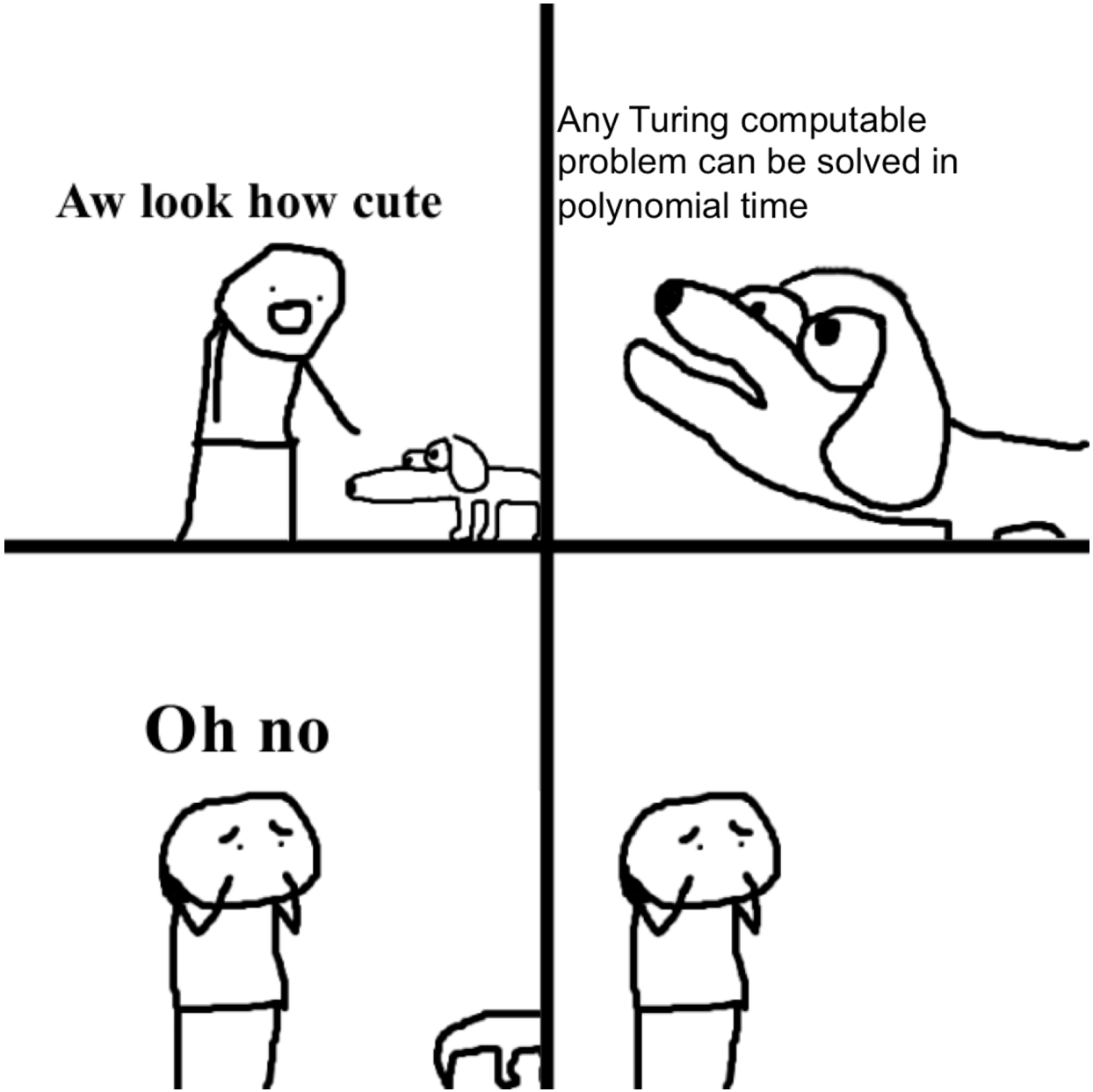
Lecture 7: More NP-Complete Problems



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- All decidable problems are either in P or NP-hard. Probably not. Some problems (e.g., graph isomorphism) are conjectured to be in $NP \setminus P$ but not NP-hard.

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k-SAT is similar to SAT, but each clause may have at most k literals. E.g.,

$$(p \vee \overline{p} \vee q) \wedge (p \vee q \vee r) \wedge (\overline{p} \vee \overline{q} \vee \overline{r}) \wedge (p \vee \overline{q} \vee r)$$

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Yes-instance: if it is satisfiable.
No-instance: otherwise.

SAT $\leq_p$ 3SAT

Idea

$$(l_1 \vee l_2 \vee l_3 \vee l_4)$$

$\mathbf{SAT} \leq_p \mathbf{3SAT}$

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$$(l_1 \vee l_2 \vee l_3 \vee l_4) \mapsto (l_1 \vee l_2 \vee x) \wedge (\overline{x} \vee l_3 \vee l_4)$$

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$$g(C) = (l_1 \vee p_1) \wedge (\overline{p_1} \vee l_2 \vee p_2) \wedge (\overline{p_2} \vee l_3 \vee p_3) \wedge$$
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Hence $v(g(C)) = T$.

$v(g(\phi)) = T$ holds for each clause, thus $g(\phi)$ is a yes-instance of 3-SAT.

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Easy to check that $v(\phi) = T$, so ϕ is a yes-instance of SAT.

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Runs in p-time (although much faster solutions are known!).

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We will use this to show that Independent Set (IS) and HCP are NP-complete.

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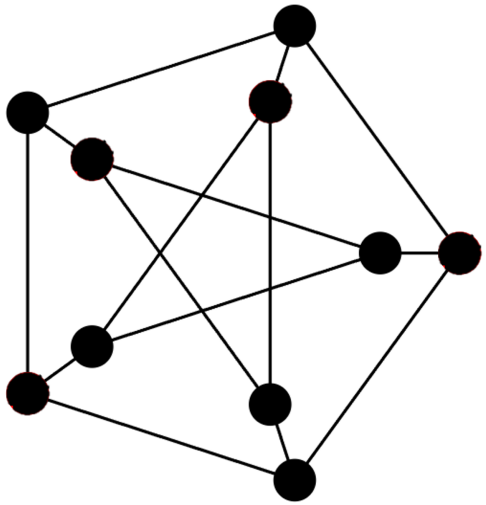

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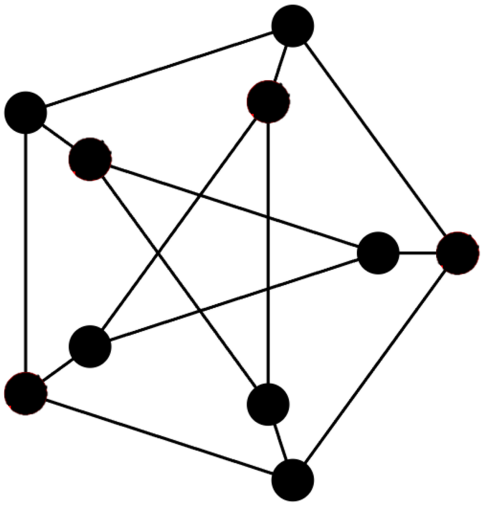

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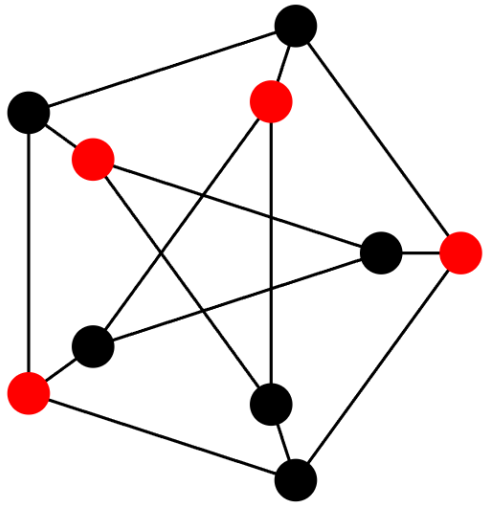

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A very simple algorithm:

(1) Guess a set of k vertices.

(2) Accept if there are no edges between the vertices.

Independent Set is *NPC* Part II: 3-*SAT* $\leq_p$ *IS*

We are given a 3-CNF formula ϕ, and need to construct an *IS* instance.

Independent Set is *NPC* Part II: 3-*SAT* $\leq_p$ *IS*

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If we are able to choose literals without picking a variable and its negation, we can satisfy the formula (e.g., using $x_1 = T, x_2 = x_3 = F$ in this example).

