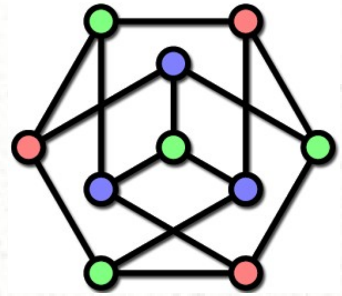

APPLIED GRAPH THEORY AND ALGORITHMS (CSC4066)

Johnson Algorithm

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Johnson Algorithm

- The problem is to find the shortest path between every pair of vertices in a given weighted directed graph and weight may be negative
- It is suitable for sparse graph
- Johnson's Algorithm uses both Dijkstra's Algorithm and Bellman-Ford Algorithm
- Johnson's Algorithm uses the technique of "reweighting."

Johnson Algorithm

- If all edge weights w in a graph G = (V, E) are nonnegative, we can find the shortest paths between all pairs of vertices by running Dijkstra's Algorithm once from each vertex.
- If G has negative - weight edges, we compute a new - set of non - negative edge weights that allows us to use the same method.

Johnson Algorithm

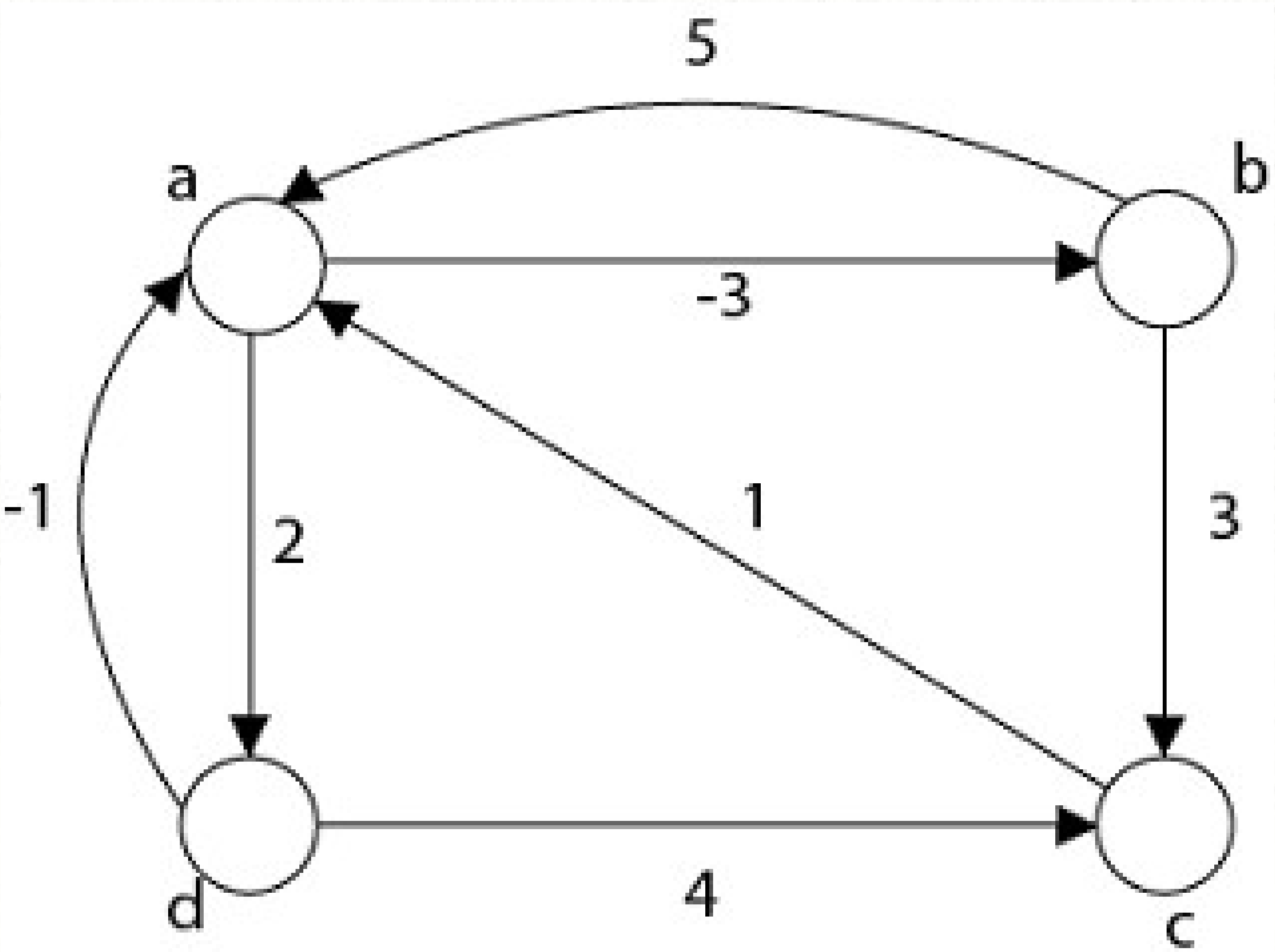
- Reweighing is done through the formula

