Fuzzy Logic in Artificial Intelligence

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Uncertainty in Data

Certainty

Uncertainty

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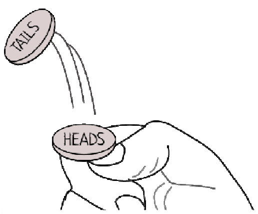
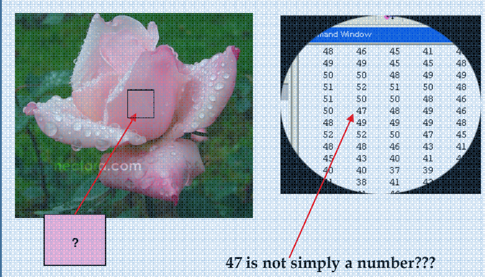
Representing the Uncertainty



$Outcomes, \boldsymbol{\Omega} = \{Head, Tail\}$

Event, X= $\begin{cases} 0, & if\ Tails \\ 1, & if\ Head \end{cases}$

$\Pr(X = 1) = f(1) = 1/2$



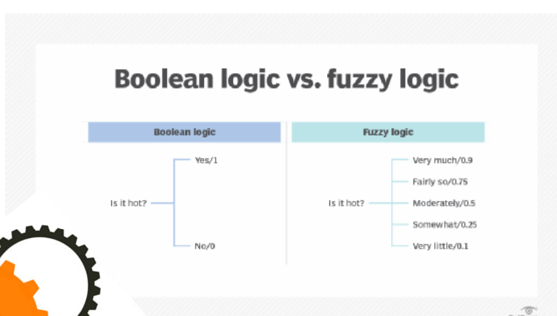
Obtained from the relationship/rules between the other surrounding numbers

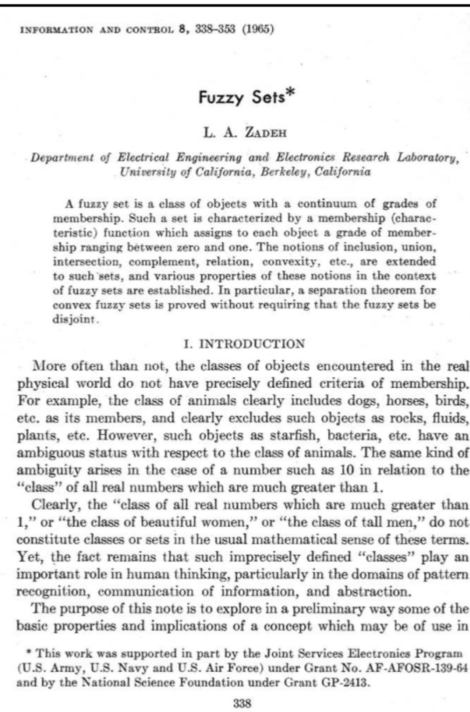
$$P(A_i|B) = \frac{P(B|A_i)\,P(A_i)}{\sum_j P(B|A_j)\,P(A_j)},$$

can be used to model those relationships

Fuzzy Logic

Fuzzy logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" (1 or 0) Boolean logic on which the modern computer is based.




INFORMATION AND CONTROL 8, 338–353 (1965)

Fuzzy Sets*

L. A. Zadeh

Department of Electrical Engineering and Electronics Research Laboratory, University of California, Berkeley, California

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint.

I. INTRODUCTION

More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes such objects as rocks, fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1.

Clearly, the "class of all real numbers which are much greater than 1," or "the class of beautiful women," or "the class of tall men," do not constitute classes or sets in the usual mathematical sense of these terms. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction.

The purpose of this note is to explore in a preliminary way some of the basic properties and implications of a concept which may be of use in

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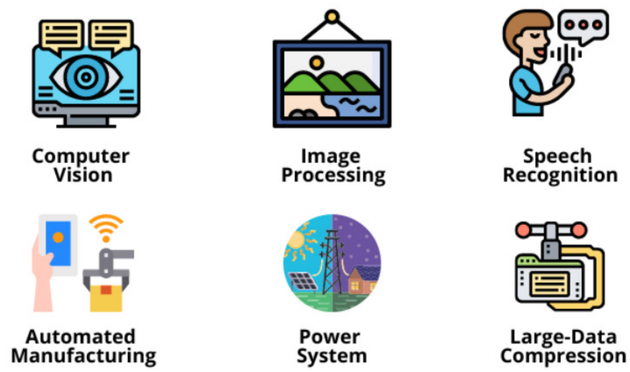
Soft Computing

