Physics 106b — Classical Mechanics

Michael Cross

California Institute of Technology

Winter Term, 2014

Outline

- Special Relativity
  (4 lectures, Hand and Finch Chapter 12)

- Parametric Resonance and Nonlinear Oscillators
  (2 lectures, Hand and Finch Chapter 10)

- Dynamical Systems and Chaos
  (4 lectures, Hand and Finch Chapter 11)

Course website: http://www.pma.caltech.edu/∼mcc/Ph106b/

Lecture 1

Relativity: Introduction

Principle of Relativity

The Principle of Relativity states:

The laws of physics are the same in all inertial frames.

A second principle is often added

Yes, really!

or more commonly

The speed of light is the same in all inertial frames.

Events

An *event* is a precise location in space and time

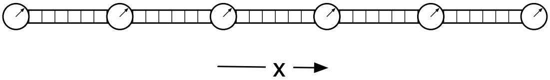
Often convenient to think of a localized physical event as defining the space-time point:

- An atom emits a flash of light (photon)
- I clap my hands
- …

To proceed

- Relate coordinates in different frames of reference for an event
- Geometric approach (next lecture)

Coordinates



Lattice of rulers and synchronized clocks

Observation of an event means noting down the ruler and clock readings coincident with the event

Units

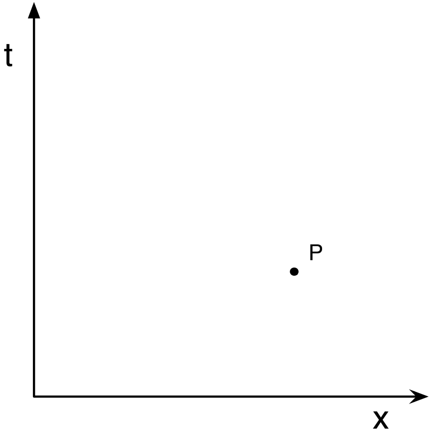
- Conventional units
  - the second is defined as the time for 9192631770 oscillations of radiation corresponding to the transition between the two hyperfine levels of Cs^{133}
  - the meter is defined as $1/(2.99792458 \times 10^8)$ of the distance traveled by electromagnetic radiation in one second
  - the speed of light is $c = 2.99792458 \times 10^8$ meters/second by definition of our units

- Relativistic units
  - the unit of time is, for example, the time for 1 oscillation of the Cs radiation
  - the unit of length is the distance traveled by the radiation in this time
  - the speed of light $c = 1$: the symbol c will not appear in any expression

To regain expressions for variables with conventional units, put in factors of c to make dimensions correct.

