6053 Final Project

Group Members

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Question

What is the influence of median income on the number of fast food restaurants in a region?

The hypothesis we want to test is that as median income (I) increases in Metropolitan Counties, there is an increase in the fast food restaurant count (F). Conversely, when the median income (I) increases in Micropolitan Counties, there is a decrease in the fast food restaurant count (F), which we think is similar to the food desert phenomenon in the US.

Data

Our data comes from a few different sources to create a simple table where we have a Core Based Statistical Area correlated with whether the area is metropolitan/micropolitan in addition to the number of fast food restaurants in the given area.

- Tigerline Core Based Statistical Area Boundaires w/ Micropolitan/Metropolitan classification for 2019
  - Source: https://www2.census.gov/geo/tiger/TIGER2019/CBSA/
- US Census Median Household Income Estimated for Core Based Statistical Areas 2019
  - Source: https://data.census.gov/table/ACSST1Y2019.S2503?q=United States&t=Financial Characteristics&g=010XX00US,$3100000&y=2019
- US Fast Food Restaurant Locations Updated as of November, 2018
  - Source: https://hub.arcgis.com/datasets/UrbanObservatory::fast-food-restaurants/explore?layer=0&showTable=true

Notebook Setup

In [40]:
```python
# imports
import httpx
import math
import geopandas as gpd
import pandas as pd
import arviz as az
import pymc as pm
import matplotlib.pyplot as plt
import numpy as np
import cartopy.crs as ccrs
import networkx as nx

from cartopy.feature import LAND
from shapely import bounds, Polygon
from scipy import stats
from os import path, mkdir

az.rcParams["stats.hdi_prob"] = 0.89
```

In [2]:
```python
# utility functions

def standardize(series):
    return (series - series.mean()) / series.std()

def polygon_to_extent(polygon: Polygon):
    b = bounds(polygon)
    return [b[0], b[2], b[1], b[3]]

def create_cartopy_plot(figsize=(12, 8), title=None):
    fig = plt.figure(figsize=figsize)
    ax = fig.add_subplot(1, 1, 1, projection=ccrs.PlateCarree())
    ax.add_feature(LAND)

    if title:
        ax.set_title(title)

    return fig, ax
```

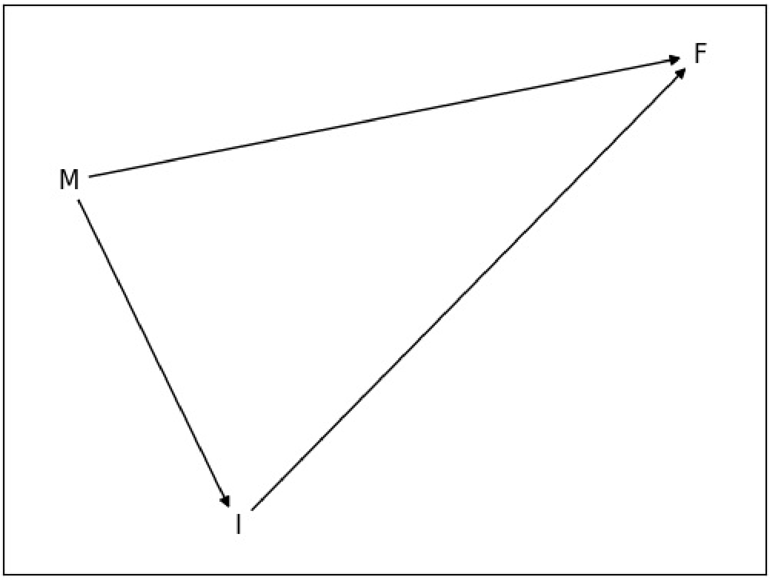
Causal Model

The DAG explains the three variables:

- Treatment Variable: Median Income (I)
- Outcome Variable: Fast Food Restaurant Count (F)
- Primary Confounder: Metropolitan/ Micropolitan Status (M)

In [45]:

```python
G = nx.DiGraph()

G.add_edge('I', 'F')
G.add_edge('M', 'F')
G.add_edge('M', 'I')

fig, ax = plt.subplots(1, 1)

nx.draw_networkx(G, node_color='white', arrows=True, ax=ax)
```



Data Prep

To create the final dataset we are interested in for this project, we need to join the three separate data sources

1. Load geospatial US county data that has both the metropolitan/micropolitan flag as well as geospatial boundary of the county
2. Load median household income estimates for the given counties by name
3. Join the above two datasets by county name, only including counties where a median household income is available
4. Load the fast food restaurant location data, which provides latitude/longitude coordinates of where each restaurant exists
5. Join the fast food restaurant location data with our county table, via a geospatial join where a lat/lon is inside a county boundary
6. Group the final table by county name, where we aggregate by the count of fast food restaurants that are within a given county

In [3]:

```python
# Load CBSA geospatial information
def get_cached_file(file_name: str, download_url: str, file_handler):
    temp_dir = './temp'
    if not path.exists(temp_dir):
        mkdir(temp_dir)
    file_path = f'{temp_dir}/{file_name}'
    if not path.exists(file_path):
        resp = httpx.get(download_url)
        if resp.status_code == 200:
            with open(file_path, 'wb') as f:
                f.write(resp.content)
        else:
            resp.raise_for_status()
    return file_handler(file_path)

# https://www2.census.gov/geo/tiger/TIGER2019/CBSA/
area_gdf = get_cached_file('tl_2019_us_cbsa.zip', 'https://www2.census.gov/geo/tiger/TIGER2019/CBSA/tl_2019_us_
area_gdf = area_gdf.rename(columns={
    'NAMELSAD': 'area_name',
    'LSAD': 'area_flag'
})
area_gdf = area_gdf[['area_name', 'area_flag', 'geometry']]
area_gdf
```

Out[3]:

	area_name	area_flag	geometry
0	Athens-Clarke County, GA Metro Area	M1	POLYGON ((-83.53739 33.96591, -83.53184 33.968...
1	Atlanta-Sandy Springs-Alpharetta, GA Metro Area	M1	POLYGON ((-85.33823 33.65312, -85.33842 33.654...
2	Atlantic City-Hammonton, NJ Metro Area	M1	POLYGON ((-74.85675 39.42076, -74.85670 39.420...
3	Atmore, AL Micro Area	M2	POLYGON ((-87.61542 31.04100, -87.61541 31.041...
4	Auburn, IN Micro Area	M2	POLYGON ((-85.19295 41.38001, -85.19296 41.381...
...	...	...	...
933	Winfield, KS Micro Area	M2	POLYGON ((-97.15084 37.30371, -97.15085 37.304...
934	Winnemucca, NV Micro Area	M2	POLYGON ((-118.87429 40.96103, -118.87600 40.9...
935	Winona, MN Micro Area	M2	POLYGON ((-92.07851 44.02033, -92.07860 44.023...
936	Winston-Salem, NC Metro Area	M1	POLYGON ((-80.45170 36.26150, -80.45170 36.261...
937	Wisconsin Rapids-Marshfield, WI Micro Area	M2	POLYGON ((-90.31566 44.51277, -90.31562 44.515...

938 rows × 3 columns

In [4]:

```
# Load census data for CBSAs
# NOTE: Median Household Income Estimate column: S2503_C01_013E

# NOTE: zip downloaded from source and unzipped to dir
income_df = pd.read_csv('data/income_data_2019/ACSST1Y2019.S2503-Data.csv').loc[1:].rename(columns={
    'S2503_C01_013E': 'median_household_income_est',
    'NAME': 'area_name'
})
income_df = income_df[['area_name', 'median_household_income_est']]
income_df
```

Out[4]:

	area_name	median_household_income_est
1	United States	65712
2	Aberdeen, WA Micro Area	61026
3	Abilene, TX Metro Area	54808
4	Adrian, MI Micro Area	53865
5	Aguadilla-Isabela, PR Metro Area	16311
...	...	...
515	York-Hanover, PA Metro Area	69172
516	Youngstown-Warren-Boardman, OH-PA Metro Area	48558
517	Yuba City, CA Metro Area	61307
518	Yuma, AZ Metro Area	46419
519	Zanesville, OH Micro Area	51844

519 rows × 2 columns

In [5]:

```
# NOTE: inner join will drop any counties that do not have a median income estimate
area_gdf = pd.merge(area_gdf, income_df, how='inner')
area_gdf
```

Out[5]:

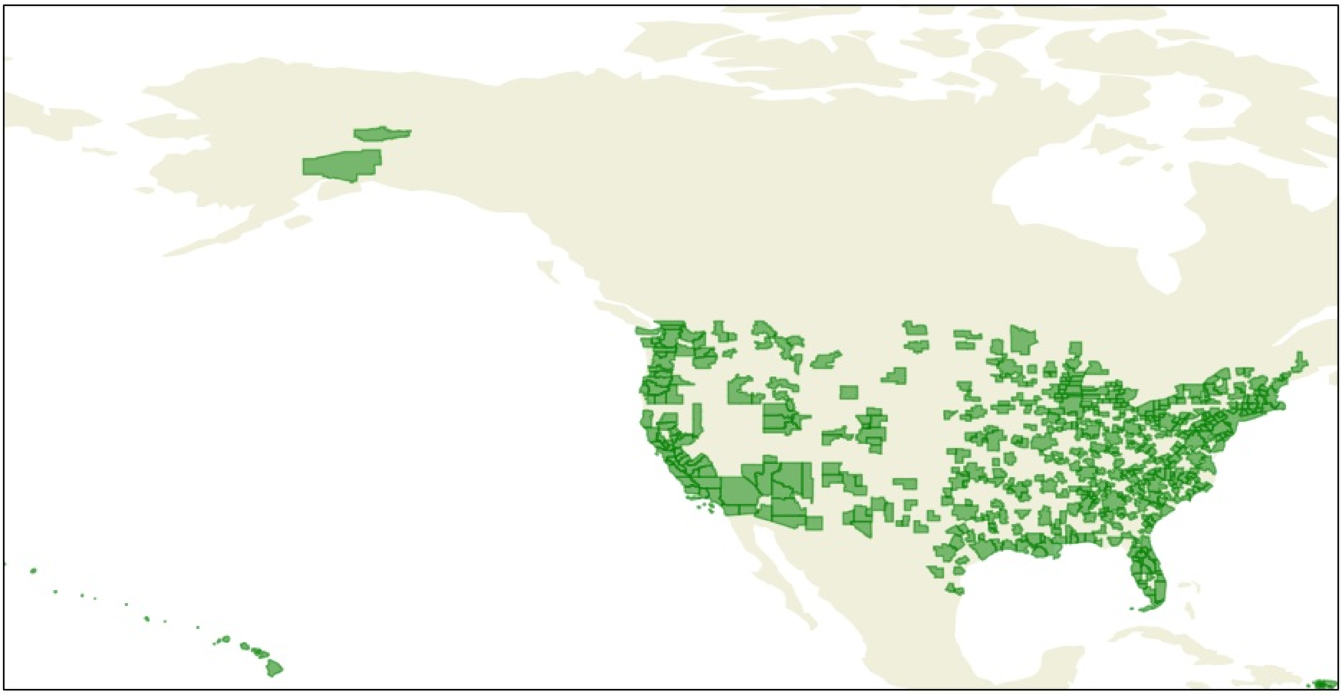
	area_name	area_flag	geometry	median_household_income_est
0	Athens-Clarke County, GA Metro Area	M1	POLYGON ((-83.53739 33.96591, -83.53184 33.968...	50962
1	Atlanta-Sandy Springs-Alpharetta, GA Metro Area	M1	POLYGON ((-85.33823 33.65312, -85.33842 33.654...	71742
2	Atlantic City-Hammonton, NJ Metro Area	M1	POLYGON ((-74.85675 39.42076, -74.85670 39.420...	63389
3	Auburn, NY Micro Area	M2	POLYGON ((-76.73797 42.96129, -76.73771 42.961...	58665
4	Auburn-Opelika, AL Metro Area	M1	POLYGON ((-85.29322 32.73073, -85.28826 32.730...	53712
...	...	...	...	...
513	Wilmington, NC Metro Area	M1	POLYGON ((-78.02992 34.33177, -78.03074 34.331...	57667
514	Wilson, NC Micro Area	M2	POLYGON ((-78.03802 35.78752, -78.03697 35.787...	38830
515	Winchester, VA-WV Metro Area	M1	POLYGON ((-78.50813 39.08863, -78.50853 39.088...	76583
516	Winston-Salem, NC Metro Area	M1	POLYGON ((-80.45170 36.26150, -80.45170 36.261...	52322
517	Wisconsin Rapids-Marshfield, WI Micro Area	M2	POLYGON ((-90.31566 44.51277, -90.31562 44.515...	56278