Independent Set is *NPC* Part II: 3-*SAT* $\leq_p$ *IS*

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Independent Set is *NPC* Part II: 3-*SAT* $\leq_p$ *IS*

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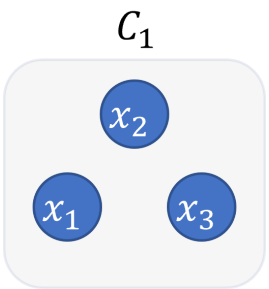

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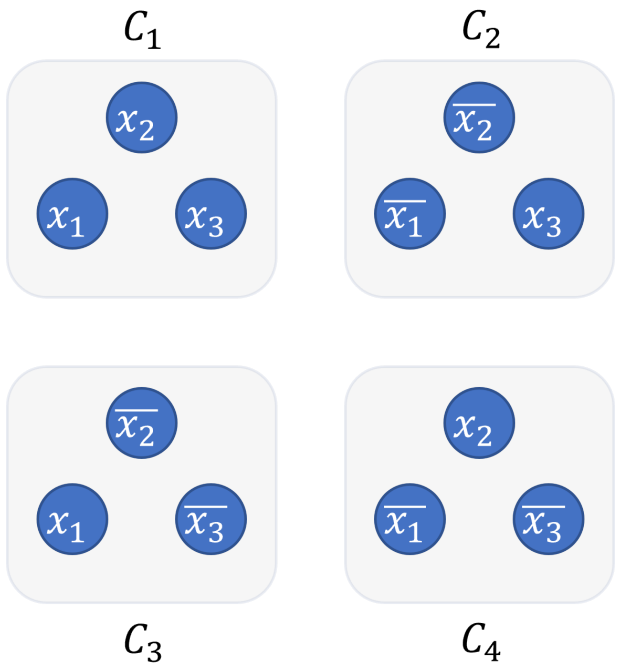

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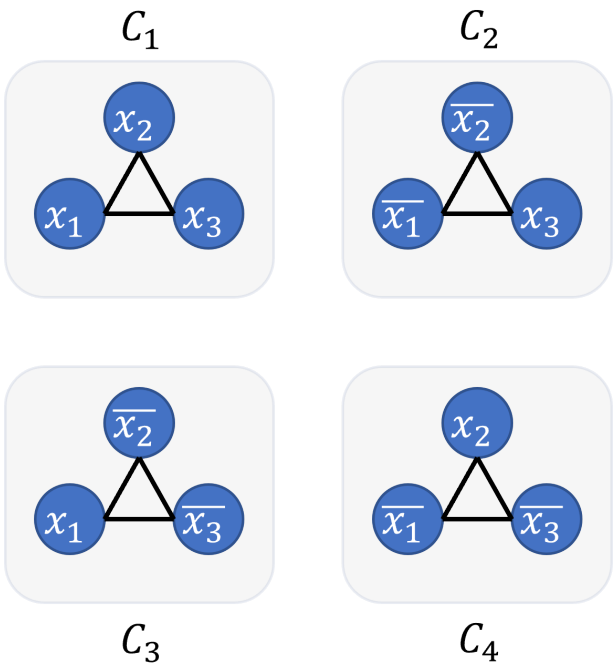

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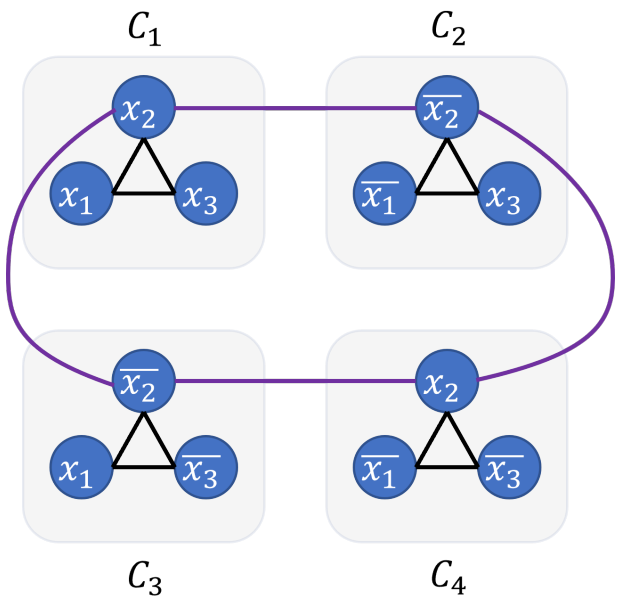

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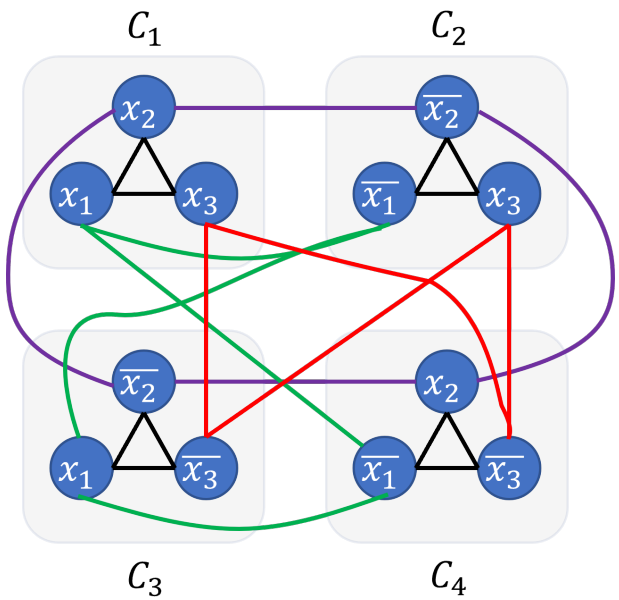

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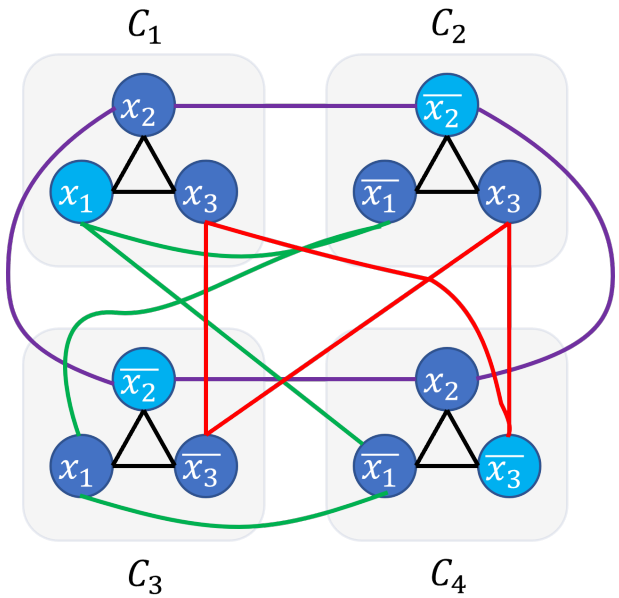

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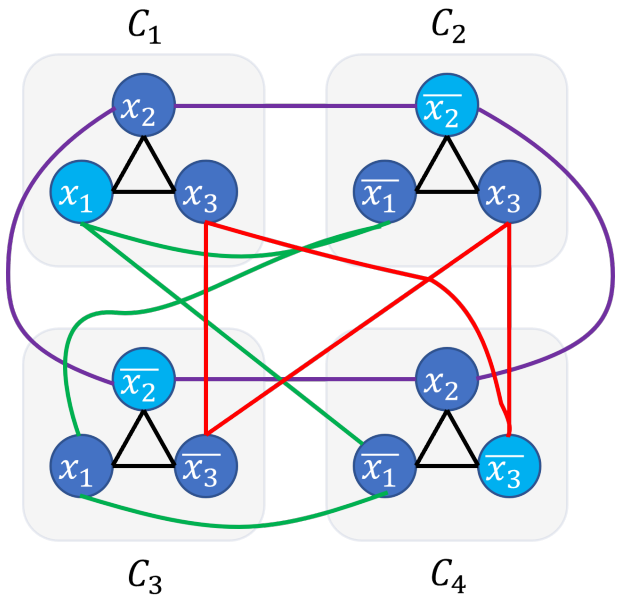
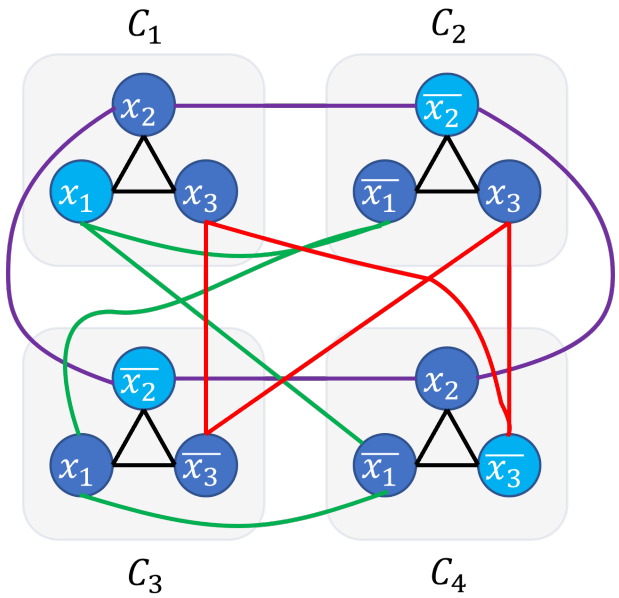


We found an independent set with m vertices, giving us the assignment $v(x_1) = T, v(x_2) = v(x_3) = F$.

Independent Set is *NPC* Part II: 3-*SAT* $\leq_p$ *IS*



(Not a formal proof!) We have that an IS of size m corresponds to choosing a T representative literal for each clause and thus to a valid assignment for the variables.

