Physics 106a 1 October, 2013

Lecture 1: Review of Newtonian Mechanics

This lecture reviews some of the basic ideas of Newtonian mechanics, just to get things going. I will go over the concepts rapidly, and leave it up to you to fill in the details if there are parts you are not familiar with or do not remember. Hand and Finch do not have a review chapter, so you can consult your favorite elementary textbook, or *Taylor* Chapters 1 and 3 or *Goldstein, Poole, and Safko* §1.1-2 (different levels of terseness!).

Newton's Laws

1. In the absence of forces, a particle moves in a straight line with constant speed v (or, equivalently, moves with a constant *velocity* $\vec{v}$ where $\vec{v} = v\hat{v}$, with $\hat{v}$ a unit vector giving the direction of the straight line motion).

2. For a particle of mass m, the *acceleration* $\vec{a}$ for an imposed force $\vec{F}$ is given by

$$\vec{F} = m\vec{a} \tag{1}$$

An equivalent formulation is in terms of the *momentum* $\vec{p} = m\vec{v}$

$$\vec{F} = \frac{d\vec{p}}{dt} \equiv \dot{\vec{p}} \tag{2}$$

3. If object 1 exerts a force $\vec{F}_{12}$ on object 2, then object 2 exerts a reaction force $\vec{F}_{21}$ on object 1 given by

$$\vec{F}_{21} = -\vec{F}_{12} \tag{3}$$

or "action and reaction are equal and opposite."

Some points to discuss:

- Think about the genius of Galileo and Newton in coming up with the first law in a familiar terrestrial world where all undriven motion eventually ceases, or the astronomical world studied by scientists where objects move in nearly perfect circles. Historically, accepting the first law implied accepting the possibility of action at a distance (gravity) not just pushes and pulls from direct contact.

- Are the concepts of mass and force defined (qualitatively, quantitatively) outside of Eq. 1? Is the equation a law of a physics or a definition of force?

- Do you know of a simple example where the third law is violated? How do you reconcile this with the accepted "truth" of Newton's laws?

The book *Isaac Newton* by *James Gleick* is interesting to read for the struggles Newton had in arriving at the right ideas and words to state the laws of motion.

Other Concepts

The formulation of the laws of motion rely on a number of underlying concepts:

- The notions of the framework of space and time. The Newtonian concepts, particularly the idea of absolute time, need to be modified in special relativity.
- An observer system or *reference frame* (e.g. a set of rulers and clocks) for quantifying separations and time intervals.
- Inertial frames: the special set of reference frames ("nonaccelerating frames") for which the laws of motion hold. Different inertial frames may be in relative motion, but only with a constant relative velocity $\vec{V}$.
- Vectors, such as $\vec{a}$, $\vec{v}$, $\vec{p}$, which take advantage of the physical principle of the rotational symmetry of space to write the equations in a form independent of coordinate basis.
- A system of coordinates (Cartesian, polar ...) to evaluate the consequence of the laws of motion.

An important invariance or symmetry known as *Galilean invariance* is that Newton's laws of motion are unchanged by transforming to a different inertial frame. Under such a transformation

$$t \to t' \quad \text{with} \quad t' = t \tag{4}$$

$$\vec{r} \to \vec{r}' \quad \text{with} \quad \vec{r}' = \vec{r} - \vec{V}t \tag{5}$$

$$\vec{v} \to \vec{v}' \quad \text{with} \quad \vec{v}' = \vec{v} - \vec{V} \tag{6}$$

$$\vec{a} \to \vec{a}' \quad \text{with} \quad \vec{a}' = \vec{a} \tag{7}$$

and it is assumed that the force is the same as measured in the two frames. Note, of course, that Newton's laws are *not* true in non-inertial (accelerating frames) — such as on the surface of the Earth! We will discuss some consequences of this (e.g. hurricanes) in later lectures.

Newton's laws of motion were profoundly important historically in establishing science as a quantitative pursuit, and the concepts introduced still shape how we think about and quantify the everyday world around us. However as science pushed into new regimes, it became clear that concepts introduced by Newton such as force are not the best way to think about small scales (quantum mechanics), very large scales (general relativity, curved space time), or motion at high speeds (special relativity). The alternative formulation of Newtonian physics known as *Lagrangian mechanics* that we will study in detail provides a more direct route to the extensions needed in these regimes.

Further Developments

Conservative forces and energy

Define the *work* done by the external force $\vec{F}$ acting on a particle in going from point 1 to point 2 by

$$W_{12} = \int_1^2 \vec{F} \cdot \vec{ds}. \tag{8}$$

Use Newton's 2nd law to write $\vec{F} = m d\vec{v}/dt$, and replace the line integral by an integral over time using $\vec{ds} = \vec{v} dt$ to find

$$W_{12} = m \int_{t_1}^{t_2} \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt = \frac{m}{2} \int_{t_1}^{t_2} \frac{d}{dt}(v^2)\, dt = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2. \tag{9}$$

We call the quantity $\frac{1}{2} m v^2$ the *kinetic energy*.

