Unit 1

- Ecomonics is study of maximizing some function $T(x)$; for us, $T'(x) = 0, T''(x) < 0$ is necessary/sufficient maximum condition.
- Oftentimes $T(x) = B(x) - C(x)$ benifits - costs with $B(x)$ strictly/weakly concave and $C(x)$ strictly/weakly convex ($B''(x) < / \leq 0, C''(x) > / \geq 0$). We are usually interested only in $x \geq 0$ domain.
- Given this, global optimum of $T(x)$ exists and is unique; if $B'(0) < C'(0)$ then $x^* = 0$ the maximum, else $B'(x^*) = C'(x^*)$. Crossing conditions $B'(x \to \infty) \to 0$ or $C'(x \to \infty) \to \infty$ guarantees uniqueness of global *maximum*.
- Constrained optimization subject to $x \in [L, B]$ is at $x_0 : T'(x_0) = 0$ if $x_0 \in [L, B]$ else it is a corner solution at $x* = L, B$.
- Sometimes a two variable problem can be eliminated to a one-variable problem when constraint exists, i.e. "Optimize $U(x) + V(y)$ subject to $px + qy = C$."

Unit 2

- "Experienced utility" function $U(x, m)$ measures utility experienced when purchasing x of desired good and m of other stuff. We will assume quasi-linear $U(x, m) = B(x) + m$.
- If then consumer has wealth W and price of x is p, then utility maximization becomes maximizing $U(x, W - xp) = B(x) + W - xp = B(x) - xp$ (since constant W doesn't factor into extremum problem) which is just benefit minus cost. So p introduces a linear $C(x)$ in the earlier $T(x) = B(x) - C(x)$, then if $B'(0) > C'(0) = p$ with concavity we know interior solution exists (as $\exists x > 0, B'(x) = C'(x)$).
- We notate the $x * (p)$ the maximum of $U(x, m)$ at some price p; this is the demand function. Plot price on vertical axis, quantity on horizontal.
- * — If we then imagine $U(x, p)$ a 3-D plot "coming out of" a P, Q or x, p plot, we can imagine than taking planes of constant p produces an upside down parabola (like $T(x)$ earlier) with maximum at $x^* > 0$ if $B'(0)_{p=p_0} > 0$. The plot of $x^*(p)$ is the line such that $U'(x^*) = B'(x^*) - p = 0$, and it is a line in the x, p plane. This equation also implies that $B'(x^*) = p^*$.
- Note no income effects $\dfrac{dx^*}{dW} = 0$. Law of demand $\dfrac{dx^*}{dp} \leq 0$ with equality only when $x^* = 0$; see by differentiating $\dfrac{d}{dp}(B'(x^*) - p) \Rightarrow B''\dfrac{dx^*}{dp} = 1$, then since concavity says $B'' < 0$ QED.
- Consumer surplus is defined as net benefit of buying optimally at price p, or $CS(p) = B(x^*(p)) - B(0) - px^*(p)$. Increase in experienced utility when buying at some price p, so $U(x^*, p) - U(0, p)$.
- Can compute CS function under assumption $p^*(x) = B'(x)$ to be $CS(p) = \int\limits_0^{x^*(p)} p^*(x) - p\ dx$. We can use fitting to determine $x^*(p)$ and its inverse, so this is just determining an $x^*(p)$ empirically and using a variable p to find CS as a function of x^* which is a function of p.
- Consumer mistakes happen where $B'(x)$ the "decision utility" is different (usually too large, i.e. addiction) from $B(x)$ the experienced utility.

Unit 3

- Production function $F(k, l)$, k capital and l labor. Define *marginal product of capital* $MPK = \dfrac{\partial F}{\partial k}$ and *marginal product of labor* $MPL = \dfrac{\partial F}{\partial l}$. Assume $\dfrac{d}{dk}MPK, \dfrac{d}{dl}MPL < 0$ as $k, l \to \infty$ respectively, i.e. optimum exists.
- Taxonomy production functions satisfied at all (k, l)
  - CRS: constant returns to scale $F(\lambda k, \lambda l) = \lambda F(k, l)$
  - DRS: decreasing returns to scale $F(\lambda k, \lambda l) < \lambda F(k, l)$
  - IRS: increasing returns to scale $F(\lambda k, \lambda l) > \lambda F(k, l)$