518 rows × 4 columns

In [6]:

```
# plot of all areas with median income estimate
fig, ax = create_cartopy_plot(title='Plot of all Core Based Statistical Areas\nwith Median Household Income Data
ax.add_geometries(area_gdf.geometry, color='green', alpha=.5, crs=ccrs.PlateCarree())
ax.set_extent(polygon_to_extent(area_gdf.geometry.unary_union))
```

Plot of all Core Based Statistical Areas
with Median Household Income Data



In [7]:

```
# Load fast food location data

# NOTE: downloaded from source and saved to data directory as geojson
fast_food_gdf = gpd.read_file('data/fast_food_restaurants_20191107.geojson').set_crs(4326)
fast_food_gdf
```

Out[7]:

	OBJECTID	ID	Letter	Address	City	State	Zip	Phone	Lat	Long_	Restaurant	geometry
0	1	1	b	3601 N.W. 27th Avenue	Miami	FL	33142	(305) 638-3838	25.8092	-80.2400	Burger King	POINT (-80.24000 25.80920)
1	2	2	b	8995 N. W. 7th Avenue	Miami	FL	33150	(305) 754-8453	25.8587	-80.2094	Burger King	POINT (-80.20940 25.85870)
2	3	3	b	30390 South Dixie Highway	Homestead	FL	33030	(305) 247-7181	25.4849	-80.4610	Burger King	POINT (-80.46100 25.48490)
3	4	4	b	7975 N. W. 27th Avenue	Miami	FL	33147	(305) 836-8152	25.8471	-80.2415	Burger King	POINT (-80.24150 25.84710)
4	5	5	b	9201 South Dixie Highway	Miami	FL	33156	(305) 666-1130	25.6849	-80.3125	Burger King	POINT (-80.31250 25.68490)
...	...	...	...	...	...	...	...	...	...	...	...	...
49997	49998	49998	w	Uscd Q-076 Price Center	La Jolla	CA	92093	None	32.8789	-117.2310	Wendy's	POINT (-117.23100 32.87890)
49998	49999	49999	w	Village At Wexford #278	Hilton Head Island	SC	29928	None	32.2162	-80.7098	Wendy's	POINT (-80.70980 32.21620)
49999	50000	50000	w	W V Rt 7	Morgantown	WV	26505	None	39.6321	-79.9568	Wendy's	POINT (-79.95680 39.63210)
50000	50001	50001	w	Westfall Town Ctr Rt 209	Matamoras	PA	18336	None	41.3709	-74.6981	Wendy's	POINT (-74.69810 41.37090)
50001	50002	50002	w	Willowbrook Mall	Wayne	NJ	7470	None	40.8902	-74.2567	Wendy's	POINT (-74.25670 40.89020)

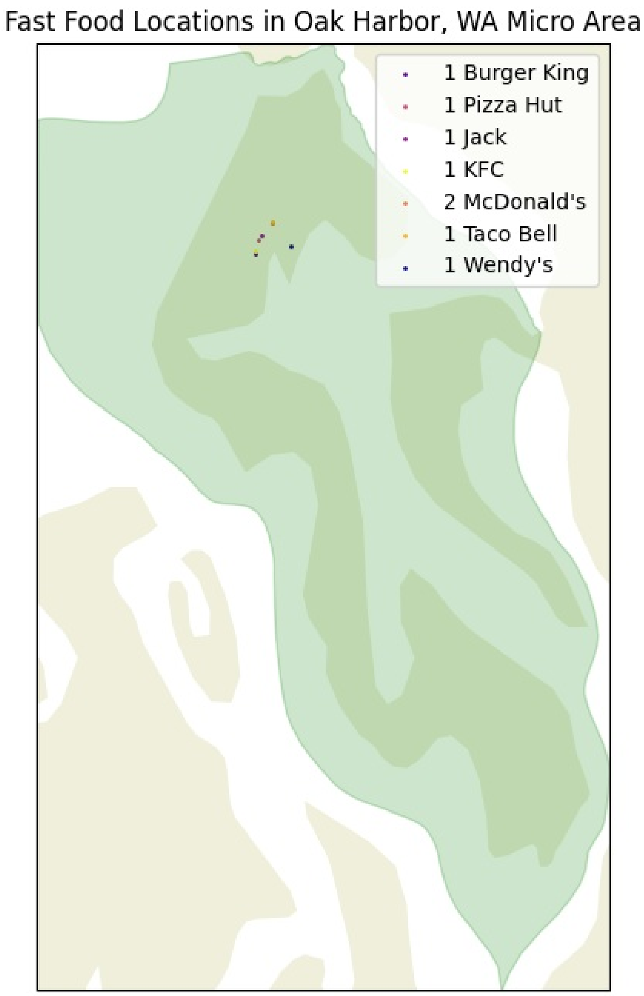
50002 rows × 12 columns

In [8]:

```
# quick sanity check that geospatial join will work
area = 'Oak Harbor, WA Micro Area'
t = area_gdf[area_gdf.area_name == area].iloc[0]
area_ff = fast_food_gdf[fast_food_gdf.geometry.within(t.geometry)]
fig, ax = create_cartopy_plot(title=f'Fast Food Locations in {area}')

ax.add_geometries([t.geometry], color='green', alpha=.2, crs=ccrs.PlateCarree())
ax.set_extent(polygon_to_extent(t.geometry))

restaurants = area_ff.Restaurant.unique()
colors = np.random.choice(len(restaurants), len(restaurants), replace=False)
cmap = plt.cm.get_cmap('plasma', len(colors))
for (r, c) in zip(restaurants, colors):
    t = area_ff[area_ff.Restaurant == r]
    t.plot(ax=ax, markersize=1, color=cmap(c), label=f'{len(t)} {r}')

ax.legend()
```

```
/var/folders/l3/_57qjm850c7chfs8l26s8c500000gn/T/ipykernel_7843/708150327.py:12: MatplotlibDeprecationWarning: T
he get_cmap function was deprecated in Matplotlib 3.7 and will be removed two minor releases later. Use ``matplo
tlib.colormaps[name]`` or ``matplotlib.colormaps.get_cmap(obj)`` instead.
  cmap = plt.cm.get_cmap('plasma', len(colors))
```

Out[8]: <matplotlib.legend.Legend at 0x128adf190>



In [9]:

```
# geospatial join of fast food on all areas
final_gdf = area_gdf.sjoin(fast_food_gdf, how='inner', predicate='covers')
final_gdf
```

Out[9]:

	area_name	area_flag	geometry	median_household_income_est	index_right	OBJECTID	ID	Letter	Address	Ci
0	Athens-Clarke County, GA Metro Area	M1	POLYGON ((-83.53739 33.96591, -83.53184 33.968...	50962	28613	28614	28614	m	2063 Experiment Station Rd	Watkinsvi
0	Athens-Clarke County, GA Metro Area	M1	POLYGON ((-83.53739 33.96591, -83.53184 33.968...	50962	44146	44147	44147	w	1011 Jordan Drive	Athe
0	Athens-Clarke County, GA Metro Area	M1	POLYGON ((-83.53739 33.96591, -83.53184 33.968...	50962	27760	27761	27761	m	1791 Epps Bridge Parkway	Athe
0	Athens-Clarke County, GA Metro Area	M1	POLYGON ((-83.53739 33.96591, -83.53184 33.968...	50962	45846	45847	45847	w	1980 Barnett Shoals Rd	Athe
0	Athens-Clarke County, GA Metro Area	M1	POLYGON ((-83.53739 33.96591, -83.53184 33.968...	50962	20675	20676	20676	k	2150 Barnett Shoals Rd	Athe
...	...	...	...	...	...	...	...	...	...	
517	Wisconsin Rapids-Marshfield, WI Micro Area	M2	POLYGON ((-90.31566 44.51277, -90.31562 44.515...	56278	39552	39553	39553	t	1750 8th Street South	Wiscons Rapi
517	Wisconsin Rapids-Marshfield, WI Micro Area	M2	POLYGON ((-90.31566 44.51277, -90.31562 44.515...	56278	20155	20156	20156	k	1750 8th Street South	Wiscons Rapi
517	Wisconsin Rapids-Marshfield, WI Micro Area	M2	POLYGON ((-90.31566 44.51277, -90.31562 44.515...	56278	31992	31993	31993	m	400 Daly Ave	Wiscons Rapi
517	Wisconsin Rapids-Marshfield, WI Micro Area	M2	POLYGON ((-90.31566 44.51277, -90.31562 44.515...	56278	1607	1608	1608	b	940 Eighth Street South	Wiscons Rapi
517	Wisconsin Rapids-Marshfield, WI Micro Area	M2	POLYGON ((-90.31566 44.51277, -90.31562 44.515...	56278	48372	48373	48373	w	555 W. Grand Avenue	Wiscons Rapi

