic Analyses

	Forward	Backward
Must	Available Expressions	Very Busy Expressions
May	Reaching Definitions	**Live Variables**

Live Variables

A **variable is live** at the exit from a label if there exists a path from the label to a use of the variable that does not re-define the variable.

Live variables analysis determines for each program point, which variables may be live at the exit from the point.

It is *backward may* analysis.

Applications: Optimization (don't store variables that aren't live, eliminate assignments where variables are dead)

Example

$[x := 2]^1;$
$[y := 4]^2;$
$[x := 1]^3;$
$if\ [y > x]^4\ then$
$\quad [z := y]^5;$
$else$
$\quad [z := y * y]^6;$
$[x := z]^7$

The variable x is not live at the exit from label 1 (the assignment is redundant).

Both x and y are live at the exit from label 3.

Killed Variables

A variable is killed by an assignment if it appears on the left hand side of it:

$$kill_{LV}: Blocks_* \rightarrow P(Var_*)$$

$$kill_{LV}\left([x := a]^l\right) = \{x\}$$
$$kill_{LV}\left([skip]^l\right) = \emptyset$$
$$kill_{LV}\left([b]^l\right) = \emptyset$$

Generated Variables

A variable is generated in the block:

$$gen_{LV} : Blocks_* \rightarrow P(Var_*)$$

$$gen_{LV}([x := a]^l) = Vars(a)$$
$$gen_{LV}([skip]^l) = \emptyset$$
$$gen_{LV}([b]^l) = Vars(b)$$

if it appears on the **right-hand side** of an assignment or in some **condition.**

Analysis

The goal of the analysis is to compute the smallest set satisfying the equation for LV_{exit}:

$$LV_{exit}(l) = \begin{cases} \emptyset & if\ l = final(program) \\ \cup \left\{ LV_{entry}(l') \mid (l', l) \in flow^R(program) \right\} & otherwise \end{cases}$$

$$LV_{entry}(l) = \left(LV_{exit}(l) \setminus kill\left(B^l\right) \right) \cup gen_{LV}\left(B^l\right)$$

where $B^l \in blocks(program)$

Example

$[x := 2]^1;$
$[y := 4]^2;$
$[x := 1]^3;$
$if\ [y > x]^4\ then$
$\quad [z := y]^5;$
$else$
$\quad [z := y * y]^6;$
$[x := z]^7$

l	$kill_{AE}(l)$	$gen_{AE}(l)$
1	$\{x\}$	$\emptyset$
2	$\{y\}$	$\emptyset$
3	$\{x\}$	$\emptyset$
4	$\emptyset$	$\{x, y\}$
5	$\{z\}$	$\{y\}$
6	$\{z\}$	$\{y\}$
7	$\{x\}$	$\{z\}$

Example

$LV_{entry}(1) = LV_{exit}(1) \setminus \{x\}$
$LV_{entry}(2) = LV_{exit}(2) \setminus \{y\}$
$LV_{entry}(3) = LV_{exit}(3) \setminus \{x\}$
$LV_{entry}(4) = LV_{exit}(4) \cup \{x, y\}$
$LV_{entry}(5) = (LV_{exit}(5) \setminus \{z\}) \cup \{y\}$
$LV_{entry}(6) = (LV_{exit}(6) \setminus \{z\}) \cup \{y\}$
$LV_{entry}(7) = \{z\}$

$LV_{exit}(1) = LV_{entry}(2)$
$LV_{exit}(2) = LV_{entry}(3)$
$LV_{exit}(3) = LV_{entry}(4)$
$LV_{exit}(4) = LV_{entry}(5) \cup LV_{entry}(6)$
$LV_{exit}(5) = LV_{entry}(7)$
$LV_{exit}(6) = LV_{entry}(4)$
$LV_{exit}(7) = \emptyset$

Example

Equations for entry and exit functions:

$$LV_{exit}(1) = LV_{entry}(2)$$
$$LV_{exit}(2) = LV_{entry}(3)$$
$$LV_{exit}(3) = LV_{entry}(4)$$
$$LV_{exit}(4) = LV_{entry}(5) \cup LV_{entry}(6)$$
$$LV_{exit}(5) = LV_{entry}(7)$$
$$LV_{exit}(6) = LV_{entry}(4)$$
$$LV_{exit}(7) = \emptyset$$

$$LV_{entry}(1) = LV_{exit}(1) \setminus \{x\}$$
$$LV_{entry}(2) = LV_{exit}(2) \setminus \{y\}$$
$$LV_{entry}(3) = LV_{exit}(3) \setminus \{x\}$$
$$LV_{entry}(4) = LV_{exit}(4) \cup \{x, y\}$$
$$LV_{entry}(5) = (LV_{exit}(5) \setminus \{z\}) \cup \{y\}$$
$$LV_{entry}(6) = (LV_{exit}(6) \setminus \{z\}) \cup \{y\}$$
$$LV_{entry}(7) = \{z\}$$

Live variables at exit of Block 4 = union of live variables at entry of Block 5 and Block 6, corresponding to both the branches (if the variables are used somewhere, they are going to be live also in the if condition).

Example

$[x := 2]^1;$
$[y := 4]^2;$
$[x := 1]^3;$
$if\ [y > x]^4\ then$
$\quad [z := y]^5;$
$else$
$\quad [z := y * y]^6;$
$[x := z]^7$

l	$LV_{entry}\ (l)$	$LV_{exit}\ (l)$
1	$\emptyset$	$\emptyset$
2	$\emptyset$	$\{y\}$
3	$\{y\}$	$\{x, y\}$
4	$\{x, y\}$	$\{y\}$
5	$\{y\}$	$\{z\}$
6	$\{y\}$	$\{z\}$
7	$\{z\}$	$\emptyset$