Week 2 – Camera & Robot Calibration

ELEC0144 Machine Learning for Robotics

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Schedule

Week	Lecture	Workshop	Assignment Deadlines
1	Introduction; Image Processing	Image Processing	
2	Camera and Robot Calibration	Camera and Robot Calibration	
3	Introduction to Neural Networks	Camera and Robot Calibration	Friday: Camera and Robot Calibration
4	MLP and Backpropagation	MLP and Backpropagation	
5	CNN and Image Classification	MLP and Backpropagation	
6	Object Detection	MLP and Backpropagation	Friday: MLP and Backpropagation
7	Path Planning	Path Planning	
8	Kalman Filter SLAM	Path Planning	
9	Extended Kalman Filter SLAM	Path Planning	
10	Particle Filter SLAM	Path Planning	Friday: Path Planning

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- Introduction
- Camera Calibration
  - Image Formation and Intrinsic Parameters
  - Extrinsic Parameters
  - Camera Matrix H
  - Camera Calibration and Calculation of H
  - Recovering Intrinsic Parameters from H
  - Recovering Extrinsic Parameters from H
  - Summary and Discussions
- Robot Calibration

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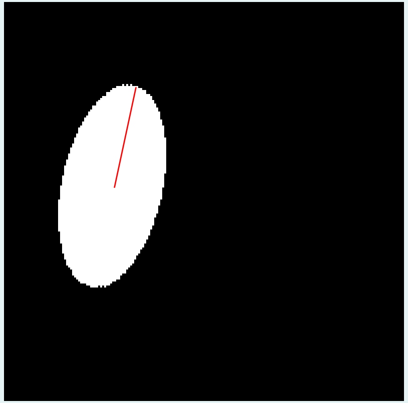
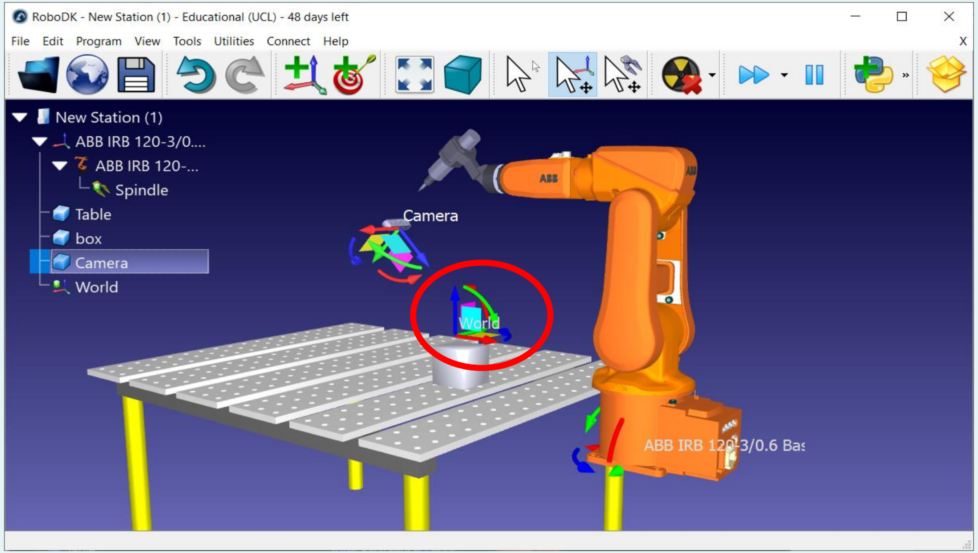
Introduction (1)



- So far, you have learnt how to determine the position and orientation of an object with respect to the 2D pixel frame.
- However, in order for the robot to come approach the object, it needs to know the position and orientation of the object with respect to the 3D robot frame.
- How can this be done?
  - In other words, what is the relationship between the 2D pixel frame and the 3D robot frame?

Introduction (2)

- The idea is to introduce an intermediary frame called the "world frame".
- Find:
  - Relationship between the 2D pixel frame with the 3D world frame – "Camera Calibration".
  - Relationship between the 3D robot frame with the 3D world frame – "Robot Calibration".
- Then the relationship between the 2D pixel frame and the 3D robot frame can be calculated.



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Camera Calibration

- The purpose of camera calibration is to determine:
  - The intrinsic parameters of the camera:
    - Focal length;
    - Scaling factor;
    - Distortion etc.
  - The extrinsic parameters of the camera:
    - Position of world frame with respect to camera frame;
    - Orientation of world frame with respect to camera frame.
- The extrinsic parameters are exactly what we want, but the intrinsic parameters are important as well.

Image Formation (1)

- We will use the pinhole projection model to describe the mathematical relationship between the coordinates of a point in 3D space and its projection onto the image plane of an ideal pinhole camera.

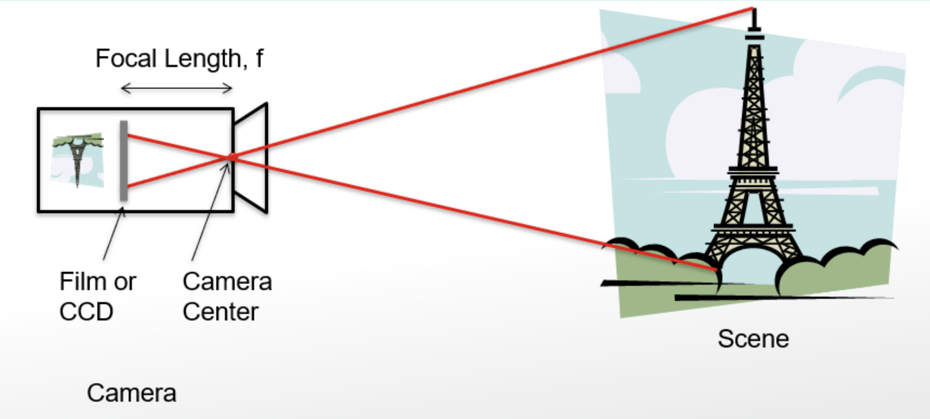
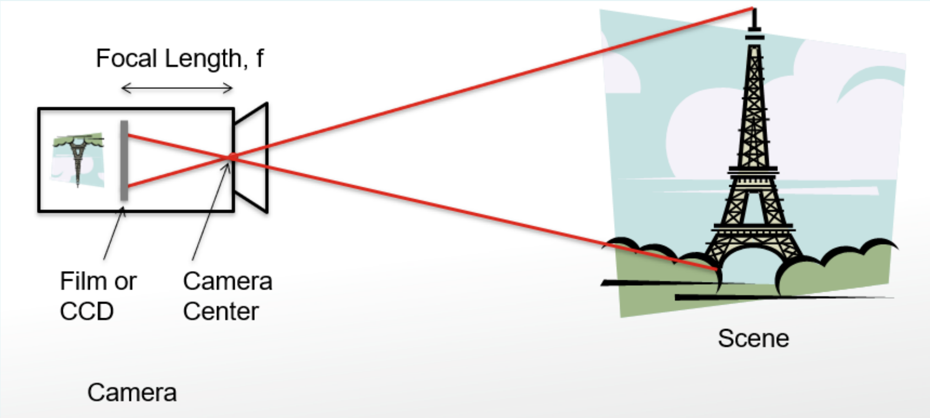


Image Formation (2)

- The light ray comes through the pinhole (camera centre), and is projected onto the film or CCD, which is at focal length, f, distance away from pinhole.



- It is obvious that the image will become inverted.

Image Formation (3)

- To simplify calculations, researchers imagine a “virtual” image plane at a distance f in front of the camera instead, so that the image is not inverted.

