KINEMATICS AND DYNAMICS OF ROBOTS

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INTRODUCTION

The study of how these robots move is divided into two main branches: kinematics and dynamics.

KINEMATICS is the branch of mechanics that describes the motion of objects without considering the forces that cause the motion. In robotics, kinematics is concerned with the study of the position, velocity, and acceleration of the robot's center of mass as it moves through its environment. This information is important for planning and controlling the robot's motion.

DYNAMICS, on the other hand, is the branch of mechanics that deals with the forces that cause motion. In robotics, dynamics is concerned with the study of the forces and torques that act on the robot as it moves. This information is important for designing the robot's structure and ensuring that it can move safely and efficiently in its environment.

When designing and controlling the robots, it is important to consider both kinematics and dynamics. Kinematic models can be used to plan the robot's motion, while dynamic models can be used to ensure that the robot can move safely and efficiently. Furthermore, both kinematics and dynamics are important for understanding the behavior of the robot and how it interacts with its environment.

There are several formulas that are commonly used in the kinematics and dynamics of robots. Here are some of the most important ones:

KINEMATICS:

POSITION: x = x0 + v*t, where x is the current position, x0 is the initial position, v is the velocity, and t is time.

VELOCITY: v = dx/dt, where v is the velocity, and dx/dt is the derivative of the position with respect to time.

ACCELERATION: a = dv/dt = d^2x/dt^2, where a is the acceleration, and dv/dt is the derivative of the velocity with respect to time.

DYNAMICS:

Newton's Second Law: **FORCE:** $F = m*a$, where F is the force acting on the robot, m is the mass of the robot, and a is the acceleration of the robot.

TORQUE: $\tau = r \times F$, where τ is the torque, r is the moment arm, and F is the force.

MOMENT OF INERTIA: $I = m*r^2$, where I is the moment of inertia, m is the mass, and r is the distance from the axis of rotation.

These formulas can be used to model the motion of a mobile robot and to design control systems that can drive the robot's motion. The specific formulas used will depend on the type of robot being studied and the specific application in which it is being used.

INVERSE DYNAMICS MATRIX: This matrix is used to compute the torques required to achieve a desired robot motion. It is an Nx1 matrix, where N is the number of degrees of freedom of the robot.

These matrices can be used to model the robot's motion and dynamics in a compact and efficient manner. They are often used in robotics control algorithms to generate and control the robot's motion.

Background

- To accelerate a robot's links, it is necessary to have actuators capable of exerting large enough forces and torques on the links and joints to move them at a desired acceleration and velocity.
- Otherwise, the links may not be moving as fast as necessary and, consequently, the robot may not maintain its desired positional accuracy.
- To calculate how strong each actuator must be, it is necessary to determine the dynamic relationships that govern the motions of the robot. These relationships are the force-mass-acceleration and the torque-inertia-angular-acceleration equations.
- Based on these equations, and considering the external loads on the robot, the designer can calculate the largest loads to which the actuators may be subjected, thereby designing the actuators to be able to deliver the necessary forces and torques.

FORWARD AND INVERSE KINEMATICS

Forward kinematics and inverse kinematics are two important concepts in robotics used to describe the ***position and motion of robots***.

FORWARD KINEMATICS:

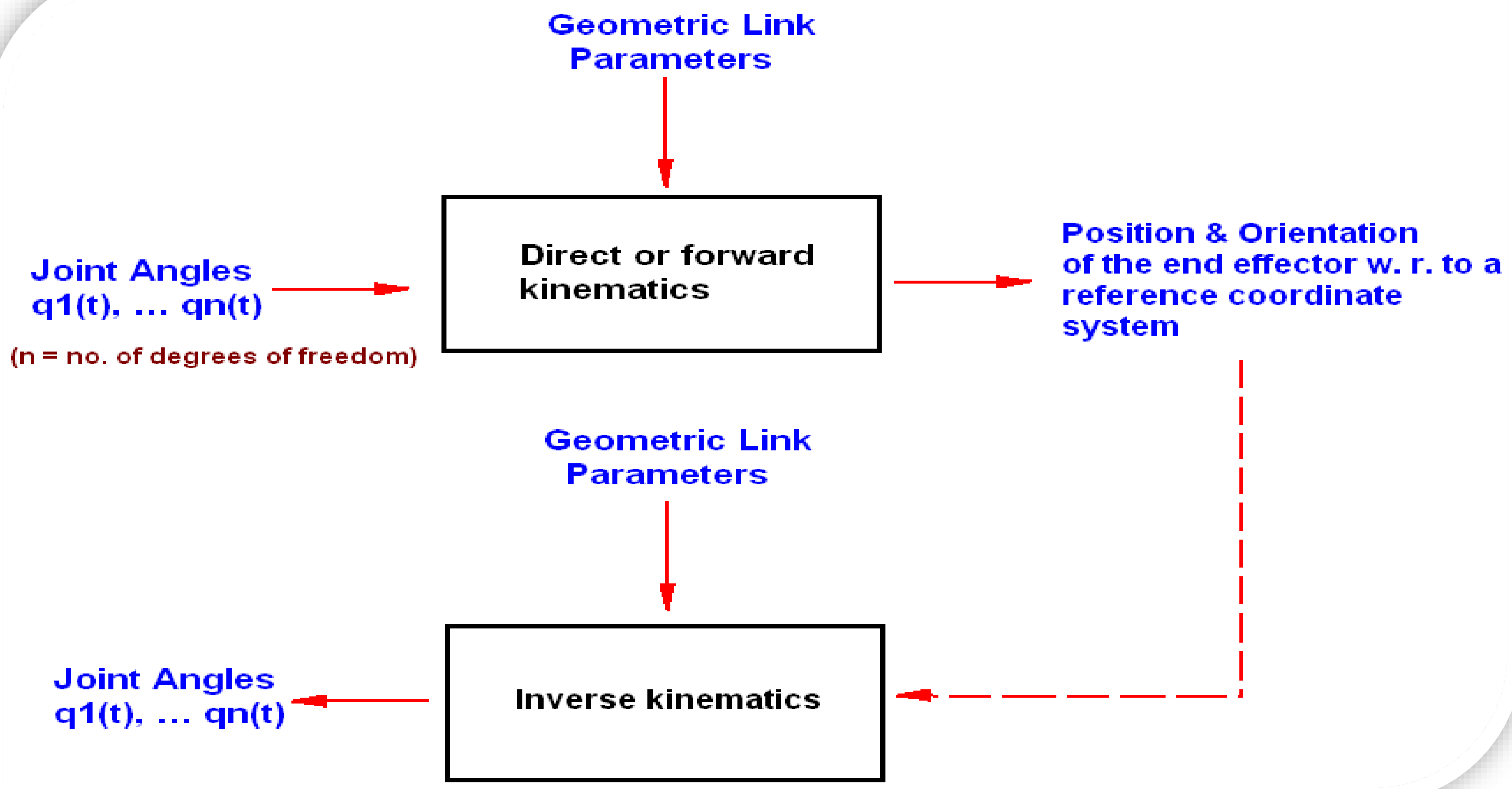
Forward kinematics is the process of determining the ***position and orientation of the end effector*** in Cartesian space with the help of the ***joint angles***.