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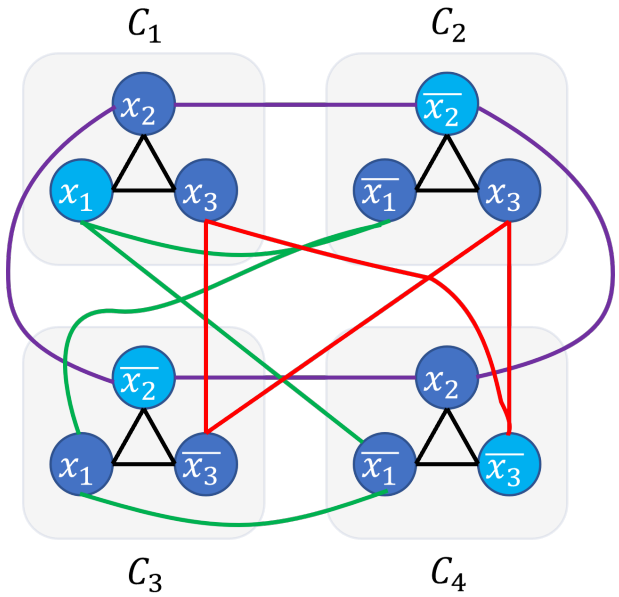

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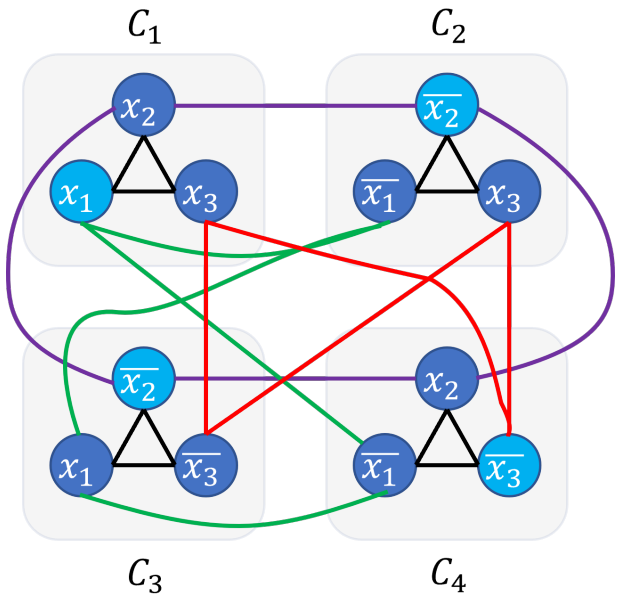

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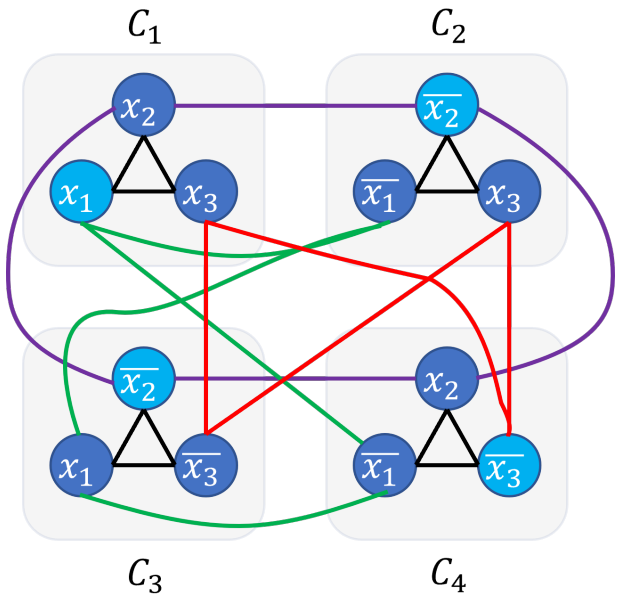
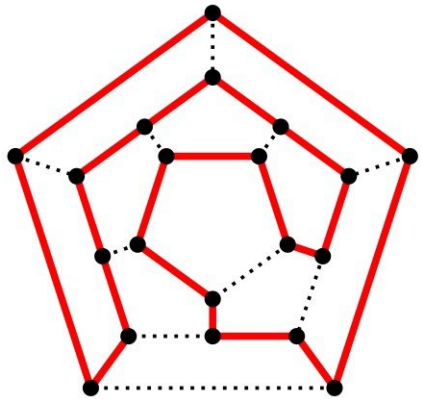


(Not a formal proof!) We have that an IS of size m corresponds to choosing a T representative literal for each clause and thus to a valid assignment for the variables.

We need to analyze the complexity. What is the resulting graph size? We get $3m$ vertices, and $O(m^2)$ edges, so the reduction is polynomial.

HCP is *NPC* Part I: 3-*SAT* $\leq_p$ *DHCP*

Reminder: in the HCP problem we get an (undirected) graph and need to determine if it has a Hamiltonian circuit.



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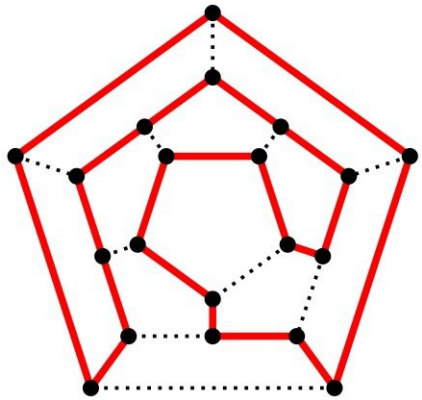

Instead of directly showing that 3-$SAT \leq_p$ *HCP*, we will go through DHCP (Directed HCP) and show (I) 3-$SAT \leq_p$ *DHCP* and (II) *DHCP* $\leq_p$ *HCP*.

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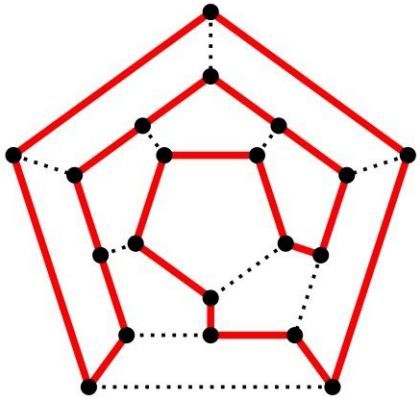

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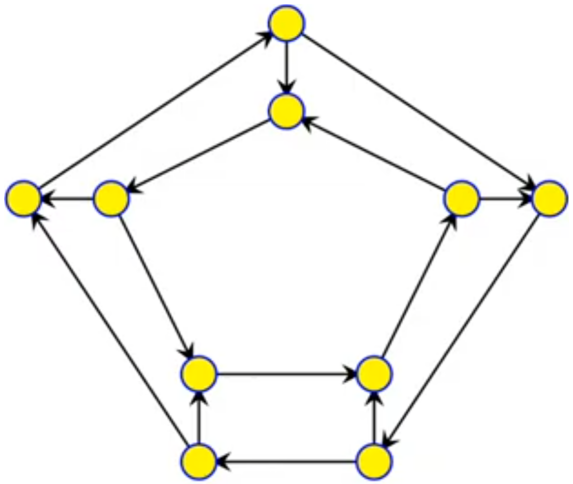

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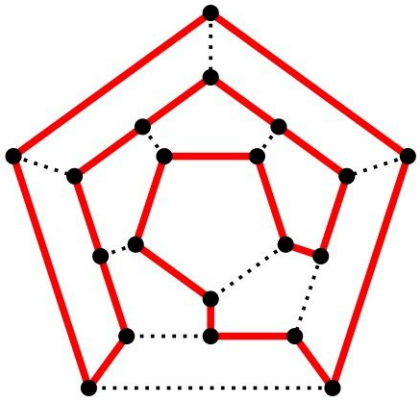

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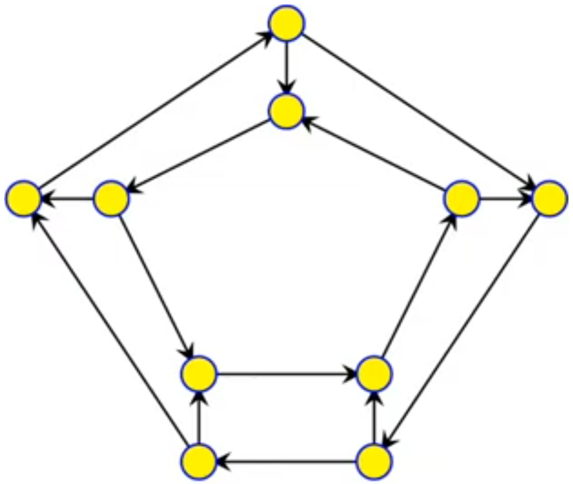

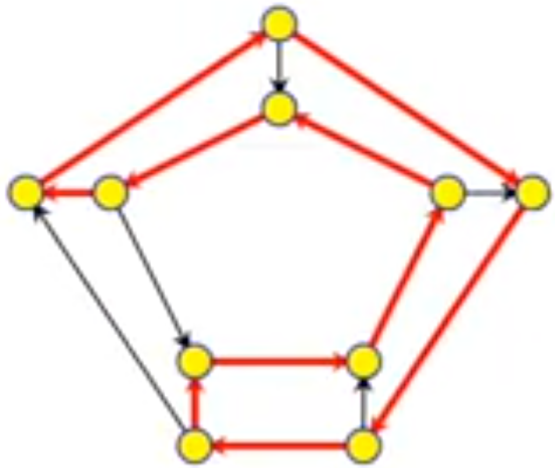

HCP is *NPC* Part I: 3-*SAT* $\leq_p$ *DHCP*

We start from a 3-CNF formula ϕ with variables $x_1, \ldots, x_n$ and clauses $C_1, \ldots, C_m$ and construct a graph.