$$\boxed{w(u, v) = w(u, v) + h(u) - h(v)}$$

```
JOHNSON (G)
 1. Compute G' where V [G'] = V[G] ∪ {S} and
E [G'] = E [G] ∪ {(s, v): v ∈ V [G] }

 2. If BELLMAN-FORD (G',w, s) = FALSE
    then "input graph contains a negative weight cycle"
  else
    for each vertex v ∈ V [G']
     do h (v) ← δ(s, v)
  Computed by Bellman-Ford algorithm
 for each edge (u, v) ∈ E[G']
   do w (u, v) ← w (u, v) + h (u) - h (v)
 for each vertex u ∈ V [G]
 do run DIJKSTRA (G, w, u) to compute
    δ (u, v) for all v ∈ V [G]
  for each vertex v ∈ V [G]
 do d_uv← δ (u, v) + h (v) - h (u)
Return D.
```

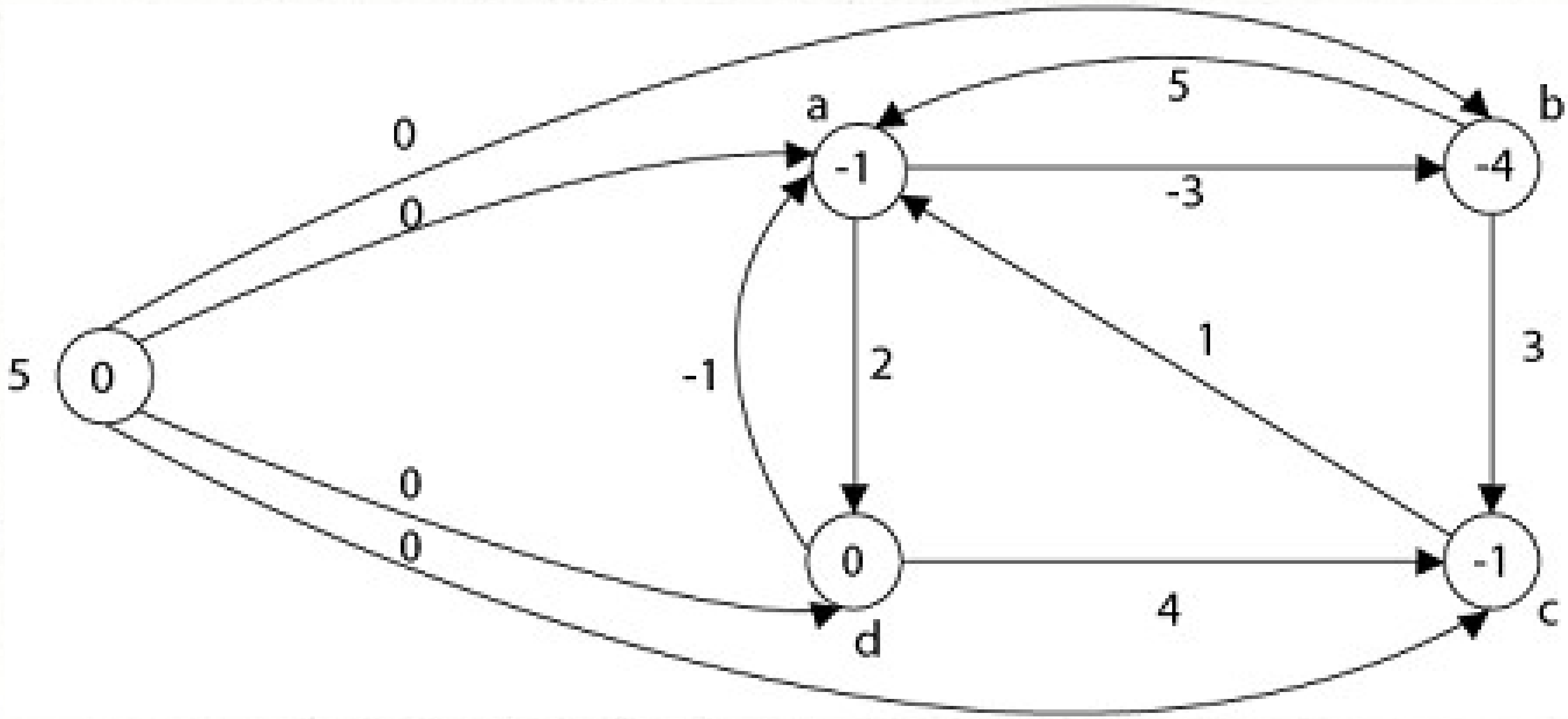
Johnson Algorithm



Johnson Algorithm

- Step1: Take any source vertex's' outside the graph and make distance from's' to every vertex '0'.
- Step2: Apply Bellman-Ford Algorithm and calculate minimum weight on each vertex.

Johnson Algorithm



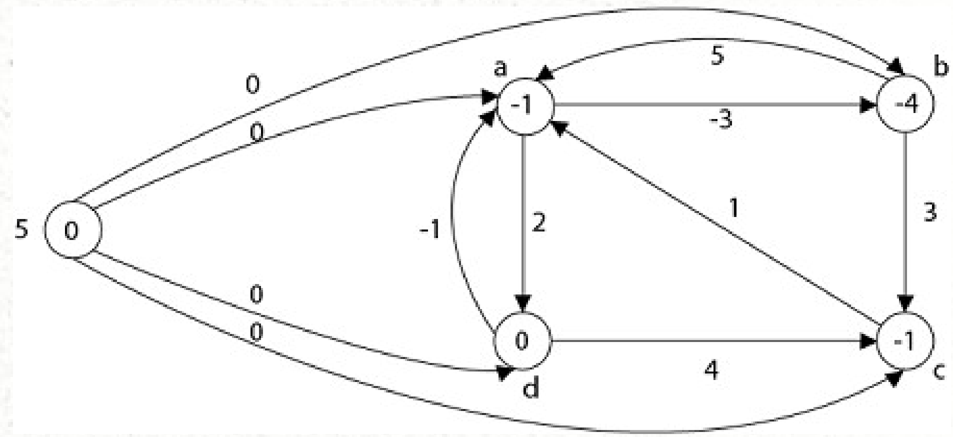
Johnson Algorithm

- Step3:

w (a, b) = w (a, b) + h (a) - h (b)

= -3 + (-1) - (-4)

= 0



w (b, a) = w (b, a) + h (b) - h (a)

= 5 + (-4) - (-1)

