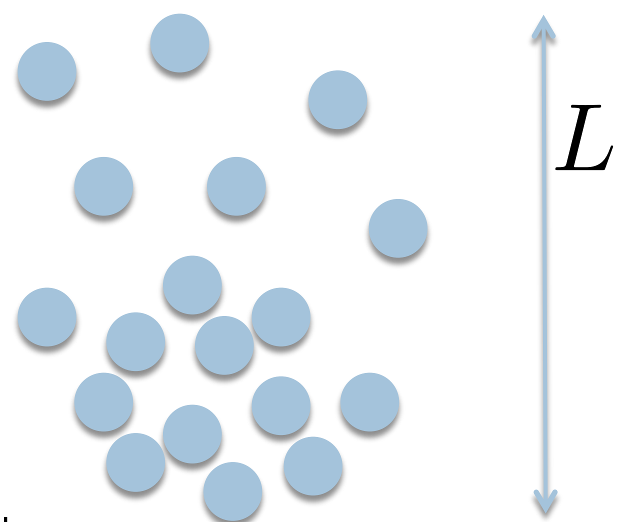
Lecture 2

- Fluid properties (cont'd)
  - Last time: viscosity and the Reynolds number
  - Today:
    - The continuum
    - The Knudsen number
    - Compressibility and the Mach number
    - Fluid/solid and fluid/fluid interfaces

Flow with whatever may happen
and let your mind be free.
Zhuangzi (4th century, BCE)

The continuum (1)

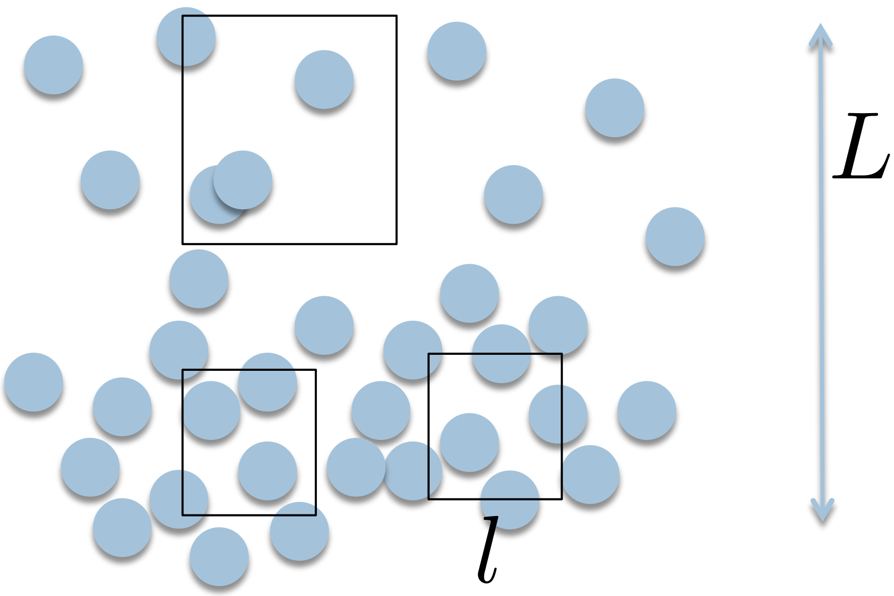
- Suitably averaged, we wish to consider fluid materials to be
  - Continuous (infinitely divisible) such that all properties may be defined *pointwise*
  - Contiguous (filling up all of space)
- How can this be?



Particles on bottom are compressed by weight of everything above

The continuum (2)

- How can this be?

