Introduction to Quantum Information and Computing - Lecture 1

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1.1 Introduction

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1.7 Outline of the Course

1. Postulates
2. Everything is a quantum channel
3. Entanglement, Separability, Nonlocality
4. Teleportation, No Cloning
5. Entropy, Trace Distance

2 Finite Dimensional Hilbert Spaces

A d-dimensional Hilbert space $\mathcal{H}$ ($1 \leq d < \infty$) is a complex vector space with an inner product defined on it. A vector in the Hilbert space $\mathcal{H}$ is denoted by $|\psi\rangle$. The inner product $\langle ., . \rangle : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$ has the following properties:

- *Non negativity* - $\langle \psi, \psi \rangle \geq 0 \ \forall \ |\psi\rangle \in \mathcal{H}$. $\langle \psi, \psi \rangle = 0$ if and only if $\langle \psi \rangle = 0$.
- *Linearity in Second Argument* - $\langle \psi, \alpha\phi_1 + \beta\phi_2 \rangle = \alpha\langle \psi, \phi_1 \rangle + \beta\langle \psi, \phi_2 \rangle$
- *Conjugate Linearity in First Argument* - $\langle \alpha\psi_1 + \beta\psi_2, \phi \rangle = \bar{\alpha}\langle \psi_1, \phi \rangle + \bar{\beta}\langle \psi_2, \phi \rangle$
- *Conjugate Symmetry* - $\langle \psi, \phi \rangle = \overline{\langle \phi, \psi \rangle}$

3 Describing a Closed Physical System

The complete description of a closed physical system is given by its state $|\psi\rangle$ where $|\psi\rangle \in \mathcal{H}$ ($\mathcal{H}$ is a Hilbert Space) and norm of $|\psi\rangle$ is 1 ($\langle \psi, \psi \rangle = 1$). For every state $|\psi\rangle \in \mathcal{H}$, $\exists \ \langle \psi|$ in the dual vector space of $\mathcal{H}$. Also, $\langle \psi| = (|\psi\rangle)^\dagger$.

For $|\psi\rangle$ to represent a closed system, the Hilbert Space it belongs to must have dimension $d \geq 2$, $d \in \mathbb{N}$.