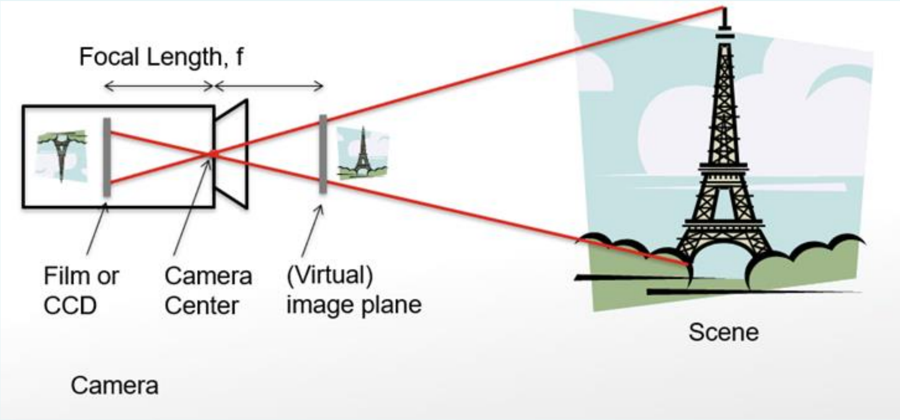
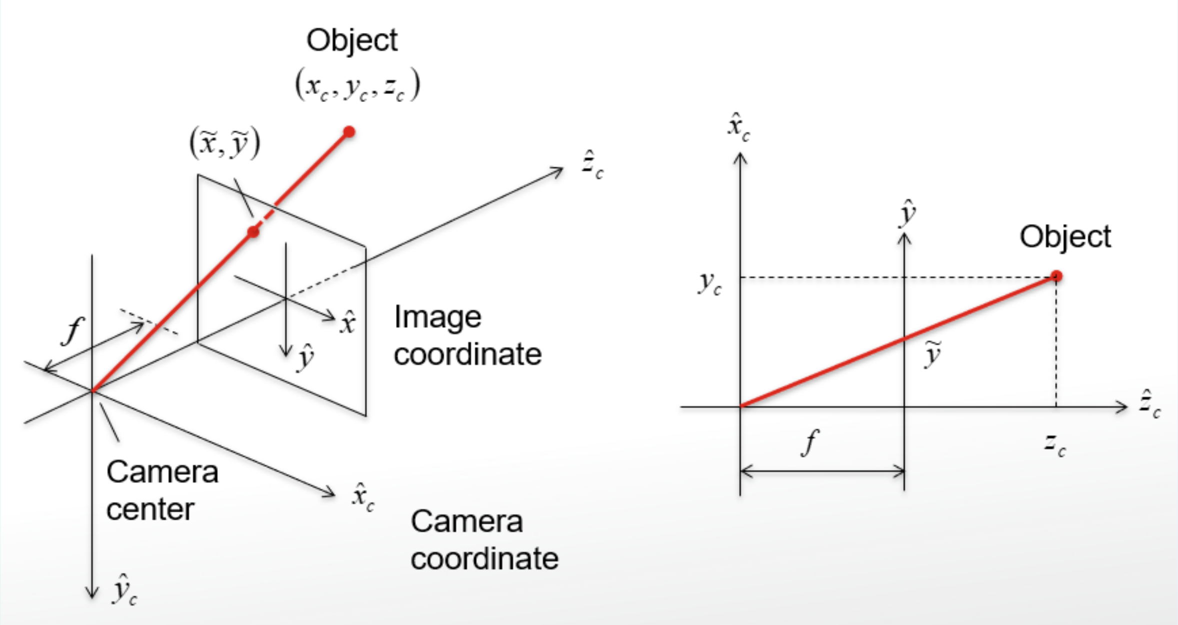


Image Formation (4)

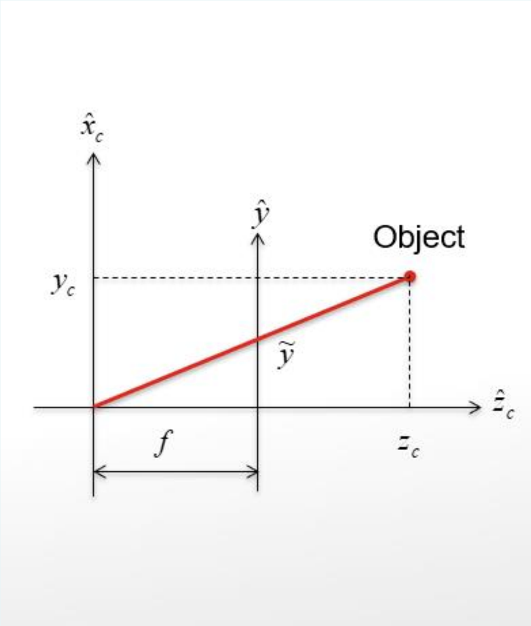
- The scenario is thus as shown below:



Intrinsic Parameters (1)

- From the 2D image on the right, it is easy to see that, due to similar triangles:

$$\boxed{\frac{\tilde{y}}{f} = \frac{y_c}{z_c} \Rightarrow \tilde{y} = f\,\frac{y_c}{z_c}}$$



- where $\tilde{y}$ means location in image plane. Similarly, we will have:

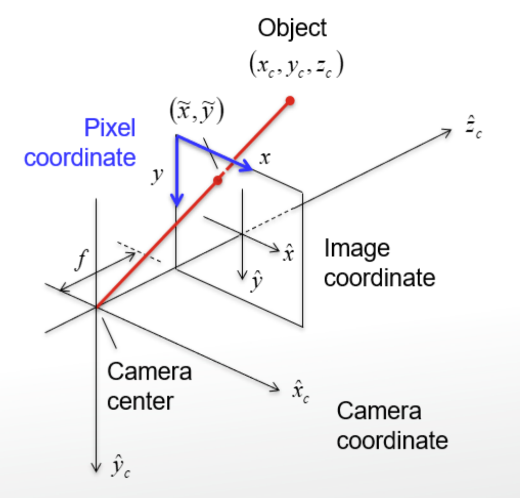
$$\boxed{\tilde{x} = f\,\frac{x_c}{z_c}}$$

Intrinsic Parameters (2)

- We next need to express the point location in image (camera centre) to the pixel frame (at the corner):
- The relationship is:

$$\boxed{x = \frac{\tilde{x}}{dx} + x_0, y = \frac{\tilde{y}}{dy} + y_0}$$

  - where x (or y) is the location in pixel coordinate,
  - dx (or dy) is the scaling factor by the physical dimension of pixel, and
  - x_0 (or y_0) is used to shift the centre of image to the corner.



Intrinsic Parameters (3)

- Combining all the above equations, we have:

$$\boxed{x = \frac{\tilde{x}}{dx} + x_0 = \frac{f\frac{x_c}{z_c}}{dx} + x_0 = \frac{1}{z_c}\left(\frac{f}{dx}x_c + x_0 z_c\right)}$$

$$\boxed{y = \frac{\tilde{y}}{dy} + y_0 = \frac{f\frac{y_c}{z_c}}{dy} + y_0 = \frac{1}{z_c}\left(\frac{f}{dy}y_c + y_0 z_c\right)}$$

Intrinsic Parameters (4)

- These two equations can be written in a matrix form:

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \frac{1}{z_c} \begin{bmatrix} \frac{f}{dx} & 0 & x_0 \\ 0 & \frac{f}{dy} & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \frac{1}{z_c} \begin{bmatrix} \alpha & 0 & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

  - where $\alpha = \frac{f}{dx}$ and $\beta = \frac{f}{dy}$.