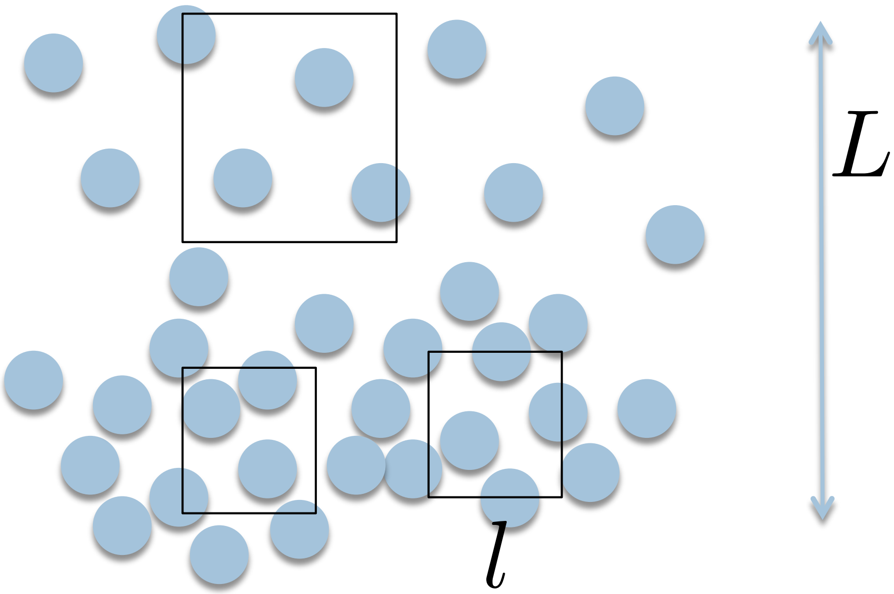


Macroscopic variation in density

Averaging box envisioned to hold many particles

The continuum (3)

- The averaging box must be **large enough** to hold many molecules, but much **smaller than** the *imposed* macroscopic variation

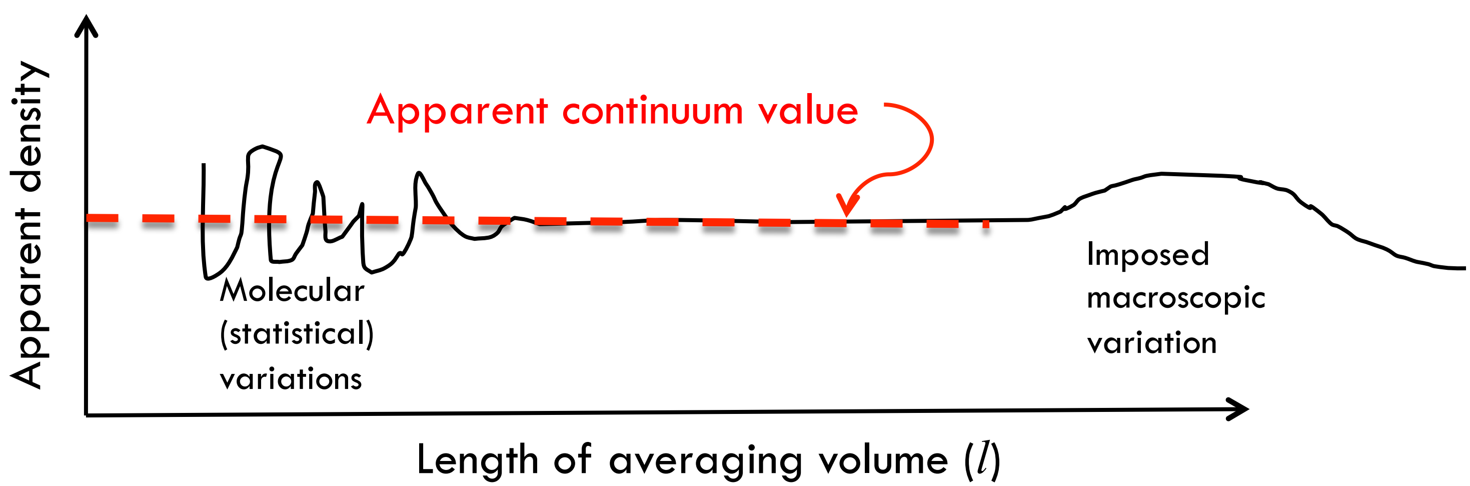


$$? << l << L$$

The continuum (4)

- Number of molecules in averaging volume

@STP	Air	Water
1 m^3	2.5×10^{25}	3×10^{28}
1 mm^3	2.5×10^{16}	3×10^{19}
1 μm^3	2.5×10^{7}	3×10^{10}