Soft Computing	Hard Computing
It can be evolve its own programs	It requires program to be written
It uses fuzzy logic	It uses two valued logic
It can deal with noisy data	It can only deal with exact data
It allows parallel computing	It allows sequential computing
It gives approximate answers	It gives exact answers
It needs robustness	It needs accuracy
It is also known as computation intelligence	It is also known as Conventional intelligence



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Fuzzy Deals with…

Uncertain Data:

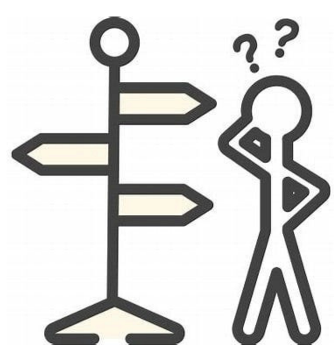
1. **Definition:** Uncertain data refers to information for which the level of confidence or probability is not precisely known.
2. **Characteristics:** It involves a degree of doubt or lack of confidence in the accuracy of the data.
3. **Example:** A weather forecast predicting a 60% chance of rain. The uncertainty lies in the probability of the event occurring.

Vague Data:

1. **Definition:** Vague data is information that lacks clarity or precision and is not well-defined.
2. **Characteristics:** It often arises from ambiguous or poorly specified terms, making interpretation challenging.
3. **Example:** Describing a person as "tall" without specifying a height range. The term "tall" is vague and can mean different things to different people.

Imprecise Data:

1. **Definition:** Imprecise data lacks exactness or accuracy and involves approximations or estimations.
2. **Characteristics:** It may result from rounded values, qualitative descriptions, or incomplete information.
3. **Example:** Providing an estimate like "about 20 minutes" instead of specifying an exact time.



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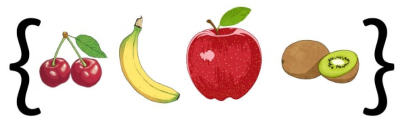
Crisp Sets

Set **A**



Crisp sets are the sets that we have used most of our life. A collection of well defined items will be there in a crisp set. For example, an apple belongs in the set of fruits and potatoes do not.

Characteristic Function

$$\chi_A(x) = \begin{cases} 1, if\ x \in A \\ 0, if\ x \notin A \end{cases}$$

$\chi_A : X = \{1,0\}$

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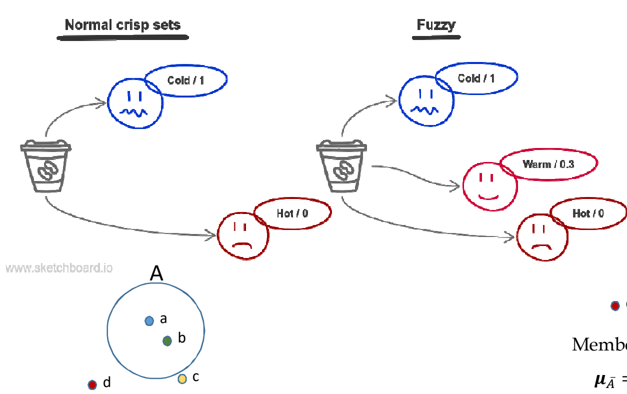
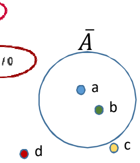
Fuzzy Sets





Membership Function

$\mu_{\bar{A}} = \{x, \mu_{\bar{A}}(x)/x \in X\}$
$\mu_{\bar{A}}$: [0,1]

$\chi_A(a) = 1 \quad \chi_A(c) = 0$

$\chi_A(b) = 1 \quad \chi_A(d) = 0$

$\mu_{\bar{A}}(a) = 0.98 \quad \mu_{\bar{A}}(c) = 0.07$

$\mu_{\bar{A}}(b) = 0.76 \quad \mu_{\bar{A}}(d) = 0$

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Fuzzy Sets…

Example 1

$X = \{1,2,3,4,5,6,7 \ldots\}$

$\bar{A}$={'Approximately 4'}

$\mu_{\bar{A}}(1) = 0$
$\mu_{\bar{A}}(2) = 0.25$
$\mu_{\bar{A}}(3) = 0.5$
$\mu_{\bar{A}}(4) = 1$
$\mu_{\bar{A}}(5) = 0.5$
$\mu_{\bar{A}}(6) = 0$