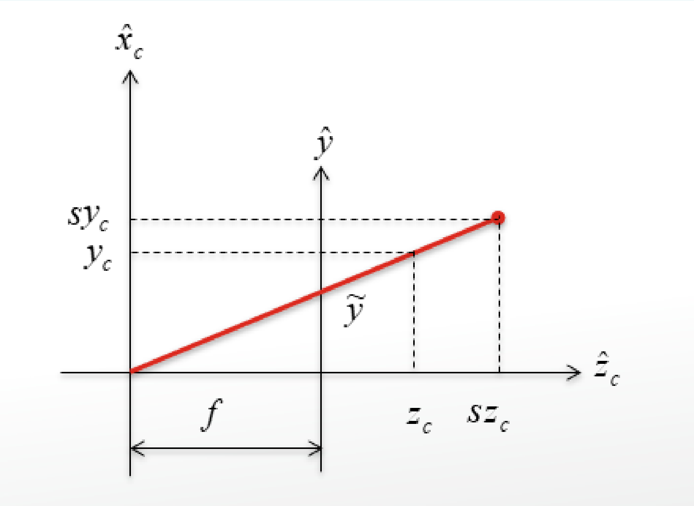
Intrinsic Parameters (5)

- It is also common to add a skewness parameter γ in the equation leading to:

$$\boxed{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \frac{1}{z_c} \begin{bmatrix} \alpha & \gamma & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}}$$

Intrinsic Parameters (6)

- The right hand side of the equation is scale invariant, i.e. if (x_c, y_c, z_c) are all scaled by the same factor s, the image point (x, y) would still be the same:



Intrinsic Parameters (7)

- Therefore, the equation is defined only up to a scale using the “proportionality” sign.

$$\boxed{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \underbrace{\begin{bmatrix} \alpha & \gamma & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{K} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = K \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}}$$

- The parameters $\alpha, \beta, \gamma, x_0$ and y_0 are called the intrinsic parameters of the camera.

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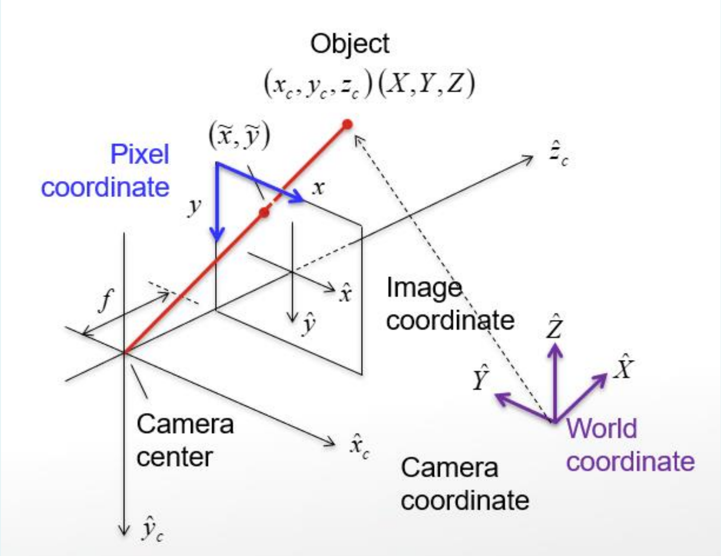
Extrinsic Parameters (1)

- The extrinsic parameters give the relationship between the World Coordinate System and the Camera Coordinate System.
- The object point has coordinates (x_c, y_c, z_c) in Camera coordinate system,
- and also has coordinates (X, Y, Z) in World coordinate system.

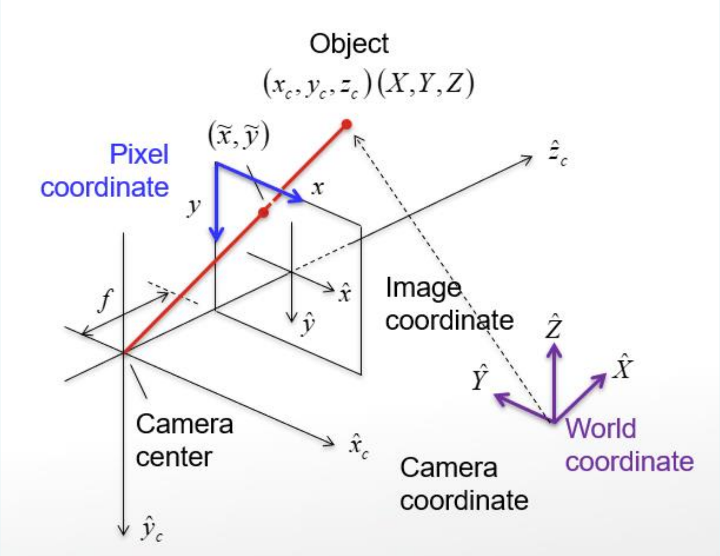


Extrinsic Parameters (2)

- You have already learnt how to relate the same point in both coordinate systems:

$$\boxed{{}^{C}P = {}^{C}_{W}R \cdot {}^{W}P + {}^{C}P_{Worg}}$$

$$\boxed{\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = {}^{C}_{W}R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + {}^{C}P_{Worg}}$$



- ${}^{C}_{W}R$ and ${}^{C}P_{Worg}$ are called the extrinsic parameters of the camera.

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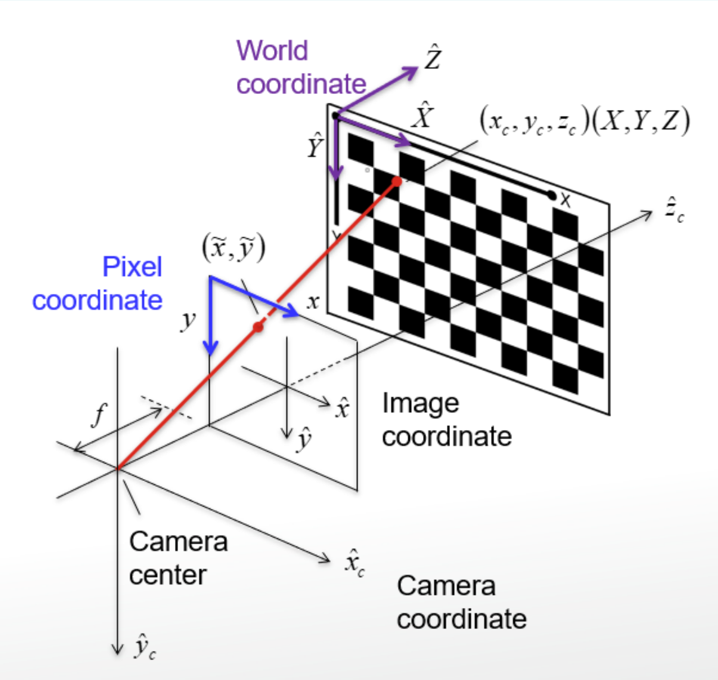
Camera Matrix (1)

- Combining the equations for intrinsic parameters and extrinsic parameters leads to:

$$\boxed{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim K \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = K\left({}_W^C R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + {}^C P_{Worg} \right) = K\left[{}_W^C R \quad {}^C P_{Worg} \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}$$

Camera Matrix (2)

- Later, we will use a planar checkerboard to perform the calibration.
- The X and Y plane of the world coordinate frame is assumed to lie on the checkerboard.
- Therefore, all the points on this checkerboard will have the Z value of zero.



Camera Matrix (3)

- The previous equation thus simplifies to:

$$\boxed{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim K\begin{bmatrix} r_1 & r_2 & r_3 & {}^{C}P_{Worg} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \underbrace{K\begin{bmatrix} r_1 & r_2 & {}^{C}P_{Worg} \end{bmatrix}}_{H}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = H\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}}$$

- H is called the camera matrix.

