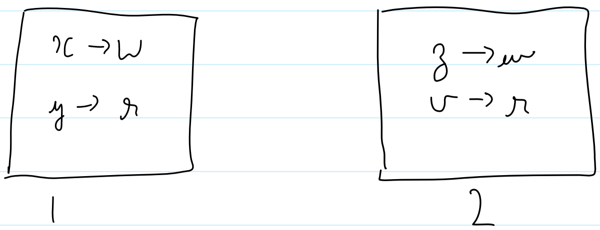
3.1. Urn 1 contains x white and y red balls. Urn 2 contains z white and v red balls. A ball is chosen at random from urn 1 and put into urn 2. ***Then*** a ball is chosen at random from urn 2. What is the probability that this ball is white?



$$P(w \text{ from } 2) = P(w \text{ from } 1) \times P(w \text{ from new } 2) + P(r \text{ from } 1) \times P(w \text{ from new } 2)$$

$$= \frac{x}{x+y} \times \frac{z+1}{z+v+1} + \frac{y}{x+y} \times \frac{z}{z+y+1}$$

$$= \frac{x(z+1) + zy}{(x+y)(z+v+1)} \quad \text{Ans}$$

3.2. Two defective tubes get mixed up with two good ones. The tubes are tested, one by one, until both defectives are found.

(a) What is the probability that the last defective tube is obtained on the second test?

(b) What is the probability that the last defective tube is obtained on the third test?

(c) What is the probability that the last defective tube is obtained on the fourth test?

(d) Add the numbers obtained for (a), (b), and (c) above. Is the result surprising?

2 → Defective Tubes
2 → Good Tubes

(a) That means both first and second tubes are defective

$P(\text{first defective}) = 2/4 = 1/2$

$$P(\text{second defective}) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

(b) Last defective found on 3rd test.

This means that in 1st and 2nd tests we have already found one defective and one non defective tube.

We can have following cases possible

DND or NDD

$$= \frac{\cancel{2}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} + \frac{\cancel{2}^{1}}{4_{2}} \times \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} = \frac{2}{6} = \frac{1}{3}$$

(c) Last defective found on 4^{th} test

Following cases are possible

DNND + NDND + NNDD

$$= \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \times 1 + \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \times 1 + \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \times 1$$

$$= \frac{3}{6} = \frac{1}{2}$$ Ans

(d) On adding results of a, b and c we get

$$\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = \frac{2+3+1}{6} = 1$$

$$\frac{1}{3}+\frac{1}{2}+\frac{1}{6} = \frac{2+3+1}{6} = 1$$

3.3. A box contains 4 bad and 6 good tubes. Two are drawn out together. One of them is tested and found to be good. What is the probability that the other one is also good?

4 → Bad

6 → Good

P(1st is Good) = 1 (Given)

$$P(2^{nd} \text{ is Good} \mid 1^{st} \text{ is Good}) = P(1st \text{ is Good})\, P(2^{nd} \text{ is Good})$$

$$= 1 \times \frac{5}{9} = \frac{5}{9}$$

3.4. In the above problem the tubes are checked by drawing a tube at random, testing it and repeating the process until all 4 bad tubes are located. What is the probability that the fourth bad tube will be located

(a) on the fifth test?

(b) on the tenth test?

(a) □ □ □ □ □

G	B	B	B	B	$6/10 \times 4/9 \times 3/8 \times 2/7 \times 1/6$
B	G	B	B	B	$4/10 \times 6/9 \times 3/8 \times 2/7 \times 1/6$
B	B	G	B	B	$4/10 \times 3/9 \times 6/8 \times 2/7 \times 1/6$
B	B	B	G	B	$4/10 \times 3/9 \times 2/8 \times 6/7 \times 1/6$

$$\cancel{4} \times \frac{\cancel{6}}{\cancel{10}_5} \times \frac{\cancel{4}^2}{\cancel{9}_3} \times \frac{\cancel{3}^1}{\cancel{8}} \times \frac{\cancel{2}}{7} \times \frac{1}{\cancel{6}} = \boxed{\frac{2}{105}} \text{ Ans}$$

(b) On tenth test

Arrange 3 bad and 6 good tubes in 9 tests

$$\frac{9!}{3!\,6!} = \frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7}{\cancel{3} \times \cancel{2}} = 84$$

$$\frac{\cancel{6}}{10} \times \frac{\cancel{5}}{\cancel{9}} \times \frac{\cancel{4}}{\cancel{8}} \times \frac{\cancel{3}}{\cancel{7}} \times \frac{\cancel{2}}{\cancel{6}} \times \frac{1}{\cancel{5}} \times \cancel{3 \times 4 \times 7} = \frac{4}{10} = \frac{2}{5} \text{ Ans}$$

3.5. Suppose that A and B are independent events associated with an experiment. If the probability that A or B occurs equals 0.6, while the probability that A occurs equals 0.4, determine the probability that B occurs.