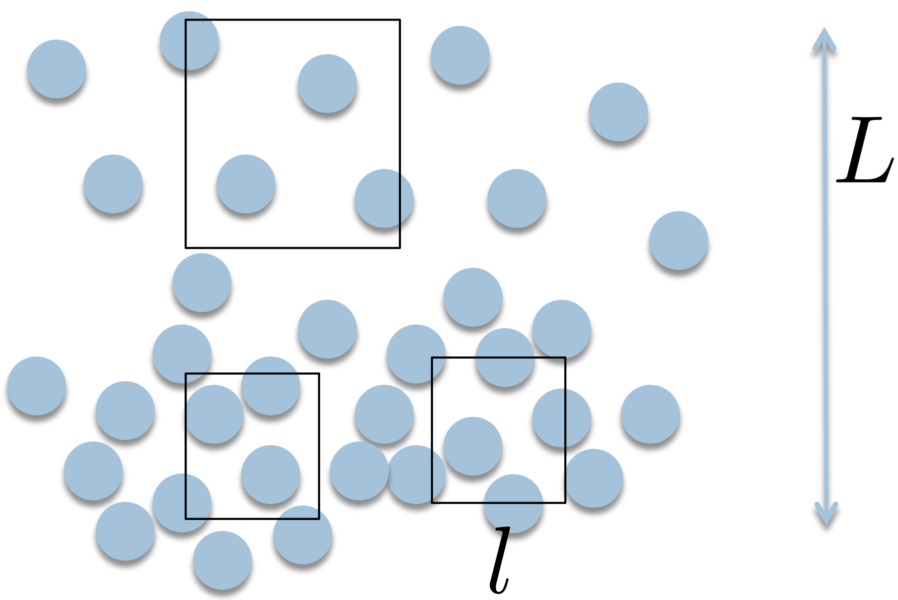


The continuum (5)

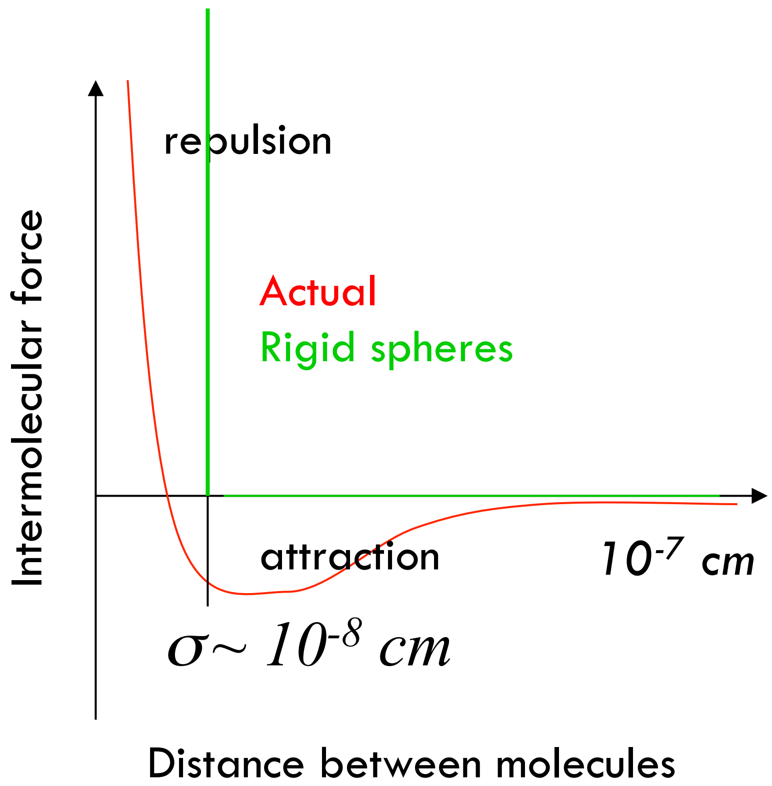
	Microscopic	Macroscopic
Velocity	Velocity of a molecule	Average velocity of many molecules (non-random part of motion)
Mass/density	Mass of a molecule	Average mass of molecules per unit volume
Temperature	-	Root-mean-square velocity of molecules (random or 'thermal' part of motion)
Pressure	-	Average normal force due to molecules colliding with a (real or imagined) surface
Viscous stress	-	Average force on a (real or imagined surface) due to gradients in average velocity

The continuum (6)

- When does the continuum approximation apply?
- Non-dimensional quantity: what molecular property should be compared to L?



Intermolecular forces



Gases $d_{avg} \sim 10\ \sigma$

Liquids & solids $d_{avg} \sim \sigma$

[Must be why density of gas about ~ 1/1000 times density of liquid/solid !]

