ACM 100b
Fourier series - part 1

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February 11, 2014

Complex form of the Fourier series

- The full Fourier series

$$f(x) = \sum_{n=0}^{\infty} B_n \cos(n\pi x/L) + \sum_{n=1}^{\infty} A_n \sin(n\pi x/L)$$

can be written in a complex form that is useful for various manipulations.

- Recall that

$$\sin(n\pi x/L) = \frac{\exp(in\pi x/L) - \exp(-in\pi x/L)}{2i}$$
$$\cos(n\pi x/L) = \frac{\exp(in\pi x/L) + \exp(-in\pi x/L)}{2}$$

- Substitute these into the series

Complex form of the Fourier series

- We see that we can write the full Fourier series as

$$f(x) = \sum_{n=-\infty}^{\infty} C_n \exp(in\pi x/L)$$

where

$$C_n = \begin{cases} \dfrac{B_n - iA_n}{2} & n > 0 \\[2ex] \dfrac{B_n + iA_n}{2} & n < 0 \\[2ex] B_0 & n = 0 \end{cases}$$

- Note this has a much more uniform look in that the sum extends from $n = -\infty$ to $n = \infty$