INVERSE KINEMATICS:

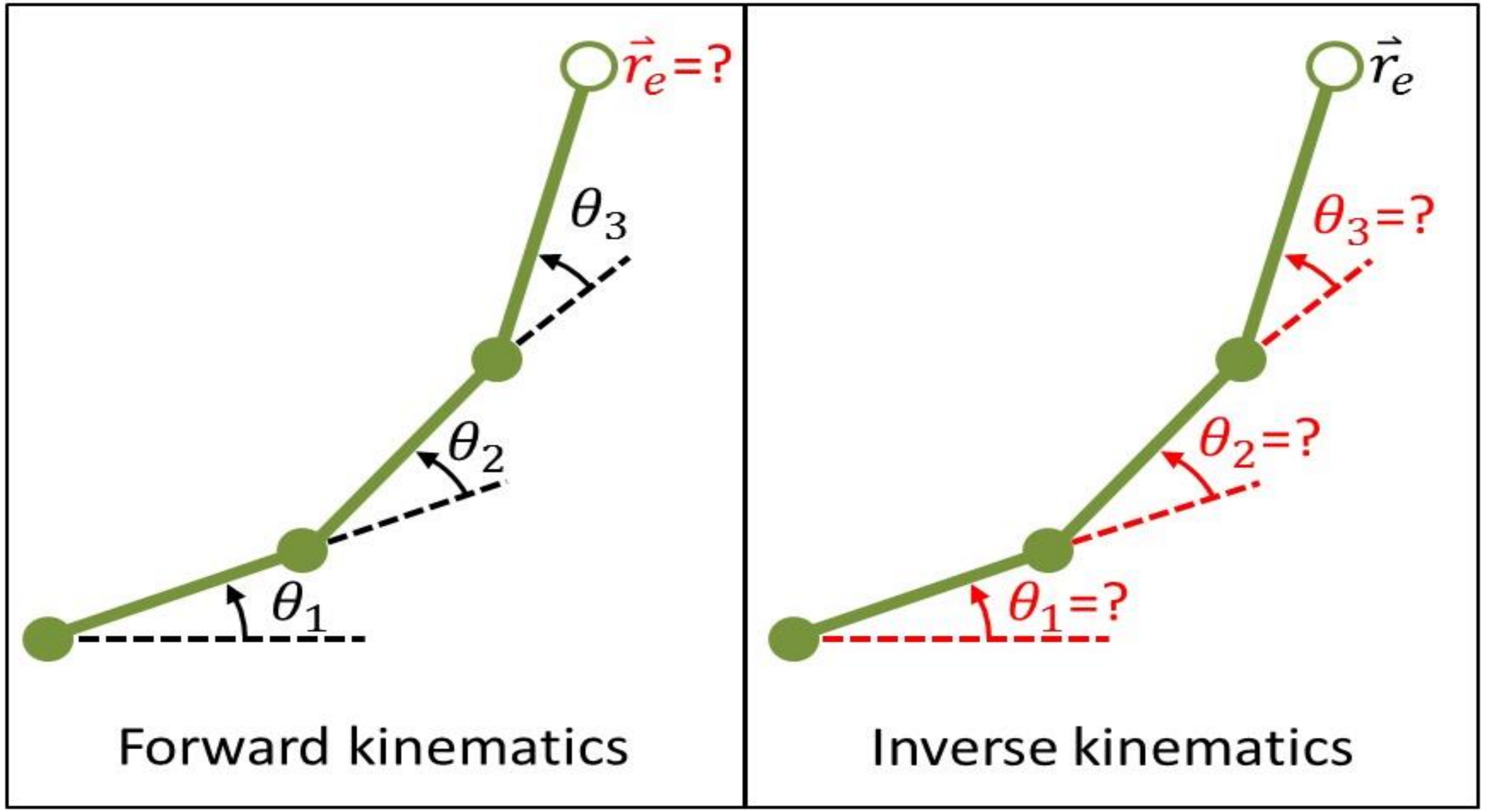
Inverse kinematics is the process of calculating the ***joint angles*** in Joint space with the help of the ***position and orientation of the end-effector***.