Collisions

- Any difference in average velocity of molecules is equilibrated through molecular collisions
- Mean-free-path: average distance a molecule travels before colliding with another molecule
- Liquids: mean-free-path ≈ d_o
- Gases: mean-free-path from kinetic theory:

$$\lambda = \frac{m}{\pi \rho \sigma^2 \sqrt{2}}$$

Mass of a molecule

Effective diameter of molecule

- Mean free path in air at STP: $\lambda \approx 10^{-7}$ m
- Mean free path of air at 120 km: ~ 4m

Knudsen number

- Non-dimensional parameter representing microscopie (mean-free-path) macroscopically imposed length scale

$$\mathrm{Kn} = \lambda/L$$

$\mathrm{Kn} \ll 1$	Continuum flow	[Kn < 0.1 a good rule of thumb]
$\mathrm{Kn} \approx 1$	Transition/slip flow	
$\mathrm{Kn} \gg 1$	Free molecule flow	

- In this class, we will always assume that Kn << 1

Compressibility and the Mach number

- Another related and very important property of fluids is their compressibility

Percent change in volume for a given change in pressure (generally positive)

$$\beta_T = -\frac{1}{v}\left.\frac{\partial v}{\partial p}\right|_T$$

Isothermal compressibility

$$\beta_s = -\frac{1}{v}\left.\frac{\partial v}{\partial p}\right|_s$$

Isentropic compressibility

- Exercise: find an expression for β_s in a perfect gas.
- Ans: $\beta_s = -\frac{1}{v}\left.\frac{\partial v}{\partial p}\right|_s = \frac{1}{\gamma p}$

Compression and expansion waves

- Play a key role in establishing fluid motion, equilibrating pressure differences
- We hear them as acoustic waves
- Propagate with a finite speed equal to *a* (called the speed of sound), where a is the positive root of

$$a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s = \frac{1}{\rho \beta_s}$$

- For a perfect gas

$$a^2 = \gamma R T = \frac{\gamma P}{\rho}$$

How compressible?

- Need a nondimensional parameter to answer
- Mach number

$$M = \frac{U}{a}$$

  - M << 1 : Material may be idealized as 'incompressible' (M < 0.3 as a rule of thumb)
  - M < 1: subsonic
  - M ~ 1: transonic
  - M > 1: supersonic
  - M >> 1: hypersonic
- We will study this more in Me19b

