

## Canonical Cover

In the case of updating the database, the responsibility of the system is to check whether the existing functional dependencies are getting violated during the process of updating. In case of a violation of functional dependencies in the new database state, the rollback of the system must take place.

A canonical cover or irreducible a set of functional dependencies FD is a simplified set of FD that has a similar closure as the original set FD.

A set of FD FC is called canonical cover of F if each FD in FC is a –

- Simple FD.
- Left reduced FD.
- Non-redundant FD.

## Extraneous attributes

An attribute of an FD is said to be extraneous if we can remove it without changing the closure of the set of FD.

**Example:** Given a relational Schema R( A, B, C, D) and set of Function Dependency FD = {  $B \rightarrow A$ ,  $AD \rightarrow BC$ ,  $C \rightarrow ABD$  }. Find the canonical cover?

**Solution:** Given FD = {  $B \rightarrow A$ ,  $AD \rightarrow BC$ ,  $C \rightarrow ABD$  }, now decompose the FD using decomposition rule( Armstrong Axiom ).

1.  $B \rightarrow A$
2.  $AD \rightarrow B$  ( using decomposition inference rule on  $AD \rightarrow BC$  )
3.  $AD \rightarrow C$  ( using decomposition inference rule on  $AD \rightarrow BC$  )
4.  $C \rightarrow A$  ( using decomposition inference rule on  $C \rightarrow ABD$  )
5.  $C \rightarrow B$  ( using decomposition inference rule on  $C \rightarrow ABD$  )
6.  $C \rightarrow D$  ( using decomposition inference rule on  $C \rightarrow ABD$  )

Now set of FD = {  $B \rightarrow A$ ,  $AD \rightarrow B$ ,  $AD \rightarrow C$ ,  $C \rightarrow A$ ,  $C \rightarrow B$ ,  $C \rightarrow D$  }

The next step is to find closure of the left side of each of the given FD by including that FD and excluding that FD, if closure in both cases are same then that FD is redundant and we remove that FD from the given set, otherwise if both the closures are different then we do not exclude that FD.

**Calculating closure of all FD {  $B \rightarrow A$ ,  $AD \rightarrow B$ ,  $AD \rightarrow C$ ,  $C \rightarrow A$ ,  $C \rightarrow B$ ,  $C \rightarrow D$  }**

1a. Closure  $B^+ = BA$  using FD = {  $B \rightarrow A$ ,  $AD \rightarrow B$ ,  $AD \rightarrow C$ ,  $C \rightarrow A$ ,  $C \rightarrow B$ ,  $C \rightarrow D$  }

1b. Closure  $B^+ = B$  using FD = {  $AD \rightarrow B$ ,  $AD \rightarrow C$ ,  $C \rightarrow A$ ,  $C \rightarrow B$ ,  $C \rightarrow D$  }

From 1 a and 1 b, we found that both the Closure( by including  $B \rightarrow A$  and excluding  $B \rightarrow A$  ) are not equivalent, hence FD  $B \rightarrow A$  is important and cannot be removed from the set of FD.

2 a. Closure  $AD^+ = ADBC$  using FD = {  $B \rightarrow A$ ,  $AD \rightarrow B$ ,  $AD \rightarrow C$ ,  $C \rightarrow A$ ,  $C \rightarrow B$ ,  $C \rightarrow D$  }

2 b. Closure  $AD^+ = ADCB$  using  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow A, C \rightarrow B, C \rightarrow D \}$

From 2 a and 2 b, we found that both the Closure (by including  $AD \rightarrow B$  and excluding  $AD \rightarrow B$ ) are equivalent, hence FD  $AD \rightarrow B$  is not important and can be removed from the set of FD.

**Hence resultant FD =  $\{ B \rightarrow A, AD \rightarrow C, C \rightarrow A, C \rightarrow B, C \rightarrow D \}$**

3 a. Closure  $AD^+ = ADCB$  using  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow A, C \rightarrow B, C \rightarrow D \}$

3 b. Closure  $AD^+ = AD$  using  $FD = \{ B \rightarrow A, C \rightarrow A, C \rightarrow B, C \rightarrow D \}$

From 3 a and 3 b, we found that both the Closure (by including  $AD \rightarrow C$  and excluding  $AD \rightarrow C$ ) are not equivalent, hence FD  $AD \rightarrow C$  is important and cannot be removed from the set of FD.

**Hence resultant FD =  $\{ B \rightarrow A, AD \rightarrow C, C \rightarrow A, C \rightarrow B, C \rightarrow D \}$**

4 a. Closure  $C^+ = CABD$  using  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow A, C \rightarrow B, C \rightarrow D \}$

4 b. Closure  $C^+ = CBDA$  using  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow B, C \rightarrow D \}$

From 4 a and 4 b, we found that both the Closure (by including  $C \rightarrow A$  and excluding  $C \rightarrow A$ ) are equivalent, hence FD  $C \rightarrow A$  is not important and can be removed from the set of FD.

**Hence resultant FD =  $\{ B \rightarrow A, AD \rightarrow C, C \rightarrow B, C \rightarrow D \}$**

5 a. Closure  $C^+ = CBDA$  using  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow B, C \rightarrow D \}$

5 b. Closure  $C^+ = CD$  using  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow D \}$

From 5 a and 5 b, we found that both the Closure (by including  $C \rightarrow B$  and excluding  $C \rightarrow B$ ) are not equivalent, hence FD  $C \rightarrow B$  is important and cannot be removed from the set of FD.

**Hence resultant FD =  $\{ B \rightarrow A, AD \rightarrow C, C \rightarrow B, C \rightarrow D \}$**

6 a. Closure  $C^+ = CDBA$  using  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow B, C \rightarrow D \}$

6 b. Closure  $C^+ = CBA$  using  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow B \}$

From 6 a and 6 b, we found that both the Closure (by including  $C \rightarrow D$  and excluding  $C \rightarrow D$ ) are not equivalent, hence FD  $C \rightarrow D$  is important and cannot be removed from the set of FD.

**Hence resultant FD =  $\{ B \rightarrow A, AD \rightarrow C, C \rightarrow B, C \rightarrow D \}$**

- Since  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow B, C \rightarrow D \}$  is resultant FD, now we have checked the redundancy of attribute, since the left side of FD  $AD \rightarrow C$  has two attributes, let's check their importance, i.e. whether they both are important or only one.

Closure  $AD^+ = ADCB$  using  $FD = \{ B \rightarrow A, \mathbf{AD} \rightarrow \mathbf{C}, C \rightarrow B, C \rightarrow D \}$

Closure  $A^+ = A$  using  $FD = \{ B \rightarrow A, \mathbf{AD} \rightarrow \mathbf{C}, C \rightarrow B, C \rightarrow D \}$

Closure  $D^+ = D$  using  $FD = \{ B \rightarrow A, \mathbf{AD} \rightarrow \mathbf{C}, C \rightarrow B, C \rightarrow D \}$

Since the closure of  $AD^+$ ,  $A^+$ ,  $D^+$  that we found are not all equivalent, hence in  $FD$   $AD \rightarrow C$ , both  $A$  and  $D$  are important attributes and cannot be removed.

Hence resultant  $FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow B, C \rightarrow D \}$  and we can rewrite as

**$FD = \{ B \rightarrow A, AD \rightarrow C, C \rightarrow BD \}$  is Canonical Cover of  $FD = \{ B \rightarrow A, AD \rightarrow BC, C \rightarrow ABD \}$ .**