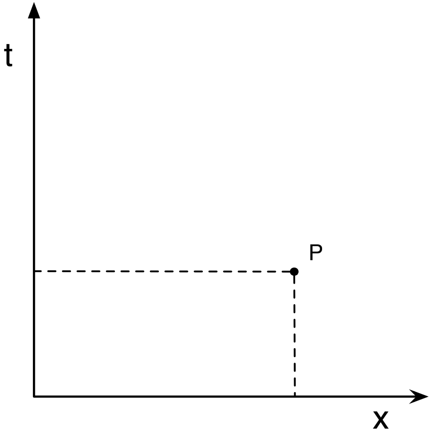
Space-time diagram

Event



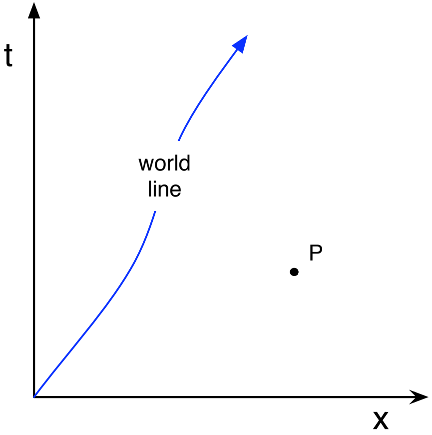
Space-time diagram

Event



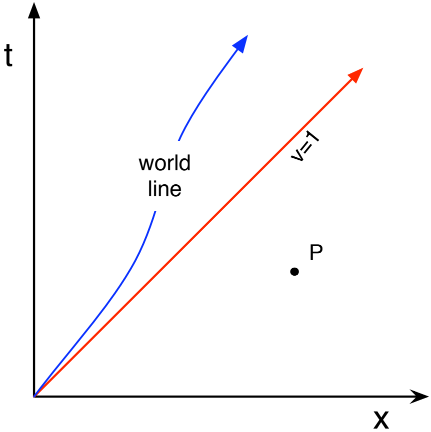
Space-time diagram

Particle worldline



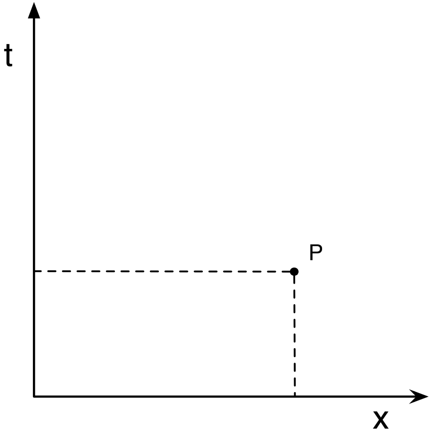
Space-time diagram

Photon worldline



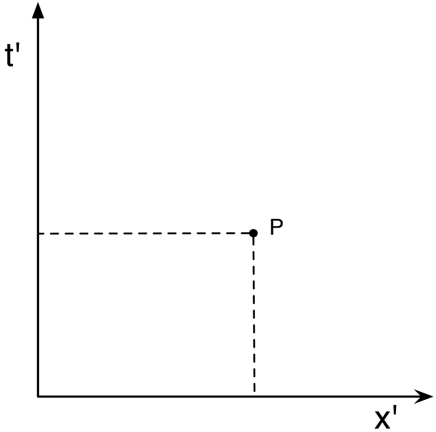
Space-time diagram

Coordinates in S of event $\mathcal{P}$



Space-time diagram

Coordinates in S' of event $\mathcal{P}$



Lorentz transformation

Consider two inertial frames:

- S with coordinates t, x, y, z
- S' with coordinates t', x', y', z'

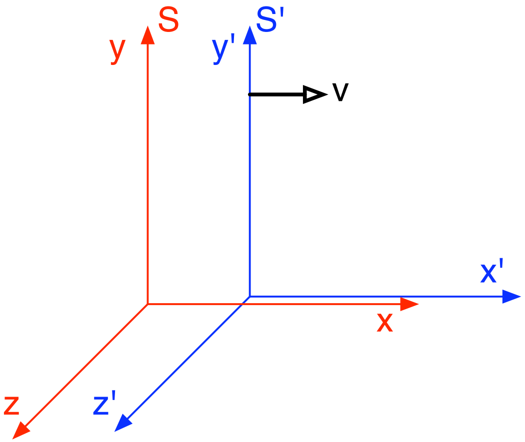
The transformation between the coordinates in two inertial frames of an event $\mathcal{P}$ is called a *Lorentz transformation*.

We initially choose a "standard configuration"

- coordinate axes are aligned
- coordinate origins coincide at times $t = 0$, $t' = 0$, i.e. the event "coordinate origins coincide" has the coordinates $t = x = y = z = 0$ in S and $t' = x' = y' = z' = 0$ in S'
- the frame S' moves along the $+x$ axis of S with speed $v < 1$

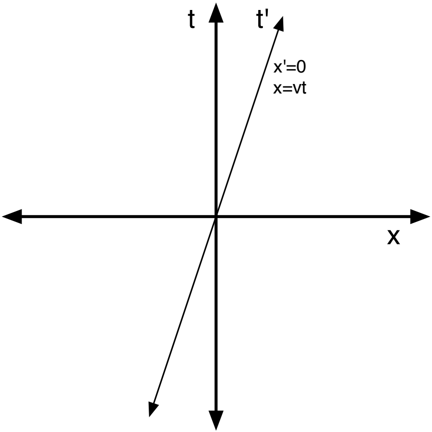
Lorentz transformation

Standard configuration



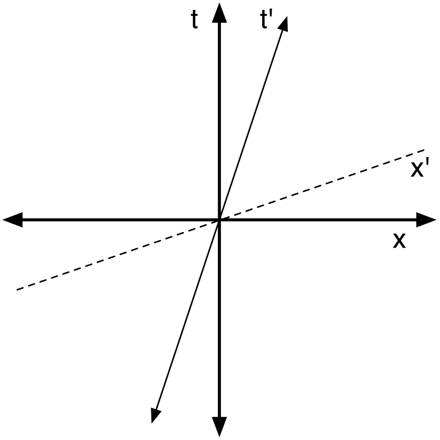
Lorentz transformation: derivation

S frame: t' axis ($x' = 0$)



