COMP0174 Practical Program Analysis
Reaching Definition Analysis

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Four Classic Analyses

	Forward	Backward
Must	Available Expressions	Very Busy Expressions
May	**Reaching Definitions**	Live Variables

Reaching Definitions

Reaching definitions analysis determines for each program point, which assignments may have been made and not overwritten, when program execution reaches this point along some path.

It is *forward may* analysis.

Applications: Bug-finding (uninitialized variables), optimization (constant propagation)

Example

$[x := 5]^1;$
$[y := 1]^2;$
$while\ [x > 1]^3\ do$
$\quad [y := x * y]^4;$
$\quad [x := x - 1]^5;$

All assignments reach the entry of 4; only the assignments 1,4,5 reach the entry of 5.

because the assignment 2 is overwritten by the assignment 4

Killed Assignments

An assignment is killed by a block if the block assigns a new value to the variable:

$$kill_{RD} : Blocks_* \rightarrow \boxed{P(Var_* \times Lab_*)}$$

Set of pairs of variables and labels corresponding to the place where the variables are assigned

Killed Assignments

An assignment is killed by a block if the block assigns a new value to the variable:

$$kill_{RD}([x := a]^l) = \{(x, ?)\} \cup \{(x, l') \mid B^{l'} \mathit{is\ an\ assignment\ to\ } x\}$$
$$kill_{RD}([skip]^l) = \emptyset$$
$$kill_{RD}([b]^l) = \emptyset$$

where ? is the label for uninitialised variables.

Generated Assignments

Assignments that appear in the block:

$$gen_{RD}\left([x := a]^l\right) = \{(x, l)\}$$
$$gen_{RD}\left([skip]^l\right) = \emptyset$$
$$gen_{RD}\left([b]^l\right) = \emptyset$$

Analysis

The goal of the analysis is to compute the smallest set satisfying the equation for RD_{entry}:

$$RD_{entry}(l) = \begin{cases} \{(x, ?) \mid x \in Vars\} & if\ l = init(program) \\ \cup \{RD_{exit}(l') \mid (l', l) \in flow\ (program)\} & otherwise \end{cases}$$

$$RD_{exit}(l) = \left(RD_{entry}(l) \setminus kill_{RD}(B^l)\right) \cup gen_{RD}(B^l)$$

where $B^l \in blocks(program)$

Example

$$
\begin{aligned}
&[x := 5]^1;\\
&[y := 1]^2;\\
&while\ [x > 1]^3\ do\\
&\qquad [y := x * y]^4;\\
&\qquad [x := x - 1]^5;
\end{aligned}
$$

$\boldsymbol{l}$	$\boldsymbol{kill_{RD}}(\boldsymbol{l})$	$\boldsymbol{gen_{RD}}(\boldsymbol{l})$
1	$\{(x, ?), (x, 1), (x, 5)\}$	$\{(x, 1)\}$
2	$\{(y, ?), (y, 2), (y, 4)\}$	$\{(y, 2)\}$
3	$\emptyset$	$\emptyset$
4	$\{(y, ?), (y, 2), (y, 4)\}$	$\{(y, 4)\}$
5	$\{(x, ?), (x, 1), (x, 5)\}$	$\{(x, 5)\}$

Example

$RD_{entry}(1) = \{(x, ?\,), (y, ?\,)\}$
$RD_{entry}(2) = RD_{exit}(1)$
$RD_{entry}(3) = RD_{exit}(2) \cup RD_{exit}(5)$
$RD_{entry}(4) = RD_{exit}(3)$
$RD_{entry}(5) = RD_{exit}(4)$

$RD_{exit}(1) = \left(RD_{entry}(1) \setminus \{(x, ?\,), (x, 1), (x, 5)\}\right) \cup \{(x, 1)\}$
$RD_{exit}(2) = \left(RD_{entry}(2) \setminus \{(y, ?\,), (y, 2), (y, 4)\}\right) \cup \{(y, 2)\}$
$RD_{exit}(3) = RD_{entry}(3)$
$RD_{exit}(4) = \left(RD_{entry}(4) \setminus \{(y, ?\,), (y, 2), (y, 4)\}\right) \cup \{(y, 4)\}$
$RD_{exit}(5) = \left(RD_{entry}(5) \setminus \{(x, ?\,), (x, 1), (x, 5)\}\right) \cup \{(x, 5)\}$

Example

$[x := 5]^1;$
$[y := 1]^2;$
$while\ [x > 1]^3\ do$
$\quad [y := x * y]^4;$
$\quad [x := x - 1]^5;$

$\boldsymbol{l}$	$\boldsymbol{RD_{entry}\ (l)}$	$\boldsymbol{RD_{exit}\ (l)}$
1	$\{(x, ?), (y, ?)\}$	$\{(y, ?), (x, 1)\}$
2	$\{(y, ?), (x, 1)\}$	$\{(x, 1), (y, 2)\}$
3	$\{(x, 1), (y, 2), (y, 4), (x, 5)\}$	$\{(x, 1), (y, 2), (y, 4), (x, 5)\}$
4	$\{(x, 1), (y, 2), (y, 4), (x, 5)\}$	$\{(x, 1), (y, 4), (x, 5)\}$
5	$\{(x, 1), (y, 4), (x, 5)\}$	$\{(y, 4), (x, 5)\}$