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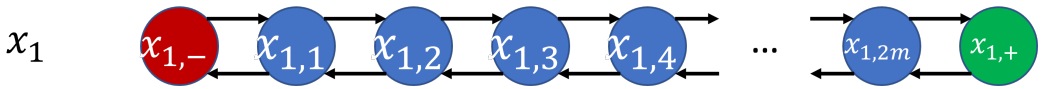
Basic graph: (1) A bi-directional path of length $2m+2$ for each variable.



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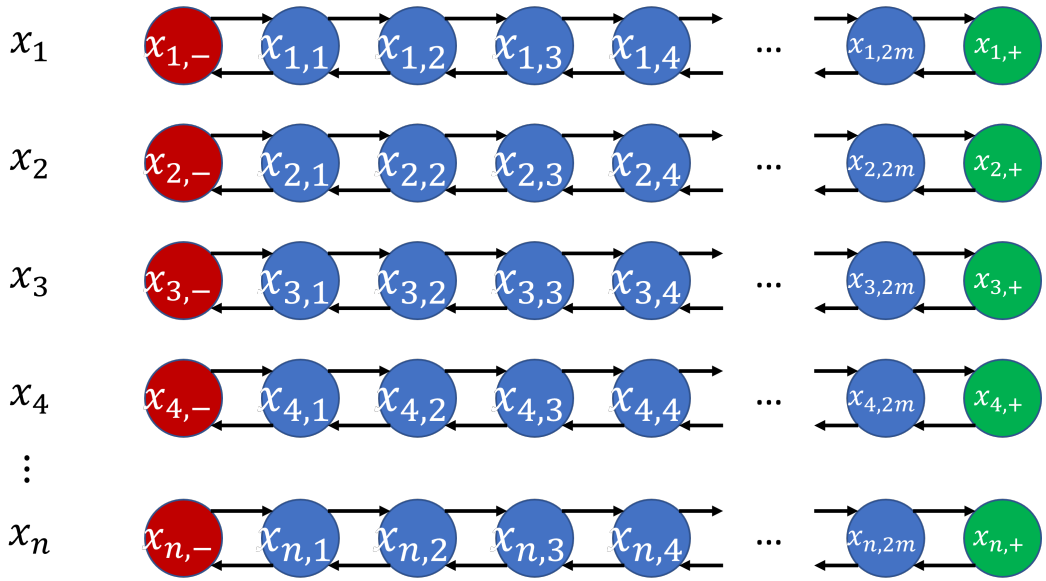

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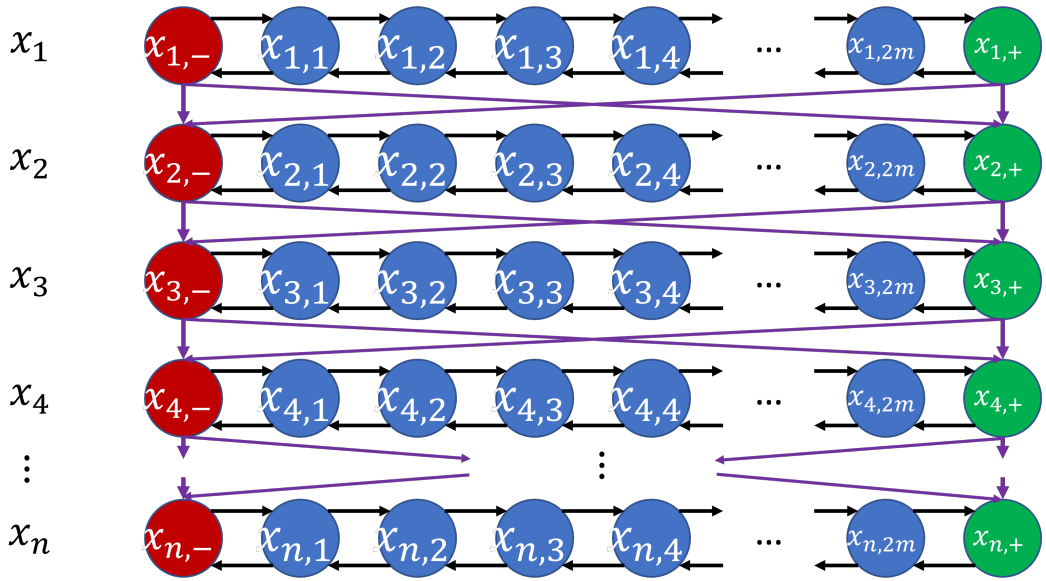

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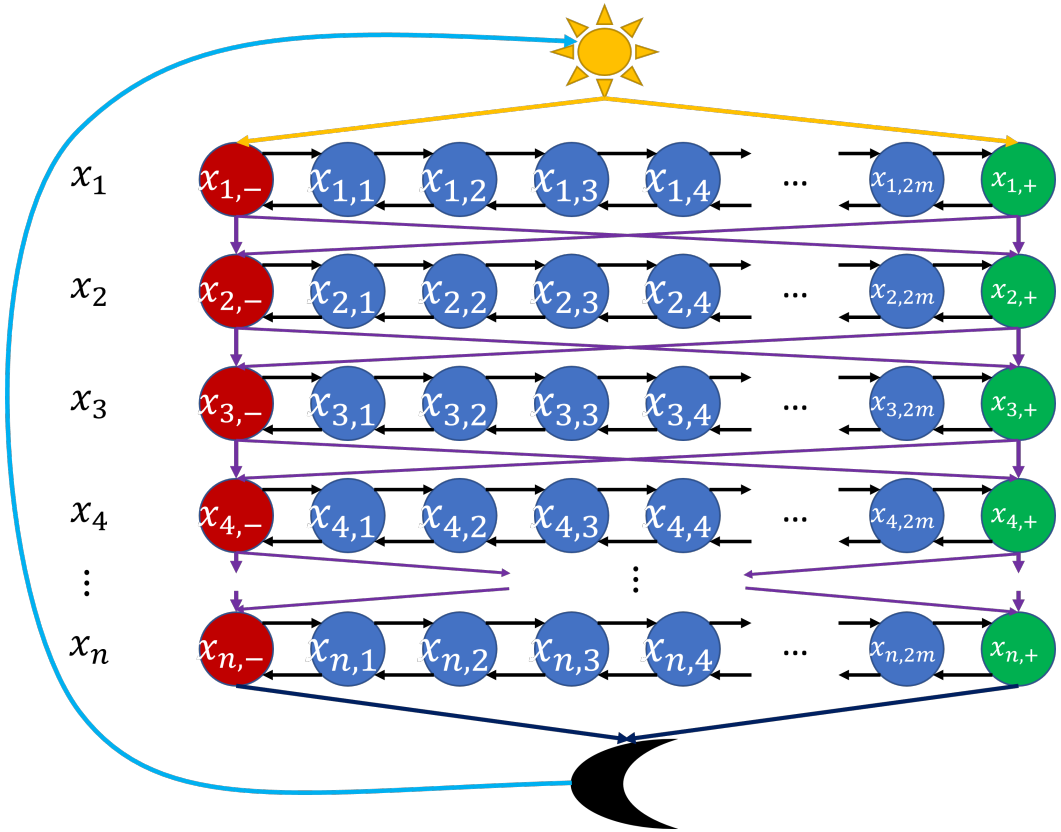
Basic graph: (3) Add a "sun" and "moon" vertices.



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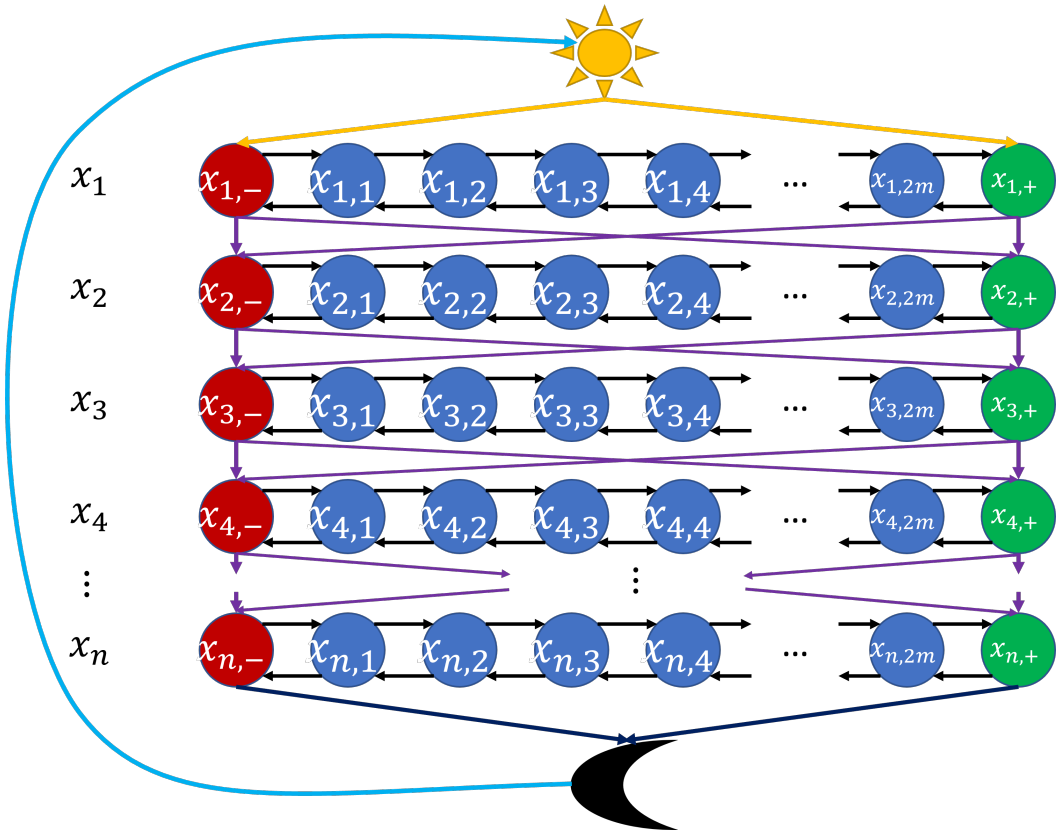