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Camera Calibration (1)

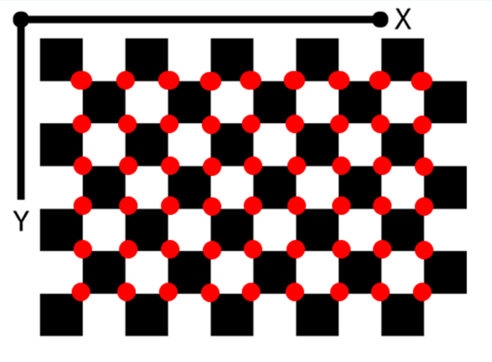
- The final equation is repeated here for convenience:

$$\boxed{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \underbrace{K\begin{bmatrix} r_1 & r_2 & {}^{C}P_{Worg} \end{bmatrix}}_{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = H \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}}$$

- Remember that (x, y, X, Y) are known, whereas H isn't.
- However, if we put values of (x, y, X, Y) into the above equation, we could solve for H.
- With H, we can then recover the intrinsic parameters K and the extrinsic parameters r_1, r_2 and ${}^{C}P_{Worg}$.

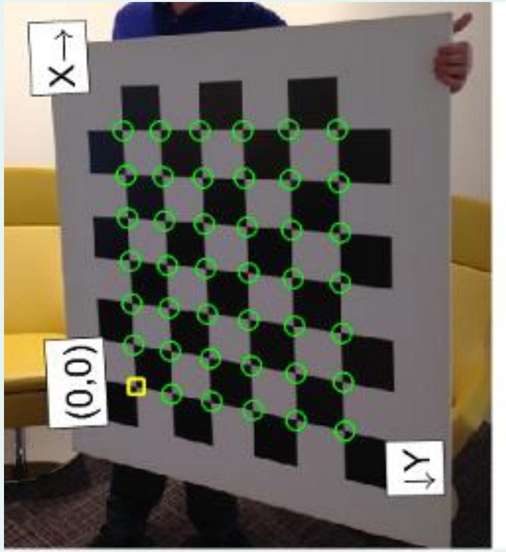
Camera Calibration (2)

- So how do we obtain H?
- Firstly, we print out a physical copy of the checkerboard, and measure the coordinates of at least four corner points of the black/white boxes accurately.
- These would give us (X_i, Y_i) coordinates with respect to the world frame which lies on the checkerboard, where the subscript i is given to each corner point.



Camera Calibration (3)

- Next, we take the first picture of the checkerboard.
- From the image, we find out where the pixel positions of the corners are.
- These would give us (x_i, y_i) values for each corner point i.



Camera Calibration (4)

- For each point i, we thus have the equation:

$$\boxed{\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix}}$$

- However, because of the “proportional” sign, we cannot calculate the h values directly.

Camera Calibration (5)

- Fortunately, "proportional" also means that the left hand side is scalar multiple of the right hand side, and therefore their cross product is zero:

$$\boxed{\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$\boxed{\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \times \begin{bmatrix} h_{11}X_i + h_{12}Y_i + h_{13} \\ h_{21}X_i + h_{22}Y_i + h_{23} \\ h_{31}X_i + h_{32}Y_i + h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

Camera Calibration (6)

- Giving:

$$\boxed{y_i(h_{31}X_i + h_{32}Y_i + h_{33}) - h_{21}X_i - h_{22}Y_i - h_{23} = 0}$$

$$\boxed{x_i(h_{31}X_i + h_{32}Y_i + h_{33}) - h_{11}X_i - h_{12}Y_i - h_{13} = 0}$$

Camera Calibration (7)

- This can be written in the linear-in-parameter form:

$$\boxed{\begin{bmatrix} 0 & 0 & 0 & X_i & Y_i & 1 & -y_iX_i & -y_iY_i & -y_i \\ X_i & Y_i & 1 & 0 & 0 & 0 & -x_iX_i & -x_iY_i & -x_i \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

Camera Calibration (8)

- We stack the above equation for all points of the checkerboard, and will get the following matrix equation:

$$\boxed{\underbrace{\begin{bmatrix} 0 & 0 & 0 & X_1 & Y_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1 \\ X_1 & Y_1 & 1 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1 \\ 0 & 0 & 0 & X_2 & Y_2 & 1 & -y_2X_2 & -y_2Y_2 & -y_2 \\ X_2 & Y_2 & 1 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & X_n & Y_n & 1 & -y_nX_n & -y_nY_n & -y_n \\ X_n & Y_n & 1 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_n \end{bmatrix}}_{\phi} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}$$

- where n is the number of corner points we have measured.

Camera Calibration (9)

- We still have the problem that the right hand side of the equation is zero, and thus all the elements of h-vector being zero is a trivial solution.
- To avoid the trivial solution, we first notice that h is the null space of ϕ.
- We can thus use singular value decomposition (SVD) to calculate the null space of the matrix ϕ.

Camera Calibration (10)

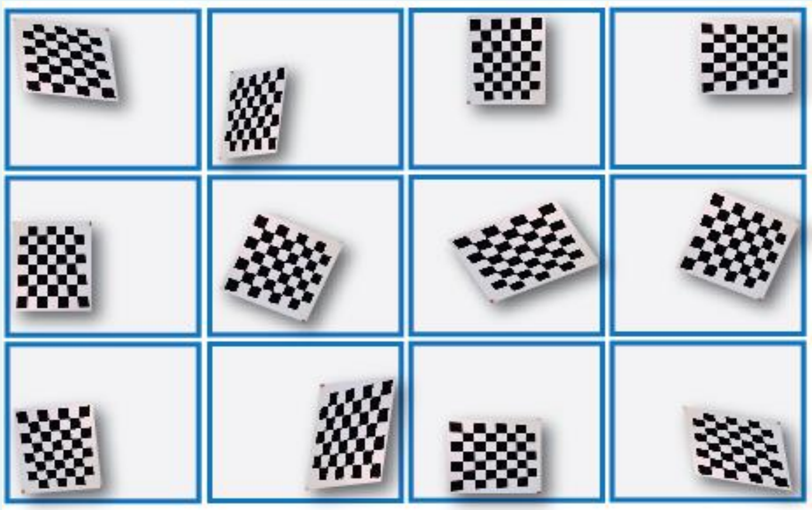
- If the SVD of ϕ is:

$$\boxed{\phi = U\Sigma V^T}$$

- then the vector h will be the column of V corresponding to the smallest singular value in Σ.
  - If you use Matlab's SVD function, h will automatically be the last column of V because the singular values will be ordered in a descending order.
- With this, the matrix H is calculated successfully for one image of the checkboard.

Camera Calibration (11)

- We repeat the same process above but with at least 3 images of the checkboard placed differently (in terms of position and orientation), each time obtaining a different H.



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Recovering Intrinsic Parameters (1)

- After obtaining H's, we will try to recover the intrinsic (and later the extrinsic) parameters. The method here is adopted from a famous paper by Zhengyou Zhang [Zhang2000].
- The equation relating H with the parameters is:

$$\boxed{H = [h_1 \quad h_2 \quad h_3] = K\left[r_1 \quad r_2 \quad {}^{C}P_{Worg}\right]}$$

Recovering Intrinsic Parameters (2)

- Because r_1 and r_2 are vectors of a rotation matrix, we know that they are orthonormal, i.e.

$$\boxed{r_1^T r_2 = 0}$$

$$\boxed{r_1^T r_1 = r_2^T r_2 (= 1)}$$

- Using the three equations above, we get:

$$\boxed{h_1^T K^{-T} K^{-1} h_2 = 0}$$

$$\boxed{h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2}$$

Recovering Intrinsic Parameters (3)

- Define:

$$B = K^{-T}K^{-1} = \begin{bmatrix} \frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2\beta} & \frac{y_0\gamma - x_0\beta}{\alpha^2\beta} \\ -\frac{\gamma}{\alpha^2\beta} & \frac{\gamma^2}{\alpha^2\beta^2}+\frac{1}{\beta^2} & -\frac{\gamma(y_0\gamma - x_0\beta)}{\alpha^2\beta^2} - \frac{y_0}{\beta^2} \\ \frac{y_0\gamma - x_0\beta}{\alpha^2\beta} & -\frac{\gamma(y_0\gamma - x_0\beta)}{\alpha^2\beta^2} - \frac{y_0}{\beta^2} & \frac{(y_0\gamma - x_0\beta)^2}{\alpha^2\beta^2} + \frac{y_0^2}{\beta^2} + 1 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix}$$

- Notice that the matrix is symmetric and has only 6 parameters $(b_{11}, b_{12}, b_{22}, b_{13}, b_{23}, b_{33})$.