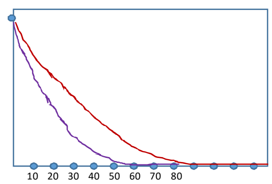
Example 2

$\overline{A1}$={'Ramesh is young'}

$X = [0,80]$

$\overline{A2}$={'Suresh is very young'}

$X = [0,50]$



$\mu_{\overline{A1}}(30) = 0.75$
$\mu_{\overline{A2}}(30) = 0.4$

Example 3

$X = \{Real\ numbers\}$

$\overline{A1}$={'Real numbers close to zero'}

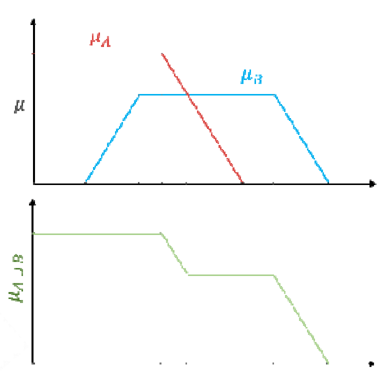
$\mu_{\bar{A}1}(x) = \dfrac{1}{1+x^2}$

$\overline{A2}$={'Real numbers very close to zero'}

$\mu_{\bar{A}2}(x) = \dfrac{1}{(1+x^2)^2}$

Fuzzy Set Operations

The **union** of two fuzzy sets $\underline{A}$ and $\underline{B}$ is a fuzzy set $\underline{C}$, written as $\underline{C} = \underline{A} \cup \underline{B}$
$\underline{C} = \underline{A} \cup \underline{B} = \{(x, \mu_{\underline{A} \cup \underline{B}}(x)) \mid \forall x \in X\}$
$\mu_{\underline{C}}(x) = \mu_{\underline{A} \cup \underline{B}}(x) = \mu_{\underline{A}}(x) \vee \mu_{\underline{B}}(x) = \max(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)), \forall x \in X$



$\underline{C} = \underline{A} \cup \underline{B} = \{(x, \mu_{\underline{A} \cup \underline{B}}(x)) \mid \forall x \in X\}$
$\underline{A} = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \}$
$\underline{B} = \{ (x_1, 0.8), (x_2, 0.6), (x_3, 0.4), (x_4, 0.2), (x_5, 0.1) \}$
$\mu_{\underline{A} \cup \underline{B}}(x_1) = \max(\mu_{\underline{A}}(x_1), \mu_{\underline{B}}(x_1)) = \max\{0.2, 0.8\} = 0.8$
$\mu_{\underline{A} \cup \underline{B}}(x_2) = \max(\mu_{\underline{A}}(x_2), \mu_{\underline{B}}(x_2)) = \max\{0.5, 0.6\} = 0.6$
$\mu_{\underline{A} \cup \underline{B}}(x_3) = \max(\mu_{\underline{A}}(x_3), \mu_{\underline{B}}(x_3)) = \max\{0.6, 0.4\} = 0.6$
$\mu_{\underline{A} \cup \underline{B}}(x_4) = \max(\mu_{\underline{A}}(x_4), \mu_{\underline{B}}(x_4)) = \max\{0.8, 0.2\} = 0.8$
$\mu_{\underline{A} \cup \underline{B}}(x_5) = \max(\mu_{\underline{A}}(x_5), \mu_{\underline{B}}(x_5)) = \max\{1.0, 0.1\} = 1.0$
So, $\underline{A} \cup \underline{B} = \{ (x_1, 0.8), (x_2, 0.6), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \}$

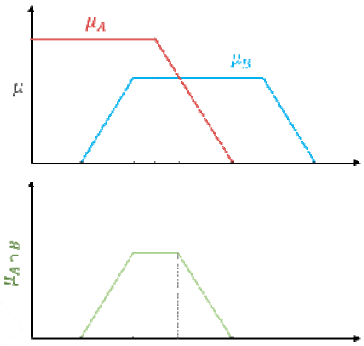
Fuzzy Set Operations…

The **intersection** of two fuzzy sets $\underline{A}$ and $\underline{B}$ is a fuzzy set $\underline{C}$, written as $\underline{C} = \underline{A} \cap \underline{B}$