- Both forward and inverse kinematics are essential for ***controlling the motion of robotic arms*** in a variety of applications, such as ***manufacturing, automation, and medical robotics***.

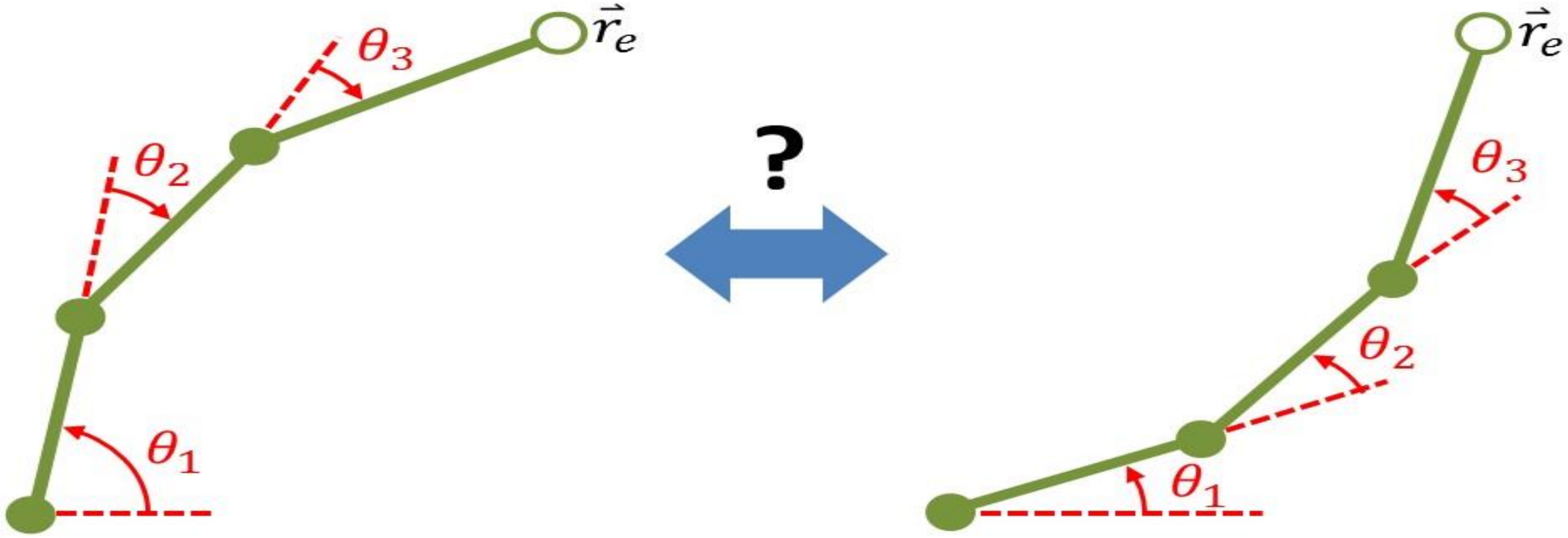
Direct(Forward) & Inverse(Reverse)Kinematics