  Not all functions are classifiable
- Call r, w cost per unit capital/labol. Then firm maximizes $qp - (rk + wl), q = F(k, l)$. Note then that q^* is supply, k^* is demand for capital, l^* is demand for labor. Two ways to solve
  - Choice over inputs: maximize $pF(k, l) - (rk + wl)$ WRT k, l and $q^* = F(k, l)$.
  - Choice over output: minimize $rk + wl$ WRT k, l subject to $F(k, l) = q$. Then define $C(q|w, l) = rk^{min}(q|w, r) + wl^{min}(q|w, r)$ cost of q units WRT w, l. We then maximize $pq - C(q|w, r)$ to obtain q^* and plug into k^{min}, l^{min}.
- Key: Profit maximization requires cost minimization! i.e. Profit maximized at $q^*, k^*, l^* \Rightarrow k^*, l^*$ minimize cost of production.
- Define *isoquant* (quantity is invariant): $\dfrac{\partial F}{\partial k}dk + \dfrac{\partial F}{\partial l}dl = 0$, *isocost* (cost is invariant): $l = \dfrac{c}{w} - \dfrac{r}{w}k$. Cost is minimized when isocost and isoquant tangent, i.e. $\frac{\frac{dF}{dk}}{\frac{dF}{dl}} = \dfrac{r}{w}$. Intuitively, $\frac{\frac{dF}{dk}}{\frac{dF}{dl}}$ $(\dfrac{MPK}{MPL})$ is just the marginal rate of technical substitution while $\dfrac{r}{w}$ is just the relative prices of capital/labor.

- Firm then tries to maximize $pq - c(q)$, $q^*(p)$ is the supply.
- Producer surplus $PS(\theta) = \Pi(\theta) - \Pi_{no-trade}$ with $\Pi_{no-trade} = FC$, FC being the fixed cost of production, i.e. difference in profits!.

Unit 4

- Aggregate demand is always the sum of the consumers' individual demands $X^D(p) = \sum_i x_i^D(p)$. Key: sum consumers' quantities not prices!! Then $P^D(x)$ is the marginal benefit of any consumer buying any more.
- Aggregate supply is $\sum_j x_j^F(p)$ so total supply at price p.
- Prices such that $X^D(p^*) = X^S(p^*)$; called competitive market equilibrium cME, where AS/AD intersect.
- Comparative statics: If some parameter a changes then solve for $AD(a,p), AS(a,p)$ and find $p^*(a)$. Explicitly

$$\frac{\partial p^*}{\partial a} = \frac{\frac{\partial X^S}{\partial a} - \frac{\partial X^D}{\partial a}}{\frac{\partial X^D}{\partial p} - \frac{\partial X^S}{\partial p}}$$

- Social surplus is consumer + producer surplus. It is easy to show that for some feasible allocation α that $SS(\alpha) = \sum_i U_i(\alpha) + C$ with constant C, the U_i being the consumer utility functions.

Unit 5

- An allocation α is called *Pareto optimal* if no other allocation Pareto improves over α; Pareto improving is "improve at least one dude while keeping everybody else at least no worse."
- First Welfare Theorem: any competitive market equilibrium allocation is Pareto Optimal.
- Deadweight loss is defined as $SS^{opt} - SS(\alpha)$ at some allocation α.
- Two types of taxes, lump-sum (fixed amount regardless of actions) and non-lump sum. Former does not introduce inefficiencies,

Unit 6

- Consumers in the endogenous market have utility function $U = q - \frac{l^2}{2\theta} + m$ with q goods purchased and l units of labor provided; θ is a measure of the disutility of providing labor (cost of effort).
- Since generally $m = lw - pq$ up to some constant, we see that for good q we have $MB = 1, MC = p$, and so the demand curve is a step function at $p = 1$ from 0 to ∞, as is the AD.
- For labol l then $MB = w$ and $MC = \frac{1}{\theta}$ so labol supply is $l = \theta w$ and AS is $w \sum_i \theta_i$ for each laborer's θ_i.
- On the other side firms have production function $F(l) = \gamma l$ and want to maximize $p\gamma l - wl$ resulting in a step function for laborer demand at $w = p\gamma$.
- Then in this super-simplified model our CME equilibrium is given

$$p^* = 1 \tag{1}$$
$$w^* = \gamma \tag{2}$$
$$q^* \propto \bar{\theta}\gamma^2 \tag{3}$$
$$l^* \propto \bar{\theta}\gamma \tag{4}$$

for $\bar{\theta}$ average θ. Note one CME per q.