43709 rows × 16 columns

In [10]:

```
# final groupby + aggregation to get restaurant count
final_gdf = final_gdf.groupby(['area_name', 'area_flag', 'median_household_income_est', 'geometry'])['Address']
final_gdf = final_gdf.rename(columns={'Address': 'restaurant_count'})
final_gdf
```

Out[10]:

	area_name	area_flag	median_household_income_est	geometry	restaurant_count
0	Aberdeen, WA Micro Area	M2	61026	POLYGON ((-124.30542 47.24464, -124.30637 47.2...	10
1	Abilene, TX Metro Area	M1	54808	POLYGON ((-100.14654 32.52279, -100.14642 32.5...	22
2	Adrian, MI Micro Area	M2	53865	POLYGON ((-84.36198 41.89876, -84.36198 41.898...	17
3	Akron, OH Metro Area	M1	57158	POLYGON ((-81.39169 41.34827, -81.39164 41.348...	146
4	Alamogordo, NM Micro Area	M2	39371	POLYGON ((-106.37642 32.91041, -106.37644 32.9...	9
...	...	...	...	...	...
505	York-Hanover, PA Metro Area	M1	69172	POLYGON ((-77.05440 40.02321, -77.05441 40.023...	58
506	Youngstown-Warren-Boardman, OH-PA Metro Area	M1	48558	POLYGON ((-81.00229 41.13419, -81.00232 41.145...	110
507	Yuba City, CA Metro Area	M1	61307	POLYGON ((-121.62376 39.29562, -121.62339 39.2...	23
508	Yuma, AZ Metro Area	M1	46419	POLYGON ((-114.76378 32.64340, -114.76342 32.6...	33
509	Zanesville, OH Micro Area	M2	51844	POLYGON ((-82.19479 40.02069, -82.19477 40.021...	19

510 rows × 5 columns

In [11]:

```
# sanity check: make sure top/bottom restaurant counts make sense
final_gdf.sort_values('restaurant_count', ascending=False).head(5)
```

Out[11]:

	area_name	area_flag	median_household_income_est	geometry	restaurant_count
267	Los Angeles-Long Beach-Anaheim, CA Metro Area	M1	77774	MULTIPOLYGON (((-118.12590 33.69715, -118.1557...	2116
325	New York-Newark-Jersey City, NY-NJ-PA Metro Area	M1	83160	POLYGON ((-74.88982 40.78773, -74.88971 40.787...	1564
82	Chicago-Naperville-Elgin, IL-IN-WI Metro Area	M1	75379	POLYGON ((-88.60224 41.63139, -88.61185 41.631...	1315
108	Dallas-Fort Worth-Arlington, TX Metro Area	M1	72265	POLYGON ((-97.92164 33.00128, -97.92153 33.008...	993
201	Houston-The Woodlands-Sugar Land, TX Metro Area	M1	69193	POLYGON ((-95.80431 30.24557, -95.80429 30.247...	944

In [12]:

```
final_gdf.sort_values('restaurant_count', ascending=True).head(5)
```

Out[12]:

	area_name	area_flag	median_household_income_est	geometry	restaurant_count
451	Sunbury, PA Micro Area	M2	47349	POLYGON ((-76.86143 41.03318, -76.86143 41.033...	7
362	Port Angeles, WA Micro Area	M2	57126	POLYGON ((-123.15163 47.86688, -123.15261 47.8...	8
425	Shelton, WA Micro Area	M2	63983	POLYGON ((-122.81343 47.30681, -122.81423 47.3...	8
26	Auburn, NY Micro Area	M2	58665	POLYGON ((-76.73797 42.96129, -76.73771 42.961...	8
330	Oak Harbor, WA Micro Area	M2	72066	POLYGON ((-122.86272 48.26269, -122.86273 48.2...	8

In [13]:

```
# save final table to csv
final_table = 'data/n_fast_food_by_cbsa.csv'
if not path.exists(final_table):
    final_gdf[['area_flag', 'median_household_income_est', 'restaurant_count']].to_csv(final_table, index=False
```

Data Exploration

In [14]:

```
df = pd.read_csv(final_table)
df = df.rename(columns={
    # rename to shorter names for convenience
    'median_household_income_est': 'median_income',
    'restaurant_count': 'ff_count'
```

```
})
df
```

Out[14]:

	area_flag	median_income	ff_count
0	M2	61026	10
1	M1	54808	22
2	M2	53865	17
3	M1	57158	146
4	M2	39371	9
...	...	...	...
505	M1	69172	58
506	M1	48558	110
507	M1	61307	23
508	M1	46419	33
509	M2	51844	19

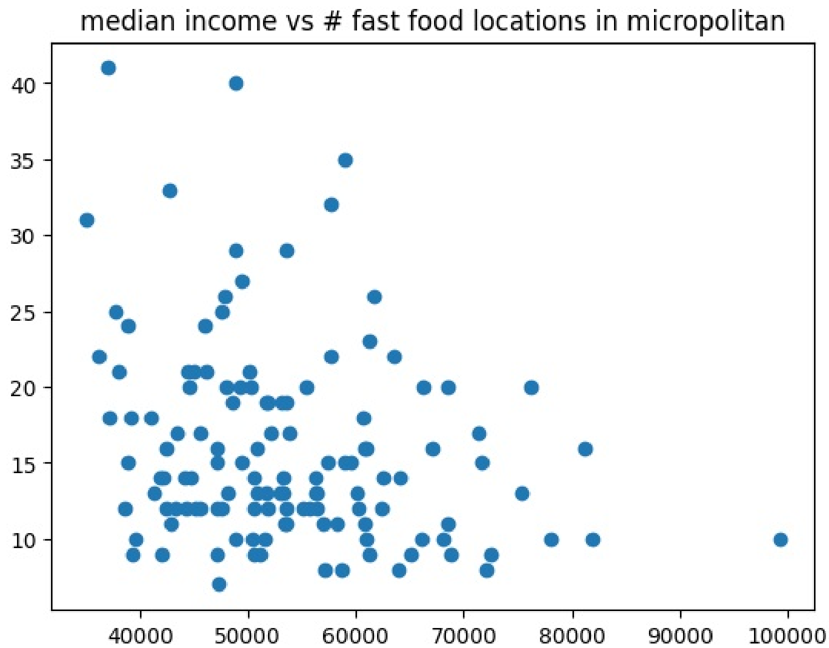
510 rows × 3 columns

In [15]:

```
fig, ax = plt.subplots(1, 1)

temp = df[df.area_flag == 'M2']
ax.scatter(temp.median_income, temp.ff_count)
ax.set_title('median income vs # fast food locations in micropolitan')
```

Out[15]:

```
Text(0.5, 1.0, 'median income vs # fast food locations in micropolitan')
```

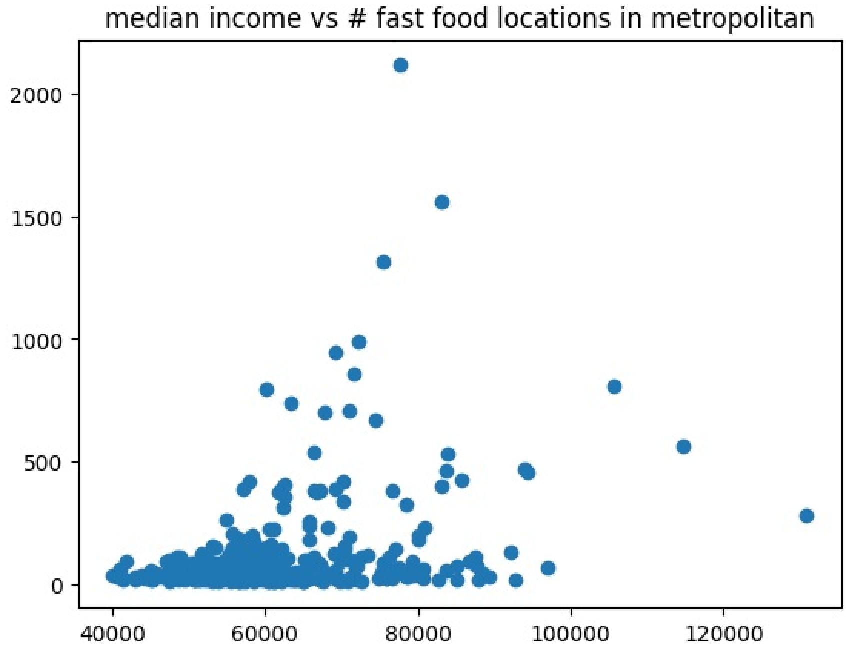


In [16]:

```
fig, ax = plt.subplots(1, 1)

temp = df[df.area_flag == 'M1']
ax.scatter(temp.median_income, temp.ff_count)
ax.set_title('median income vs # fast food locations in metropolitan')
```

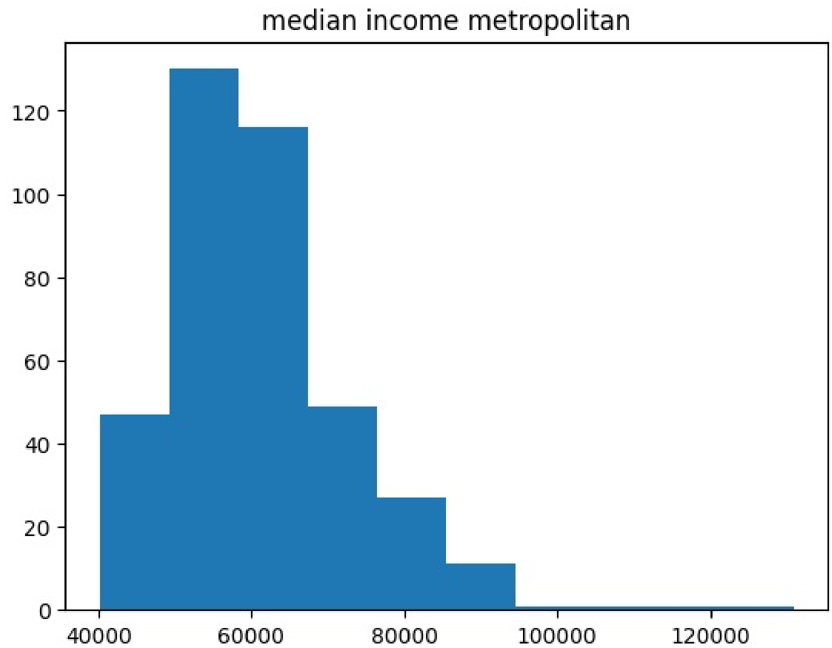
Out[16]:

```
Text(0.5, 1.0, 'median income vs # fast food locations in metropolitan')
```