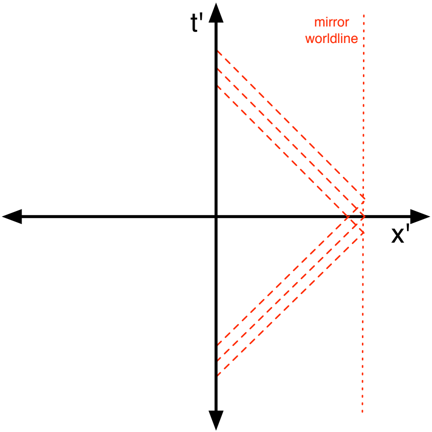
Lorentz transformation: derivation

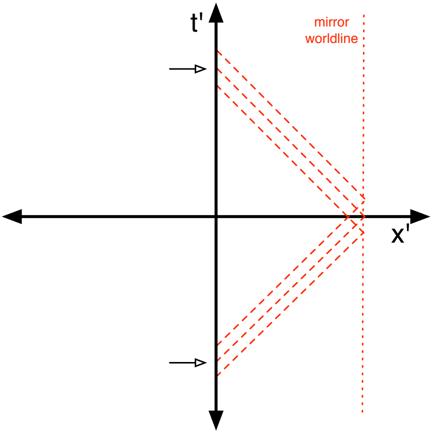
S frame: what is x' axis ($t' = 0$)?



Lorentz transformation: derivation

S' frame

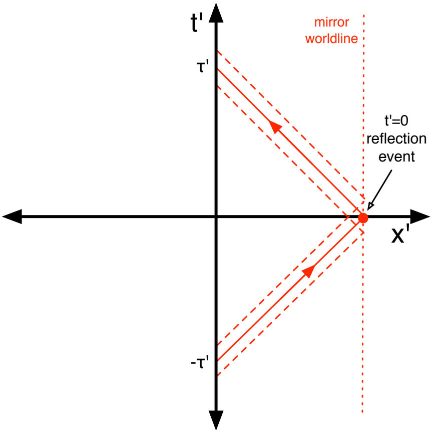




t'
mirror
worldline
x'