Recovering Intrinsic Parameters (4)

- Let the ith column of H be $h_i = \begin{bmatrix} h_{1i} \\ h_{2i} \\ h_{3i} \end{bmatrix}$. Then:

$$\boxed{\begin{aligned} h_i^T B h_j &= \begin{bmatrix} h_{1i} & h_{2i} & h_{3i} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix} \begin{bmatrix} h_{1j} \\ h_{2j} \\ h_{3j} \end{bmatrix} \\ &= \underbrace{\begin{bmatrix} h_{1i}h_{1j} & \begin{pmatrix} h_{2i}h_{1j} \\ +h_{1i}h_{2j} \end{pmatrix} & h_{2i}h_{2j} & \begin{pmatrix} h_{3i}h_{1j} \\ +h_{1i}h_{3j} \end{pmatrix} & \begin{pmatrix} h_{3i}h_{2j} \\ +h_{2i}h_{3j} \end{pmatrix} & h_{3i}h_{3j} \end{bmatrix}}_{v_{ij}^T} \begin{bmatrix} b_{11} \\ b_{12} \\ b_{22} \\ b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} \end{aligned}}$$

Recovering Intrinsic Parameters (5)

- Equation repeated here for convenience:

$$h_i^T B h_j = \underbrace{\begin{bmatrix} h_{1i}h_{1j} & \begin{pmatrix} h_{2i}h_{1j} \\ +h_{1i}h_{2j} \end{pmatrix} & h_{2i}h_{2j} & \begin{pmatrix} h_{3i}h_{1j} \\ +h_{1i}h_{3j} \end{pmatrix} & \begin{pmatrix} h_{3i}h_{2j} \\ +h_{2i}h_{3j} \end{pmatrix} & h_{3i}h_{3j} \end{bmatrix}}_{v_{ij}^T} \begin{bmatrix} b_{11} \\ b_{12} \\ b_{22} \\ b_{13} \\ b_{23} \\ b_{33} \end{bmatrix}$$

- Remember that all the h's are already known, and we want to find the b's.

Recovering Intrinsic Parameters (6)

- Combining the equations above, we have:

$$\boxed{h_1^T K^{-T} K^{-1} h_2 = h_1^T B h_2 = v_{12}^T b = 0}$$

$$\boxed{h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2 \Rightarrow h_1^T B h_1 = h_2^T B h_2 \Rightarrow v_{11}^T b = v_{22}^T b}$$

- These two equations are combined into matrix form:

$$\boxed{\begin{bmatrix} v_{12}^T \\ (v_{11}^T - v_{22}^T) \end{bmatrix} b = 0}$$

Recovering Intrinsic Parameters (7)

- The above equation is obtained from one H (for one image of checkerboard).
- Since we have several H's (for different checkerboard positions & orientations), we stack each equation and get:

$$\underbrace{\begin{array}{r} \text{From first image} \rightarrow \\ \\ \\ \text{From last image} \rightarrow \\ \\ \end{array} \begin{bmatrix} v_{12}^T \\ (v_{11}^T - v_{22}^T) \\ \vdots \\ v_{12}^T \\ (v_{11}^T - v_{22}^T) \end{bmatrix}}_{v} \begin{bmatrix} b_{11} \\ b_{12} \\ b_{22} \\ b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = 0$$

Recovering Intrinsic Parameters (8)

- We can again solve for b using SVD:

$$\boxed{v = U\Sigma V^T}$$

- and b will be the column of V corresponding to the smallest singular value in Σ.
  - If you use Matlab's SVD function, b will automatically be the last column of V because the singular values will be ordered in a descending order.

Recovering Intrinsic Parameters (9)

- Recall that:

$$B = K^{-T}K^{-1} = \begin{bmatrix} \frac{1}{\alpha^2} & -\frac{\gamma}{\alpha^2\beta} & \frac{y_0\gamma - x_0\beta}{\alpha^2\beta} \\ -\frac{\gamma}{\alpha^2\beta} & \frac{\gamma^2}{\alpha^2\beta^2}+\frac{1}{\beta^2} & -\frac{\gamma(y_0\gamma - x_0\beta)}{\alpha^2\beta^2} - \frac{y_0}{\beta^2} \\ \frac{y_0\gamma - x_0\beta}{\alpha^2\beta} & -\frac{\gamma(y_0\gamma - x_0\beta)}{\alpha^2\beta^2} - \frac{y_0}{\beta^2} & \frac{(y_0\gamma - x_0\beta)^2}{\alpha^2\beta^2} + \frac{y_0^2}{\beta^2} + 1 \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix}$$

Recovering Intrinsic Parameters (10)

- The elements of B are now known, but only up to a scale factor, i.e.

$$B = \lambda K^{-T} K^{-1} = \begin{bmatrix} \frac{\lambda}{\alpha^2} & -\frac{\lambda\gamma}{\alpha^2\beta} & \frac{\lambda y_0\gamma - \lambda x_0\beta}{\alpha^2\beta} \\ -\frac{\lambda\gamma}{\alpha^2\beta} & \frac{\lambda\gamma^2}{\alpha^2\beta^2} + \frac{\lambda}{\beta^2} & -\frac{\lambda\gamma(y_0\gamma - x_0\beta)}{\alpha^2\beta^2} - \frac{\lambda y_0}{\beta^2} \\ \frac{\lambda y_0\gamma - \lambda x_0\beta}{\alpha^2\beta} & -\frac{\lambda\gamma(y_0\gamma - x_0\beta)}{\alpha^2\beta^2} - \frac{\lambda y_0}{\beta^2} & \frac{\lambda(y_0\gamma - x_0\beta)^2}{\alpha^2\beta^2} + \frac{\lambda y_0^2}{\beta^2} + \lambda \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix}$$

Recovering Intrinsic Parameters (11)

- We can thus recover the intrinsic parameters of the camera as:

- $$\boxed{y_0 = \frac{(b_{12}b_{13} - b_{11}b_{23})}{(b_{11}b_{22} - b_{12}^2)}}$$
- $$\boxed{\lambda = b_{33} - \frac{\left(b_{13}^2 + y_0(b_{12}b_{13} - b_{11}b_{23})\right)}{b_{11}}}$$
- $$\boxed{\alpha = \sqrt{\frac{\lambda}{b_{11}}}}$$
- $$\boxed{\beta = \sqrt{\frac{\lambda b_{11}}{(b_{11}b_{22} - b_{12}^2)}}}$$
- $$\boxed{\gamma = -\frac{b_{12}\alpha^2\beta}{\lambda}}$$
- $$\boxed{x_0 = \frac{\gamma y_0}{\alpha} - \frac{b_{13}\alpha^2}{\lambda}}$$

Recovering Intrinsic Parameters (12)

- The individual intrinsic parameters then forms the K matrix as:

$$\boxed{K = \begin{bmatrix} \alpha & \gamma & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Content

- Introduction
- Camera Calibration
  - Image Formation and Intrinsic Parameters
  - Extrinsic Parameters
  - Camera Matrix H
  - Camera Calibration and Calculation of H
  - Recovering Intrinsic Parameters from H
  - Recovering Extrinsic Parameters from H
  - Summary and Discussions
- Robot Calibration

Recovering Extrinsic Parameters (1)

- Once K is known, we can recover the extrinsic parameters in a straightforward manner.
- This is however valid for one image at a time, because each image gives us one H.
- This is intuitive, since each image was taken with different position and orientation of the checkerboard.