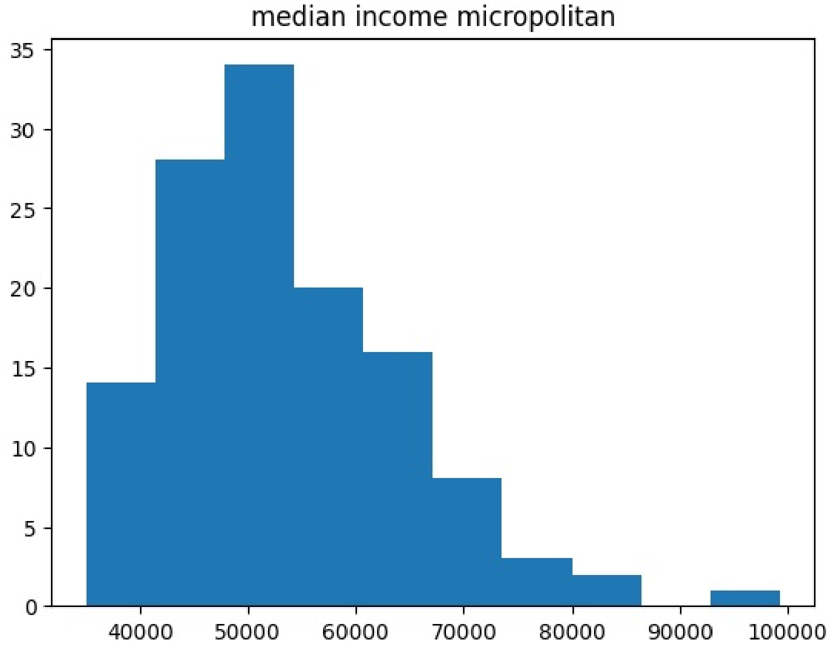
In [17]:
```
fig, ax = plt.subplots(1, 1)

temp = df[df.area_flag == 'M1']
ax.hist(temp.median_income)
ax.set_title('median income metropolitan')
```

Out[17]: Text(0.5, 1.0, 'median income metropolitan')



In [18]:
```
fig, ax = plt.subplots(1, 1)

temp = df[df.area_flag == 'M2']
ax.hist(temp.median_income)
ax.set_title('median income micropolitan')
```

Out[18]: Text(0.5, 1.0, 'median income micropolitan')



In [19]:
```
df.area_flag.value_counts()
```

Out[19]:
```
area_flag
M1    384
M2    126
Name: count, dtype: int64
```

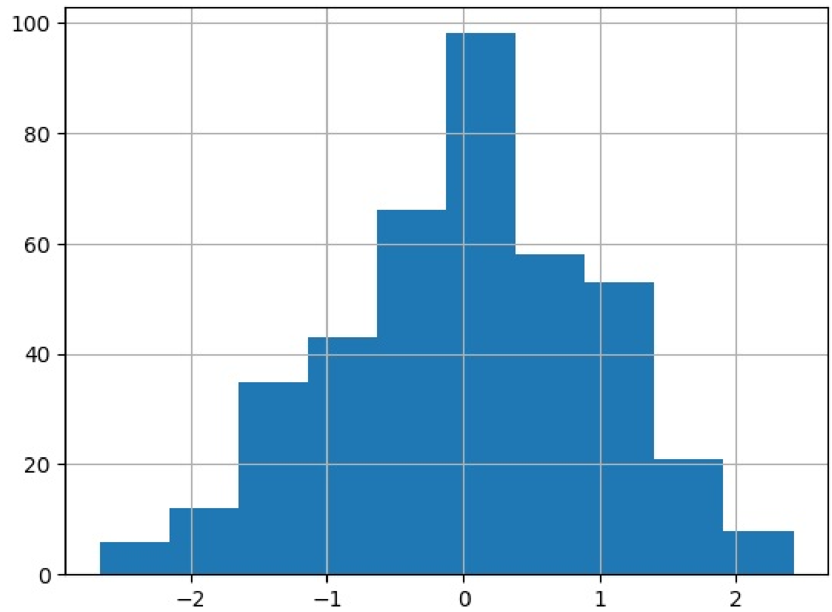
Prior Predictive Simulation

For our priors, we are pulling in the following assumptions

- Median income will follow a standard normal distribution, regardless of metropolitan/micropolitan, based on general knowledge of income in US
- There are more metropolitan areas collected vs micropolitan (3 - 1), as there is more population and better data for these regions
- Fast food restaurants will change depending on micro/metro. Due to food desserts in the US, we predict that micropolitan areas with lower income will have higher # of fast food locations, where as metro areas will follow a more standard increase. Both of these scenarios will be distributed in a Log Normal Distribution, where many of the areas will have standard number, and a few locations will have large number of restaurants

In [20]:
```
# Median income prior

income_sim = pd.Series(stats.norm.rvs(size=400))
income_sim.hist()
```

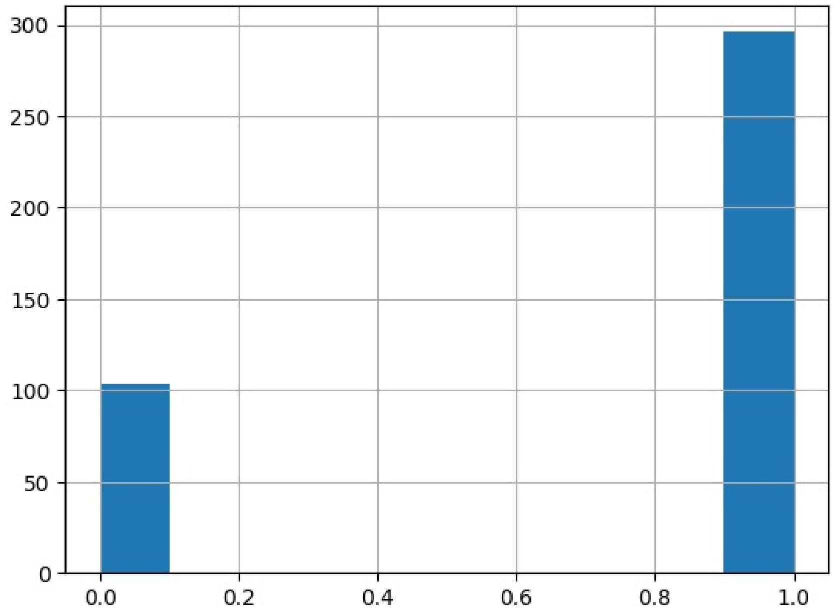
Out[20]: <Axes: >



In [21]:
```
# Metro/Micro flag prior
```

```
flag_sim = pd.Series(stats.bernoulli.rvs(p=.75, size=400))
flag_sim.hist()
```

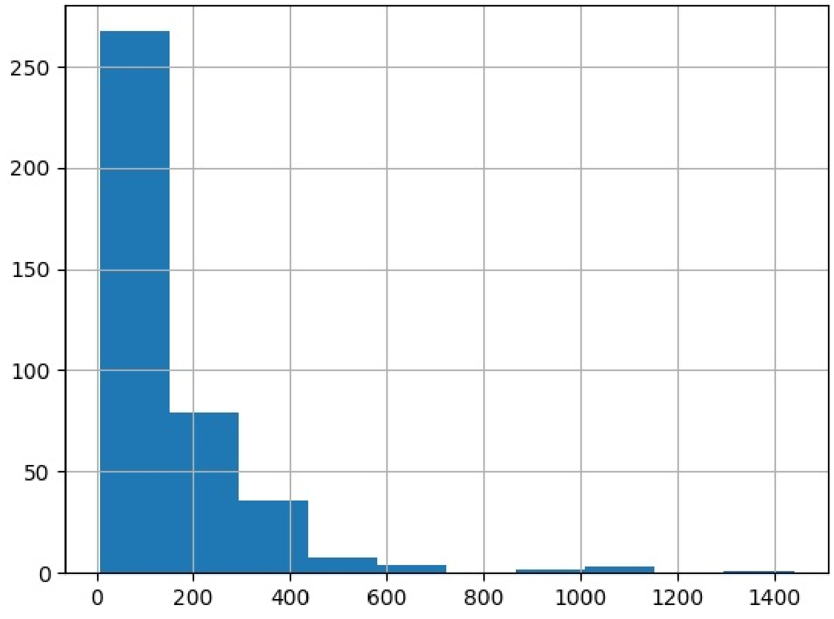
Out[21]: <Axes: >



In [22]:

```
# Metro Restaurant count

# NOTE: higher variance in micro areas
metro_food_sim = pd.Series(stats.lognorm(s=1, scale=math.exp(4.5)).rvs(size=400))
metro_food_sim.hist()
```

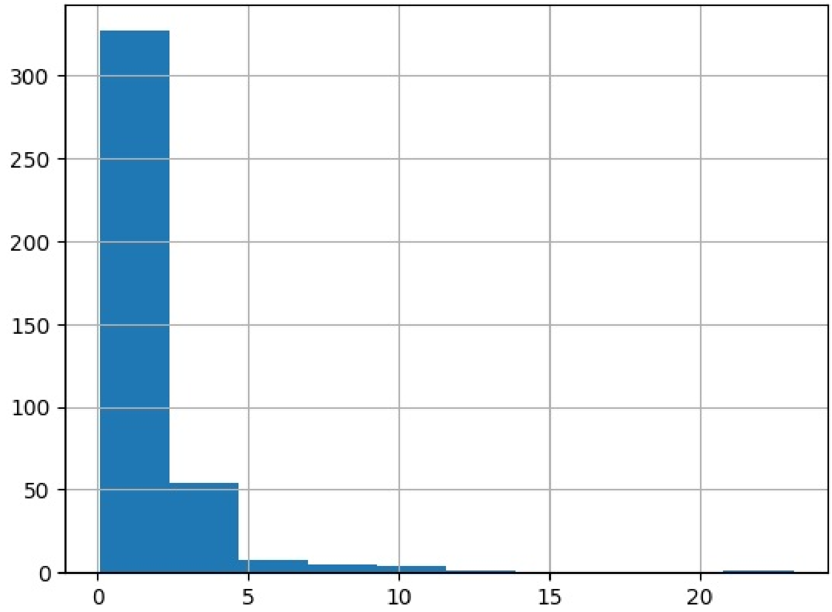
Out[22]: <Axes: >



In [23]:

```
# Micro Restaurant count

# NOTE: lower variance in micro areas
metro_food_sim = pd.Series(stats.lognorm(s=1, scale=1).rvs(size=400))
metro_food_sim.hist()
```

Out[23]: <Axes: >



In [24]:
```
def generate_data(size=400):
    i = pd.Series(stats.norm.rvs(size=size))
    m = pd.Series(stats.bernoulli.rvs(p=.75, size=size))
    f = []
    for flag in m:
        if flag == 1:
            f.append(stats.lognorm(s=1, scale=math.exp(4.5)).rvs(size=1)[0])
        else:
            f.append(stats.lognorm(s=1, scale=1).rvs(size=1)[0])

    df = pd.DataFrame({
        'I_sim': i,
        'M_sim': m,
        'F_sim': f
    })
    return df

sim = generate_data(size=500)
sim
```

