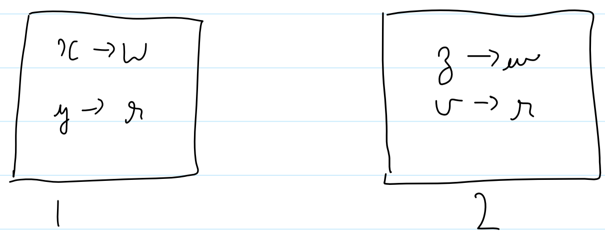
3.1. Urn 1 contains x white and y red balls. Urn 2 contains z white and v red balls. A ball is chosen at random from urn 1 and put into urn 2. *Then* a ball is chosen at random from urn 2. What is the probability that this ball is white?



$$P(w \text{ from } 2) = P(w \text{ from } 1) \times P(w \text{ from new } 2) + P(r \text{ from } 1) \times P(w \text{ from new } 2)$$

$$= \frac{x}{x+y} \times \frac{z+1}{z+v+1} + \frac{y}{x+y} \times \frac{z}{z+y+1}$$

$$= \frac{x(z+1) + zy}{(x+y)(z+v+1)} \quad \text{Ans}$$

3.2. Two defective tubes get mixed up with two good ones. The tubes are tested, one by one, until both defectives are found.

(a) What is the probability that the last defective tube is obtained on the second test?

(b) What is the probability that the last defective tube is obtained on the third test?

(c) What is the probability that the last defective tube is obtained on the fourth test?

(d) Add the numbers obtained for (a), (b), and (c) above. . Is the result surprising?

2 → Defective Tubes
2 → Good Tubes

(a) That means both first and second tubes are defective

$P(\text{first defective}) = 2/4 = 1/2$

$P(\text{second defective}) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

(b) Last defective found on 3rd test.

This means that in 1st and 2nd tests we have already found one defective and one non defective tube.

We can have following cases possible

DND or NDD

$$= \frac{\cancel{2}^1}{\cancel{4}_2} \times \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} + \frac{\cancel{2}^1}{4_2} \times \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} = \frac{2}{6} = \frac{1}{3}$$

(c) Last defective found on 4^{th} test

Following cases are possible

DNND + NDND + NNDD

$$= \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \times 1 + \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \times 1 + \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \times 1$$

$$= \frac{3}{6} = \frac{1}{2} \text{ Ans}$$

(d) On adding results of a, b and c we get

$$\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = \frac{2+3+1}{6} = 1$$

$$\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = \frac{2+3+1}{6} = 1$$

3.3. A box contains 4 bad and 6 good tubes. Two are drawn out together. One of them is tested and found to be good. What is the probability that the other one is also good?

$4 \rightarrow$ Bad

$6 \rightarrow$ Good

$P(\text{1st is Good}) = 1$ (Given)

$$P(\text{2nd is Good} \mid \text{1st is Good}) = P(\text{1st is Good})\, P(\text{2nd is Good})$$

$$= 1 \times \frac{5}{9} = \frac{5}{9}$$

3.4. In the above problem the tubes are checked by drawing a tube at random, testing it and repeating the process until all 4 bad tubes are located. What is the probability that the fourth bad tube will be located

(a) on the fifth test?

(b) on the tenth test?

(a) $\square \ \square \ \square \ \square \ \square$

G	B	B	B	B	$6/10 \times 4/9 \times 3/8 \times 2/7 \times 1/6$
B	G	B	B	B	$4/10 \times 6/9 \times 3/8 \times 2/7 \times 1/6$
B	B	G	B	B	$4/10 \times 3/9 \times 6/8 \times 2/7 \times 1/6$
B	B	B	G	B	$4/10 \times 3/9 \times 2/8 \times 6/7 \times 1/6$

$$\cancel{4} \times \frac{\cancel{6}}{\cancel{10}_{5}} \times \frac{\cancel{4}^{2}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{8}} \times \frac{\cancel{2}}{7} \times \frac{1}{\cancel{6}} = \boxed{\frac{2}{105}} \text{ Ans}$$

(b) On tenth test

Arrange 3 bad and 6 good tubes in 9 tests

$$\frac{9!}{3!\,6!} = \frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7}{\cancel{3} \times \cancel{2}} = 84$$

$$\frac{\cancel{6}}{10} \times \frac{\cancel{5}}{\cancel{9}} \times \frac{\cancel{4}}{\cancel{8}} \times \frac{\cancel{3}}{\cancel{7}} \times \frac{\cancel{2}}{\cancel{6}} \times \frac{1}{\cancel{8}} \times \cancel{3 \times 4 \times 7} = \frac{4}{10} = \frac{2}{5} \text{ Ans}$$

3.5. Suppose that A and B are independent events associated with an experiment. If the probability that A or B occurs equals 0.6, while the probability that A occurs equals 0.4, determine the probability that B occurs.

$P(A \cup B) = 0.6$

$P(A) = 0.4$

As we know that for two independent events $P(A \cap B)$ is given by

$$P(A \cap B) = P(A)\,P(B)$$

and we also know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$

$\Rightarrow 0.6 = 0.4 + P(B)(1 - 0.4)$

$\Rightarrow \dfrac{0.2}{0.6} = P(B)$

$P(B) = \dfrac{1}{}$ Ans

3

3.6. Twenty items, 12 of which are defective and 8 nondefective, are inspected one after the other. If these items are chosen at random, what is the probability that:

(a) the first two items inspected are defective?
(b) the first two items inspected are nondefective?
(c) among the first two items inspected there is one defective and one nondefective?

Total = 20
Defective = 12 Nondefective = 8

(a) P(First two items are defective) = $\frac{\cancel{12}^{3}}{\cancel{20}_{5}} \times \frac{11}{19} = \frac{33}{95}$ Ans

(b) P(First two items are non defective) = $\frac{8}{20} \times \frac{7}{19} = \frac{14}{95}$ Ans

(c) P(One defective and one non defective) = $\frac{\cancel{12}^{3}}{\cancel{20}_{5}} \times \frac{8}{19} \times 2 = \frac{48}{95}$ Ans

2 is multiplied because there could be two scenarios
1st Defective and 2nd Non Defective
1st Non Defective and 2nd Defective

3.7. Suppose that we have two urns, 1 and 2, each with two drawers. Urn 1 has a gold coin in one drawer and a silver coin in the other drawer, while urn 2 has a gold coin in each drawer. One urn is chosen at random; then a drawer is chosen at random from the chosen urn. The coin found in this drawer turns out to be gold. What is the probability that the coin came from urn 2?

1G	1S

Urn 1

1G	1G

Urn 2

$$\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$$

$$P(G \mid \text{Urn } 1) = \frac{1}{2}$$