Recovering Extrinsic Parameters (2)

- For the image of interest, we knew:

$$\boxed{H = [h_1 \quad h_2 \quad h_3] = K\left[r_1 \quad r_2 \quad {}^{C}P_{Worg}\right]}$$

- Therefore:

$$\boxed{r_1 = K^{-1} h_1}$$

$$\boxed{r_2 = K^{-1} h_2}$$

$$\boxed{{}^{C}P_{Worg} = K^{-1} h_3}$$

Recovering Extrinsic Parameters (3)

- There is however no guarantee that the above values have the correct scale.
- Fortunately, $\|r_1\|$ should be one, and therefore we can define a scale to ensure that this is the case:

$$\boxed{\sigma = 1/\|K^{-1}h_1\|}$$

Recovering Extrinsic Parameters (4)

- Using this scale, the extrinsic parameters are now updated to:

$$\boxed{r_1 = \sigma K^{-1} h_1}$$

$$\boxed{r_2 = \sigma K^{-1} h_2}$$

$$\boxed{r_3 = r_1 \times r_2}$$

$$\boxed{{}^{C}P_{Worg} = \sigma K^{-1} h_3}$$

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Summary of Camera Calibration (1)

- **Step 1**: Fix the camera on the robot station.
- **Step 2**: Print out a checkboard, and measure the coordinates (X_i, Y_i) of at least four corner points, assuming that the World coordinate frame lies on the checkerboard plane.
- **Step 3**: Take at least three images of the checkerboard at different positions and orientations.
  - One of these images should be taken when the checkerboard is placed at approximately the same height as the object

Summary of Camera Calibration (2)

- **Step 4**: For each image:
  a) Find the pixel position (x_i, y_i) of the corner points.
  b) Build the matrix equation:

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & X_1 & Y_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1 \\ X_1 & Y_1 & 1 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1 \\ 0 & 0 & 0 & X_2 & Y_2 & 1 & -y_2X_2 & -y_2Y_2 & -y_2 \\ X_2 & Y_2 & 1 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & X_n & Y_n & 1 & -y_nX_n & -y_nY_n & -y_n \\ X_n & Y_n & 1 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_n \end{bmatrix}}_{\phi} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Summary of Camera Calibration (3)

c) Calculate the h vector using Singular Value Decomposition method.

- If $\phi = U\Sigma V^T$ then the vector h will be the column of V corresponding to the smallest singular value in Σ.

d) Use the known h values to calculate v_{11}^T, v_{22}^T and v_{12}^T where:

$$\boxed{v_{ij}^T = \begin{bmatrix} h_{1i}h_{1j} & \begin{pmatrix} h_{2i}h_{1j} \\ +h_{1i}h_{2j} \end{pmatrix} & h_{2i}h_{2j} & \begin{pmatrix} h_{3i}h_{1j} \\ +h_{1i}h_{3j} \end{pmatrix} & \begin{pmatrix} h_{3i}h_{2j} \\ +h_{2i}h_{3j} \end{pmatrix} & h_{3i}h_{3j} \end{bmatrix}}$$

Summary of Camera Calibration (4)

- **Step 5**: Build the following matrix equation:

$$\boxed{\underbrace{\begin{array}{r} \text{From first image} \rightarrow \\ \\ \\ \text{From last image} \rightarrow \\ \\ \end{array}\begin{bmatrix} v_{12}^T \\ (v_{11}^T - v_{22}^T) \\ \vdots \\ v_{12}^T \\ (v_{11}^T - v_{22}^T) \end{bmatrix}}_{v}\begin{bmatrix} b_{11} \\ b_{12} \\ b_{22} \\ b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = 0}$$

- **Step 6**: Calculate the b vector using Singular Value Decomposition method.
    - If $v = U\Sigma V^T$ then the vector b will be the column of V corresponding to the smallest singular value in Σ.

Summary of Camera Calibration (5)

- **Step 7**: Recover the intrinsic parameters as:

$$\boxed{K = \begin{bmatrix} \alpha & \gamma & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix}}$$

- Where:
  - $\boxed{y_0 = \frac{(b_{12}b_{13} - b_{11}b_{23})}{(b_{11}b_{22} - b_{12}^2)}}$
  - $\boxed{\lambda = b_{33} - \frac{\left(b_{13}^2 + y_0(b_{12}b_{13} - b_{11}b_{23})\right)}{b_{11}}}$
  - $\boxed{\alpha = \sqrt{\frac{\lambda}{b_{11}}}}$
  - $\boxed{\beta = \sqrt{\frac{\lambda b_{11}}{(b_{11}b_{22} - b_{12}^2)}}}$
  - $\boxed{\gamma = -\frac{b_{12}\alpha^2\beta}{\lambda}}$
  - $\boxed{x_0 = \frac{\gamma y_0}{\alpha} - \frac{b_{13}\alpha^2}{\lambda}}$

Summary of Camera Calibration (6)

- **Step 8**: For the image where the checkerboard is roughly at the same height as the object, (and therefore the corresponding H), recover the extrinsic parameters as:

$$\boxed{\sigma = 1/\left\|K^{-1}h_1\right\|}$$

$$\boxed{r_1 = \sigma K^{-1}h_1}$$

$$\boxed{r_2 = \sigma K^{-1}h_2}$$

$$\boxed{r_3 = r_1 \times r_2}$$

$$\boxed{{}^{C}P_{Worg} = \sigma K^{-1}h_3}$$

Discussion – No. of Corner Points (1)

- We mentioned that we need to measure at least four corner points. But why?
- Recall that we need to calculate the h vector for each image from:

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & X_1 & Y_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1 \\ X_1 & Y_1 & 1 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1 \\ 0 & 0 & 0 & X_2 & Y_2 & 1 & -y_2X_2 & -y_2Y_2 & -y_2 \\ X_2 & Y_2 & 1 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & X_n & Y_n & 1 & -y_nX_n & -y_nY_n & -y_n \\ X_n & Y_n & 1 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_n \end{bmatrix}}_{\phi} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Discussion – No. of Corner Points (2)

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & X_1 & Y_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1 \\ X_1 & Y_1 & 1 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1 \\ 0 & 0 & 0 & X_2 & Y_2 & 1 & -y_2X_2 & -y_2Y_2 & -y_2 \\ X_2 & Y_2 & 1 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & X_n & Y_n & 1 & -y_nX_n & -y_nY_n & -y_n \\ X_n & Y_n & 1 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_n \end{bmatrix}}_{\phi} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

- There seem to be 9 parameters in h, whereas each corner point gives us only two rows in the matrix equation.
- Shouldn't we need at least five corner points instead?

Discussion – No. of Corner Points (3)

- Not really!
- Because the H matrix is defined up to a scale only, the h vector can be scaled such that for instance $h_{33} = 1$.