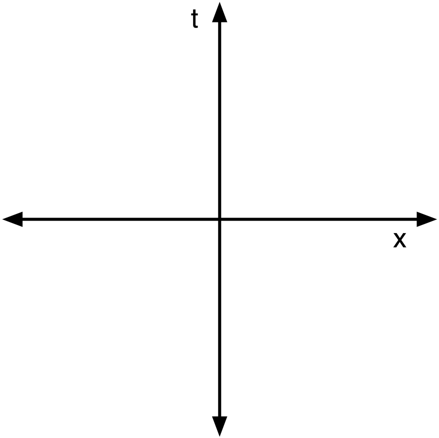
Lorentz transformation: derivation

S' frame



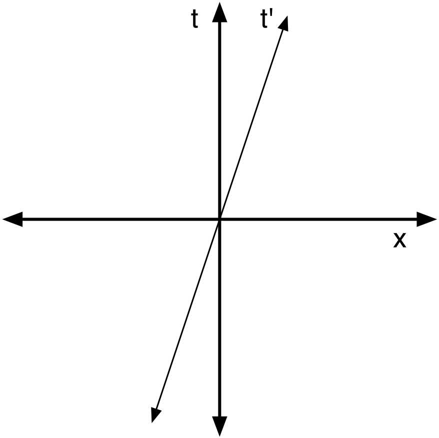
Lorentz transformation: derivation

S frame



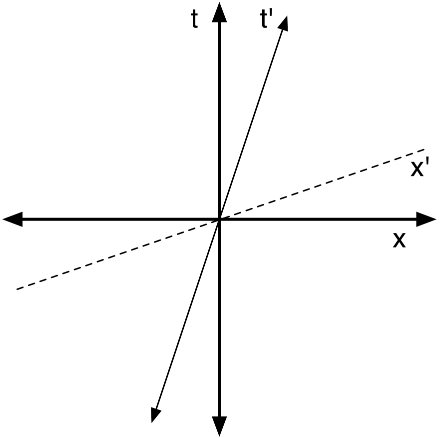
Lorentz transformation: derivation

S frame



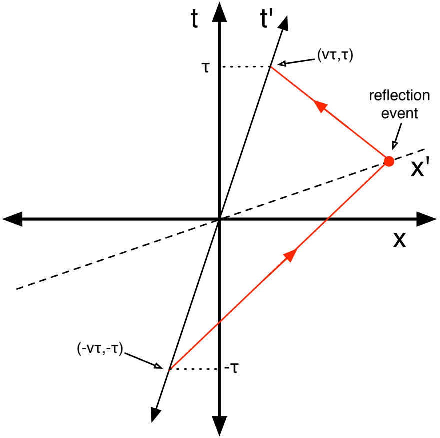
Lorentz transformation: derivation

S frame



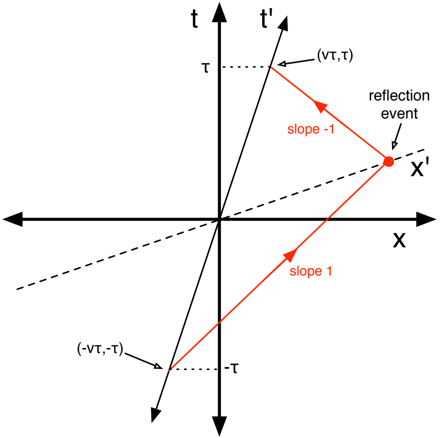
Lorentz transformation: derivation

S frame



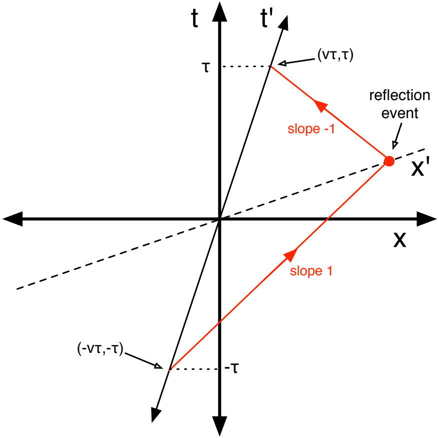
Lorentz transformation: derivation

S frame

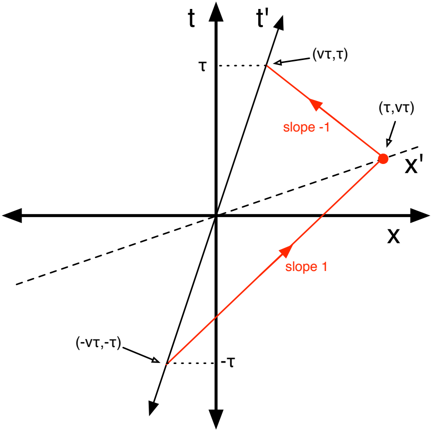


Lorentz transformation: derivation

S frame



$(-v\tau, -\tau) + r(1, 1) + s(-1, 1) = (v\tau, \tau) \quad \Rightarrow r = (1 + v)\tau, s = (1 - v)\tau$



Reflection event is at $(\tau, v\tau) \Rightarrow$ slope of x' axis is v

Lorentz transformation

- Transformation $S \to S'$ must be linear in x, t

$$x' = \gamma(x - vt), \qquad t' = \tilde{\gamma}(t - vx)$$

with $\gamma = \gamma(|v|)$, $\tilde{\gamma} = \tilde{\gamma}(|v|)$

- Inverse transformation $S' \to S$ is given by $v \to -v$

$$x = \gamma(x' + vt'), \qquad t = \tilde{\gamma}(t' + vx')$$

- Substitute second in first

$$x' = x'(\gamma^2 - \gamma\tilde{\gamma}v^2) + t'(\gamma^2 v - \gamma\tilde{\gamma}v)$$

- True for all x', t'

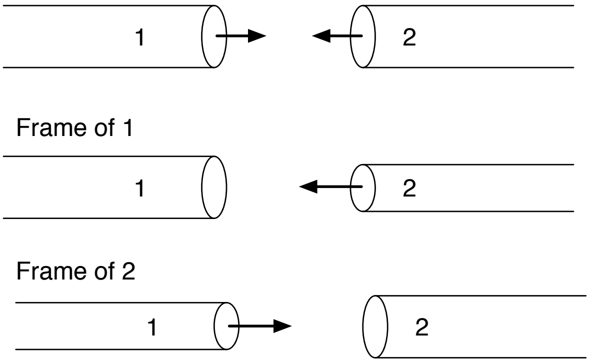
$$\gamma = \tilde{\gamma} = \frac{1}{\sqrt{1 - v^2}}$$