$\underline{C} = \underline{A} \cap \underline{B} = \{(x, \mu_{\underline{A} \cap \underline{B}}(x)) \mid \forall x \in X\}$

$\mu_{\underline{C}}(x) = \mu_{\underline{A} \cap \underline{B}}(x) = \mu_{\underline{A}}(x) \wedge \mu_{\underline{B}}(x)$

$= \min(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)), \forall x \in X$



$\underline{C} = \underline{A} \cap \underline{B} = \{(x, \mu_{\underline{A} \cap \underline{B}}(x)) \mid \forall x \in X\}$

$\underline{A} = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \}$

$\underline{B} = \{ (x_1, 0.8), (x_2, 0.6), (x_3, 0.4), (x_4, 0.2), (x_5, 0.1) \}$

$\mu_{\underline{A} \cap \underline{B}}(x_1) = \min(\mu_{\underline{A}}(x_1), \mu_{\underline{B}}(x_1)) = \max\{0.2, 0.8\} = 0.2$

$\mu_{\underline{A} \cap \underline{B}}(x_2) = \min(\mu_{\underline{A}}(x_2), \mu_{\underline{B}}(x_2)) = \max\{0.5, 0.6\} = 0.5$

$\mu_{\underline{A} \cap \underline{B}}(x_3) = \min(\mu_{\underline{A}}(x_3), \mu_{\underline{B}}(x_3)) = \max\{0.6, 0.4\} = 0.4$

$\mu_{\underline{A} \cap \underline{B}}(x_4) = \min(\mu_{\underline{A}}(x_4), \mu_{\underline{B}}(x_4)) = \max\{0.8, 0.2\} = 0.2$

$\mu_{\underline{A} \cap \underline{B}}(x_5) = \min(\mu_{\underline{A}}(x_5), \mu_{\underline{B}}(x_5)) = \max\{1.0, 0.1\} = 0.1$

So, $\underline{A} \cap \underline{B} = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.4), (x_4, 0.2), (x_5, 0.1) \}$

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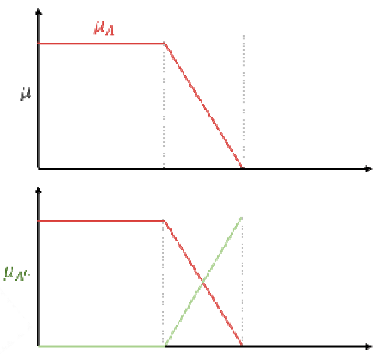
Fuzzy Set Operations…

The **complement** of fuzzy set $\underline{A}$, denoted by $\underline{A}^C$, is defined as

$\underline{A}^C = \{(x, \mu_{\underline{A}C}(x)) \mid \forall x \in X\}$

$\underline{A}^C(x) = 1 - \mu_{\underline{A}}(x)$



$\underline{A}^C(x) = 1 - \mu_{\underline{A}}(x)$

$\underline{A} = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \}$

$\underline{A}^C = \{ (x_1, 0.8), (x_2, 0.5), (x_3, 0.4), (x_4, 0.2), (x_5, 0.0) \}$

$\underline{A} \cup \underline{A}^C = \{ (x_1, 0.8), (x_2, 0.5), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0) \} \neq X$

$\underline{A} \cap \underline{A}^C = \{ (x_1, 0.2), (x_2, 0.5), (x_3, 0.4), (x_4, 0.2), (x_5, 0.0) \} \neq \Phi$

	Crisp Set	Fuzzy Set
Law of contradiction	$A \cap A^c = \phi$	$\bar{A} \cap \bar{A}^c \neq \phi$
Law of excluded middle	$A \cup A^c = X$	$\bar{A} \cup \bar{A}^c \neq X$

Practice Problem

$\underline{A} = \{ (x_1, 0.4), (x_2, 0.5), (x_3, 0.2), (x_4, 0.4), (x_5, 0.8) \}$

$\underline{B} = \{ (x_1, 1.0), (x_2, 0.3), (x_3, 0.5), (x_4, 0.7), (x_5, 0.1) \}$

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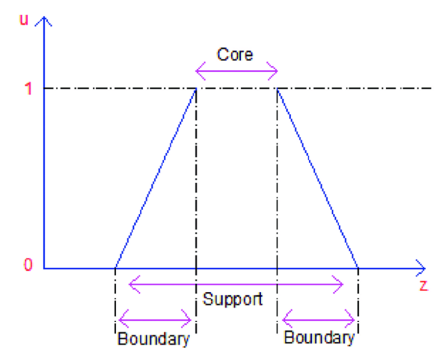
Features of Membership Function



The **core** of a membership function for some fuzzy set $\bar{A}$ is defined as that region of the universe that is characterized by complete and full membership in the set $\bar{A}$. That is, the core comprises those elements x of the universe such that $\mu\bar{A}(x) = 1$.