$P(A \cup B) = 0.6$

$P(A) = 0.4$

As we know that for two independent events $P(A \cap B)$ is given by

$$P(A \cap B) = P(A)\,P(B)$$

and we also know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$

$\Rightarrow 0.6 = 0.4 + P(B)(1 - 0.4)$

$\Rightarrow \frac{0.2}{0.6} = P(B)$

$P(B) = \frac{1}{}$ Ans

3

3.6. Twenty items, 12 of which are defective and 8 nondefective, are inspected one after the other. If these items are chosen at random, what is the probability that:

(a) the first two items inspected are defective?
(b) the first two items inspected are nondefective?
(c) among the first two items inspected there is one defective and one nondefective?

Total = 20
Defective = 12 Nondefective = 8

(a) P(First two items are defective) = $\frac{\cancel{12}^{3}}{\cancel{20}_{5}} \times \frac{11}{19} = \frac{33}{95}$ Ans

(b) P(First two items are non defective) = $\frac{8}{20} \times \frac{7}{19} = \frac{14}{95}$ Ans

(c) P(One defective and one non defective) = $\frac{\cancel{12}^{3}}{\cancel{20}_{5}} \times \frac{8}{19} \times 2 = \frac{48}{95}$ Ans

2 is multiplied because there could be two scenarios
1st Defective and 2nd Non Defective
1st Non Defective and 2nd Defective

3.7. Suppose that we have two urns, 1 and 2, each with two drawers. Urn 1 has a gold coin in one drawer and a silver coin in the other drawer, while urn 2 has a gold coin in each drawer. One urn is chosen at random; then a drawer is chosen at random from the chosen urn. The coin found in this drawer turns out to be gold. What is the probability that the coin came from urn 2?

①	②
1G	1S

Urn 1

③	④
1G	1G

Urn 2

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$P(\text{Urn}1) = 1/2 \qquad P(\text{Urn }2) = 1/2$

$P(G|\text{Urn}1) = \frac{1}{4} \qquad P(G|\text{Urn }2) = \frac{1}{2} \times 1 = \frac{1}{2}$

$$P(\text{Urn}2|G) = \frac{1/2}{1/4 + 1/2} = \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{3} = \frac{2}{3} \quad \text{Ans}$$

3.8. A bag contains three coins, one of which is coined with two heads while the other two coins are normal and not biased. A coin is chosen at random from the bag and tossed four times in succession. If heads turn up *each* time, what is the probability that this is the two-headed coin?

1	2	3
1H 1T	1H 1T	2H 1T

$P(1) = P(2) = P(3) = 1/2$

$P(4H|1) = \frac{1}{2} \times \left(\frac{1}{2}\right)^4 \qquad P(4H|2) = \frac{1}{2} \times \left(\frac{1}{2}\right)^4 \qquad P(4H|3) = \frac{1}{2} \times (1)^4$

$$P(3|4H) = \frac{1/2}{\frac{1}{2^5} + \frac{1}{2^5} + \frac{1}{2}} = \frac{1/2}{\frac{1+1+2^4}{2^5}} = \frac{1}{2} \times \frac{2^5}{18} = \frac{2^4}{18}$$

$$= \frac{16}{18} = \frac{8}{9} \quad \text{Ans}$$

3.9. In a bolt factory, machines A, B, and C manufacture 25, 35, and 40 percent of the total output, respectively. Of their outputs, 5, 4, and 2 percent, respectively, are defective bolts. A bolt is chosen at random and found to be defective. What is the probability that the bolt came from machine A? B? C?

