Introduction to Quantum Information and Computing - Lecture 6

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1 Quantum Channel

$$\mathcal{N}_{A\to B} : \mathcal{B}(\mathcal{H_A}) \to \ B(\mathcal{H_B})$$

where

$$(\mathcal{H_A}) = \text{Hilbert space of input system,}$$
$$(\mathcal{B}) = \text{Set of bounded operators}$$

Quantum channel is trace-preserving since it maps trace-class operators to trace-class operators.

A matrix with a finite trace represents a trace-class operator. A finite-dimensional matrix is a trace-class operator if it is bounded.

A **Super Operator** acts on an operator to give an operator.

1.1 Properties of Quantum Channel

For a super operator to be a quantum channel, it has to follow the following properties:

1. Completely positive

$$id_B \otimes \mathcal{N}_{A\to C}(\Phi_{AB}) \geq 0$$

where $\Phi_{AB} \geq 0$ and id = identity super operator, and ϕ_{AB} is a maximally entangled state.

Positivity: $X_A \geq 0 \Rightarrow \mathcal{N}_{A\to C}(\mathcal{X}_A) = \mathcal{Y}_C \geq 0$

2. Trace-preserving map

$$Tr[\rho_A] \Rightarrow Tr[\mathcal{N}_{A\to C}(\rho_A)]$$

2 Joint States

2.1 Product State

The density operator of the product state of A and B is the tensor product of the density operators of A and B.

$$\rho_{AB} = \rho_A \otimes \sigma_B$$

In a product state, there is no correlation between the two states. A and B are independent.

2.2 Separable States

The density operator of a separable state is obtained as:

$$\rho_{AB} = \sum_x p_x \rho_A^{(x)} \otimes \sigma_B^{(x)}$$

where p_x is the probability of each state such that

$$\sum p_x = 1, p_x \geq 0$$

Here, $\rho_A^{(x)}$ and $\sigma_B^{(x)}$ may be pure or mixed states.

2.3 Entangled State

State which cannot be written as separable states or product of states is in an entangled state.

$$\mathcal{N}_{A\to C}(\Phi_{AB})$$

Dimension, d $= min(dim(\mathcal{H}_A), dim(\mathcal{H}_B))$

Maximally entangled state: $\Phi_{AB} = \frac{1}{d}\sum_{i,j=0}^{d-1} |i\rangle_A \otimes |i\rangle_B \langle j|_A \otimes \langle j|_B$
$= \frac{1}{d}\sum_{i,j=0}^{d-1} |i\rangle \langle j|_A \otimes |i\rangle \langle j|_B$

3 Kraus Operators: K_i

$$\mathcal{N}_{A\to C}(\cdot) = \sum_i K_i(\cdot)K_i{}^\dagger$$

where

$$\sum K_i{}^\dagger K_i = \nVdash$$
$$K_i : \mathcal{H}_A \to \mathcal{H}_C$$

For any completely positive, trace-preserving quantum channel, $\exists Krausoperators$.
If an N has Kraus operators $\Rightarrow$ it is completely positive and trace-preserving.

Projective measurement is a special case of quantum channel where Ps are Kraus operators.