The **support** of a membership function for some fuzzy set $\bar{A}$ is defined as that region of the universe that is characterized by nonzero membership in the set $\bar{A}$. That is, the support comprises those elements x of the universe such that $\mu\bar{A}(x) > 0$.

The **boundaries** of a membership function for some fuzzy set A~ are defined as that region of the universe containing elements that have a nonzero membership but not complete membership. That is, the boundaries comprise those elements x of the universe such that $0 < \mu A\sim(x) < 1$.

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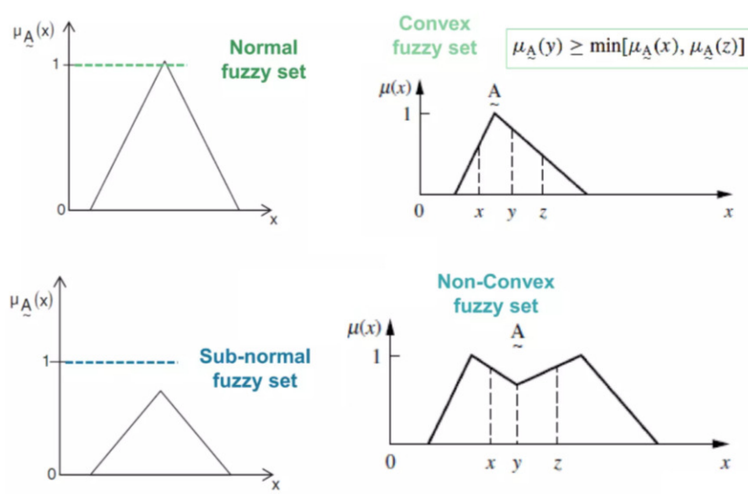
Types of Fuzzy Sets



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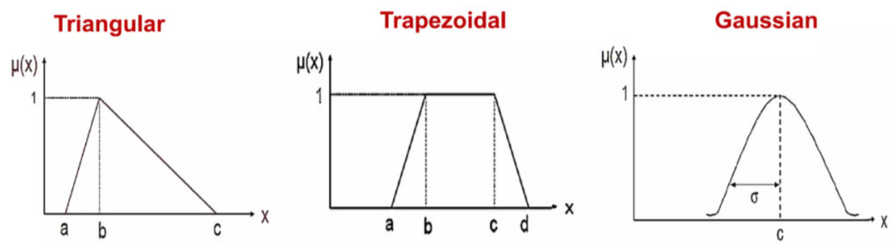
Types of Membership Functions



$$\mu(x) = \begin{cases} 0 & ,x \le a \\ \frac{x-a}{b-a} & ,a \le x \le b \\ \frac{c-x}{c-b} & ,b \le x \le c \\ 0 & ,x \ge c \end{cases}$$

$$\mu(x) = \begin{cases} 0 & ,x \le a \\ \frac{x-a}{b-a} & ,a \le x \le b \\ 1 & ,b \le x \le c \\ \frac{c-x}{c-b} & ,c \le x \le d \\ 0 & ,x \ge d \end{cases}$$

$$\mu(x) = exp\left(\frac{-(x-c)^2}{2\sigma^2}\right)$$

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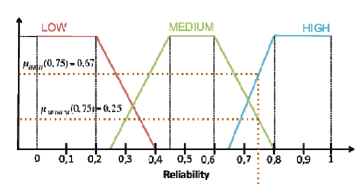
Fuzzification

Fuzzification is the process of transforming a crisp set to a fuzzy set or a fuzzy set to a fuzzier set, i.e.,crisp quantities are converted to fuzzy quantities. This operation translates accurate crisp input values into linguistic variables.

Intuition method is based upon the common intelligence of human. It is the capacity of the human to develop membership functions on the basis of their own intelligence and understanding capacity.

The **inference** method uses knowledge to perform deductive reasoning. There are various methods for performing deductive reasoning. The knowledge of geometrical shapes and geometry is used for defining membership values.