$A = 25\%$ $\quad B = 35\%$ $\quad C = 40\%$

$D_A = 5\%$ $\quad D_B = 4\%$ $\quad D_C = 2\%$

$P(A) = 25/100$ $\quad P(B) = 35/100$ $\quad P(C) = 40/100$

$P(D_A | A) = \frac{5}{100} \times \frac{25}{100}$ $\quad P(D_B | B) = \frac{4}{100} \times \frac{35}{100}$ $\quad P(D_C | C) = \frac{2}{100} \times \frac{40}{100}$

$$P(A | D_A) = \frac{5 \times 25}{100 \times 100} \times \frac{100 \times 100}{125 + 140 + 80}$$

$$= \frac{5 \times 25}{345} = \frac{25}{69} = 0.362 \quad \text{Ans}$$

$$P(B | D_B) = \frac{140}{345} = \frac{28}{69} = 0.406 \quad \text{Ans}$$

$$P(C | D_C) = \frac{80}{345} = \frac{16}{69} = 0.232 \quad \text{Ans}$$

3.10. Let A and B be two events associated with an experiment. Suppose that $P(A) = 0.4$ while $P(A \cup B) = 0.7$. Let $P(B) = p$.

(a) For what choice of p are A and B mutually exclusive?

(b) For what choice of p are A and B independent?

$P(A) = 0.4 \quad P(A \cup B) = 0.7 \quad P(B) = p$

(a) If two events are mutually exclusive, then their intersection is an empty set.

Therefore, $A \cap B = \phi \Rightarrow P(A \cap B) = 0$

Since, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.7 = 0.4 + P(B)$

$\Rightarrow P(B) = 0.3$ Ans

(b) If two events A and B are independent, then

$$P(A \cap B) = P(A)\,P(B)$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.7 = 0.4 + P(B) - P(A)\,P(B)$

$\Rightarrow 0.3 = P(B)(1 - 0.4)$

$P(B) = \frac{3}{6} = \frac{1}{2}$ Ans

3.11. Three components of a mechanism, say C_1, C_2, and C_3 are placed in series (in a straight line). Suppose that these mechanisms are arranged in a random order. Let R be the event $\{C_2$ is to the right of $C_1\}$, and let S be the event $\{C_3$ is to the right of $C_1\}$. Are the events R and S independent? Why?

C_1, C_2, C_3 are 3 components

$R = \{C_2$ is right of $C_1\}$

$S = \{C_3$ is right of $C_1\}$

Result set of $R = \{C_2 C_1 C_3,\ C_2 C_3 C_1,\ C_3, C_2, C_1\}$

Result set of $S = \{C_3 C_1 C_2,\ C_3 C_2 C_1,\ C_2 C_3 C_1\}$

$R \cap S = \{C_2 C_3 C_1,\ C_3 C_2, C_1\}$

Total arrangements possible $= 3 \times 2 \times 1 = 3! = 6$

$P(R) = \frac{1}{2}$ $\quad P(S) = \frac{1}{2}$ $\quad P(R \cap S) = \frac{2}{6} = \frac{1}{3}$

Since, $P(R \cap S) \neq P(R)\,P(S)$

Therefore, the two events are not independent.

3.12. A die is tossed, and independently, a card is chosen at random from a regular deck. What is the probability that:

(a) the die shows an even number and the card is from a red suit?
(b) the die shows an even number or the card is from a red suit?

(a) $P(\text{even number on a die}) = \frac{1}{2}$

$P(\text{red suit card}) = \frac{1}{2}$

$P(\text{even number die and red suit card}) = \frac{1}{4}$ Ans

(b) $P(\text{even number die or red suit card}) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$

$= \frac{3}{4}$ Ans

3.13. A binary number is one composed only of the digits zero and one. (For example, 1011, 1100, etc.) These numbers play an important role in the use of electronic computers. Suppose that a binary number is made up of n digits. Suppose that the probability of an incorrect digit appearing is p and that errors in different digits are independent of one another. What is the probability of forming an incorrect *number*?