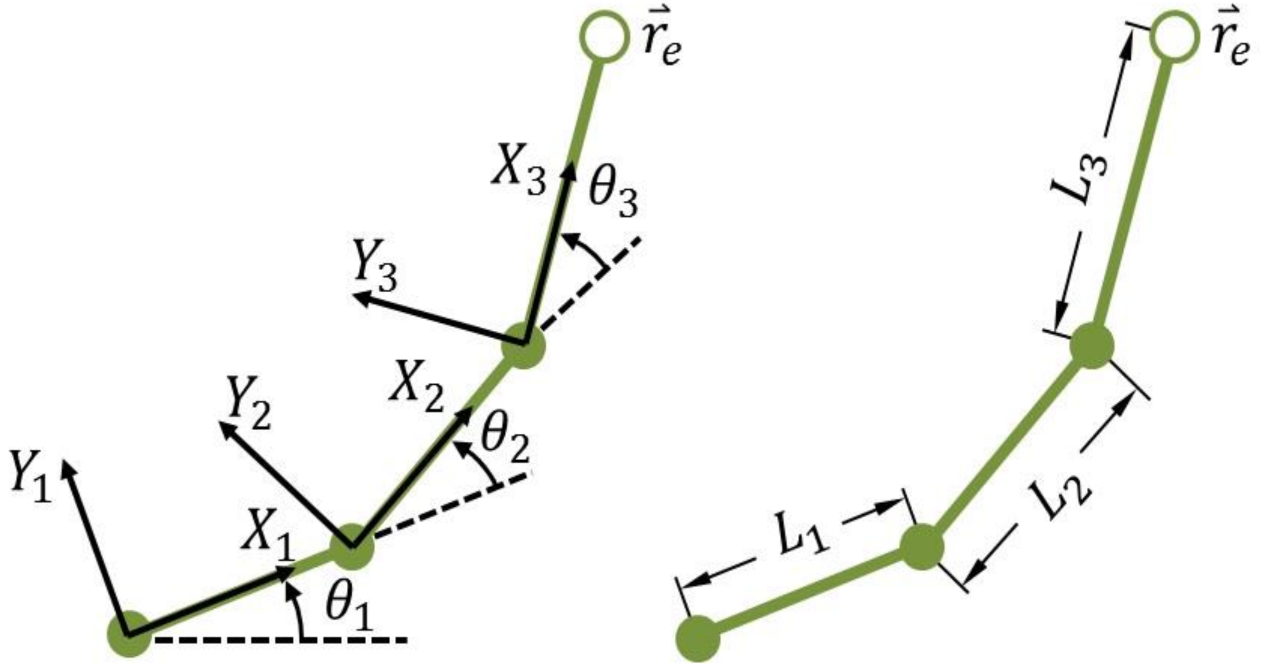
Forward & Inverse Kinematics



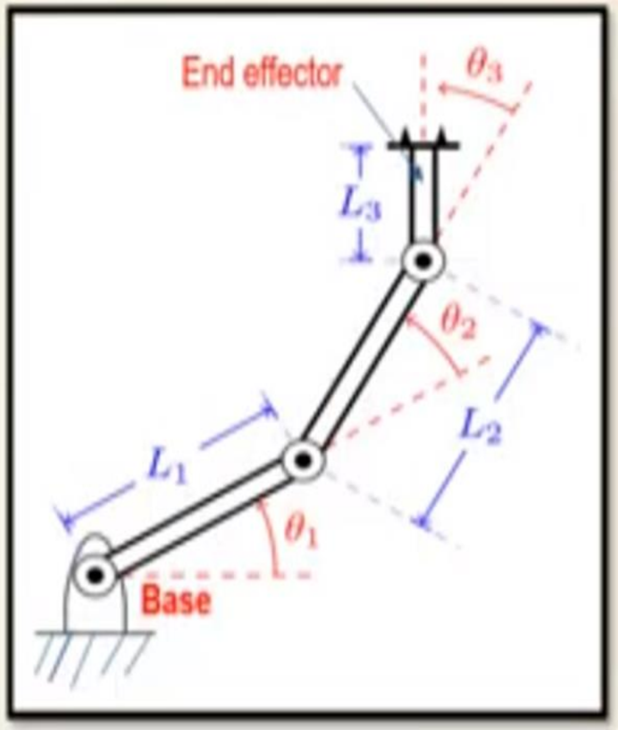
Inverse Kinematics



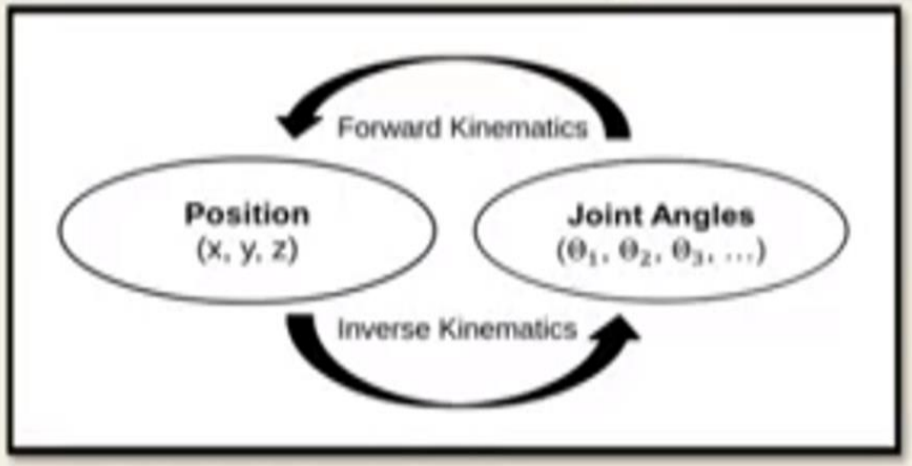
Co-ordinate System

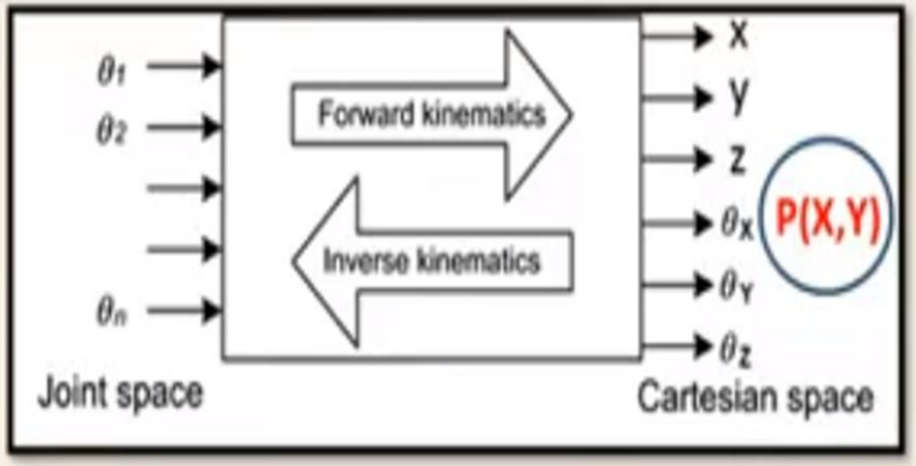


Forward and Inverse kinematics



- Forward Kinematics:
  to determine where the robot's hand is?
  (If all joint variables are known)
- Inverse Kinematics:
  to calculate what each joint variable is?
  (If we desire that the hand be located at a particular point)