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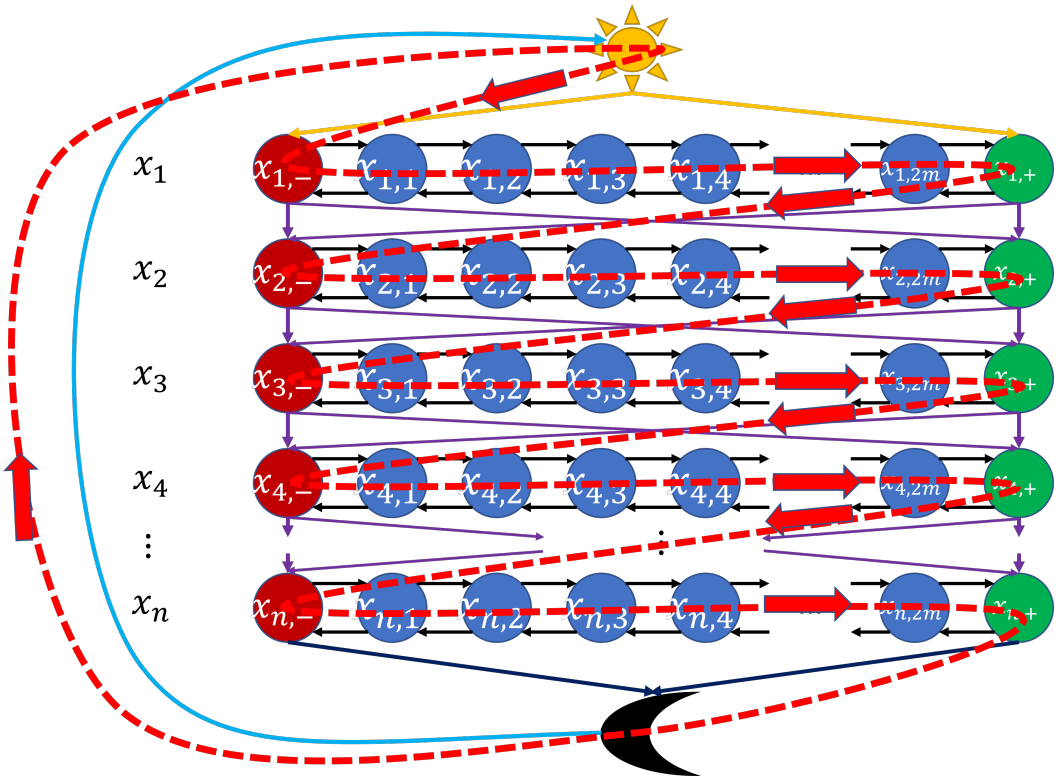

HCP is *NPC* Part I: 3-*SAT* $\leq_p$ *DHCP*

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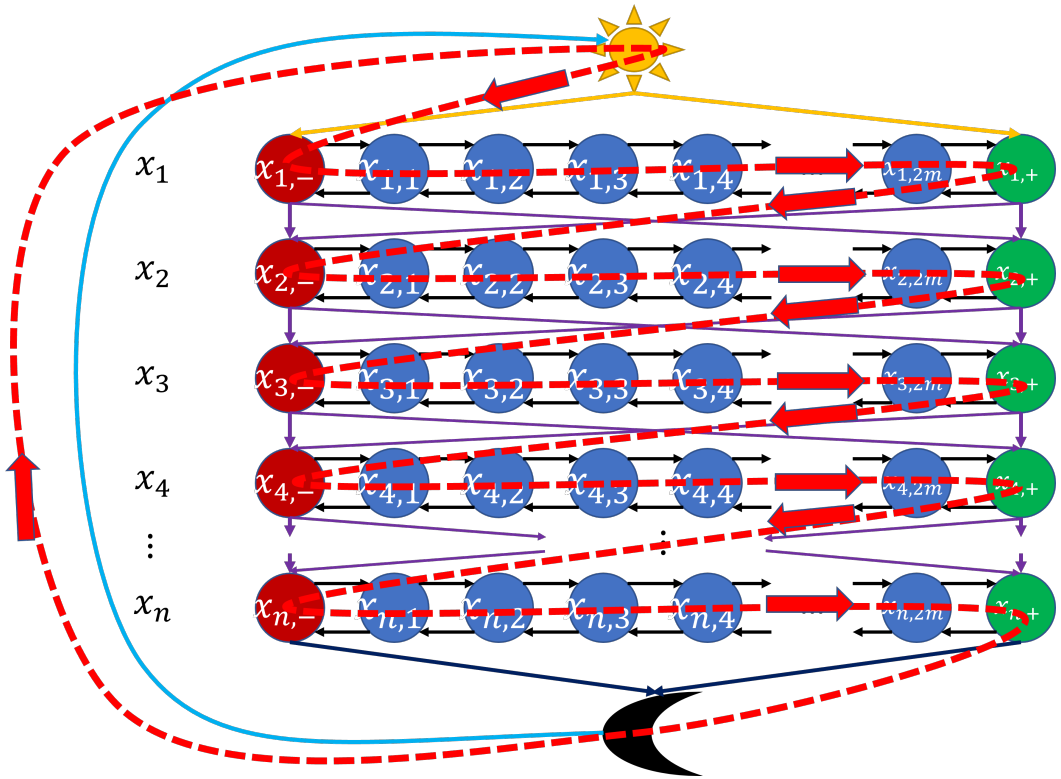

Convention: The last x_i vertex on the circuit is $x_{i,+} \iff v(x_i) = T$.

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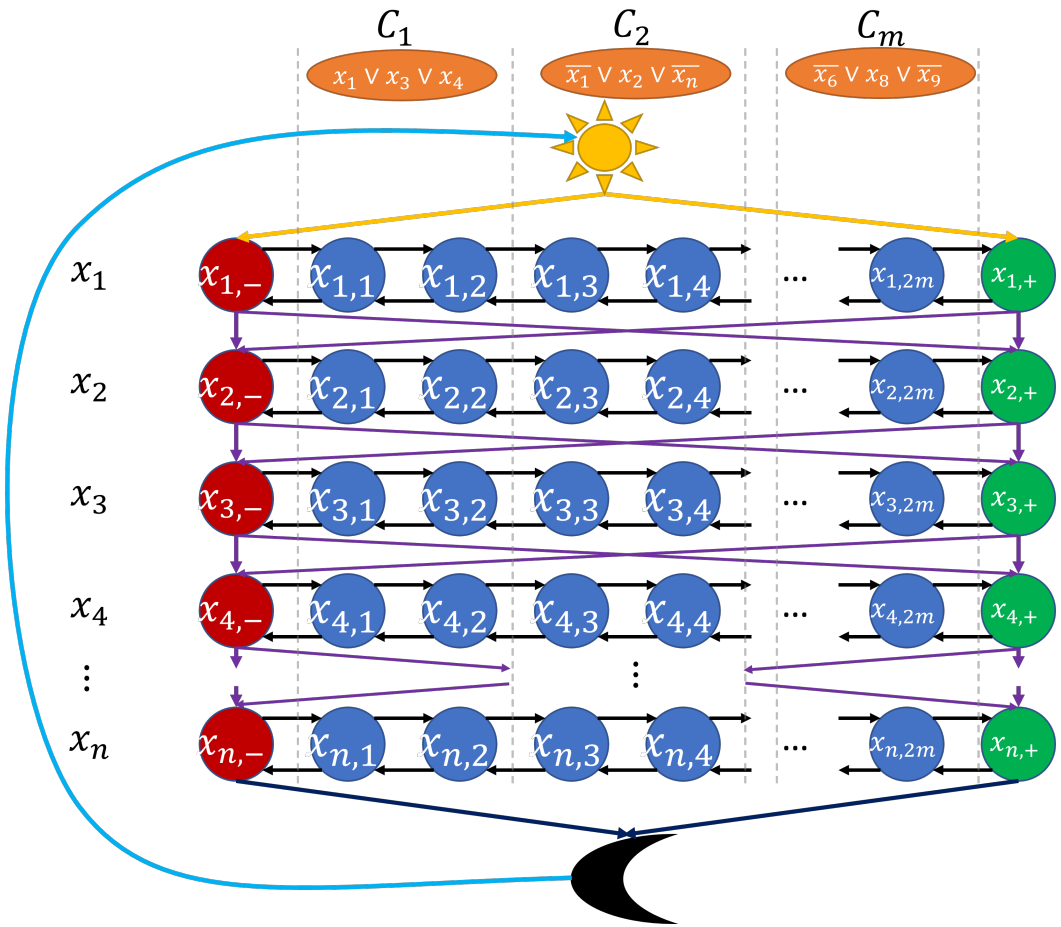
Each clause C_j has a vertex and two columns $\{x_{i,2j}, x_{i,2j+1} \mid i \in \{1, \ldots, n\}\}$.