Rank Ordering methodology can be adapted to assign membership values to a fuzzy variable. Pairwise comparisons enable us to determine preferences and this results in determining the order of the membership.

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Defuzzification

Conversion of a fuzzy quantity into a precise quantity. The output of a fuzzy process may be union of two or more fuzzy membership functions defined on the universe of discourse of the output variable.

Different Defuzzification Methods
The following are the known methods of defuzzification.

- ✓ Center of Sums Method (COS)
- ✓ Center of gravity (COG) / Centroid of Area (COA) Method
- ✓ Weighted Average Method
- ✓ Maxima Methods
  - First of Maxima Method (FOM)
  - Last of Maxima Method (LOM)
  - Mean of Maxima Method (MOM)

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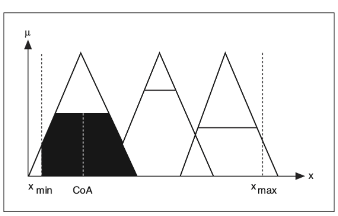
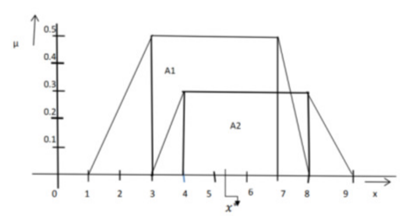
Center of Sums Method (COS)



$$x_{final} = \frac{(CoA_1 area_1 + CoA_2 area_2 + \ldots + CoA_n area_n)}{(area_1 + area_2 + \ldots + area_n)}$$

Problem -1



A_1 = ½ * [(8-1) + (7-3)] * 0.5 = ½ * 11 * 0.5 = 55/20=2.75

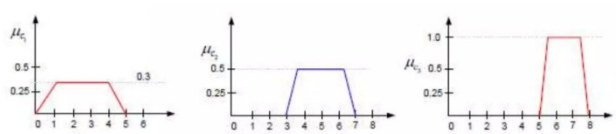
A_2 = ½ * [(9-3) + (8-4)] * 0.3 = ½ * 10 * 0.3 = 3/2 =1.5

Now the center of area of the fuzzy set C_1 is let say $\overline{x_1}$ = (7+3)/2= 5 and

the center of area of the fuzzy set C_2 is $\overline{x_2}$= (8+4)/2=6.

Now the defuzzified value $x^* = \frac{(A_1 \overline{x_1} + A_2 \overline{x_2})}{A_1 + A_2} = \frac{(2.75 * 5 + 1.5 * 6)}{(2.75 + 1.5)}$ = 22.75/4.25 = 5.35

Problem -2



$$x^* = \frac{\frac{1}{2}\times 0.3\times(3+5)\times 2.5+\frac{1}{2}\times 0.5\times(4+2)\times 5+\frac{1}{2}\times 1\times(3+1)\times 6.5}{\frac{1}{2}\times 0.3\times(3+5+\frac{1}{2}\times 0.5\times(4+2)+\frac{1}{2}\times 1\times(3+1)} = 5.00$$

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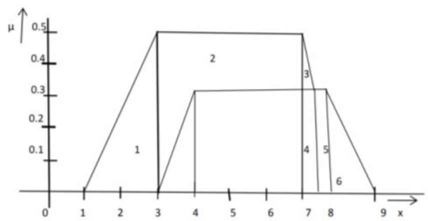
Center of gravity (COG) / Centroid of Area (COA) Method

$$x^* = \frac{\sum_{i=1}^{n} x_i \cdot \mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)}$$



The total area of the sub-area 1 is ½ * 2 * 0.5 = 0.5
The total area of the sub-area 2 is (7-3) * 0.5 = 4 * 0.5 = 2
The total area of the sub-area 3 is ½ * (7.5-7) * 0.2 = 0.5 * 0.5 *0.2 =.05
The total area of the sub-area 4 is 0.5* 0.3 = .15
The total area of the sub-area 5 is 0.5* 0.3 = .15
The total area of the sub-area 6 is ½ *1* 0.3 = .15

Centroid of sub-area1 will be (1+3+3)/3 = 7/3 =2.333
Centroid of sub-area2 will be (7+3)/2 = 10/2 = 5
Centroid of sub-area3 will be (7+7+7.5)/3 = 21.5/3 =7.166
Centroid of sub-area4 will be (7+7.5)/2 =14.5/2=7.25
Centroid of sub-are5 will be (7.5+8)/2 =15.5/2 = 7.75
Centroid of sub-area6 will be (8+8+9)/3 = 25/3 = 8.333