Size of number = n digits

Probability of incorrect digit $= p$

probability of correct digit $= 1-p$

Probability of all digits correct $= (1-p)^n$

Probability of atleast one digit incorrect $= 1-(1-p)^n$

3.14. A die is thrown n times. What is the probability that "6" comes up at least once in the n throws?

P (6 comes up in one throw) $= \frac{1}{6}$

P (6 doesnot come up in one throw) $= \frac{5}{6}$

P(No 6 shows up in n throws) $= \left(\frac{5}{6}\right)^n$

P (Atleast one 6 shows up in n throws) $= 1 - \left(\frac{5}{6}\right)^n$ Ans

---

3.15. Each of two persons tosses three fair coins. What is the probability that they obtain the same number of heads?

$$\left.\begin{aligned} 0 \text{ Heads} &= \frac{1}{8} \times \frac{1}{8} \\ 1 \text{ Head} &= \frac{3}{8} \times \frac{3}{8} \\ 2 \text{ Heads} &= \frac{3}{8} \times \frac{3}{8} \\ 3 \text{ Heads} &= \frac{1}{8} \times \frac{1}{8} \end{aligned}\right\} + = \frac{20}{64} = \frac{5}{16} \text{ Ans}$$

3.16. **Two dice are rolled. Given that the faces show different numbers, what is the probability that one face is 4?**

$$P(\text{Both showing different numbers}) = 1 - P(\text{Both showing same number})$$

$$= 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$$

$$P(\text{Getting 4 is first}) = \frac{1}{6}$$

$$P(\text{Getting 4 is second}) = \frac{5}{6} \times \frac{1}{6}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \frac{\frac{1}{6} + \frac{5}{36}}{\frac{5}{6}} = \frac{\frac{11}{\cancel{36}_{6}}}{} \times \frac{\cancel{6}^{1}}{5} = \frac{11}{30} \text{ Ans}$$

3.17. It is found that in manufacturing a certain article, defects of one type occur with probability 0.1 and defects of a second type with probability 0.05. (Assume independence between types of defects.) What is the probability that:

(a) an article does not have both kinds of defects?
(b) an article is defective?
(c) an article has only one type of defect, given that it is defective?

$P(\text{Type}1) = 0.1 \qquad P(\text{Type }2) = 0.05$

$P(\text{not Type }1) = 1-0.1 = 0.9 \qquad P(\text{not Type}2) = 1-0.05 = 0.95$

(a) $P(\text{not Type}1 \text{ and not Type } 2) = 1 - P(\text{have both kinds of defects})$
$= 1 - P(\text{Type}1)\,P(\text{Type }2)$
$= 1 - 0.1 \times 0.05$
$= 1 - 0.005$
$= 0.995$

(b) $P(\text{an article is defective}) = P(\text{Type}1 \cup \text{Type}2)$

$= P(\text{Type}1) + P(\text{Type}2) - P(\text{Type}1 \cap \text{Type}2)$

Since, both are independent.
Therefore,

$P(\text{Type}1 \cap \text{Type }2) = P(\text{Type }1)\ P(\text{Type }2)$

$= P(\text{Type}1) + P(\text{Type }2) - P(\text{Type}1)\,P(\text{Type }2)$

$= 0.1 + 0.05 - 0.1 \times 0.05$

$= 0.145$ Ans

(c) $P(\text{Only one kind of defect}) = 0.1(1-0.05) + (1-0.1)(0.05)$

$= 0.095 + 0.045$

$= 0.14$ Ans

3.18. Verify that the number of conditions listed in Eq. (3.8) is given by $2^n - n - 1$.

Definition. The n events $A_1, A_2, \ldots, A_n$ are mutually independent if and only if we have for $k = 2, 3, \ldots, n$,

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2}) \cdots P(A_{i_k}). \qquad (3.8)$$

(There are altogether $2^n - n - 1$ conditions listed; see Problem 3–18.)

Total number of non empty subsets of n mutually independent events listed are

$$2^n - 1$$

Now we need to remove all singletons from this
There are n singletons possible

Therefore,

$$2^n - n - 1$$

$\square$

Hence Proved

3.19. Prove that if A and B are independent events, so are A and $\overline{B}$, $\overline{A}$ and B, $\overline{A}$ and $\overline{B}$.

If A and B are independent events

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$
$$= P(A) + P(B)(1 - P(A))$$

$$P(A\cup B) = P(A) + P(B) - P(A)P(B)$$
$$= P(A) + P(B)(1 - P(A))$$
$$= P(A) + P(B)P(\bar{A})$$
$$= 1 - P(\bar{A}) + P(B)P(\bar{A})$$
$$= 1 - P(\bar{A})(1 - P(B))$$
$$= 1 - P(\bar{A})P(\bar{B})$$