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & X_1 & Y_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1 \\ X_1 & Y_1 & 1 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1 \\ 0 & 0 & 0 & X_2 & Y_2 & 1 & -y_2X_2 & -y_2Y_2 & -y_2 \\ X_2 & Y_2 & 1 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & X_n & Y_n & 1 & -y_nX_n & -y_nY_n & -y_n \\ X_n & Y_n & 1 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_n \end{bmatrix}}_{\phi} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Discussion – No. of Corner Points (4)

- Or:

$$\boxed{\underbrace{\begin{bmatrix} 0 & 0 & 0 & X_1 & Y_1 & 1 & -y_1X_1 & -y_1Y_1 \\ X_1 & Y_1 & 1 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 \\ 0 & 0 & 0 & X_2 & Y_2 & 1 & -y_2X_2 & -y_2Y_2 \\ X_2 & Y_2 & 1 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & X_n & Y_n & 1 & -y_nX_n & -y_nY_n \\ X_n & Y_n & 1 & 0 & 0 & 0 & -x_nX_n & -x_nY_n \end{bmatrix}}_{\phi'} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = \begin{bmatrix} y_1 \\ x_1 \\ y_2 \\ x_2 \\ \vdots \\ y_n \\ x_n \end{bmatrix}}$$

Discussion – No. of Corner Points (5)

- As can be seen, there are actually only 8 parameters, and thus 4 corner points (giving us $4 \times 2n$ equations) are adequate to solve for the scaled h vector.
- In fact, solving the above equation above via $h = \phi'^{\dagger} x$ is an alternative to the SVD method, where $\phi'^{\dagger}$ means the pseudo-inverse of ϕ'.

Discussion – No. of Images (1)

- We mentioned that we need to measure at least three images. But why?
- Recall that we needed to solve for b from:

$$\underbrace{\begin{array}{r} \text{From first image} \rightarrow \\ \\ \\ \text{From last image} \rightarrow \\ \\ \end{array}\begin{bmatrix} v_{12}^T \\ (v_{11}^T - v_{22}^T) \\ \vdots \\ v_{12}^T \\ (v_{11}^T - v_{22}^T) \end{bmatrix}}_{v}\begin{bmatrix} b_{11} \\ b_{12} \\ b_{22} \\ b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = 0$$

Discussion – No. of Images (2)

- Each image gives us 2 equations, and since there are 6 unknown parameters in b, three images will give us a total of 6 equations allowing us to solve for b.
- Note that the numbers above (four corner points and three images) are the minimum requirement.
  - The more corner points and the more images we use, the more accurate the calibration would be.

Discussion – Scaling Factor λ (1)

- Earlier, we first defined $B = K^{-T}K^{-1}$.
- However, when we try to recover K from B, we added a scaling factor λ i.e. $B = \lambda K^{-T}K^{-1}$.
- You might be wondering why λ is needed.

Discussion – Scaling Factor λ (2)

- To answer this, it would be good to show an alternative method of recovering K from B.
- In linear algebra, the Cholesky decomposition is used to decompose a Hermitian (equivalent to symmetric if matrix is real), positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose:

$$\boxed{B = LL^*}$$

Discussion – Scaling Factor λ (3)

- We will focus instead on Matlab's interpretation of the Cholesky decomposition. In Matlab, the code R = chol(B) factorizes B into an upper triangular matrix R and its transpose:

$$\boxed{B = R^T R}$$

- Comparing this with our original definition of B without λ, i.e. $B = K^{-T} K^{-1}$, we see that:

$$\boxed{R = K^{-1}}$$

Discussion – Scaling Factor λ (4)

- Therefore, we can obtain K from:

$$\boxed{K = R^{-1}}$$

- Unfortunately, the K matrix obtained using this method (R = chol(B) followed by $K = R^{-1}$) does not guarantee that the (3,3)-element of K being one:

$$\boxed{K_{\text{Cholesky}} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ 0 & k_{22} & k_{23} \\ 0 & 0 & k_{33} \end{bmatrix}}$$

Discussion – Scaling Factor λ (5)

- However, since we ultimately want K to be in the form of:

$$\boxed{K = \begin{bmatrix} \alpha & \gamma & x_0 \\ 0 & \beta & y_0 \\ 0 & 0 & 1 \end{bmatrix}}$$

- we will divide all the elements of K_{Cholesky} by k_{33}. This will make the (3,3)-element of K one.

Discussion – Scaling Factor λ (6)

- It is to be noted that k_{33} from Cholesky decomposition method and the λ from our first method are related as:

$$\boxed{k_{33} = \frac{1}{\sqrt{\lambda}}}$$

- We can now answer the question of why λ is needed.
- It is used to ensure that all the intrinsic parameters are correctly scaled with the (3,3)-element of K being one.

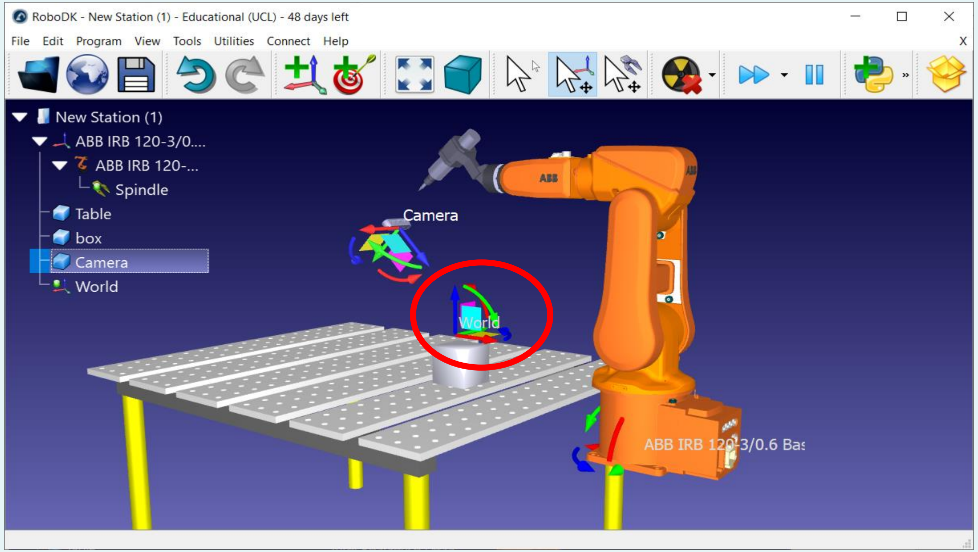
Discussion – Scaling Factor λ (7)

- Note: The Cholesky factorization cannot run if B is not positive definite.
- In this case, apply the Cholesky factorization on $-B$ instead.

Content

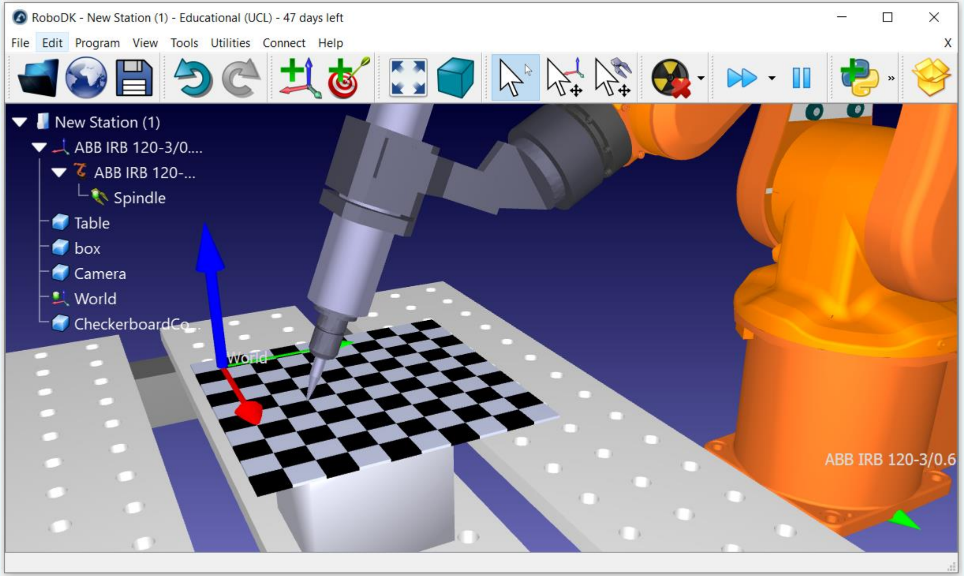
Robot Calibration (1)

- Recall that we wanted to find:
  - Relationship between the 2D pixel frame with the 3D world frame – “Camera Calibration”.
  - Relationship between the 3D robot frame with the 3D world frame – “Robot Calibration”.