- At equilibrium each person has income $I = \theta w^2 = \theta\gamma^2$ and utility $U = \frac{\theta\gamma^2}{2}$.
- With a tax τ per earned dollar on the consumer maximization becomes $\max_{q,l\geq 0} q - \frac{l^2}{2\theta} + l(1-\tau)w - pq + T$ with T the total tax return. Consumer assumes fixed T.
- Since then no change in q, l markets, $p^* = 1, w^* = \gamma$. However, MB of labor supply falls to $MB = (1-\tau)w$ so labor AS goes to $\propto (1-\tau)w\bar{\theta}$.
- Total revenue is $\propto \tau(1-\tau)\gamma C\bar{\theta}$. Laffer curve!
- Net redistribution of tax per consumer is $\tau(1-\tau)\gamma^2\left(\bar{\theta} - \theta_i\right)$, so consumers who hate work receive net transfer and those with low disutility of labor pay net tax.
- Second Welfare theorem: Let α be a Pareto optimal allocation, then there exists a set of zero-sum lump-sum transfers T_i such that α remains a CME. This is a T_i for each consumer though, which is unrealistic; too much info required!
- Price controls include ceiling/floor or simple (fixed price), complex. First three generate inefficiency when change equilibrium; produces excess demand so rationing required.
- Complex price floor: gov buys excess supply at equilibrium p^* and destroys, pulling money from equal lump-sum tax. Super inefficient!

Unit 7

- Monopolies differ in that they are price setters, so take consumer demand, plug into profits function and maximize wrt p i.e. $\max p^D(q)q - c(q)$.
- Monopolies are inefficient; generate DWL because $q^m < q^*$ the monopoly quantity and the optimal.

- Monopolies under price discrimination are slightly different; they can extract *all* consumer benefit.

- Consider two consumer types $p_A^D(q), p_B^D(q)$, then monopolist wants to max $n_A B(q_A) + n_B B(q_B) - c(n_A q_A + n_B q_B)$ (noting equivalence of consumer benefit and monopolist's revenue). Note this allocation is Pareto Optimal: PS = SS, CS = 0.

- Simple model of imperfect price discrimination is to sell $\bar{q}$ units at p_1 and additional are sold at p_2. Turns out with homogeneous consumers monopoly can still extract full SS.

- Multi-market discrimination is just maximizing with respect to each market; marginal revenue in each market is equal to marginal cost.

Unit 8

- Oligopoly lies just between between monopoly/competition. Firms maximize profits assuming fixed other firms' behavior, i.e. maximize $q_i p^D(q_i + q_j) - c_i(q_i)$; maximization from here is just like monopoly. Oligopoly equilibrium occurs when all firms predict correctly, less DWL than monopoly. Identical firms is special case, easy to solve exactly by assuming all q_i are equal.

- Monopolistic competition: Firms pay SFC to create a brand and produce at constant MC μ; brands split market equally and are monopolists within own brand. Consumer becomes loyal buyer of only one brand but demand doesn't shift upon commitment.

- Let I be number of brands, then each firm faces demand $\mu = p^{max} - Imq$ and yields $p^{MC} = \dfrac{p^{max} + \mu}{2}$. This yields profits $\Pi^{MC} = \dfrac{(p^{max})^2}{4mI} - F$ and equilibrium of firms is largest i such that $\Pi > 0$.

- Takeaway is that brands screw with perceived utility, equilibrium produced q remains at monopolistic level.

Unit 9

- Define *externality* when actions of one economic actor affect another actor; direct/market typing.

- *Public bads* are externalities that impact decision utility vs. experienced utility that impact DWL; maximize given $e(q)$ externality as a function of quantity.

- Ways to eliminate public bads DWL:

  - Pigouvian taxation: tax imposed on every action generating externality equal to marginal benefit of externality, returned to consumers in lump-sum transfer. Leads to optimal allocation but requires much information

  - Permit markets: Gov. creates q^{opt} of permits, cannot exceed permits, permit trading freely; also optimal.

- *Public goods* positive externalities to other actors. Corrective policies then include Pogouvian subsidies (opposite of taxation), gov. provision (buy some public goods).

- Pigouvian taxes are equal to marginal externality. We compute optimal tax by setting taxed consumption equal to optimal production under externality; when doing this set $e = q^*$.