Forward Kinematics (angles to position)

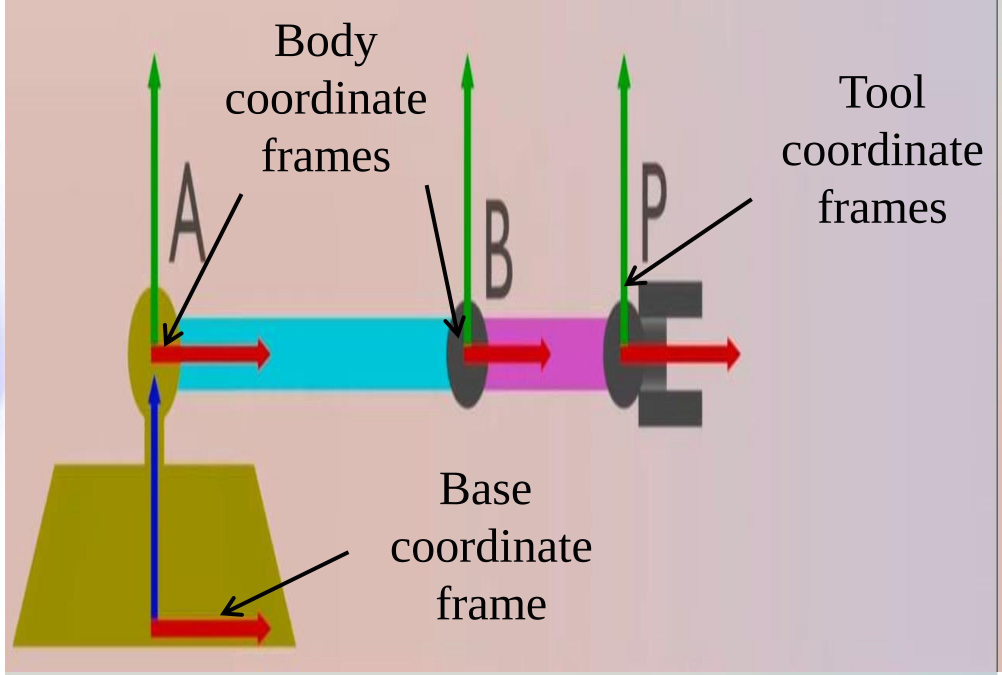
What you are given: The length of each link; The angle of each joint — (L1,L2,L3 & Ø1, Ø2, Ø3)

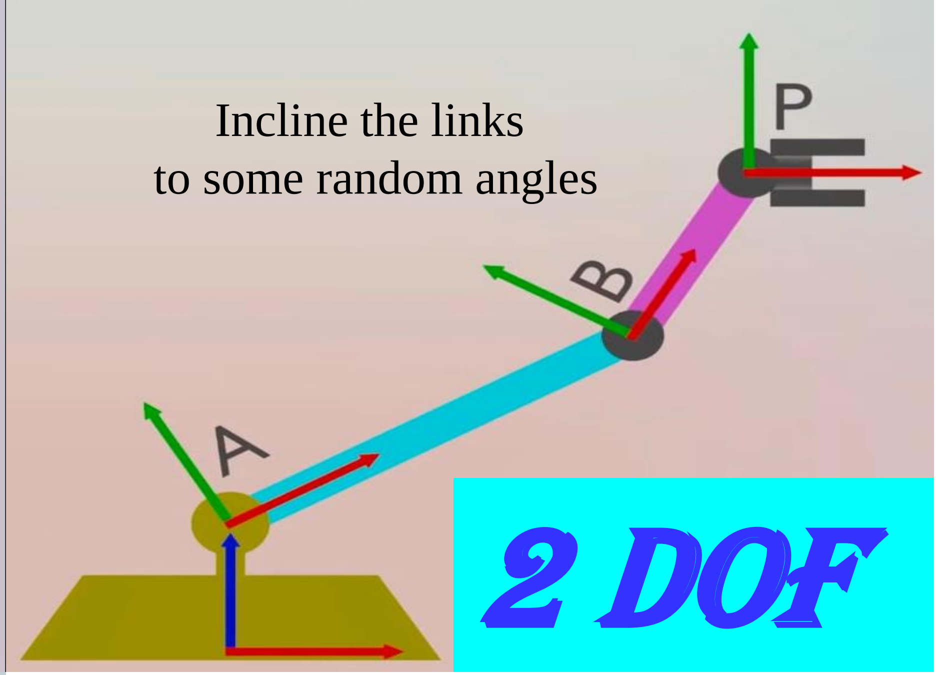
What you can find: The position of any point (i.e. it's (x, y, z) coordinates — P(X,Y,Z)