Robot Calibration (2)

- Camera calibration is already done, so we will focus on robot calibration now.
- Firstly, we place the checkerboard on the object.
- We then jog the robot such that the end-effector TCP touches the corner points on the checkerboard.



Robot Calibration (3)

- When the TCP touches the corner point i, we are able to find out the position of the TCP (X_{ri}, Y_{ri}, Z_{ri}) with respect to the robot frame by using forward kinematics.
- We also know the position of the corner point $(X_i, Y_i, 0)$ with respect to the World frame.

Robot Calibration (4)

- We therefore have the relationship between the two frames as:

$$\boxed{\begin{bmatrix} X_{ri} \\ Y_{ri} \\ Z_{ri} \\ 1 \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} & t_x \\ \rho_{21} & \rho_{22} & \rho_{23} & t_y \\ \rho_{31} & \rho_{32} & \rho_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ 0 \\ 1 \end{bmatrix}}$$

Robot Calibration (5)

- To solve for the unknown parameters (ρ and t), we use the method proposed by Cashbaugh et al. in [Cashbaugh2018].
- The individual linear equations are:

$$\boxed{\begin{aligned} X_{ri} &= \rho_{11}X_i + \rho_{12}Y_i + t_x \\ Y_{ri} &= \rho_{21}X_i + \rho_{22}Y_i + t_y \\ Z_{ri} &= \rho_{31}X_i + \rho_{32}Y_i + t_z \end{aligned}}$$

Robot Calibration (6)

- We can obtain the unknown parameters by minimizing the square of errors which are:

$$\boxed{\begin{aligned} E_X^2 &= \sum_{i=1}^{n}\left(X_{ri} - (\rho_{11}X_i + \rho_{12}Y_i + t_x)\right)^2 \\ E_Y^2 &= \sum_{i=1}^{n}\left(Y_{ri} - \left(\rho_{21}X_i + \rho_{22}Y_i + t_y\right)\right)^2 \\ E_Z^2 &= \sum_{i=1}^{n}\left(Z_{ri} - (\rho_{31}X_i + \rho_{32}Y_i + t_z)\right)^2 \end{aligned}}$$

Robot Calibration (7)

- The minimum value of the square of errors occurs when the derivatives are zero.
- For E_X^2, we have:

$$\boxed{\begin{aligned}
\frac{\partial E_X^2}{\partial \rho_{11}} &= -2\sum_{i=1}^{n}\left(X_{ri} - (\rho_{11}X_i + \rho_{12}Y_i + t_x)\right)X_i = 0 \\
\frac{\partial E_X^2}{\partial \rho_{12}} &= -2\sum_{i=1}^{n}\left(X_{ri} - (\rho_{11}X_i + \rho_{12}Y_i + t_x)\right)Y_i = 0 \\
\frac{\partial E_X^2}{\partial t_x} &= -2\sum_{i=1}^{n}\left(X_{ri} - (\rho_{11}X_i + \rho_{12}Y_i + t_x)\right) = 0
\end{aligned}}$$

Robot Calibration (8)

- The equation can be re-written as:

$$\boxed{\begin{gathered}\sum \rho_{11} X_i X_i + \sum \rho_{12} Y_i X_i + \sum t_x X_i = \sum X_{ri} X_i \\ \sum \rho_{11} X_i Y_i + \sum \rho_{12} Y_i Y_i + \sum t_x Y_i = \sum X_{ri} Y_i \\ \sum \rho_{11} X_i + \sum \rho_{12} Y_i + \sum t_x = \sum X_{ri}\end{gathered}}$$

Robot Calibration (9)

- or in matrix form as:

$$\boxed{\begin{bmatrix} \sum X_i X_i & \sum Y_i X_i & \sum X_i \\ \sum X_i Y_i & \sum Y_i Y_i & \sum Y_i \\ \sum X_i & \sum Y_i & n \end{bmatrix} \begin{bmatrix} \rho_{11} \\ \rho_{12} \\ t_x \end{bmatrix} = \begin{bmatrix} \sum X_{ri} X_i \\ \sum X_{ri} Y_i \\ \sum X_{ri} \end{bmatrix}}$$

Robot Calibration (10)

- The rotation and translation parameters can then be calculated as:

$$\boxed{\begin{bmatrix} \rho_{11} \\ \rho_{12} \\ t_x \end{bmatrix} = \begin{bmatrix} \sum X_i X_i & \sum Y_i X_i & \sum X_i \\ \sum X_i Y_i & \sum Y_i Y_i & \sum Y_i \\ \sum X_i & \sum Y_i & n \end{bmatrix}^{-1} \begin{bmatrix} \sum X_{ri} X_i \\ \sum X_{ri} Y_i \\ \sum X_{ri} \end{bmatrix}}$$

Robot Calibration (11)

- The remaining parameters can be obtained in similar manner by using the E_Y^2 and E_Z^2 equations:

$$\boxed{\begin{bmatrix}\rho_{21}\\ \rho_{22}\\ t_y\end{bmatrix} = \begin{bmatrix}\sum X_iX_i & \sum Y_iX_i & \sum X_i\\ \sum X_iY_i & \sum Y_iY_i & \sum Y_i\\ \sum X_i & \sum Y_i & n\end{bmatrix}^{-1}\begin{bmatrix}\sum Y_{ri}X_i\\ \sum Y_{ri}Y_i\\ \sum Y_{ri}\end{bmatrix}}$$

Robot Calibration (12)

$$\boxed{\begin{bmatrix}\rho_{31}\\ \rho_{32}\\ t_z\end{bmatrix} = \begin{bmatrix}\sum X_iX_i & \sum Y_iX_i & \sum X_i\\ \sum X_iY_i & \sum Y_iY_i & \sum Y_i\\ \sum X_i & \sum Y_i & n\end{bmatrix}^{-1} \begin{bmatrix}\sum Z_{ri}X_i\\ \sum Z_{ri}Y_i\\ \sum Z_{ri}\end{bmatrix}}$$

- Finally, $(\rho_{13}, \rho_{23}, \rho_{33})$ can be calculated as:

$$\boxed{\begin{bmatrix}\rho_{13}\\ \rho_{23}\\ \rho_{33}\end{bmatrix} = \begin{bmatrix}\rho_{11}\\ \rho_{21}\\ \rho_{31}\end{bmatrix} \times \begin{bmatrix}\rho_{12}\\ \rho_{22}\\ \rho_{32}\end{bmatrix}}$$

Combining Everything (1)

- We are now ready to know the 3D position of an object in robot frame (X_r, Y_r and Z_r), given the 2D position of the same image in pixel frame (x and y).
  - The latter could be determined via image processing method you learnt previously.

Combining Everything (2)

- Relationship between pixel coordinate and world coordinate:

$$\boxed{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \underbrace{K\begin{bmatrix} r_1 & r_2 & {}^{C}P_{Worg} \end{bmatrix}}_{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = H \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}}$$

- From which we can obtain:

$$\boxed{\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \sim H^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}$$

- Need to normalize such that the last row on RHS is 1.

Combining Everything (3)

- Relationship between robot coordinate and world coordinate:

$$\boxed{\begin{bmatrix} X_r \\ Y_r \\ Z_r \\ 1 \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} & t_x \\ \rho_{21} & \rho_{22} & \rho_{23} & t_y \\ \rho_{31} & \rho_{32} & \rho_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}}$$

- Putting in X and Y from previous slide into the equation above, and we will obtain X_r, Y_r and Z_r.
- Your robot can finally move to the correct position based on camera image of object!

References

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Thank you for your attention!

Any questions?