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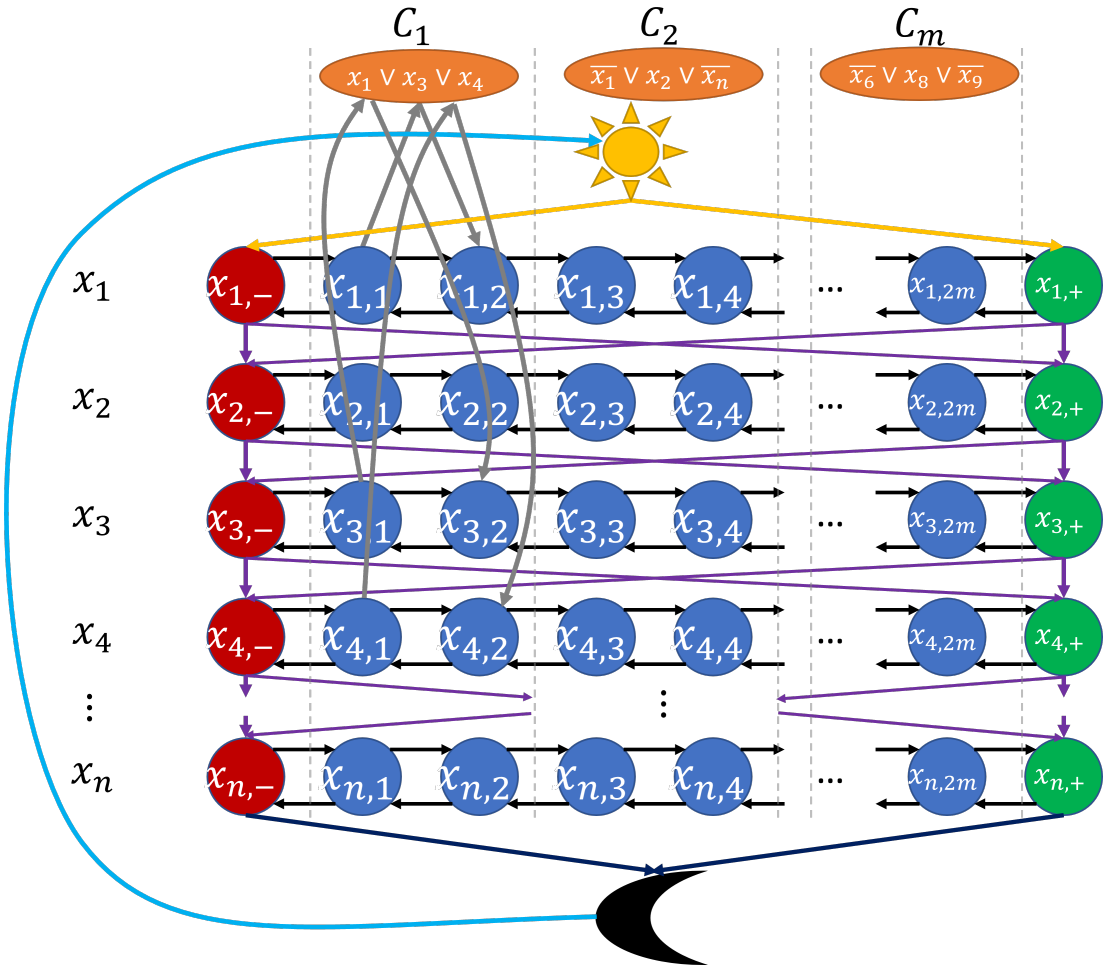

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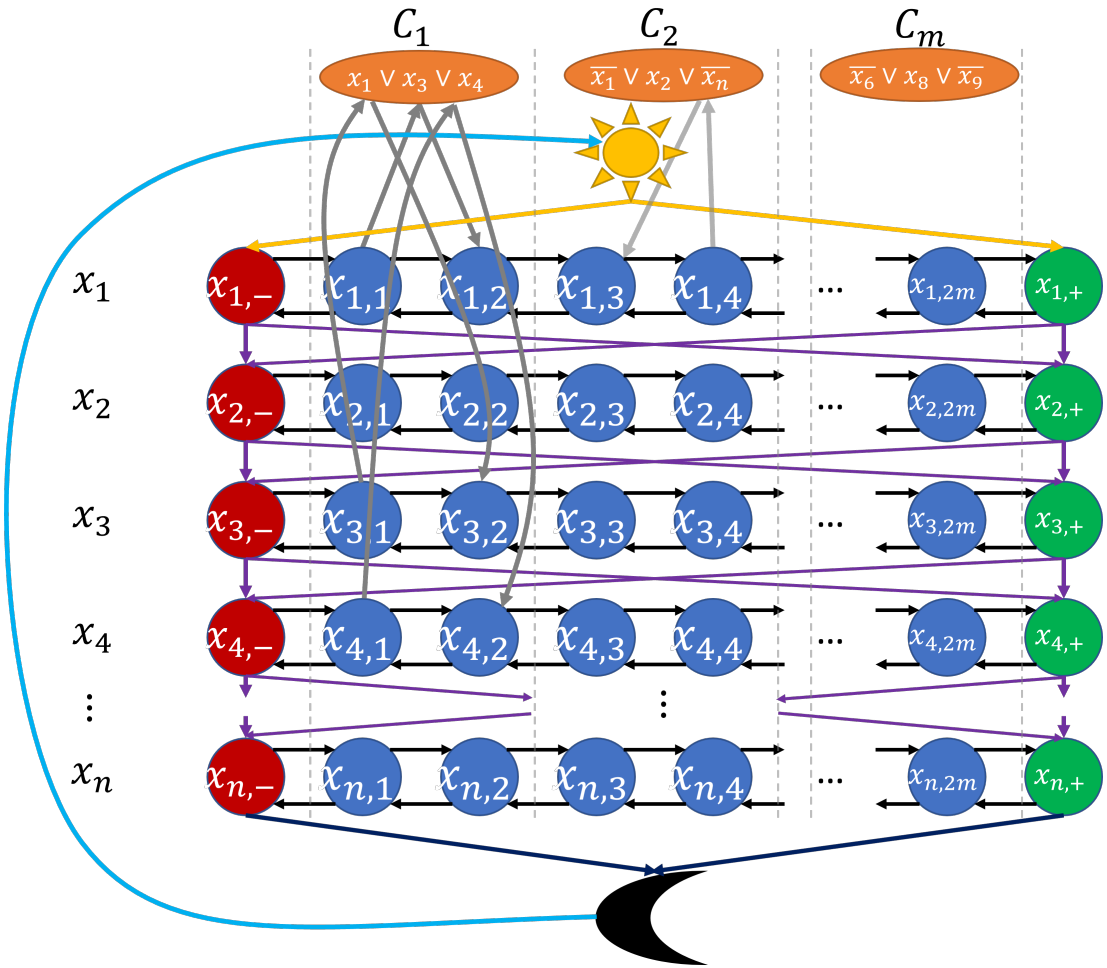
Observe that assignment for x_1 can satisfy C_1 or C_2, but not both!



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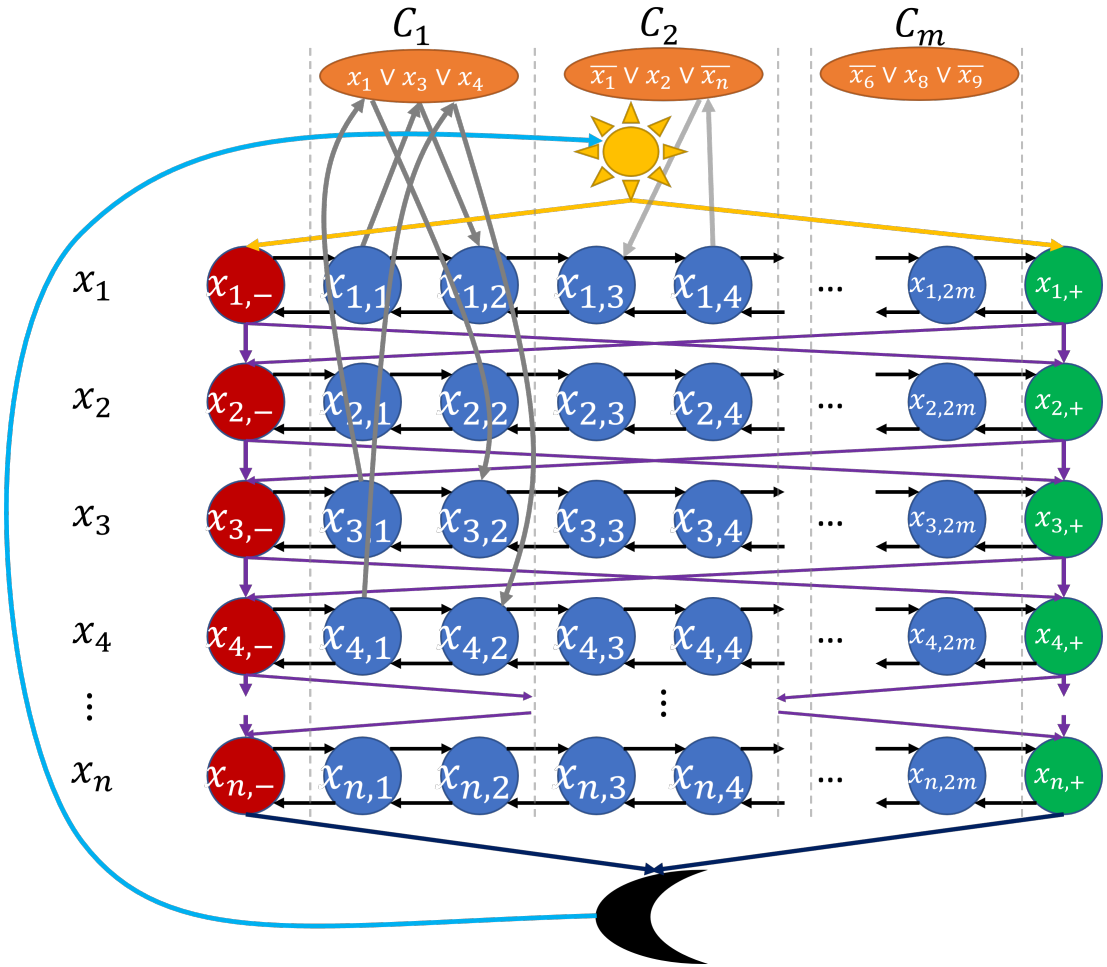
For correctness, showing that a satisfying assignment corresponds to a circuit is easy (why?).



