Introduction to Quantum Information and Computing Half 2 Lecture 7

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So S is basically

$$sin(\theta/2)]|w> + cos(\theta/2)|S_{\overline{w}}>$$

$D = 2|S><S| - I$

$$D = 2[|w><w|sin^2(\theta/2) + |w><S_{\overline{w}}|sin(\theta/2)cos(\theta/2)$$
$$+ |S_{\overline{w}}><w|sin(\theta/2)cos(\theta/2) + |S_{\overline{w}}><S_{\overline{w}}|cos^2(\theta/2)] - I$$

After all D is a 2×2 matrix
So in the basis $|S_{\overline{w}}>, |w>$ D can be represented as :

D = $\begin{bmatrix} cos(\theta) & sin(\theta) \\ sin(\theta) & -cos(\theta) \end{bmatrix}$
We are given $G = D \cdot u_f$

G = $\begin{bmatrix} cos(\theta) & sin(\theta) \\ sin(\theta) & -cos(\theta) \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
$G = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$
As we can see G is a rotation matrix.
$|S>= G^k \cdot |S>$
Let's see the product one time :
$G|S> = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} |S>$
$G|S> = \sin(\theta + \theta/2)|w> + cos(\theta + \theta/2)|S_{\overline{w}}>$

After k times :
$|S> = \sin(k\theta + \theta/2)|w> + cos(k\theta + \theta/2)|S_{\overline{w}}>$
At that time : $sin(k\theta + \theta/2) = 1$
$\implies \theta(k + 1/2) = \pi/2$
$\implies \theta = \frac{\pi}{2k+1}$ We know that
$sin(\theta/2) = \sqrt{\frac{M}{N}} \approx \theta/2$
$2k + 1 = \frac{\pi\sqrt{N}}{2\sqrt{M}}$
$k = \frac{\pi\sqrt{N}}{2\sqrt{M}} - \frac{1}{2}$
$k \approx O(\sqrt{\frac{N}{M}})$
So finally
$G^k|S >= \frac{1}{\sqrt{M}} \sum_{f(|x>=1)} |x>$

Now what if M is unknown :-
(a) Estimate 'M' before only (Quantum Counting)
(b) Randomized Quantum Search

Amplification of amplitude :
$G^k H^{\otimes n}|0^n >= G^k[sin(\theta/2)]|w> + cos(\theta/2)|S_{\overline{w}}>]$
$\sqrt{p} = sin(\theta/2)$
To amplify this term to 1 I need $\frac{1}{\sqrt{p}}$ queries.
$A^{\frac{1}{\sqrt{p}}} u|0> \approx |\psi_{good}>$

1 Models of Quantum Computing

1. Adiabatic Model :
$H(s) = (1 - s)H_0 + sH_k$
s$\in [0, 1]$
2. Quantum walks
3. MDQC
4. Topological Quantum Channel