= 2

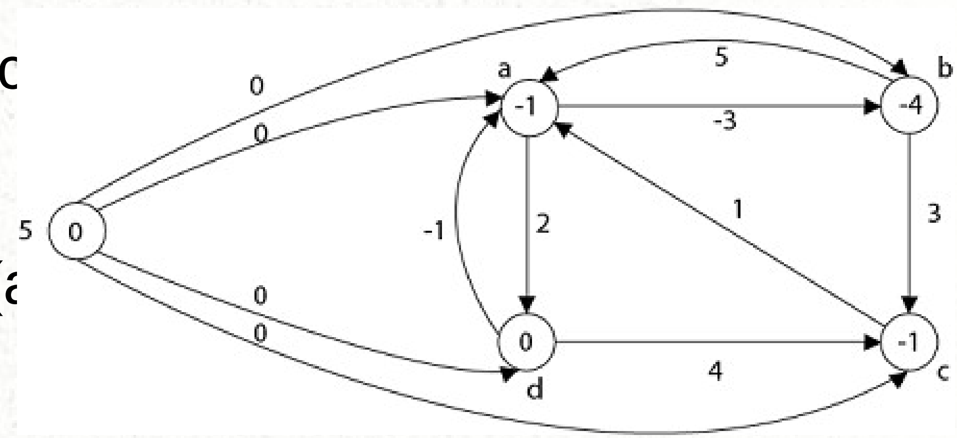
w (b, c) = w (b, c) + h (b) - h (c)

w (b, c) = 3 + (-4) - (-1)

= 0

Johnson Algorithm

w (c, a) = w (c, a) + h (c) - h (a)
w (c, a) = 1 + (-1) - (-1)
= 1
w (d, c) = w (d, c) + h (d) - h (c
w (d, c) = 4 + 0 - (-1)
= 5
w (d, a) = w (d, a) + h (d) - h (a
w (d, a) = -1 + 0 - (-1)
= 0
w (a, d) = w (a, d) + h (a) - h (d)
w (a, d) = 2 + (-1) - 0 = 1

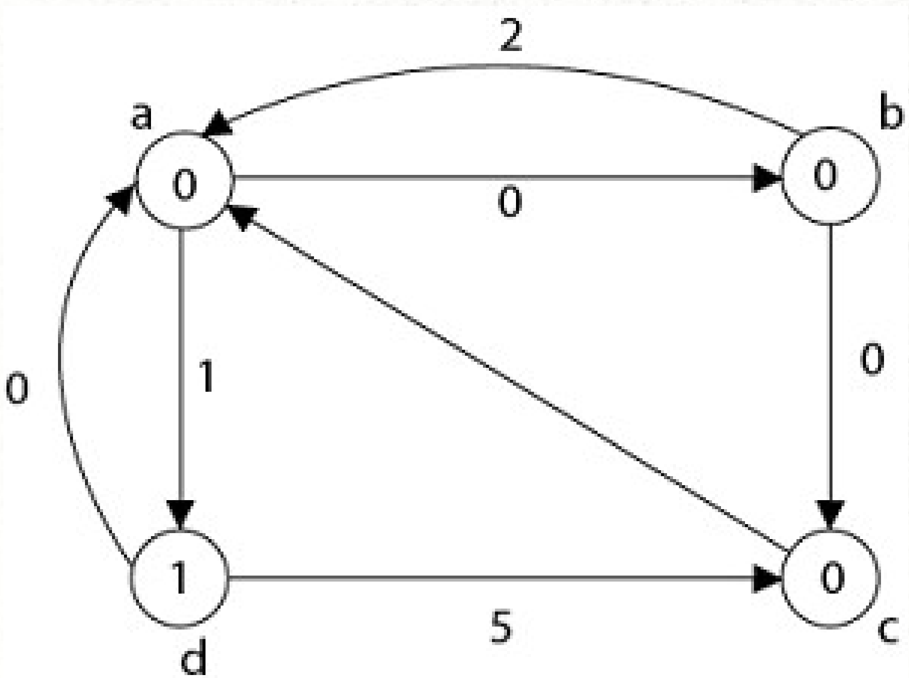


Johnson Algorithm

- Step 4: Now all edge weights are positive and now we can apply Dijkstra's Algorithm on each vertex and make a matrix corresponds to each vertex in a graph