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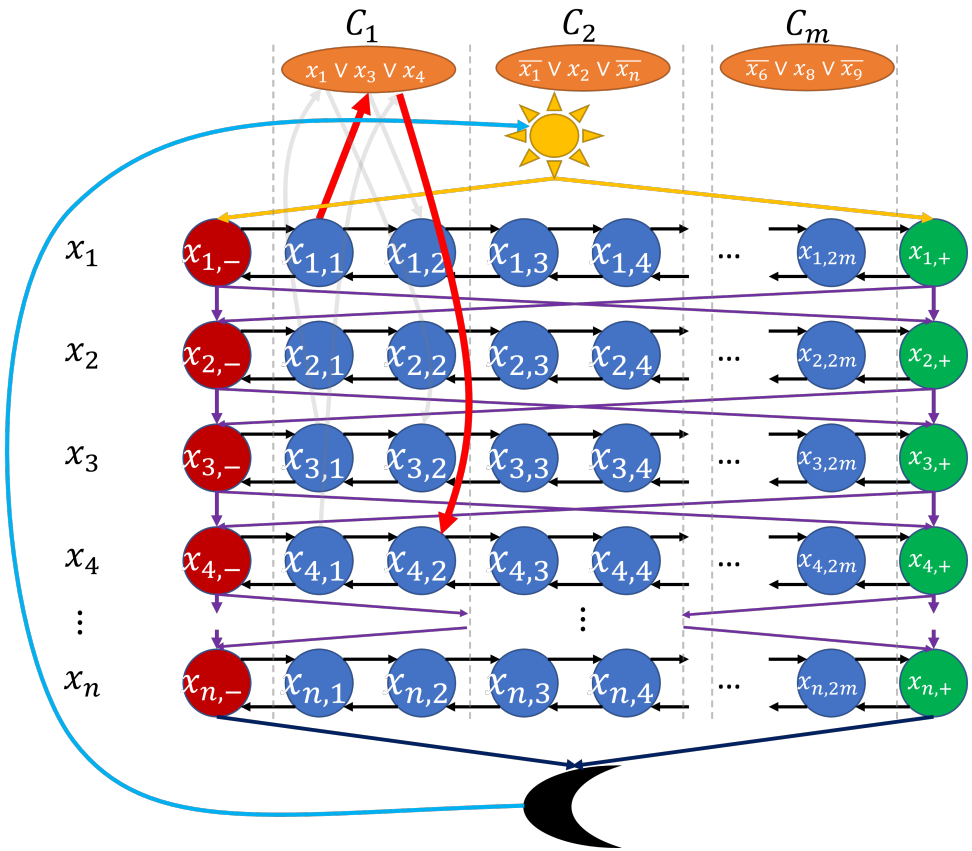
But why must a circuit correspond to a satisfying assignment?



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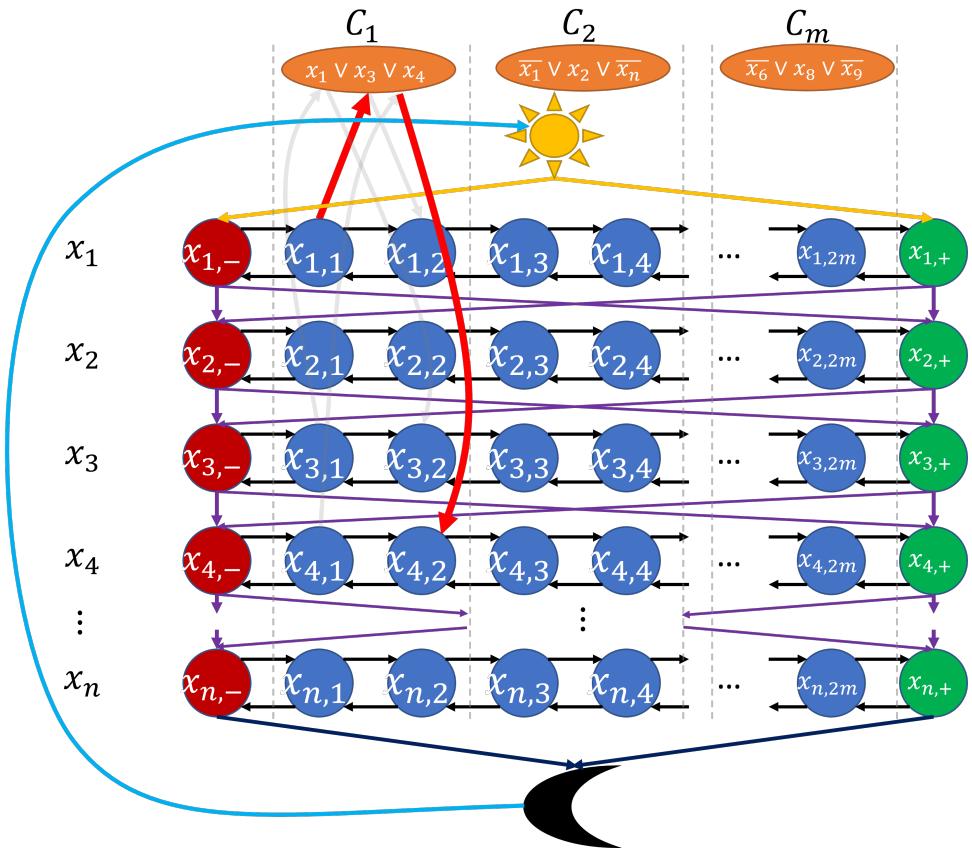

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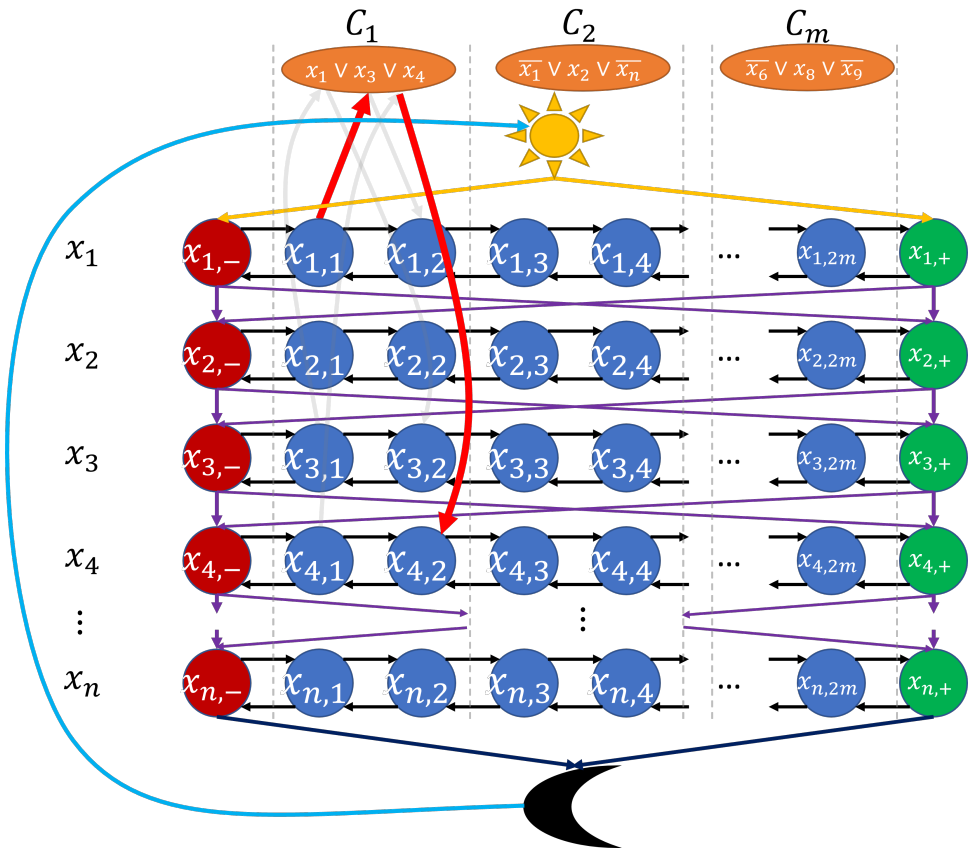

If the circuit "cheats" like that, how can it visit $x_{1,2}$?
How big is the resulting graph? $O(n \cdot m)$, which is polynomial in the formula size.