Another interpretation of the Mach number

$$M^2 = \frac{U^2}{a^2} = \frac{U^2}{\gamma RT}$$

Proportional to KE of orderly (average) motion

Proportional to KE of random (thermal) motion

So far…

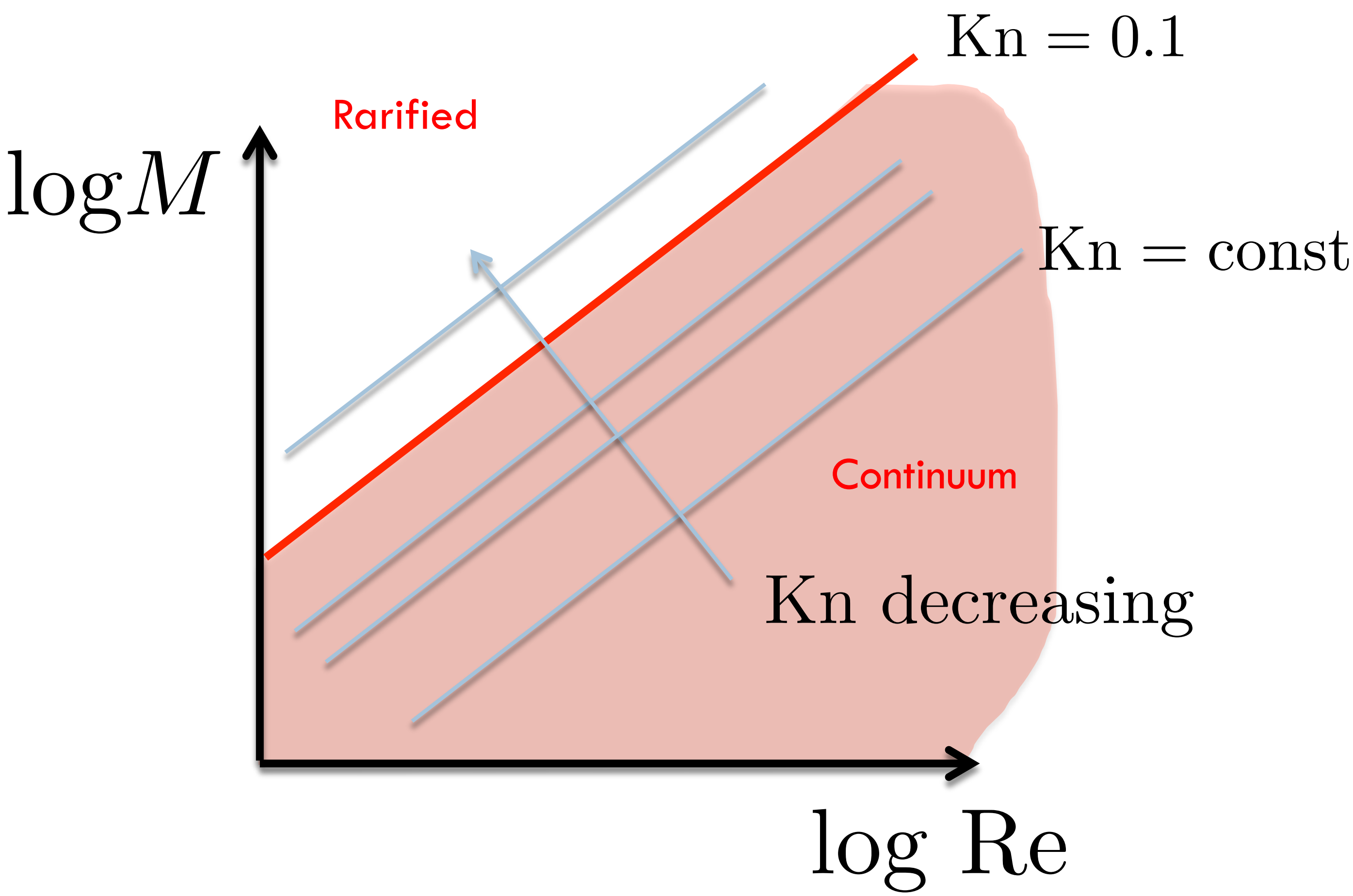
- Three important non-dimensional parameters
  - Knudsen number : how dense (continuum?)
  - Reynolds number : how viscous?
  - Mach number : how compressible?

- However: they are not all independent
- Kinetic theory:

$$\mu = \frac{1}{2} u_{rms} \lambda \sim a\lambda \qquad \mathrm{Kn} = \frac{M}{\mathrm{Re}}\sqrt{\frac{\gamma\pi}{2}}$$

(perfect gas)