Johnson Algorithm

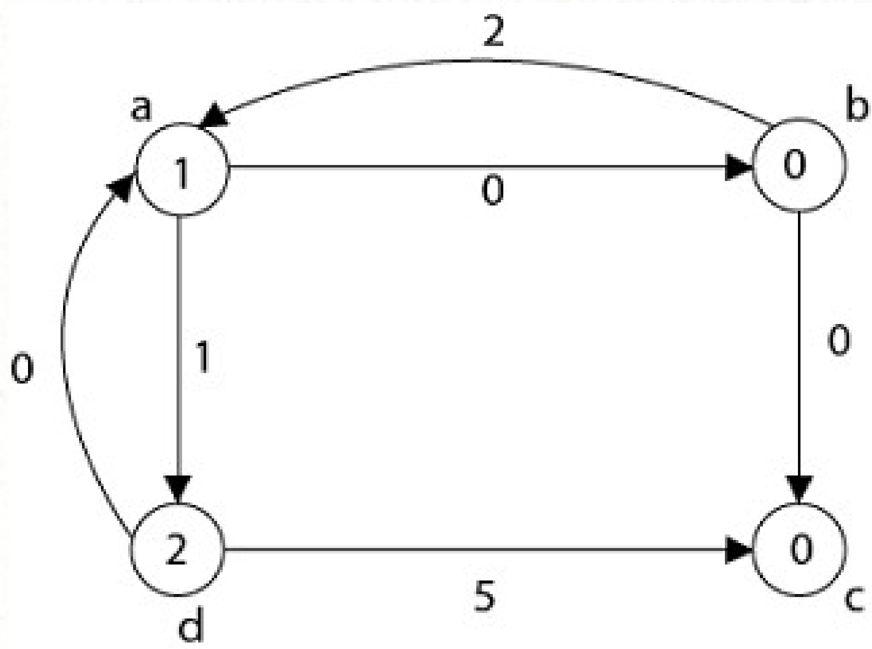
- Case 1: 'a' as a source vertex



a, a	0
a, b	0
a, c	0
a, d	1

Johnson Algorithm

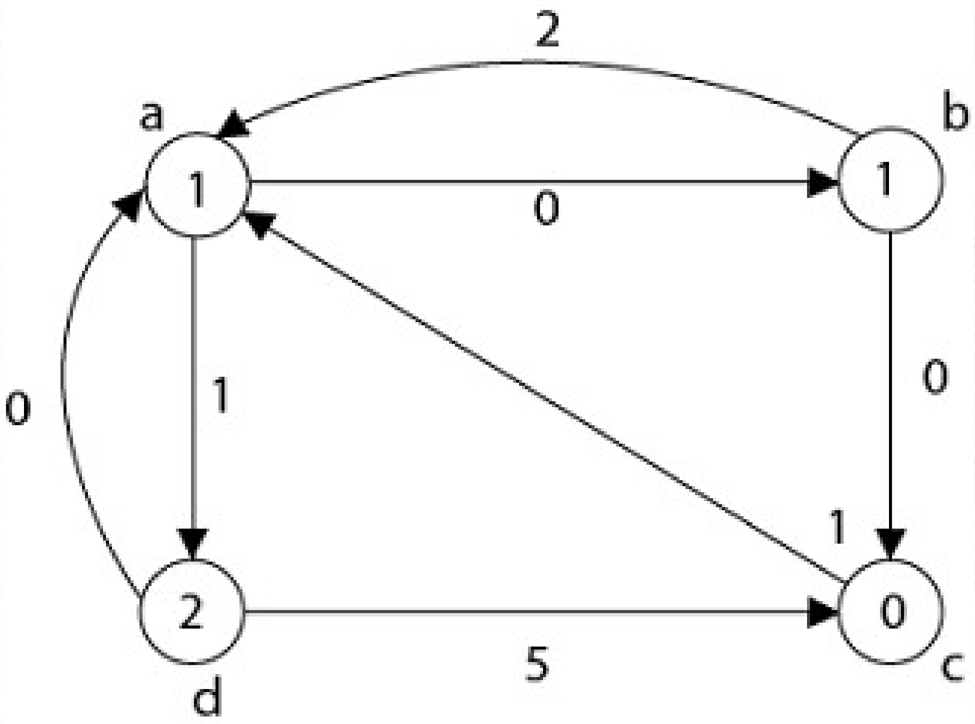
- Case 2: 'b' as a source vertex



b, a	2
b, b	0
b, c	0
b, d	2

Johnson Algorithm

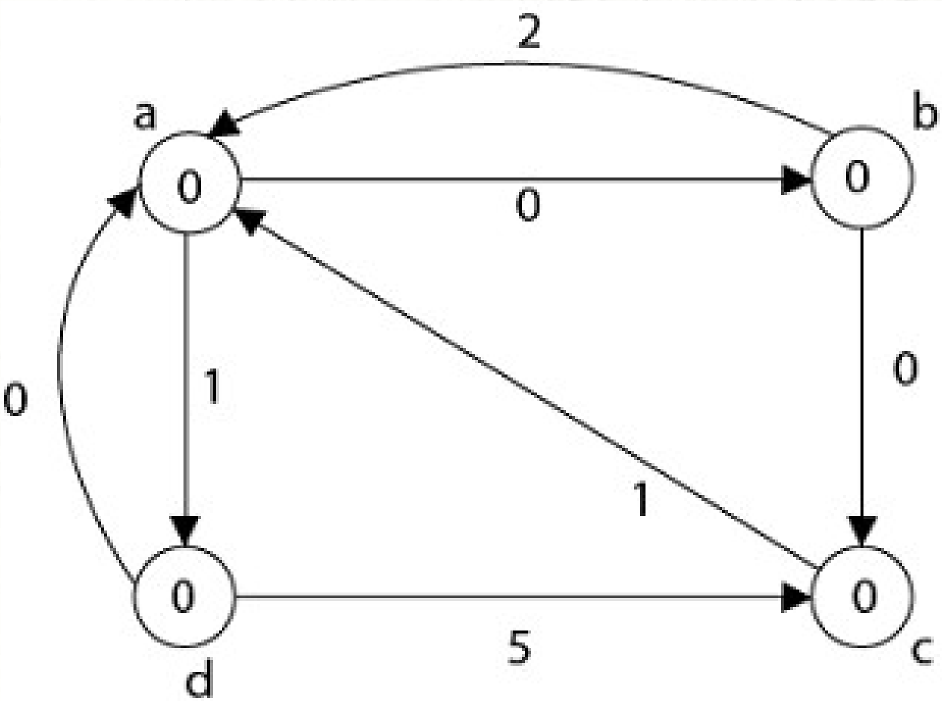
- Case 3: 'c' as a source vertex



c, a	1
c, b	1
c, c	0
c, d	2

Johnson Algorithm

- Case4:'d' as source vertex



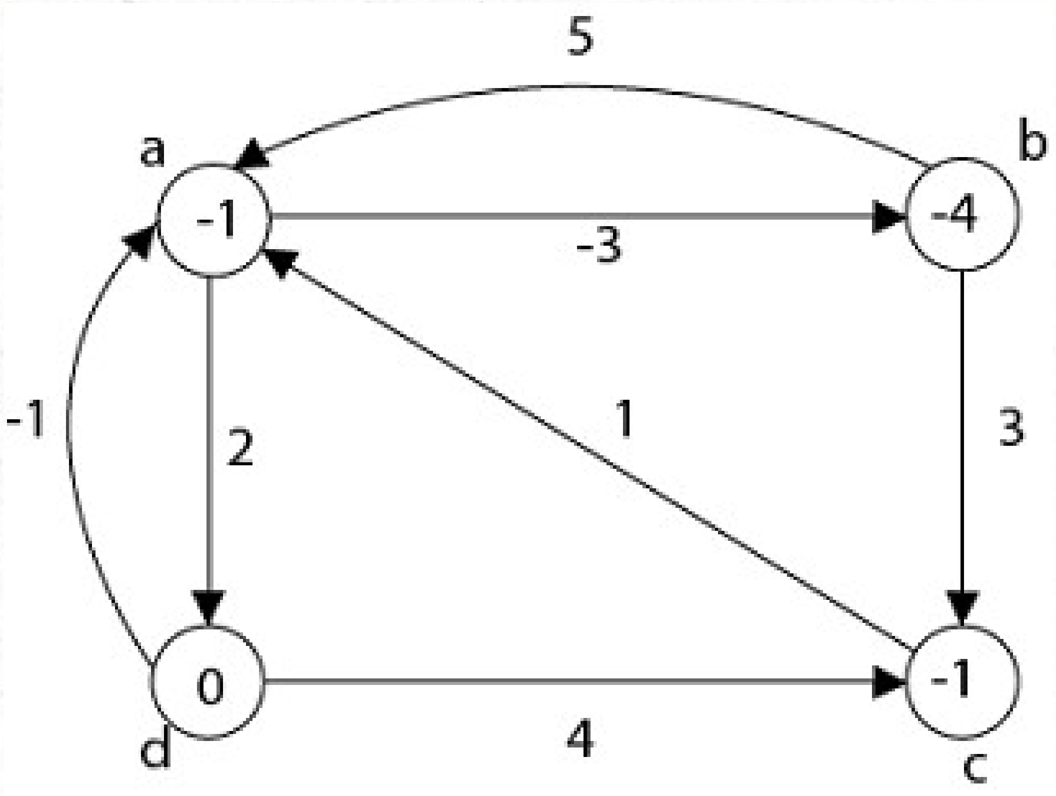
d, a	0
d, b	0
d, c	0
d, d	0