$$\boxed{P(A\cup B) = 1 - P(\bar{A})P(\bar{B})} \quad (1)$$

$$P(\bar{A}\cup\bar{B}) = P(\bar{A}) + P(\bar{B}) - P(\bar{A}\cap\bar{B})$$

From De Morgan's Law we know that

$$\bar{A}\cap\bar{B} = \overline{A\cup B} \quad \text{and} \quad \bar{A}\cup\bar{B} = \overline{A\cap B}$$

$$P(\overline{A\cap B}) = 1 - P(A) + 1 - P(B) - P(\bar{A}\cap\bar{B})$$
$$\Rightarrow 1 - P(A\cap B) = 2 - (P(A) + P(B) + P(\bar{A}\cap\bar{B}))$$
$$\Rightarrow - P(A)P(B) = 1 - (P(A) + P(B) + P(\bar{A}\cap\bar{B}))$$

$$P(\bar{A}\cap\bar{B}) = 1 - P(A) - P(B) + P(A)P(B)$$

$$= 1 - P(A) - P(B)(1 - P(A))$$

$$= (1 - P(A))(1 - P(B))$$

$$\boxed{P(\bar{A}\cap\bar{B}) = P(\bar{A})P(\bar{B})} \quad (2) \quad \square$$

Hence Proved

Replacing eqn (2) in eqn (1)

$$P(A\cup B) = 1 - P(\bar{A}\cap\bar{B}) = 1 - P(\overline{A\cup B})$$

For B and $\bar{A}$

For B and $\bar{A}$

$$A \cup B = A \cup (B \cap \bar{A})$$

$$\Rightarrow P(A \cup B) = P(A \cup (B \cap \bar{A}))$$

$$\Rightarrow \cancel{P(A)} + P(B) - P(A \cap B) = \cancel{P(A)} + P(B \cap \bar{A}) + P(A \cap B \cap \bar{A})$$

$$\Rightarrow P(B) - P(B)P(A) = P(B \cap \bar{A}) + 0$$

$$\Rightarrow P(B \cap \bar{A}) = P(B)(1 - P(A))$$

$$\Rightarrow P(B \cap \bar{A}) = P(B)P(\bar{A})$$

Hence, B and $\bar{A}$ are independent. $\square$

Hence Proved

For A and $\bar{B}$

$$A \cup B = (A \cap \bar{B}) \cup B)$$

$$\Rightarrow P(A \cup B) = P((A \cap \bar{B}) \cup B)$$

$$\Rightarrow P(A) + \cancel{P(B)} - P(A \cap B) = P(A \cap \bar{B}) + \cancel{P(B)} - P(A \cap \bar{B} \cap B)$$

$$\Rightarrow P(A) - P(A)P(B) = P(A \cap \bar{B})$$

$$P(A \cap \bar{B}) = P(A)(1 - P(B))$$

$$P(A \cap \bar{B}) = P(A)P(\bar{B})$$

Hence, A and $\bar{B}$ are independent.

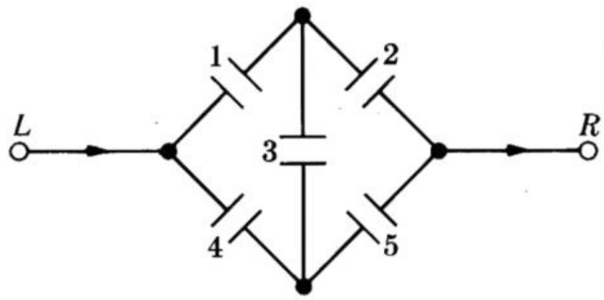
$\square$

Hence Proved

$\sqcup$

Hence Proved

3.20. In Fig. 3.11(a) and (b), assume that the probability of each relay being closed is p and that each relay is open or closed independently of any other relay. In each case find the probability that current flows from L to R.



(a)

FIGU

There are 4 possible ways in the above circuit for current to flow from L to R

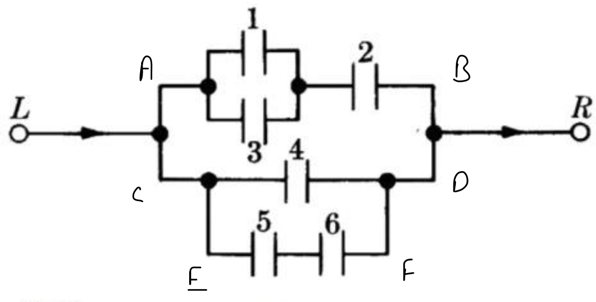
1-2 , 1-3-5 , 4-5 , 4-3-2

$$P\big(\underbrace{(1\cap 2)}_{a} \cup \underbrace{(3\cap 5\cap 1)}_{b} \cup \underbrace{(4\cap 5)}_{c} \cup \underbrace{(4\cap 3\cap 2)}_{d}\big)$$

