

# Introduction to Quantum Information and

## Computing Half 2 Lecture 7

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So S is basically

$$\sin(\theta/2)]|w\rangle + \cos(\theta/2)|S_{\bar{w}}\rangle$$

$$D = 2|S\rangle\langle S| - I$$

$$D = 2[|w\rangle\langle w|\sin^2(\theta/2) + |w\rangle\langle S_{\bar{w}}|\sin(\theta/2)\cos(\theta/2) \\ + |S_{\bar{w}}\rangle\langle w|\sin(\theta/2)\cos(\theta/2) + |S_{\bar{w}}\rangle\langle S_{\bar{w}}|\cos^2(\theta/2)] - I$$

After all D is a  $2 \times 2$  matrix

So in the basis  $|S_{\bar{w}}\rangle, |w\rangle$  D can be represented as :

$$D = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$$

We are given  $G = D \cdot u_f$

$$G = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$G = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

As we can see G is a rotation matrix.

$$|S\rangle = G^k \cdot |S\rangle$$

Let's see the product one time :

$$G|S\rangle = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} |S\rangle$$

$$G|S\rangle = \sin(\theta + \theta/2)|w\rangle + \cos(\theta + \theta/2)|S_{\bar{w}}\rangle$$

After k times :

$$|S\rangle = \sin(k\theta + \theta/2)|w\rangle + \cos(k\theta + \theta/2)|S_{\bar{w}}\rangle$$

At that time :  $\sin(k\theta + \theta/2) = 1$

$$\implies \theta(k + 1/2) = \pi/2$$

$$\implies \theta = \frac{\pi}{2k+1} \text{ We know that}$$

$$\sin(\theta/2) = \sqrt{\frac{M}{N}} \approx \theta/2$$

$$2k + 1 = \frac{\pi\sqrt{N}}{2\sqrt{M}}$$

$$k = \frac{\pi\sqrt{N}}{2\sqrt{M}} - \frac{1}{2}$$

$$k \approx O(\sqrt{\frac{N}{M}})$$

So finally

$$G^k|S\rangle = \frac{1}{\sqrt{M}} \sum_{f(|x|=1)} |x\rangle$$

Now what if M is unknown :-

(a) Estimate 'M' before only (Quantum Counting )

(b) Randomized Quantum Search

Amplification of amplitude :

$$G^k H^{\otimes n} |0^n\rangle = G^k [\sin(\theta/2)] |w\rangle + \cos(\theta/2) |S_{\bar{w}}\rangle$$

$$\sqrt{p} = \sin(\theta/2)$$

To amplify this term to 1 I need  $\frac{1}{\sqrt{p}}$  queries.

$$A^{\frac{1}{\sqrt{p}}} u |0\rangle \approx |\psi_{good}\rangle$$

## 1 Modules of Quantum Computing

1. Adiabatic Model :

$$H(s) = (1-s)H_0 + sH_k$$

$s \in [0, 1]$

2. Quantum walks

3. MDQC

4. Topological Quantum Channel