Graphical representation

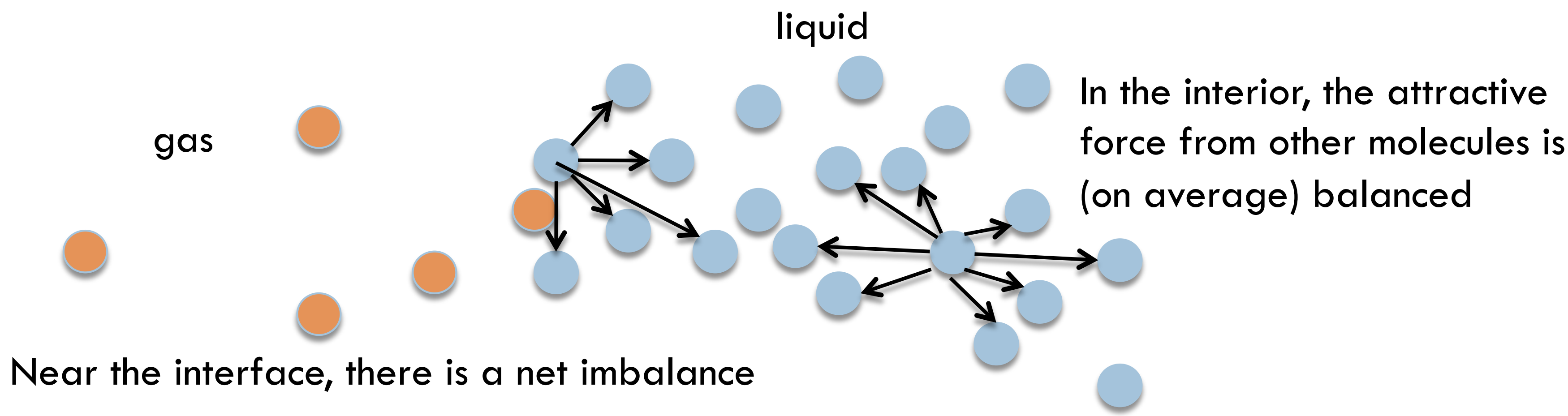


Immersed solids

- Along the solid surface in contact with the fluid, there is transfer of momentum between collisions of the molecules of the solid and fluid
- Provided the fluid is "dense enough" the average velocity (each component) of the solid and liquid are equal at all points along the interface
- For the normal component of velocity, this is referred to as the no-penetration (no flow through) condition
- For the tangent components, it is referred to as the no-slip condition
- If we idealize a fluid as frictionless (inviscid), we keep the no-penetration, but neglect the no-slip condition

Fluid-fluid interfaces and surface tension

- Fluids that are immiscible (do not mix, and idealization) retain a sharp interface between the constituent components
  - Liquids: intermolecular forces, attractive
  - Gases: much weaker attraction
- Result is a force that on average, tends to contract the interface to minimize the surface area



Surface tension (2)

- In the absence of gravity, a drop of liquid in a gas (or vacuum) will therefore be spherical
- If the drop is small enough, then the difference in elevation results a much smaller gravitational force than the surface tension effect (again, spherical droplet)
- Molecular observations are consistent with imposing, on average, a force tangent to the surface (like an imaginary membrane stretched over the surface)
- Surface tension σ
  - Force, per unit distance
  - SI : N/m = kg/s^2

Surface tension (3)

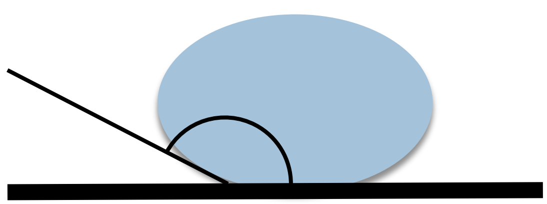
- Like viscosity, surface tension is independent of the magnitude, direction, and duration of the motion, and is principally a function of temperature

$$\sigma = \sigma(T)$$

- Unlike viscosity, σ is a property of both fluids (the liquid and the gas, or of the two liquids)
- Water/air @ STP, $\sigma = 0.073$ N/m.
- Value of surface tension is also sensitive to the accumulation of contaminant molecules (*surfactants*)
  - Detergents lower the value of surface tension, making it easier to spread the water out (to wet something)

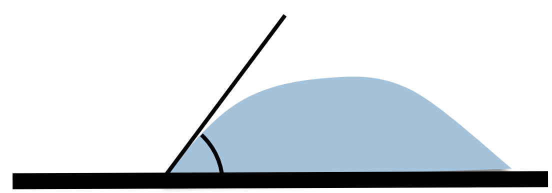
Surface tension (4)

- Similar effect at triple-junction between solid, liquid and gas (or another immiscible liquid)
- The free surface will make a *contact angle* with the surface, which can be regarded as a joint property of the 3 materials
- Like σ, contact angle is sensitive to temperature and/or surfactants, as well as surface roughness



$$\theta > 90^o$$

Non-wetting, or hydrophobic



$$\theta > 90^o$$

Wetting, or hydrophilic

Summary

- Continuum
  - For sufficiently dense fluids, averages over molecules can be reinterpreted as continuous, pointwise quantities
  - Have to be careful when density is low, or when the imposed length scale is small
    - Nondimensional parameter: Knudsen ($Kn = \lambda / L$)
    - Continuum: $Kn < 0.1$
- Compressibility
  - Determines speed at which compression and expansion waves move
  - Mach number, $M = U/a$: "how compressible"
- No slip and no penetration conditions at interface between fluid and solid
- Surface tension leads to a surface force between dissimilar, immiscible materials (and to a contact angle between those materials and a nearby solid)
  - due to unbalanced intermolecular forces near interface