Out[24]:

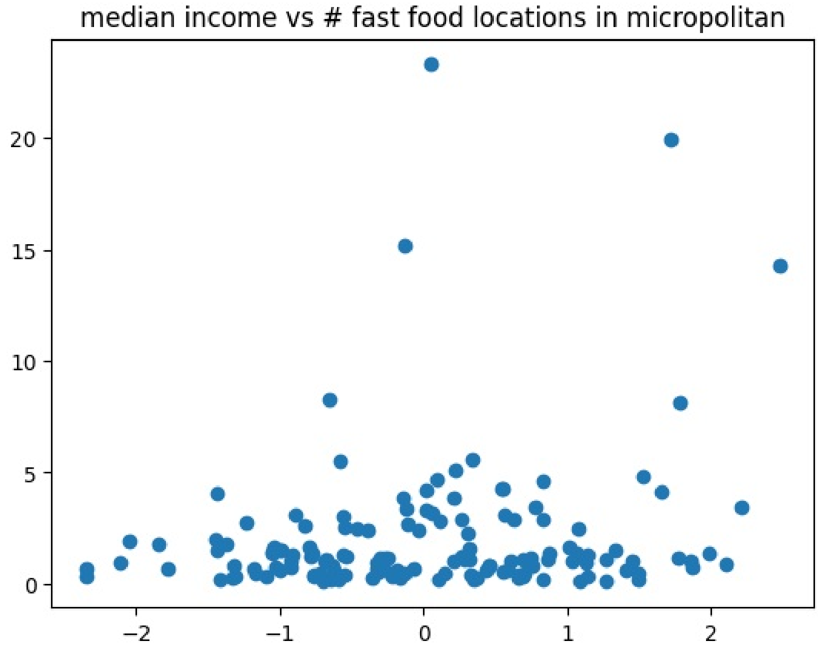
	I_sim	M_sim	F_sim
0	0.392582	1	378.228001
1	-0.874223	1	18.854181
2	-0.249876	1	124.779541
3	1.492182	0	0.443221
4	-0.992319	1	29.691767
...	...	...	...
495	0.277314	1	31.821742
496	1.357498	1	156.230658
497	1.383930	1	8.853682
498	-1.286854	1	76.995388
499	-0.712602	1	37.087726

500 rows × 3 columns

In [25]:
```
fig, ax = plt.subplots(1, 1)
```

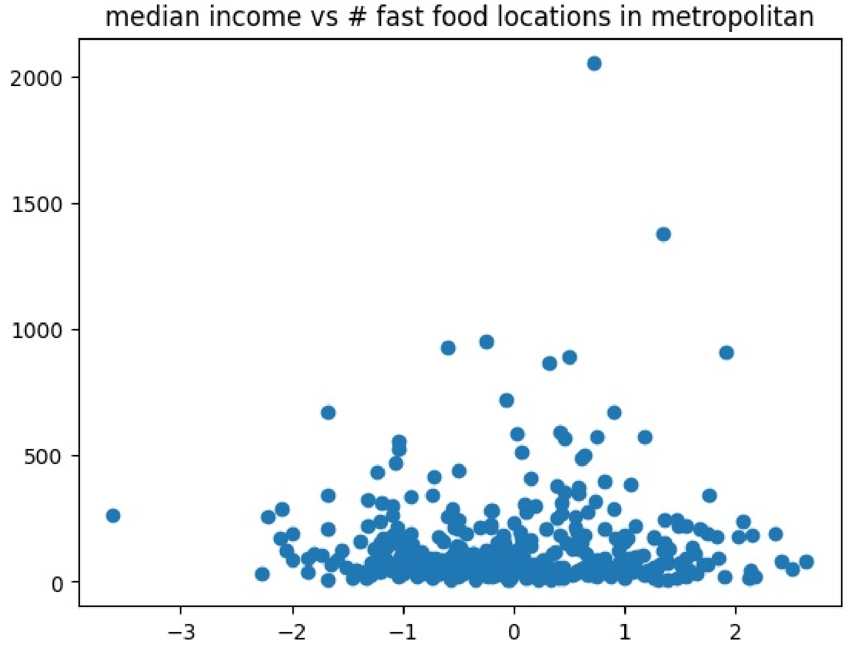
```python
temp = sim[sim.M_sim == 0]
ax.scatter(temp.I_sim, temp.F_sim)
ax.set_title('median income vs # fast food locations in micropolitan')
```

Out[25]: Text(0.5, 1.0, 'median income vs # fast food locations in micropolitan')



In [26]:
```python
fig, ax = plt.subplots(1, 1)

temp = sim[sim.M_sim == 1]
ax.scatter(temp.I_sim, temp.F_sim)
ax.set_title('median income vs # fast food locations in metropolitan')
```

Out[26]: Text(0.5, 1.0, 'median income vs # fast food locations in metropolitan')



In [27]:
```python
# test statistical model on simulated data
with pm.Model() as simulated_model:
    sigma = pm.Exponential('sigma', 1, shape=2)

    beta = pm.Normal('beta', 0, .5, shape=2)
    alpha = pm.Normal('alpha', 0, .2, shape=2)

    mu = pm.Deterministic('mu', beta[sim.M_sim] * sim.I_sim + alpha[sim.M_sim])

    f = pm.LogNormal("f", mu=mu, sigma=sigma[sim.M_sim], observed=sim.F_sim)

    simulated_idata = pm.sample(idata_kwargs={"log_likelihood": True})

az.summary(simulated_idata, var_names='~mu', kind='all')
```

```
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [sigma, beta, alpha]
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
Output()
```

```
Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws total) took 59 seconds.
```

Out[27]:

	mean	sd	hdi_5.5%	hdi_94.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
alpha[0]	0.097	0.083	-0.039	0.226	0.001	0.001	5677.0	3445.0	1.0
alpha[1]	4.130	0.063	4.026	4.228	0.001	0.001	3658.0	3222.0	1.0
beta[0]	0.134	0.087	-0.008	0.268	0.001	0.001	4509.0	3246.0	1.0
beta[1]	-0.009	0.058	-0.106	0.078	0.001	0.001	6216.0	3550.0	1.0
sigma[0]	1.053	0.063	0.957	1.154	0.001	0.001	4769.0	3116.0	1.0
sigma[1]	1.104	0.047	1.030	1.177	0.001	0.001	3736.0	2962.0	1.0

Prior Predictive comments

While not perfect, the simulation does accurately capture how metropolitan areas will have a higher intercept for fast food restaurants, with positive slope associated with # of restaurants with increas in income.

Actual model/data will reveal if our assumptions were correct.

Model

We have tested 3 different models based on our assumptions on fast food locations per area:

1. **LogNormal**
2. **Poisson**
3. **NegativeBinomial**

- The LogNormal Model proved superior, demonstrating the best balance of complexity and predictive accuracy, making it ideal for our analysis. It effectively captured the variance and skewness in restaurant counts, using median income and area type as predictors.
- The Poisson and Negative Binomial Models were considered but showed less optimal performance. The Poisson model particularly suffered a bad fit as indicated by its high predictive information criteria scores and low effective model weight in cross-validation comparison.
- Model Comparison using LOO-CV (Leave-One-Out Cross-Validation) reinforced the LogNormal model's superiority, with it showing the highest log predictive density, indicating it is the most suitable for predicting new data without overfitting.

In [28]:

```
df['I'] = standardize(df['median_income'])
df['M'] = df.area_flag.apply(lambda el: (1 if el == 'M1' else 0))
df['F'] = df.ff_count.copy()
df
```

Out[28]:

	area_flag	median_income	ff_count	I	M	F
0	M2	61026	10	0.140965	0	10
1	M1	54808	22	-0.370166	1	22
2	M2	53865	17	-0.447682	0	17
3	M1	57158	146	-0.176992	1	146
4	M2	39371	9	-1.639115	0	9
...	...	...	...	...	...	...
505	M1	69172	58	0.810580	1	58
506	M1	48558	110	-0.883927	1	110
507	M1	61307	23	0.164063	1	23
508	M1	46419	33	-1.059757	1	33
509	M2	51844	19	-0.613812	0	19

510 rows × 6 columns

In [29]:

```
with pm.Model() as lognormal_model:
```

```
    sigma = pm.Exponential('sigma', 1, shape=2)

    beta = pm.Normal('beta', 0, .5, shape=2)
    alpha = pm.Normal('alpha', 0, .2, shape=2)

    M = pm.MutableData("M", df.M)
    I = pm.MutableData("I", df.I)

    mu = pm.Deterministic('mu', beta[M] * I + alpha[M])

    f = pm.LogNormal("f", mu=mu, sigma=sigma[M], observed=df.F, shape=M.shape[0])

    lognormal_idata = pm.sample(idata_kwargs={"log_likelihood": True})

az.summary(lognormal_idata, var_names='~mu', kind='all')
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/pymc/data.py:303: F
utureWarning: MutableData is deprecated. All Data variables are now mutable. Use Data instead.
  warnings.warn(
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [sigma, beta, alpha]
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
Output()

Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws total) took 56 seconds.
```

Out[29]:

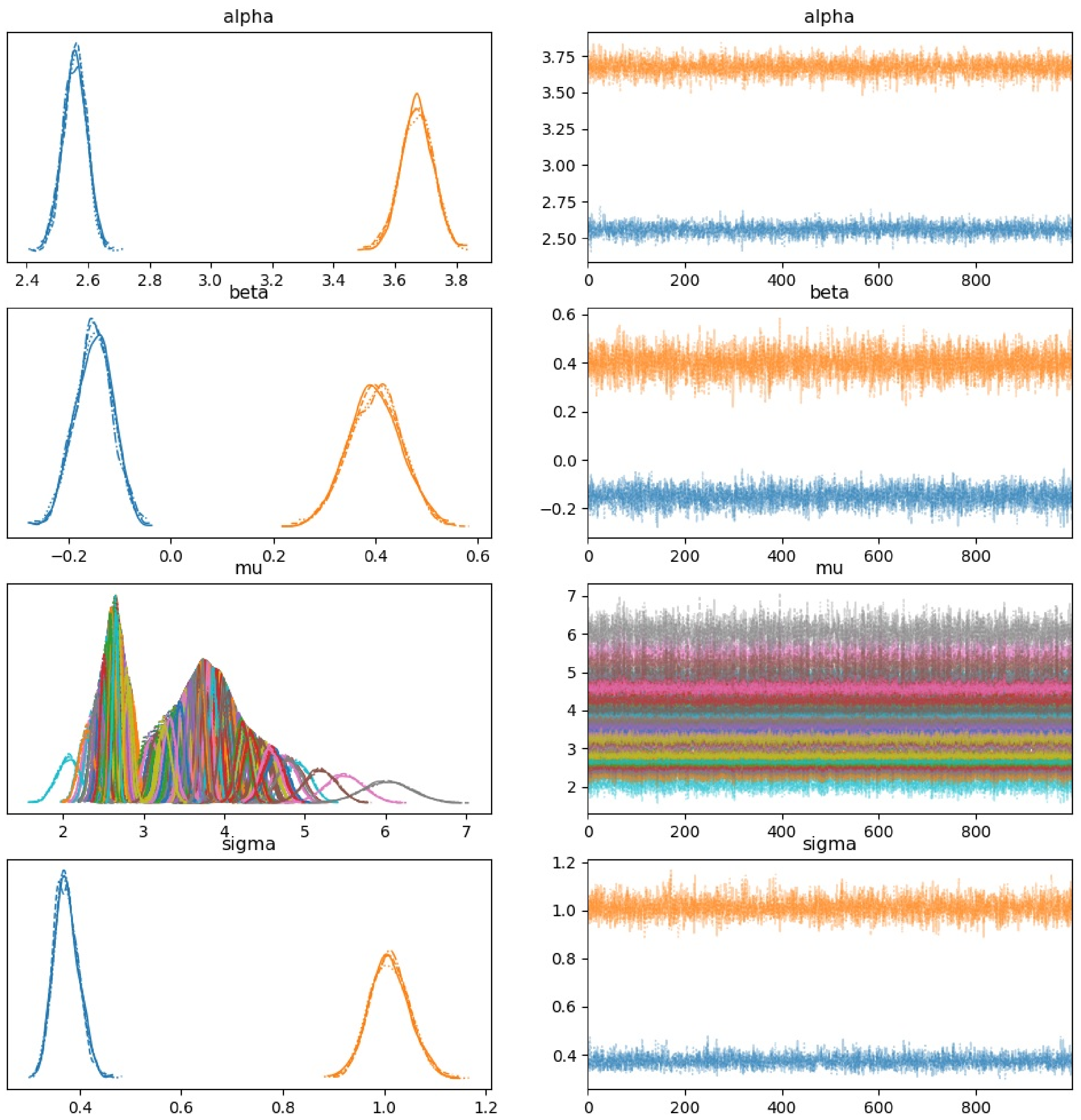
	mean	sd	hdi_5.5%	hdi_94.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
alpha[0]	2.557	0.039	2.497	2.618	0.001	0.000	3212.0	3204.0	1.0
alpha[1]	3.671	0.053	3.587	3.754	0.001	0.001	3892.0	2906.0	1.0
beta[0]	-0.151	0.037	-0.210	-0.093	0.001	0.000	3574.0	3737.0	1.0
beta[1]	0.399	0.052	0.321	0.486	0.001	0.001	4382.0	3025.0	1.0
sigma[0]	0.374	0.025	0.336	0.413	0.000	0.000	3820.0	3178.0	1.0
sigma[1]	1.011	0.039	0.947	1.071	0.001	0.000	4026.0	2998.0	1.0

In [30]:

```
az.plot_trace(lognormal_idata,  figsize= (12, 12))
```

Out[30]:

```
array([[<Axes: title={'center': 'alpha'}>,
        <Axes: title={'center': 'alpha'}>],
       [<Axes: title={'center': 'beta'}>,
        <Axes: title={'center': 'beta'}>],
       [<Axes: title={'center': 'mu'}>, <Axes: title={'center': 'mu'}>],
       [<Axes: title={'center': 'sigma'}>,
        <Axes: title={'center': 'sigma'}>]], dtype=object)
```




```
In [31]: with pm.Model() as poisson_model:
             beta = pm.Normal('beta', 0, .5, shape=2)
             alpha = pm.Normal('alpha', 0, .2, shape=2)

             M = pm.MutableData("M", df.M)
             I = pm.MutableData("I", df.I)

             lam = pm.math.exp(alpha[M] + beta[M] * I)

             f = pm.Poisson("f", lam, observed=df.F, shape=M.shape[0])

             poisson_idata = pm.sample(idata_kwargs={"log_likelihood": True})

         az.summary(poisson_idata, var_names='~mu', kind='all')
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/pymc/data.py:303: F
utureWarning: MutableData is deprecated. All Data variables are now mutable. Use Data instead.
  warnings.warn(
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [beta, alpha]
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
Output()
```

```
Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws total) took 54 seconds.
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/utils.py:142:
UserWarning: Items starting with ~: ['mu'] have not been found and will be ignored
  warnings.warn(
```

Out[31]:

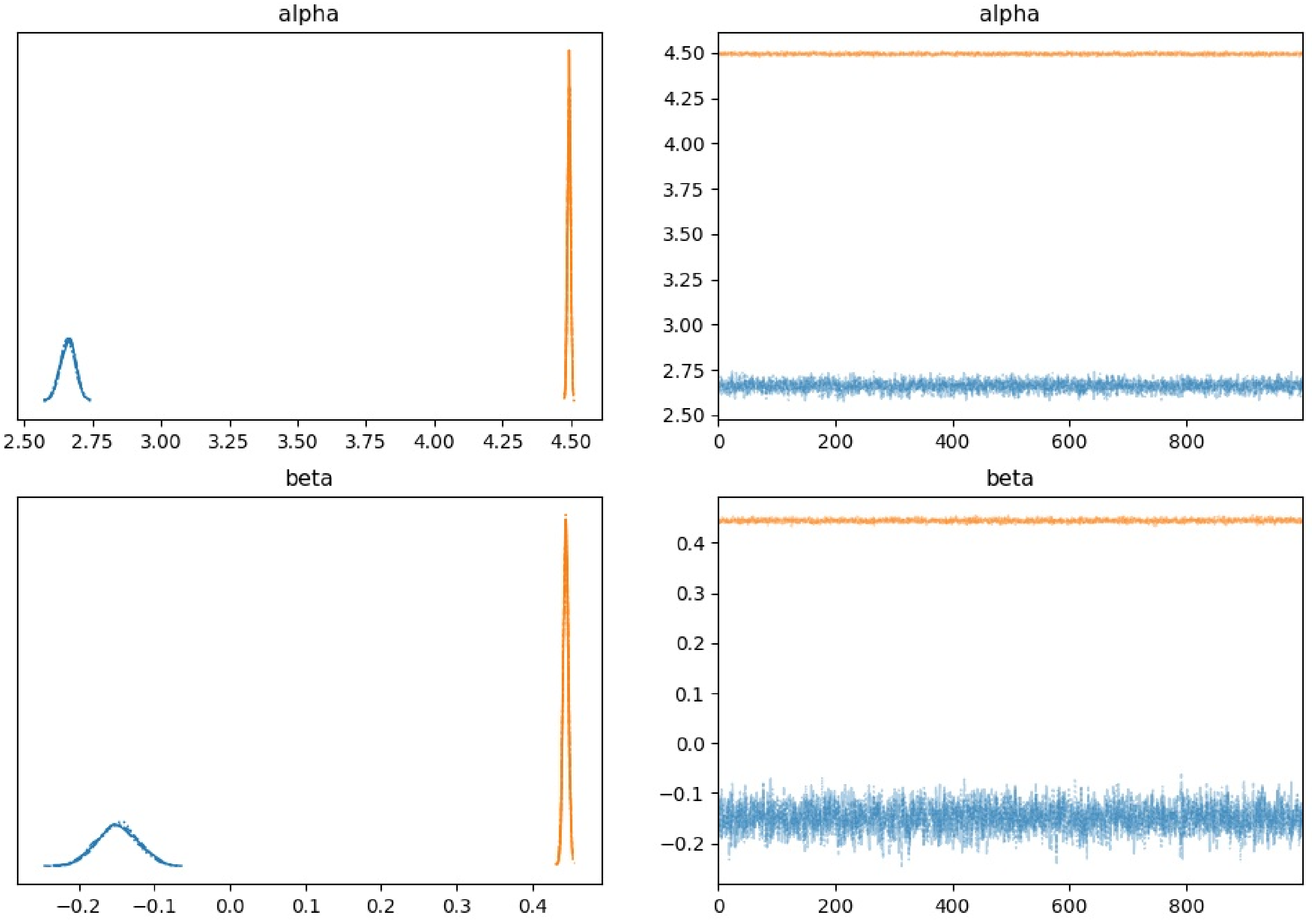
	mean	sd	hdi_5.5%	hdi_94.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
alpha[0]	2.660	0.028	2.615	2.704	0.0	0.0	3205.0	2895.0	1.0
alpha[1]	4.494	0.006	4.484	4.502	0.0	0.0	2911.0	3038.0	1.0
beta[0]	-0.149	0.027	-0.190	-0.105	0.0	0.0	2911.0	2715.0	1.0
beta[1]	0.444	0.003	0.439	0.449	0.0	0.0	3118.0	2952.0	1.0

In [32]:

```
az.plot_trace(poisson_idata, figsize= (12, 8))
```

Out[32]:

```
array([[<Axes: title={'center': 'alpha'}>,
        <Axes: title={'center': 'alpha'}>],
       [<Axes: title={'center': 'beta'}>,
        <Axes: title={'center': 'beta'}>]], dtype=object)
```



In [33]:

```
with pm.Model() as negbinom_model:
    beta = pm.Normal('beta', 0 , 0.01, shape=2)
    alpha = pm.Normal('alpha', mu=-1, sigma=1, shape=2)
    dispersion = pm.Gamma('dispersion', alpha=2, beta=0.1, shape=2)

    M = pm.MutableData("M", df.M)
    I = pm.MutableData("I", df.I)

    mu = pm.math.exp(alpha[M] + beta[M] * I)

    f = pm.NegativeBinomial('f', mu=mu, alpha=dispersion[M], observed=df.F, shape=M.shape[0])

    negbinom_idata = pm.sample(idata_kwargs={'log_likelihood': True})

az.summary(negbinom_idata, var_names='~mu', kind='all')
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/pymc/data.py:303: F
utureWarning: MutableData is deprecated. All Data variables are now mutable. Use Data instead.
  warnings.warn(
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [beta, alpha, dispersion]
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
WARNING (pytensor.tensor.blas): Using NumPy C-API based implementation for BLAS functions.
Output()
```

```
Sampling 4 chains for 1_000 tune and 1_000 draw iterations (4_000 + 4_000 draws total) took 55 seconds.
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/utils.py:142:
UserWarning: Items starting with ~: ['mu'] have not been found and will be ignored
  warnings.warn(
```

Out[33]:

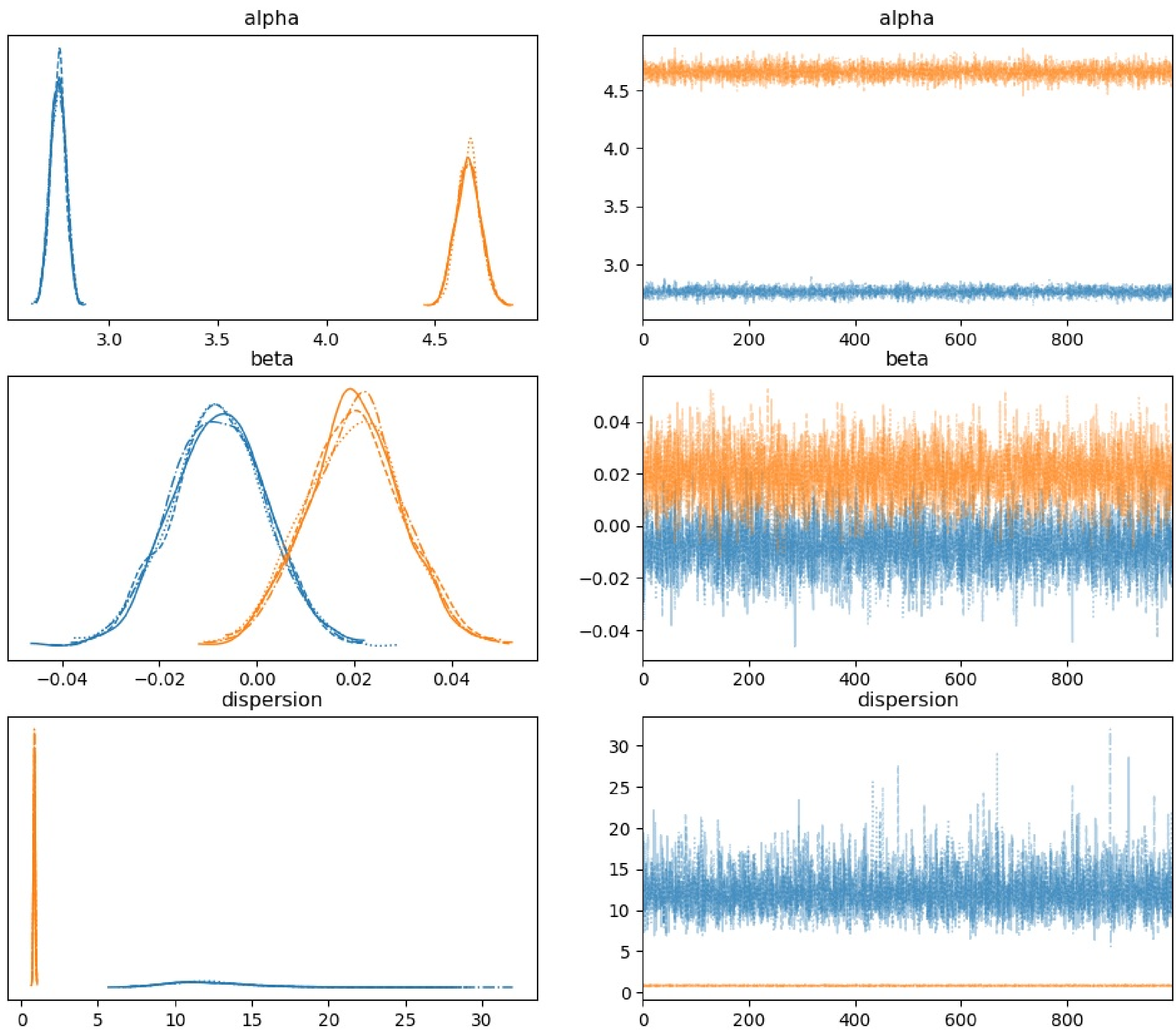
	mean	sd	hdi_5.5%	hdi_94.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat
alpha[0]	2.764	0.035	2.707	2.819	0.000	0.000	5812.0	3264.0	1.0
alpha[1]	4.656	0.056	4.562	4.739	0.001	0.001	5652.0	3446.0	1.0
beta[0]	-0.008	0.010	-0.023	0.008	0.000	0.000	6169.0	3302.0	1.0
beta[1]	0.020	0.010	0.005	0.036	0.000	0.000	4299.0	3389.0	1.0
dispersion[0]	12.306	2.740	7.744	16.034	0.035	0.027	7018.0	3286.0	1.0
dispersion[1]	0.851	0.054	0.766	0.939	0.001	0.001	5091.0	3202.0	1.0

In [34]:

```
az.plot_trace(negbinom_idata, figsize= (12, 10))
```

Out[34]:

```
array([[<Axes: title={'center': 'alpha'}>,
        <Axes: title={'center': 'alpha'}>],
       [<Axes: title={'center': 'beta'}>,
        <Axes: title={'center': 'beta'}>],
       [<Axes: title={'center': 'dispersion'}>,
        <Axes: title={'center': 'dispersion'}>]], dtype=object)
```



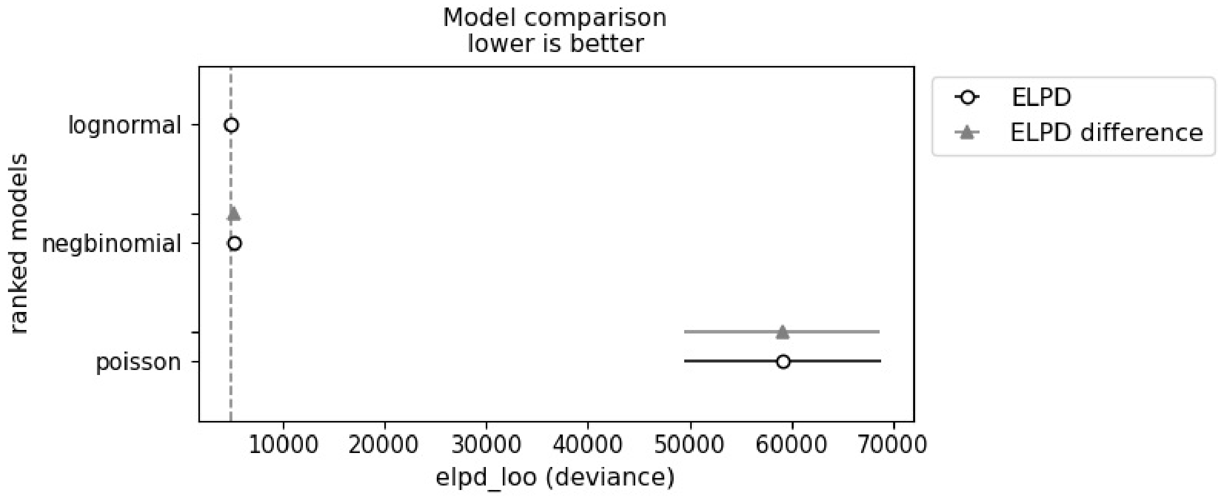
In [35]:
```
model_dict = {
    'lognormal': lognormal_idata,
    'poisson': poisson_idata,
    "negbinomial": negbinom_idata
}
compare_df_psis = az.compare(
    compare_dict=model_dict,
    ic="loo",
    scale="deviance"
)
compare_df_psis
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/stats/stats.p
y:789: UserWarning: Estimated shape parameter of Pareto distribution is greater than 0.7 for one or more samples
. You should consider using a more robust model, this is because importance sampling is less likely to work well
if the marginal posterior and LOO posterior are very different. This is more likely to happen with a non-robust
model and highly influential observations.
  warnings.warn(
```

Out[35]:

	rank	elpd_loo	p_loo	elpd_diff	weight	se	dse	warning	scale
lognormal	0	4945.575878	5.823000	0.000000	1.000000e+00	82.508741	0.000000	False	deviance
negbinomial	1	5168.948757	7.045789	223.372879	1.121708e-11	86.906463	30.609808	False	deviance
poisson	2	59094.179611	484.259575	54148.603734	7.580603e-13	9637.036837	9587.735108	True	deviance

In [36]:
```
_ = az.plot_compare(compare_df_psis)
```



Posterior Predictive Analysis

1. **Micropolitan Areas**
   - **Observation**: The graph shows a slight decrease in the predicted count of fast food restaurants as median income increases. The red line, representing the posterior mean, trends downward slightly amidst a wide confidence interval shaded in pink.
   - **Interpretation**: This indicates that in micropolitan areas, higher median incomes might be associated with a decrease in the number of fast-food restaurants, although the data points (blue dots) display considerable variability around the prediction.
2. **Metropolitan Areas**
   - **Observation**: There is a clear upward trend in the predicted number of fast food restaurants as median income increases, illustrated by the green line. The confidence interval, shaded in green, widens at higher income levels, suggesting increased uncertainty in the predictions as income rises.
   - **Interpretation**: This suggests that in metropolitan areas, higher median incomes are strongly associated with increases in the number of fast-food restaurants. The observed data points (blue dots) mostly cluster around the lower income levels but support the upward trend.
3. **Insights**:
   - The contrasting trends between micropolitan and metropolitan areas align with the estimand, affirming that area type significantly modifies the impact of median income on fast food restaurant prevalence.
   - The variability in both graphs, especially in the metropolitan data, underscores the influence of other unmodeled factors or inherent data variability that might affect restaurant counts.