Johnson Algorithm

	a	b	c	d
a	0	0	0	1
b	1	0	0	2
c	1	1	0	2
d	0	0	0	0

Johnson Algorithm

- **Step 5:**
- **duv ← δ (u, v) + h (v) - h (u)**
- **d (a, a) = 0 + (-1) - (-1) = 0**
- **d (a, b) = 0 + (-4) - (-1) = -3**
- **d (a, c) = 0 + (-1) - (-1) = 0**
- **d (a, d) = 1 + (0) - (-1) = 2**
- **d (b, a) = 1 + (-1) - (-4) = 4**
- **d (b, b) = 0 + (-4) - (-4) = 0**
- **d (c, a) = 1 + (-1) - (-1) = 1**
- **d (c, b) = 1 + (-4) - (-1) = -2**
- **d (c, c) = 0**
- **d (c, d) = 2 + (0) - (-1) = 3**
- **d (d, a) = 0 + (-1) - (0) = -1**
- **d (d, b) = 0 + (-4) - (0) = -4**
- **d (d, c) = 0 + (-1) - (0) = -1**
- **d (d, d) = 0**



Johnson Algorithm

	a	b	c	d
a	0	-3	0	2
b	4	0	3	6
c	1	-2	0	3
d	-1	-4	-1	0

Johnson Algorithm

- Time Complexity
- Time complexity of Floyd Warshal's Algorithm is $O(V^3)$
- The time complexity of Bellman ford algorithm is O(VE)
- Using Max Heap, Dijkstra's complexity is O(ElogV), otherwise it is $O(V^2)$
- The time complexity of Johnson algorithm is O(VElogV)

Thank you

Any Question ??????????