$$\begin{aligned}
&= P(a) + P(b) + P(c) + P(d) - P(a\cap b) - P(a\cap c) - P(a\cap d) \\
&\quad - P(b\cap c) - P(b\cap d) - P(c\cap d) \\
&\quad + P(a\cap b\cap c) + P(a\cap c\cap d) + P(b\cap c\cap d) \\
&\quad + P(a\cap b\cap d) - P(a\cap b\cap c\cap d)
\end{aligned}$$

$$= p^2 + p^3 + p^2 + p^3 - p^4 - p^4 - p^4 - p^4 - p^4 + p^5 + p^5 + p^5 + \cancel{p^5} - \cancel{p^5}$$

$$= 2p^2 + 2p^3 - 5p^4 + 3p^5 \quad \text{Ans}$$



: 3.11 (b)

Between AB = $(1 \cup 3) \cap 2$

$= (1 \cap 2) \cup (3 \cap 2)$

Between CD = $4 \cup (5 \cap 6)$

Let $a = 1 \cap 2$ $\quad b = 3 \cap 2$ $\quad c = 4$ $\quad d = 5 \cap 6$

$$P(\text{Between } LR) = P\big((1\cap 2) \cup (3 \cap 2) \cup 4 \cup (5 \cap 6)\big)$$

$$= P(a \cup b \cup c \cup d)$$

$$= P(a) + P(b) + P(c) + P(d) - P(a \cap b) - P(a \cap c) - P(a \cap d) - P(b \cap c)$$
$$- P(b \cap d) - P(c \cap d) + P(a \cap b \cap c) + P(a \cap c \cap d) + P(a \cap b \cap d)$$
$$+ P(b \cap c \cap d) - P(a \cap b \cap c \cap d)$$

$$= p^2 + p^2 + p + p^2 - p^3 - p^3 - p^4 - p^3 - \cancel{p^4} - p^3 + \cancel{p^4} + p^5 + p^5 + p^5 - p^6$$

$$= p + 3p^2 - 4p^3 - p^4 + 3p^5 - p^6 \quad \text{Ans}$$

TABLE 3.2

Number of breakdowns	0	1	2	3	4	5	6
A	0.1	0.2	0.3	0.2	0.09	0.07	0.04
B	0.3	0.1	0.1	0.1	0.1	0.15	0.15

3.21. Two machines, *A*, *B*, being operated independently, may have a number of breakdowns each day. Table 3.2 gives the probability distribution of breakdowns for each machine. Compute the following probabilities.

(a) A and B have the same number of breakdowns.
(b) The total number of breakdowns is less than 4; less than 5.
(c) A has more breakdowns than B.
(d) B has twice as many breakdowns as A.
(e) B has 4 breakdowns, when it is known that B has at least 2 breakdowns.
(f) The minimum number of breakdowns of the two machines is 3; is less than 3.
(g) The maximum number of breakdowns of the machines is 3; is more than 3.

a) Same number of breakdowns means that either they both have 0 breakdowns or 1 breakdown or 2 breakdown and so on

Therefore,

$$P(\text{Same number of breakdowns}) = 0.1 \times 0.3 + 0.2 \times 0.1 + 0.3 \times 0.1 + 0.2 \times 0.1 + 0.09 \times 0.1 + 0.07 \times 0.15 + 0.04 \times 0.15$$

$$= 0.1255 \text{ Ans}$$

b) $P(\text{Total breakdowns} < 4) = P(0A)P(0B) + P(0A)P(1B) + P(0A)P(2B) + P(0A)P(3B) + P(0B)P(1A) + P(0B)P(2A) + P(0B)P(3A) + P(1A)P(1B) + P(1A)P(2B) + P(1B)P(2A) + P(2A)P(2B)$

Similarly, we can compute $P(\text{Total breakdown} < 5)$.