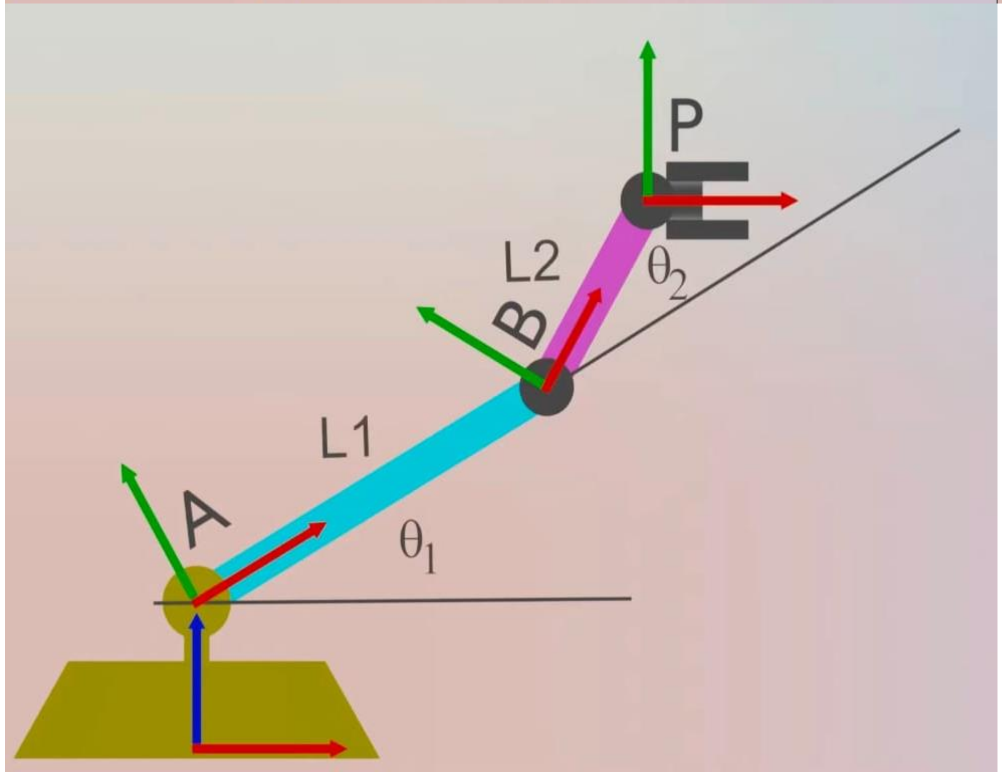
Inverse Kinematics (position to angles)

What you are given: The length of each link (L1,L2,L3); The position of some point on the robot P(X,Y,Z)

What you can find: The angles of each joint needed to obtain that position (Ø1, Ø2, Ø3)