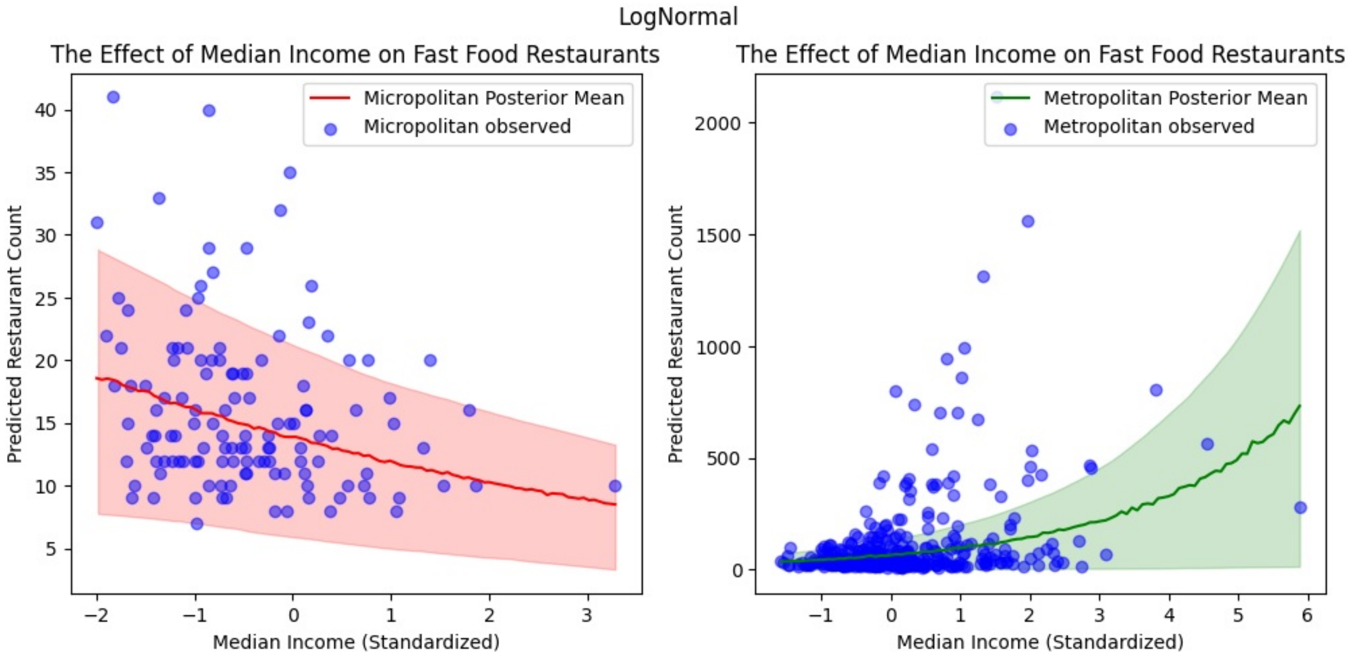
In [37]:
```
def plot_posterior_predictions(model, idata, title):
    fig, axs = plt.subplots(1, 2, figsize=(12, 5))
    args = zip(
        ['Micropolitan', 'Metropolitan'],
        [0, 1],
        axs,
        ['red', 'green']
    )

    for arg in args:

        t, f, ax, g = arg

        ns = 100
        I = df[df.M == f].I
        I_seq = np.array(np.linspace(I.min(), I.max(), ns))
        M_seq = np.array([f] * ns)


        with model:
            pm.set_data({"M": M_seq, "I": I_seq})
            f_pred = az.extract(
                    pm.sample_posterior_predictive(idata, var_names=["f"]),
                    group="posterior_predictive"
                )["f"]

        f_pred_mean = f_pred.mean("sample")

        obs = df[df.M == f]

        az.plot_hdi(I_seq, f_pred.T, color=g, fill_kwargs={"alpha": 0.2}, ax=ax)
        ax.plot(I_seq, f_pred_mean, label=f'{t} Posterior Mean', color=g)

        ax.scatter(obs.I, obs.F, color='blue', alpha=.5, label=f'{t} observed')
        ax.legend()
        ax.set_xlabel("Median Income (Standardized)")
```

```
        ax.set_ylabel("Predicted Restaurant Count")
        ax.set_title("The Effect of Median Income on Fast Food Restaurants")
        fig.suptitle(title)

plot_posterior_predictions(lognormal_model, lognormal_idata, 'LogNormal')
None
```

```
Sampling: [f]
Output()
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/plots/hdiplot
.py:161: FutureWarning: hdi currently interprets 2d data as (draw, shape) but this will change in a future relea
se to (chain, draw) for coherence with other functions
  hdi_data = hdi(y, hdi_prob=hdi_prob, circular=circular, multimodal=False, **hdi_kwargs)
Sampling: [f]
Output()
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/plots/hdiplot
.py:161: FutureWarning: hdi currently interprets 2d data as (draw, shape) but this will change in a future relea
se to (chain, draw) for coherence with other functions
  hdi_data = hdi(y, hdi_prob=hdi_prob, circular=circular, multimodal=False, **hdi_kwargs)
```

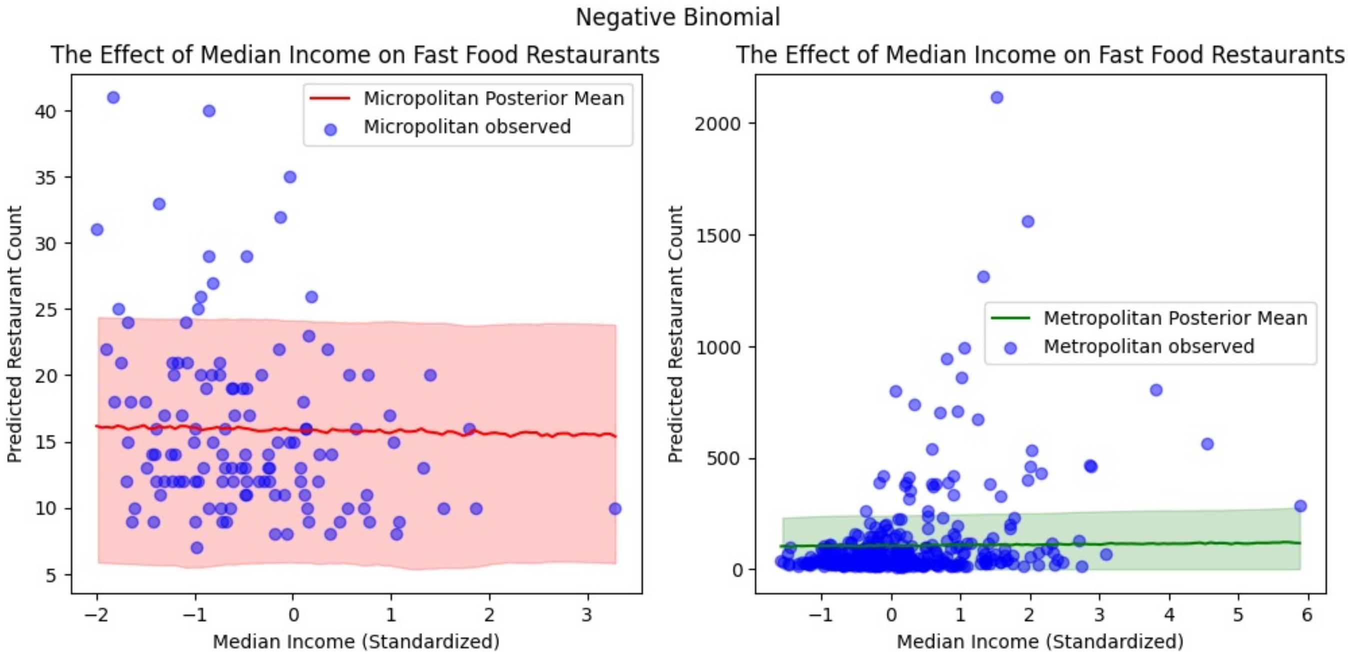


In [38]:
```
plot_posterior_predictions(negbinom_model, negbinom_idata, 'Negative Binomial')
None
```

```
Sampling: [f]
Output()
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/plots/hdiplot
.py:161: FutureWarning: hdi currently interprets 2d data as (draw, shape) but this will change in a future relea
se to (chain, draw) for coherence with other functions
  hdi_data = hdi(y, hdi_prob=hdi_prob, circular=circular, multimodal=False, **hdi_kwargs)
Sampling: [f]
Output()
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/plots/hdiplot
.py:161: FutureWarning: hdi currently interprets 2d data as (draw, shape) but this will change in a future relea
se to (chain, draw) for coherence with other functions
  hdi_data = hdi(y, hdi_prob=hdi_prob, circular=circular, multimodal=False, **hdi_kwargs)
```



In [39]:
```
plot_posterior_predictions(poisson_model, poisson_idata, 'Poisson')
None
```

```
Sampling: [f]
Output()
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/plots/hdiplot
.py:161: FutureWarning: hdi currently interprets 2d data as (draw, shape) but this will change in a future relea
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  hdi_data = hdi(y, hdi_prob=hdi_prob, circular=circular, multimodal=False, **hdi_kwargs)
Sampling: [f]
Output()
```

```
/Users/willemmirkovich/.venv/6053-final-project-lyJDXofz-py3.11/lib/python3.11/site-packages/arviz/plots/hdiplot
.py:161: FutureWarning: hdi currently interprets 2d data as (draw, shape) but this will change in a future relea
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```

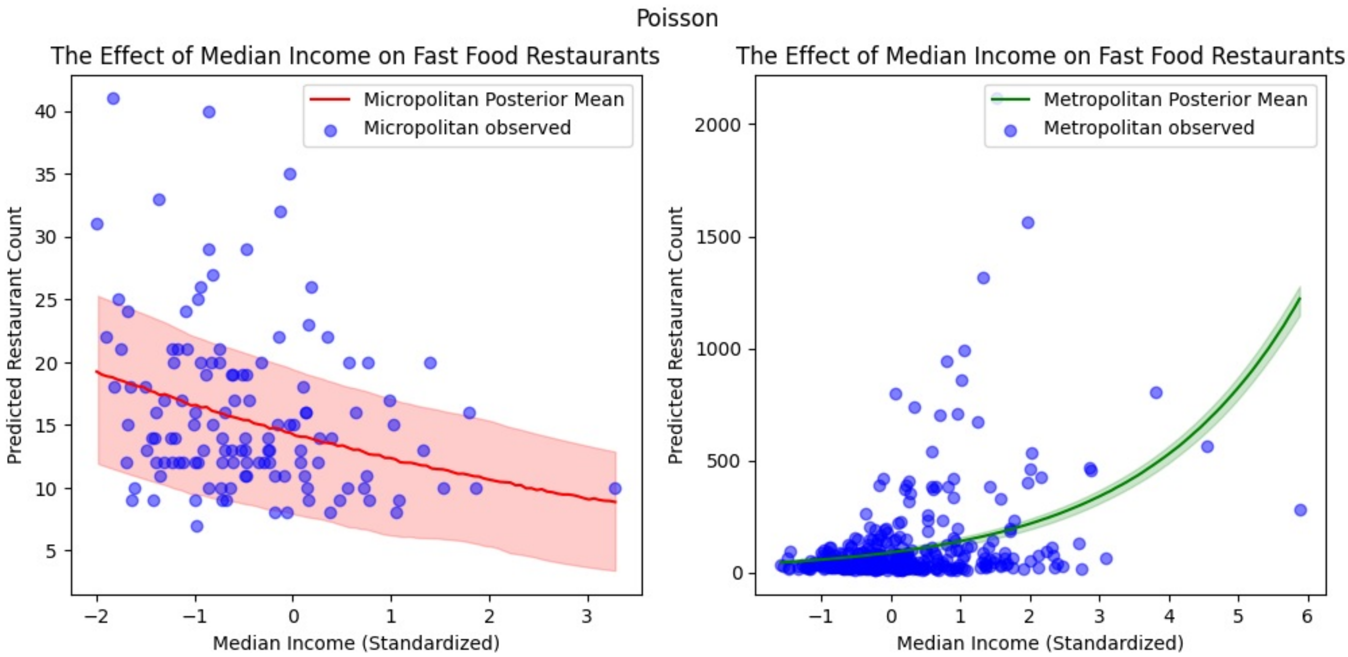

Final Discussion

The estimand can be observed by the posterior predictive simulation plots, especially the ones done using the LogNormal Model. The model effectively reflects the estimand positive relationship in metropolitan areas and a negative trend in micropolitan areas, which seems to suggest that fast food deserts do exist in lower income areas.