Sub-area number	Area(A_i)	Centroid of area($\bar{x}_i$)	$A_i \cdot \bar{x}_i$
1	0.5	2.333	1.1665
2	02	5	10
3	.05	7.166	0.3583
4	.15	7.25	1.0875
5	.15	7.75	1.1625
6	.15	8.333	1.2499

The defuzzified value x^* will be $\frac{\sum_{i=1}^{N} A_i \times \bar{x}_i}{\sum_{i=1}^{N} A_i}$

$$= \frac{(1.1665+10+0.3583+1.0875+1.1625+1.2499)}{(0.5+2+.05+.15+.15+.15)}$$

= (15.0247)/3 =5.008

$x^* = 5.008$

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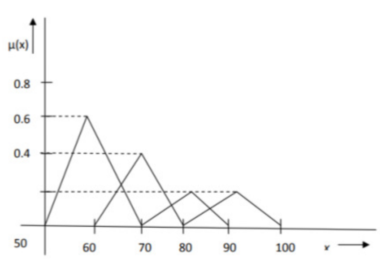
Weighted Average Method

$$x^* = \frac{\sum \mu(x).x}{\sum \mu(x)}$$



$$x^* = \frac{(60*0.6+70*0.4+80*0.2+90*0.2+100*0)}{0.6+0.4+0.2+0.2+0}$$

= 98/1.4=70

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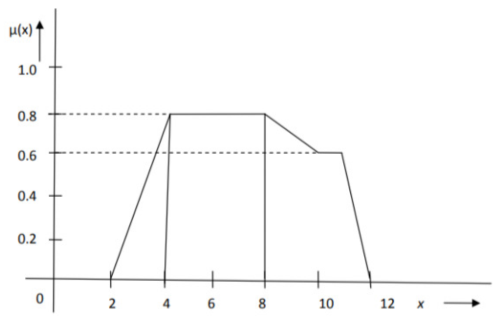
Weighted Average Method



First of Maxima Method (FOM)
The defuzzified value of the given fuzzy set will be 4

Last of Maxima Method (LOM)
LOM method will be 8

Mean of Maxima Method (MOM)

According to MOM method, $x^* = \frac{\sum_{x_i \in M} x_i}{|M|}$

Now the defuzzified value x^* will be $x^* = \frac{4+6+8}{3} = \frac{18}{3} = 6.$

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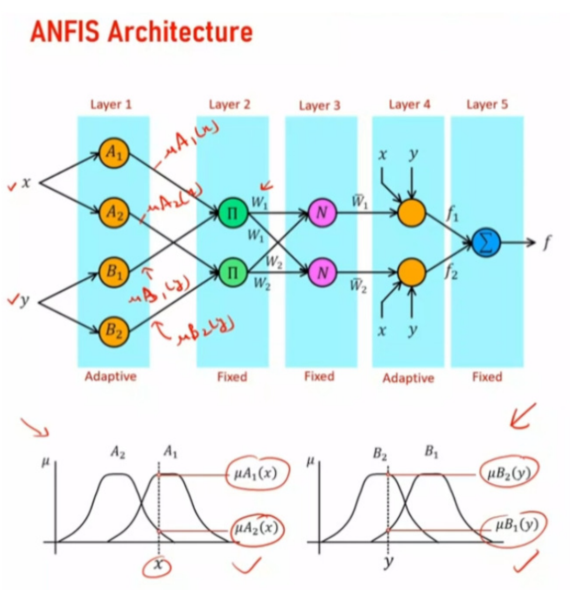

$w_1 = \mu A_1(x) \cdot \mu B_1(y) = \mu B_1(y)$

$w_2 = \mu A_2(x) \cdot \mu B_2(y) = \mu A_2(x)$

$\bar{w}_1 = \frac{w_1}{w_1+w_2} = \frac{\mu B_1(y)}{\mu A_2(x) + \mu B_1(y)}$

$\bar{w}_2 = \frac{w_2}{w_1+w_2} = \frac{\mu A_2(x)}{\mu A_2(x) + \mu B_1(y)}$

$f_1 = \bar{w}_1 \times (p_1 x + q_1 y + r_1)$

$f_2 = \bar{w}_2 \times (p_2 x + q_2 y + r_2)$

$f = f_1 + f_2$