c) $P(\text{atleast 2 breakdowns}) = 1 - P(0 \text{ or } 1 \text{ breakdown})$

$$= 1 - (P(0) + P(1) - P(0 \cap 1))$$
$$= 1 - 0.3 - 0.1 + 0.3 \times 0.1$$
$$= 1 - 0.37$$
$$= 0.63$$

$$P(4 \text{ breakdown} \cap \text{atleast 2 breakdowns}) = \frac{0.1}{0.63} = 0.159$$

P (4 breakdown $\cap$ atleast 2 breakdowns) $= \dfrac{0.1}{0.63} = \underline{0.159}$

3.22. By verifying Eq. (3.2), show that for fixed A, $P(B \mid A)$ satisfies the various postulates for probability.

$$P(B_1 \cup B_2 \mid A) = P(B_1 \mid A) + P(B_2 \mid A) \quad \text{if} \quad B_1 \cap B_2 = \emptyset, \qquad (3.2)$$

$$P(B_1 \cup B_2 \mid A) = \frac{P((B_1 \cup B_2) \cap A)}{P(A)}$$

$$= \frac{P((B_1 \cap A) \cup (B_2 \cap A))}{P(A)}$$

$$= \frac{P(B_1 \cap A) + P(B_2 \cap A) - P(B_1 \cap B_2 \cap A)}{P(A)}$$

If $B_1 \cap B_2 = \phi$, then $P(B_1 \cap B_2 \cap A) = 0$

Therefore,

$$P(B_1 \cup B_2 \mid A) = \frac{P(B_1 \cap A)}{P(A)} + \frac{P(B_2 \cap A)}{P(A)}$$

$$= P(B_1 \mid A) + P(B_2 \mid A)$$

Hence Proved

3.23. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$.)

Second Order Determinant = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ = $ad - bc$

$ad - bc > 0$

$\Rightarrow ad > bc$

Each of a, b, c, d has a $\frac{1}{2}$ probability for each value 0 and 1

So total combinations of $a, b, c, d = 2^4 = 16$

Values possible for determinant = $1, 0, -1$

So, for $ad > bc$ to be true

$ad = 1$ and $bc = 0$

For $ad = 1$ both $a = 1$ and $d = 1$

and For $bc = 0$ both $b = 0$ and $c = 0$

$b = 1$ and $c = 0$

$b = 0$ and $c = 0$

Total number of possible combinations $= 1 \times 3$

$= 3$

$P(D > 0) = \frac{3}{16}$ Ans

3.24. Show that the multiplication theorem $P(A \cap B) = P(A \mid B)P(B)$, established for two events, may be generalized to three events as follows:

$$P(A \cap B \cap C) = P(A \mid B \cap C)P(B \mid C)P(C).$$

Given $\rightarrow$ $P(A \cap B) = P(A \mid B)\, P(B)$ is true

To Prove $\rightarrow$ $P(A \cap B \cap C) = P(A|B \cap C)\, P(B|C)\, P(C)$

$$P(A \cap B \cap C) = P(A \cap (B \cap C))$$

$$= P(A|B \cap C)\, P(B \cap C)$$

$$= P(A|B \cap C)\, P(B|C)\, P(C) = RHS$$

Hence Proved

$\square$

3.25. An electronic assembly consists of two subsystems, say *A* and *B*. From previous testing procedures, the following probabilities are assumed to be known:

$$P(A \text{ fails}) = 0.20,$$

$$P(B \text{ fails } alone) = 0.15.$$

$$P(A \text{ and } B \text{ fail}) = 0.15,$$

Evaluate the following probabilities.

(a) $P(A \text{ fails} \mid B \text{ has failed})$,

(b) $P(A \text{ fails alone})$.

(a)

$$P(A \text{ fails alone}) = P(A \text{ fails}) - P(A \text{ and } B \text{ fails})$$

$$= 0.20 - 0.15 = 0.05$$

$$P(B \text{ fails}) = P(B \text{ fails alone}) + P(A \text{ and } B \text{ fails})$$

$$= 0.15 + 0.15 = 0.30$$

$$P(A \text{ fails} \mid B \text{ has failed}) = \frac{P(A \text{ and } B \text{ fail})}{P(B \text{ fails})} = \frac{0.15}{0.30} = \frac{1}{2} = 0.5$$

Ans

(b) $P(A \text{ fails alone}) = 0.05$ Ans

3.26. Finish the analysis of the example given in Section 3.2 by deciding which of the types of candy jar, *A* or *B*, is involved, based on the evidence of two pieces of candy which were sampled.