Introduction to Quantum Information and Computing - Lecture 1

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1 Introduction and Motivation

1.1 Introduction

Quantum mechanics is a fundamental theory in physics that provides a description of the physical properties of nature at the scale of atoms and subatomic particles. It is a superset of classical mechanics and can explain behaviour in experiments that are not explained by only classical mechanics laws. One such experiment is the Stern Gerlach experiment.

The mathematical framework for quantum mechanics is linear algebra. States are described as vectors in a complex vector space, while measurements and operators are linear operators that act on the space.

1.2 Stern-Gerlach Experiment

Silver atoms are heated in an oven and projected at a screen through a hole. The stream of silver atoms are subjected to a homogeneous magnetic field. This causes them to bend in trajectory upwards or downwards.

The magnetic moment of the atom is proportional to the spin of the atom's unpaired electron: $\mu \propto S$

Because the interaction energy of the magnetic moment with the magnetic field is just $-\mu \cdot B$, the z-component of the force experienced by the atom is given by,

$$\mathbf{F}_z = \frac{\partial}{\partial z}(-\mu \cdot B) \simeq \mu_z \frac{\partial B_z}{\partial z}$$

The expected pattern is that of a band of silver, however only two spots are observed. This implies the spin of the atoms along the Z-axis is quantized to two values, S_z+ and S_z-. This goes against the classical prediction.

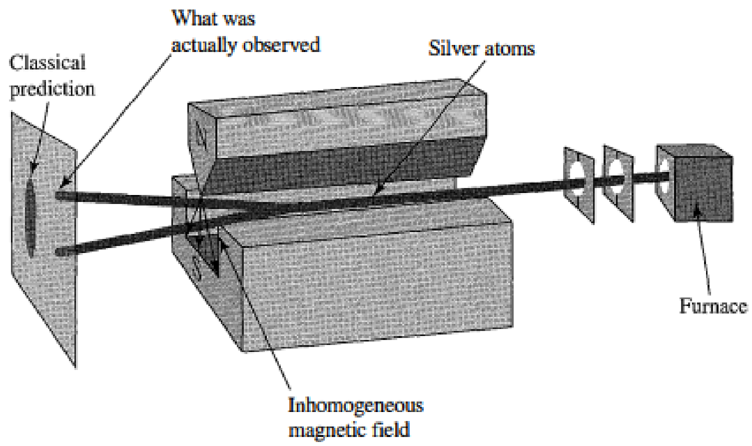

FIGURE 1.1 The Stern-Gerlach experiment.

Moreover, on sequential Stern-Gerlach experiments, more non classical behaviour was seen.

In the experiment, S_z was measured and the atoms with S_z- spin are blocked. Then the rest are sent through another Stern-Gerlach for measuring S_x. Again, the atoms with spin S_x- are blocked. The final leftover atoms are passed through an experiment measuring S_z again.

It is found that even though all atoms with S_z- were blocked, there still are atoms with S_z- spin after the third experiment. This phenomenon cannot be explained by classical mechanics. Measuring S_x destroys any information obtained about the S_z component of the atoms.

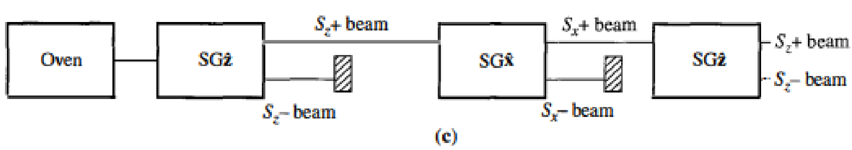

FIGURE 1.3 Sequential Stern-Gerlach experiments.

1.3 Shannon's Theory of Information and Entropy

Claude Shannon created the modern information theory. He justified use of entropy as measure of information about a system. The units for information were taken as bits. In terms of quantum information, the units are qubits.

Shannon defined entropy as the average measure of randomness or uncertainty in a system. The Shannon entropy is defined as entropy H (Greek capital letter eta) of a discrete random variable X, which takes values in the alphabet

$\mathcal{X}$ and is distributed according to

$$p : \mathcal{X} \longrightarrow [0, 1]$$

such that

$$p(x) := \mathbb{P}[X = x]$$

And H is given by

$$H(\mathcal{X}) = \mathbb{E}[-logp(X)] = -\sum_{x \in \mathbb{X}} p(x) logp(x)$$

1.4 No Cloning Theorem

In physics, the no-cloning theorem states that it is impossible to create an independent and identical copy of an arbitrary unknown quantum state. This has profound implications in quantum computation. The theorem follows from the fact that all quantum operations must be unitary linear transformation on the state.

1.5 Outline of the Course

In the course we will be exploring the following topics:

1. Postulates
2. Everything is a quantum channel
3. Entanglement, Separability, Non-locality
4. Teleportation, No Cloning
5. Entropy, Trace Distance

2 Finite Dimensional Hilbert Spaces

A d-dimensional Hilbert space $\mathcal{H}$ $(1 \leq d < \infty)$ is a complex vector space with an inner product defined on it. A vector in the Hilbert space $\mathcal{H}$ is denoted by $|\psi\rangle$. The inner product $\langle ., . \rangle : \mathcal{H} \times \mathcal{H} \rightarrow \mathbb{C}$ has the following properties:

- *Non negativity* - $\langle \psi, \psi \rangle \geq 0 \; \forall \; |\psi\rangle \in \mathcal{H}$. $\langle \psi, \psi \rangle = 0$ if and only if $\langle \psi \rangle = 0$.
- *Linearity in Second Argument* - $\langle \psi, \alpha\phi_1 + \beta\phi_2 \rangle = \alpha\langle \psi, \phi_1 \rangle + \beta\langle \psi, \phi_2 \rangle$
- *Conjugate Linearity in First Argument* - $\langle \alpha\psi_1 + \beta\psi_2, \phi \rangle = \bar{\alpha}\langle \psi_1, \phi \rangle + \bar{\beta}\langle \psi_2, \phi \rangle$
- *Conjugate Symmetry* - $\langle \psi, \phi \rangle = \overline{\langle \phi, \psi \rangle}$

3 Describing a Closed Physical System

The complete description of a closed physical system is given by its state $|\psi\rangle$ where $|\psi\rangle \in \mathcal{H}$ ($\mathcal{H}$ is a Hilbert Space) and norm of $|\psi\rangle$ is 1 ($(\langle\psi,\psi\rangle = 1)$). For every state $|\psi\rangle \in \mathcal{H}$, $\exists$ $\langle\psi|$ in the dual vector space of $\mathcal{H}$. Also, $\langle\psi| = (|\psi\rangle)^\dagger$.

For $|\psi\rangle$ to represent a closed system, the Hilbert Space it belongs to must have dimension $d \geq 2$, $d \in \mathbb{N}$.