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We saw that 3-$SAT \leq_p DHCP$, and have that $DHCP \in NP$ (exercise!).

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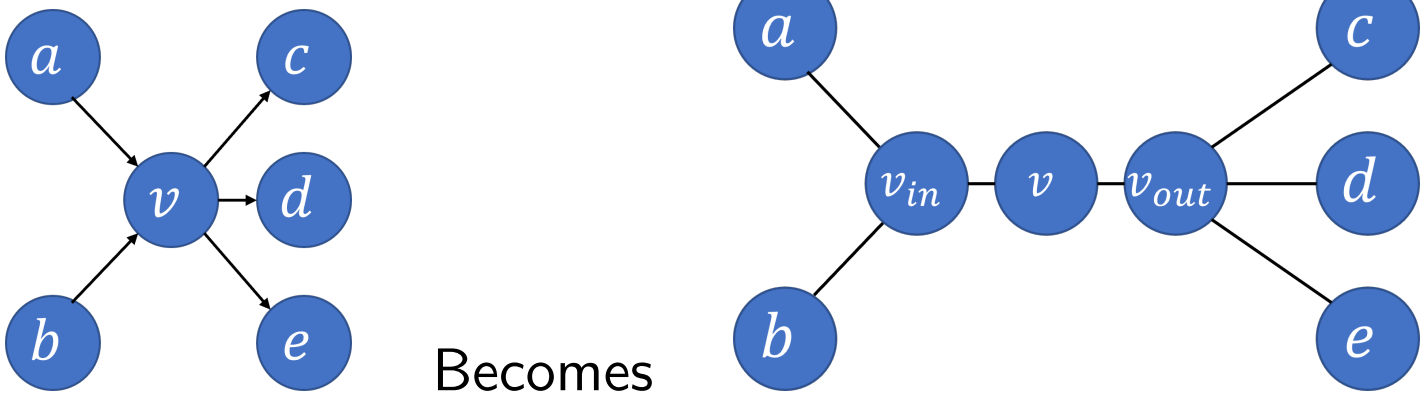

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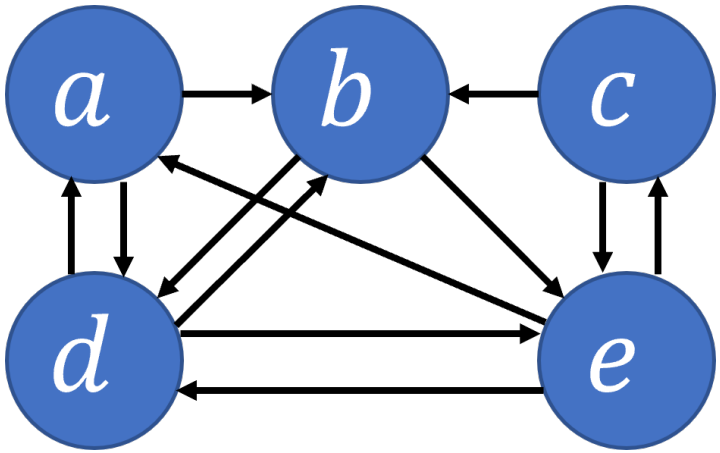

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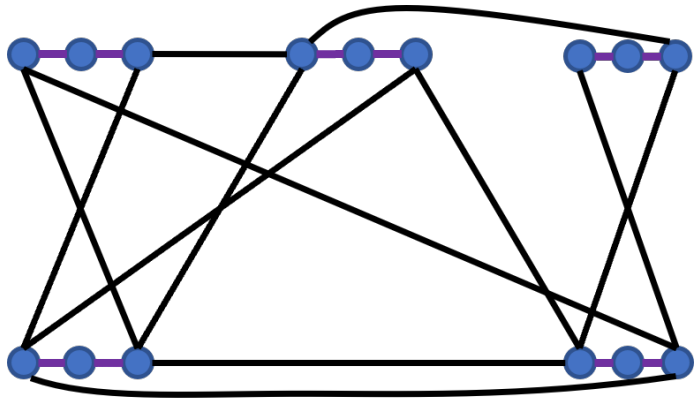

Correctness proof is left as an exercise.

NP-complete problems we saw so far, a summary

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Let's see one more, which is a bit different.

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A vertex cover $C \subseteq V$ of a graph (V, E) is a set such that each edge has an endpoint in C.

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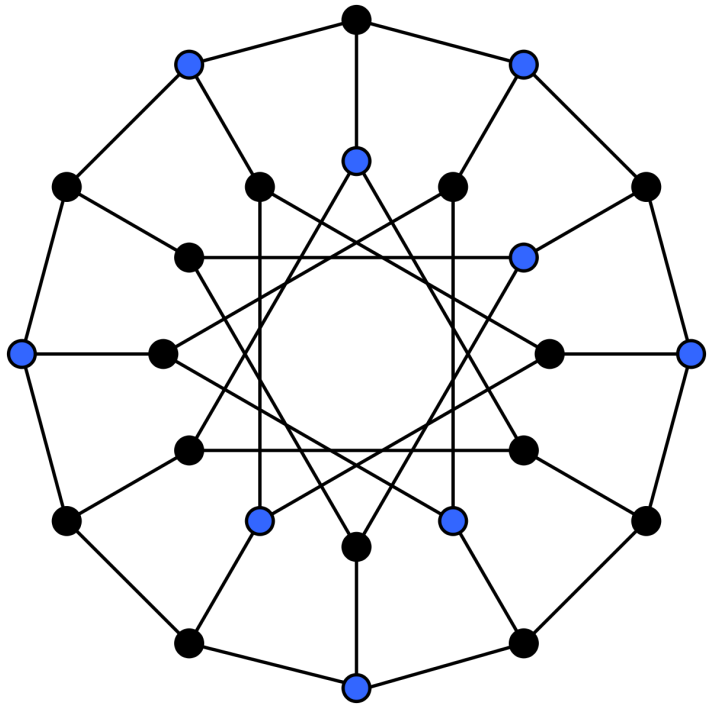

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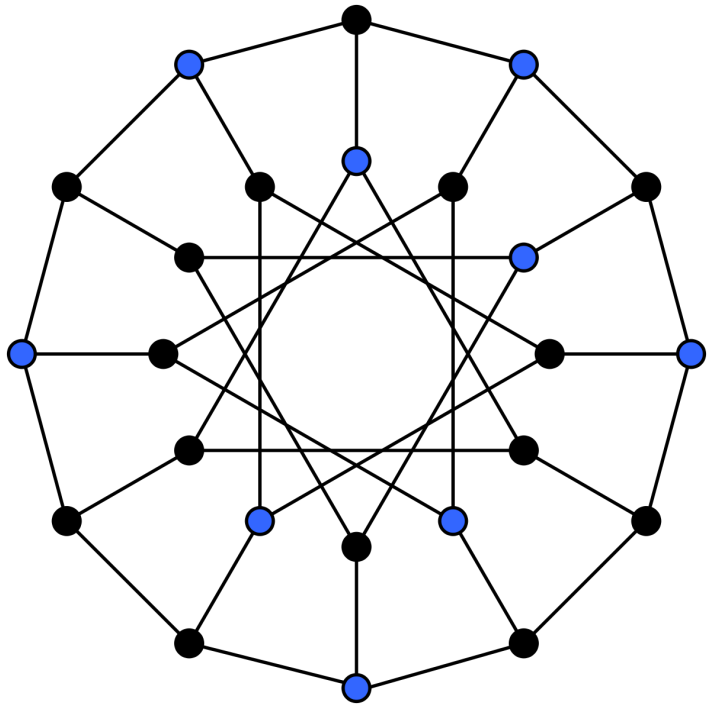

Any ideas?

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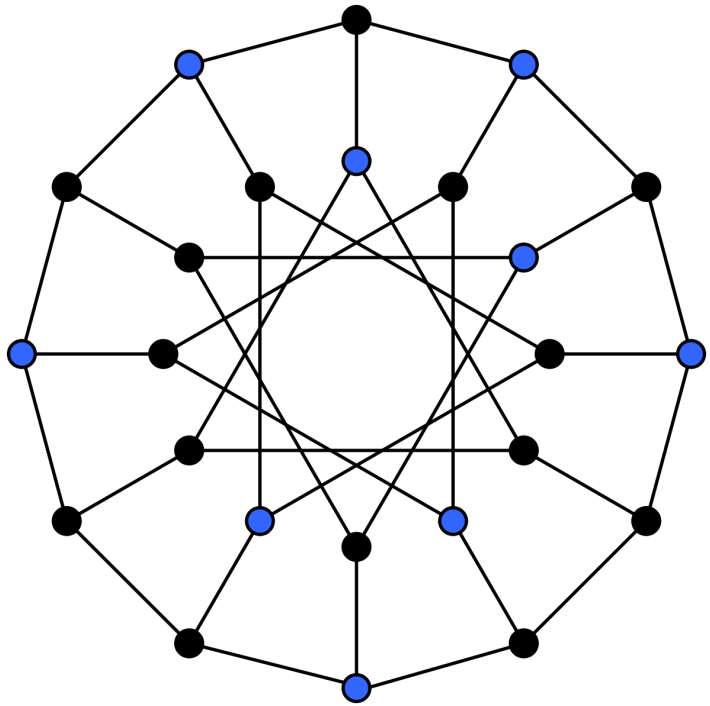

Any ideas? G has a VC of size $k \iff$ it has an IS of size $|V| - k$.

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