Introduction to Quantum Information and Computing - Lecture 4

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1 Observables

- Each observable can be represented by Hermitian operators.
- Expectation value of an observable $\hat{O}$ for a quantum state ρ is $\mathrm{Tr}[\hat{O}\rho]$
- If ρ is pure state $\{|\psi\rangle\langle\psi|\}$, then:

  $\mathrm{Tr}[\hat{O}\rho]=\mathrm{Tr}[\hat{O}|\psi\rangle\langle\psi|]=\mathrm{Tr}[\langle\psi|\hat{O}|\psi\rangle]=\langle\psi|\hat{O}|\psi\rangle$
- If ρ is not a pure state, i.e., $\rho=\sum_i p_i|\psi_i\rangle\langle\psi_i|$, then:

  $\mathrm{Tr}[\hat{O}\rho]=\mathrm{Tr}[\hat{O}\sum_i p_i|\psi_i\rangle\langle\psi_i|]=\sum_i p_i\mathrm{Tr}[\hat{O}|\psi_i\rangle\langle\psi_i|]=\sum_i p_i\langle\psi|\hat{O}|\psi_i\rangle$

2 Mixed State

Mixed state is said to be the mixture of the different pure states.
Mixed states can be written as: $\sigma = \sum_i p_i\rho_i, where \sum_i p_i = 1$
Here σ: is the mixed state , ρ_i:pure state and p_i: probability of state ρ_i to be present in the mixed state.

- A state is pure if $\rho^2 = \rho$

Example1:- Let us consider a system which has 5 qubits of $|0\rangle$ and 5 qubits of $|1\rangle$ where $\{|0\rangle, |1\rangle\}$ are orthonormal basis. Find the mixed state of the system.
Answer1:- $\sigma = \frac{1}{2}\{|0\rangle\langle 0| + |1\rangle\langle 1|\}$

Example2:- Let us consider a system which has 5 qubits of $|+\rangle$ and 5 qubits of $|-\rangle$ where $\{|+\rangle, |-\rangle\}$ are orthonormal basis. Find the mixed state of the system.
Answer2:- $\sigma = \frac{1}{2}\{|+\rangle\langle +| + |-\rangle\langle -|\}$

Note:- Both σ obtained in the above two examples are one and the same.
Proof:-
$\sigma = \frac{1}{2}\{|0\rangle\langle 0| + |1\rangle\langle 1|\} = \frac{1}{2}\{|+\rangle\langle +| + |-\rangle\langle -|\}$

$\sigma = \frac{1}{2}(\sum_i |i\rangle\langle i|) = \frac{1}{2}(\sum_i |i\rangle\langle i|)$

$\sigma = \frac{1}{2}\mathbb{1} = \frac{1}{2}\mathbb{1}$

3 Superposition

The linear combination of states is called superposition.
A superpositioned state can be represented as:
$|\psi\rangle = \sum_i c_i |\psi_i\rangle$,
where $\sum_i c_i^2 = 1$.