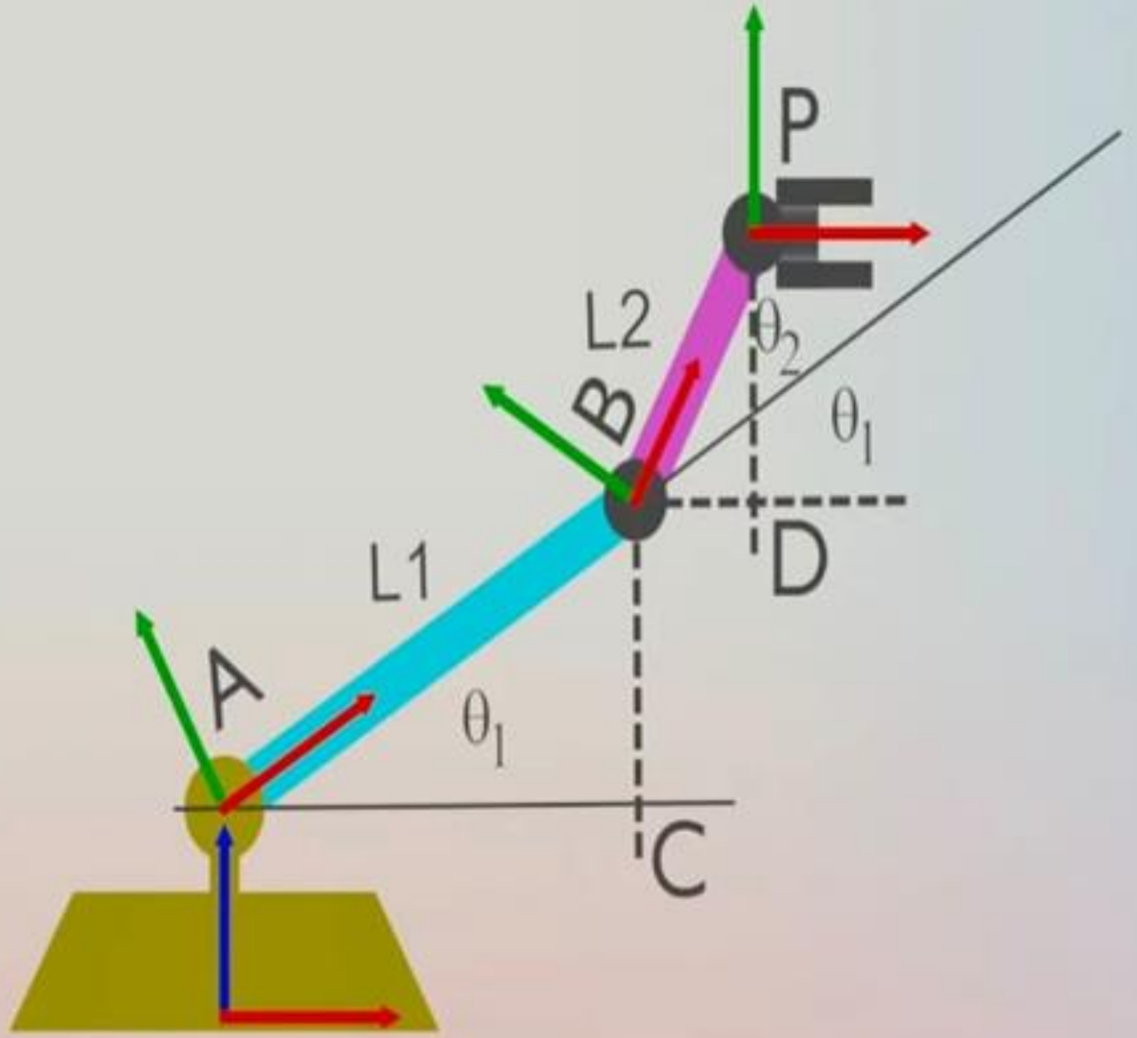


Θ_1 - Inclination of Link 1 w.r.t base coordinate frame

Θ_2 - Inclination of Link 2 w.r.t body coordinate frame of Link 1

L_1, L_2 – length of finite links

Resolve the components w.r.t X-axis and Y-axis



For link 1

$AB = L_1$

$AC = L_1 \cos\theta_1$

$BC = L_1 \sin\theta_1$

For link 2

$BP = L_2$

$BD = L_2 \cos(\theta_1 + \theta_2)$

$DP = L_2 \sin(\theta_1 + \theta_2)$

$$\text{Total } x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$\text{Total } y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

Problem on Forward kinematics of 2 Dof's robot :-

2. Consider the forward transformation of the two joint manipulator as shown in the fig.

Given that the length of joint - 1 L1 =10 inch, the length of joint 2, L2 = 10 inch , the angle ϴ1=35° and angle ϴ2 = 30 °, compute the co ordinate position (x , & y co-ordinate) for the end of the arm P.

Answer :-

Given : L1 = 10"

L2 = 10"

ϴ1 = 35 °

ϴ2 = 30 °

x = ?

y = ?

We know that formula for forward kinematics

x = L1cosϴ1 + L2cos(ϴ1 + ϴ2)

y = L1 sinϴ1 + L2sin (ϴ1 + ϴ2)

x = 10 cos 35 +10 cos (35+30)

x = 10*0.154 + 10 cos(35+30)

x = 8.19 +2.58

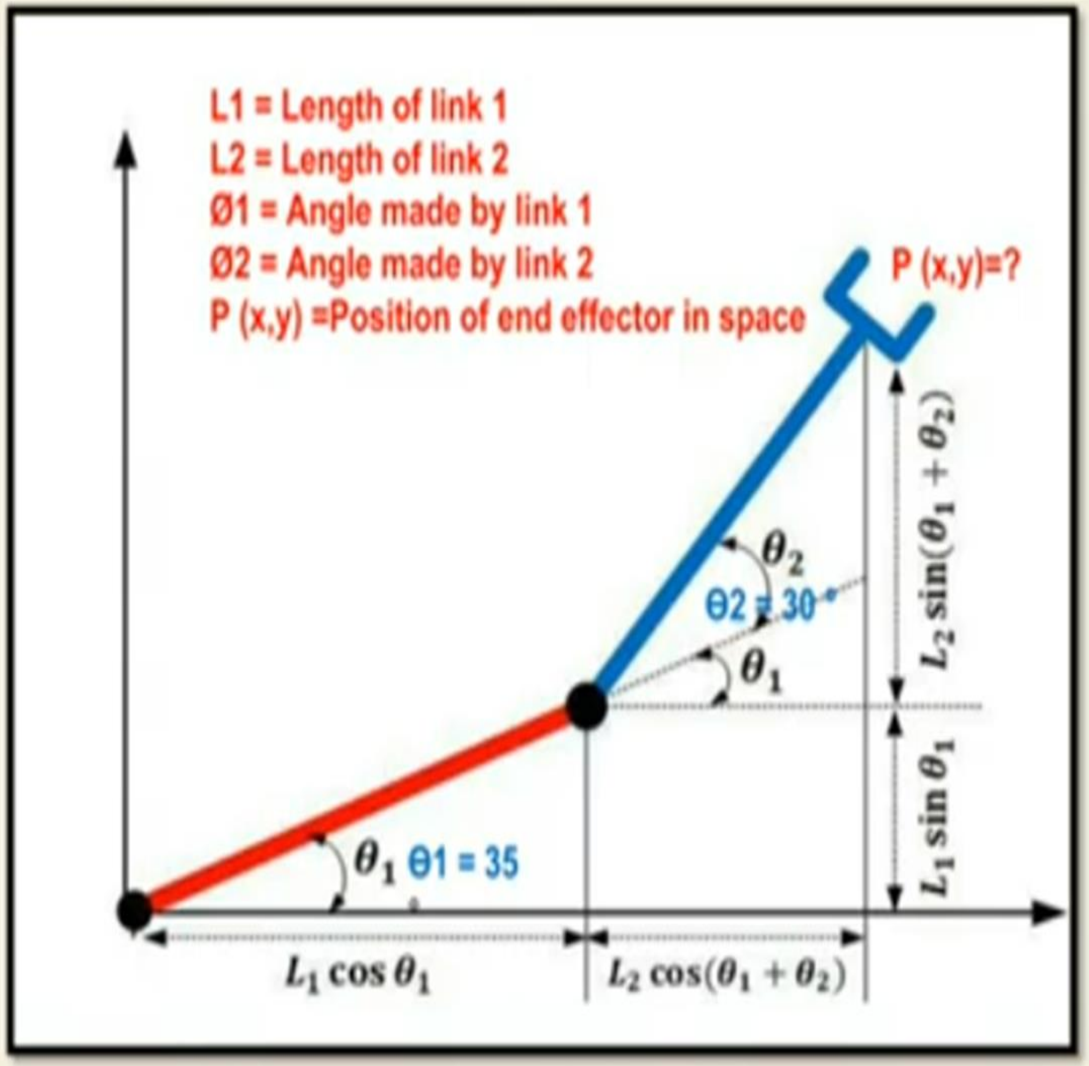
x = 10.77

x = 10.77 "

y = 10 sin 35 +10 sin(30+35)

y = 5.73+9.65

y = 15.38 "