If W_{12} is *independent* of the path taken from 1 to 2 we say the force is *conservative*. Equivalently

$$\oint \vec{F} \cdot \vec{ds} = 0 \quad \text{or} \quad F = -\vec{\nabla} V(\vec{r}), \tag{10}$$

with $V(\vec{r})$ the *potential energy* (review an elementary mechanics text or the introduction of potential in electrostatics if you are not happy with these steps). For a conservative force

$$\tfrac{1}{2} m v_1^2 + V(\vec{r}_1) = \tfrac{1}{2} m v_2^2 + V(\vec{r}_2) \tag{11}$$

and the *total energy* (kinetic plus potential) is conserved in the dynamics. Not all familiar forces are conservative: electrostatic forces are; magnetic forces and friction are not.

Many particle dynamics, conservation of momentum and angular momentum

For a set of many particles i (not necessarily of the same mass) Newton's second law reads

$$\dot{\vec{p}}_i = \sum_j \vec{F}_{ji} + \vec{F}_i^{(e)} \tag{12}$$

with $\vec{F}_{ji}$ the interparticle forces and $\vec{F}_i^{(e)}$ the forces from external sources. Introducing the *total momentum* $\vec{P} = \sum_i \vec{p}_i$ and summing over all particles i gives

$$\dot{\vec{P}} = \sum_{ij} \vec{F}_{ji} + \sum_i \vec{F}_i^{(e)}, \tag{13}$$

If Newton's third law applies, the first (double) sum vanishes, and we get Newton's law for the total momentum in terms of the total external force $\vec{F}^{(e)} = \sum_i \vec{F}_i^{(e)}$

$$\dot{\vec{P}} = \vec{F}^{(e)}. \tag{14}$$

For zero external force

$$\dot{\vec{P}} = 0, \tag{15}$$

and the total momentum is conserved. We can also write the total momentum as

$$\vec{P} = M \frac{d\vec{R}}{dt} \tag{16}$$

with $M = \sum_i m_i$ the total mass, and $\vec{R} = \sum_i m_i \vec{r}_i / M$ the *center of mass* coordinate. Equation (14) for the case when Newton's third law is true, can be written

$$M \frac{d^2 \vec{R}}{dt^2} = \vec{F}^{(e)}. \tag{17}$$

These equations show that if Newton's third law holds, the internal forces cancel, and we can write the same laws of motion for the center of mass coordinate of a composite body.

For a set of particles with vector positions $\vec{r}_i$ measured from origin O, the *angular momentum* about O is defined as

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i. \tag{18}$$

Taking the time derivative

$$\dot{\vec{L}} = \sum_i \dot{\vec{r}}_i \times \vec{p}_i + \sum_i \vec{r}_i \times \dot{\vec{p}}_i. \tag{19}$$

The first term is zero, since $\vec{p}_i$ is parallel to $\dot{\vec{r}}_i$. Now using Newton's second law

$$\dot{\vec{L}} = \sum_i \vec{r}_i \times \vec{F}_i^{(e)} + \sum_{ij} \vec{r}_i \times \vec{F}_{ji}. \tag{20}$$

The terms in the last sum can be paired up, such as $\vec{r}_1 \times \vec{F}_{21} + \vec{r}_2 \times \vec{F}_{12} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{21}$, where Newton's third law is used in the last equality. For the special case of *central forces*, the force $\vec{F}_{21}$ is along the vector separation of the particles $\vec{r}_1 - \vec{r}_2$ and so the last term in Eq. (20) vanishes. This gives the equation of motion of the total angular momentum

$$\dot{\vec{L}} = \sum_i \vec{r}_i \times \vec{F}_i^{(e)} \equiv \vec{N}^{(e)}, \tag{21}$$

with $\vec{N}^{(e)}$ the total external *torque*. If the external torque is zero, angular momentum is conserved. Note that these results for the angular momentum were derived with the rather restrictive assumption of central interparticle forces. As we will see, they apply for rigid bodies more generally.

The state of a *rigid body* is defined by its center of mass coordinate and its orientation — six coordinates. The dynamics of the rigid body can be obtained from the total momentum equation Eq. (14) and the total angular momentum equation Eq. (21) — six equations. Rigid body dynamics can be quite complicated and interesting, and we will return to this in later lectures.

Example

I looked at a simple example of applying Newton's laws of motion, a block sliding on an inclined plane. This is discussed in Hand and Finch §1.1, and will be used as a recurring example as we proceed.

Important lessons from the example on the practical application of the Newtonian approach are:

1. the "natural" coordinates or *generalized coordinates* $\{q_k\}$ may not be Cartesian coordinates with respect to some inertial frame;
2. we needed to introduce "forces of constraint" that are usually not of interest;
3. complicated algebra may be needed to eliminate the "extra" coordinates and forces introduced.

Many of these difficulties are eliminated using the *Lagrangian* approach introduced in the next lecture.

Michael Cross, November 30, 2013