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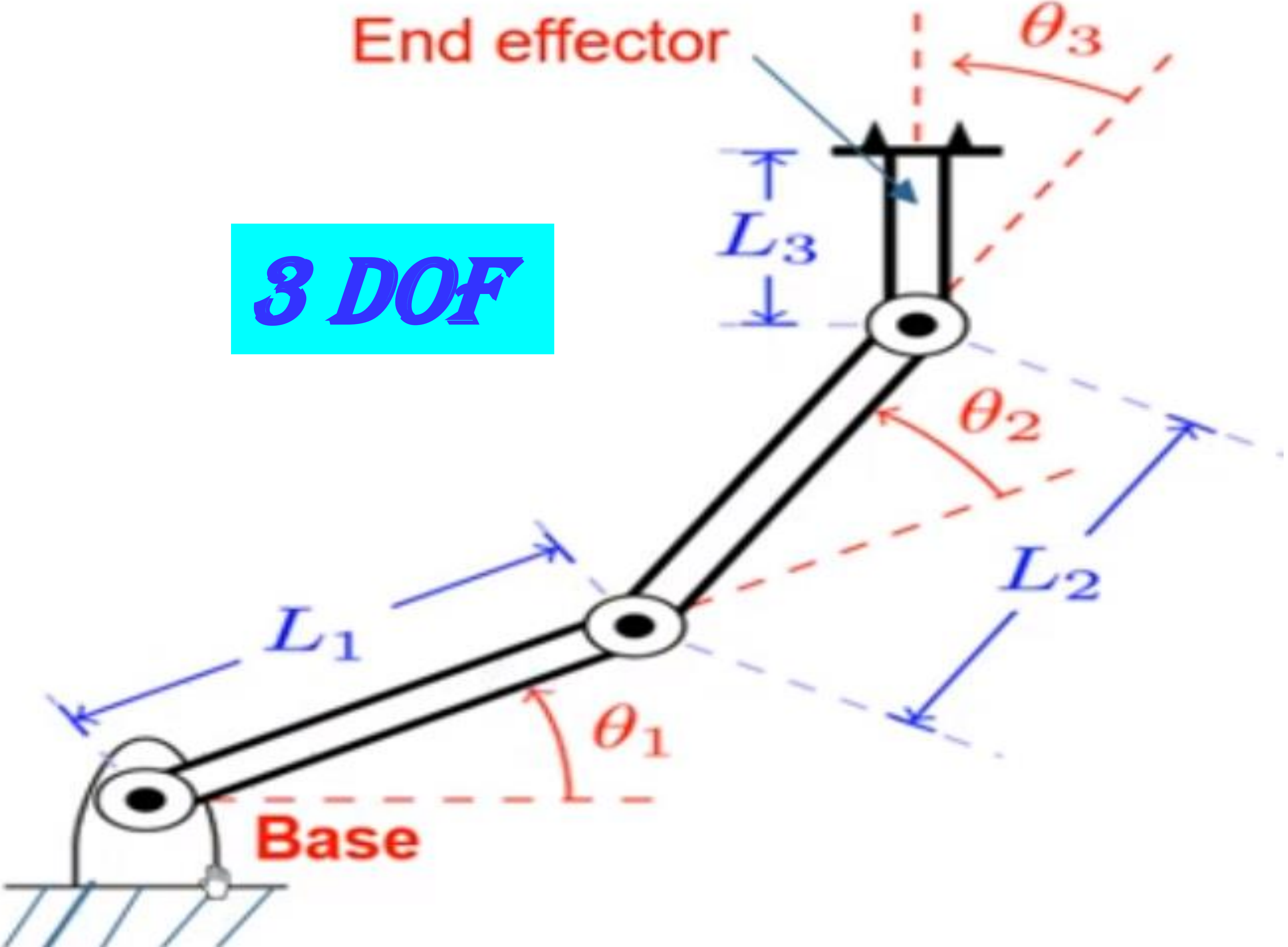
3 DOF



Problem on Forward kinematics of 3 Dof's robot :-

1. Consider the forward transformation of the three joint manipulator as shown in the fig.
Given that the length of joint - 1 L1 =10 inch, the length of joint 2, L2 = 12 inch,L3=8 inch the angle Θ1=30° , Θ2=30° Θ3 = 45 °, compute the co ordinate position (x , & y co-ordinate) for the end of the arm P.

Answer :-

Given : L1 = 10"
L2 = 12" L3 = 8"
Θ1 = 30 °
Θ2 = 30 ° Θ3 = 45 °
x = ? y = ?

We know that formula for forward kinematics of 3 Dofs robot

x = L1cosΘ1 + L2cos(Θ1 + Θ2)+ L3cos(Θ1+Θ2+ Θ3)
y = L1 sinΘ1 + L2sin (Θ1 + Θ2))+ L3sin(Θ1+Θ2+ Θ3)

x = 10cos 30 +12 cos (30+30)+8cos(30+30+45)

x = 8.66+6-2.07

$\boxed{x = 12.69"}$

y = 10sin30 +12 sin (30+30)+8sin(30+30+45)

y = 5+10.39+7.72

$\boxed{y = 23.11"}$