Transverse coordinates

Transverse coordinates unchanged

$$y' = y$$
$$z' = z$$

Contraction of transverse coordinates would violate the principle of relativity



Lorentz transformation

$S \to S'$

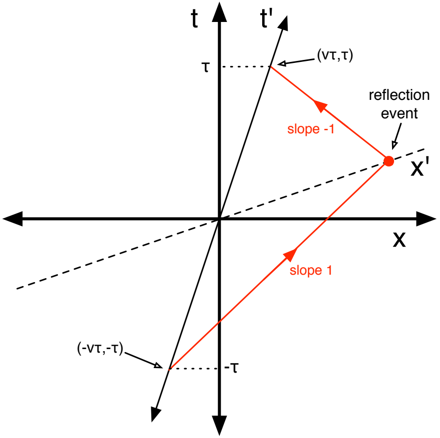
$$
\begin{aligned}
x' &= \gamma(x - vt) \\
y' &= y \\
z' &= z \\
t' &= \gamma(t - vx)
\end{aligned}
$$

$S' \to S$

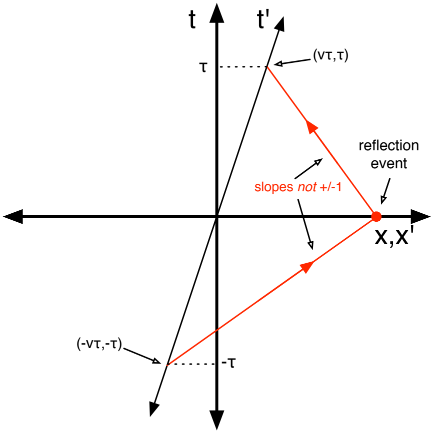
$$
\begin{aligned}
x &= \gamma(x' + vt') \\
y &= y' \\
z &= z' \\
t &= \gamma(t' + vx')
\end{aligned}
$$

- Describe physical process in terms of events
- Lorentz transformation relates coordinates of each event

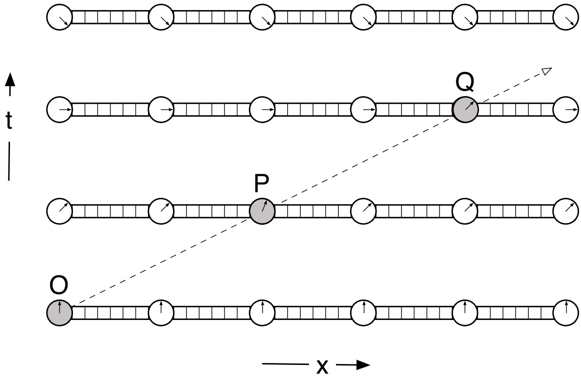
Lorentz transformation



Galilean transformation



Time dilation



Clock at the origin of the S' frame (grey) moving through the S frame. The S frame clock at the position of the grey clock is not shown, but would, of course read the same time as the other clocks in the lattice.

Velocity transformation or addition

Particle moves at velocity $\vec{u}$ in S frame. What is velocity $\vec{u}'$ in S' frame.

Calculate as uniform motion between $(0, 0, 0)$ at $t = 0$ to (x, y, z) at time t.

$$
\begin{aligned}
u'_x &= \frac{x'}{t'} = \frac{\gamma(x - vt)}{\gamma(t - vx)} = \frac{u_x - v}{1 - u_x v} \\
u'_y &= \frac{y'}{t'} = \frac{y}{\gamma(t - vx)} = \frac{u_y}{\gamma(1 - u_x v)}, \qquad \gamma \equiv \gamma_v = \frac{1}{\sqrt{1 - v^2}} \\
u'_z &= \frac{z'}{t'} = \frac{z}{\gamma(t - vx)} = \frac{u_z}{\gamma(1 - u_x v)}
\end{aligned}
$$

Inverse

$$
u_x = \frac{u'_x + v}{1 + u'_x v}, \quad u_y = \frac{u'_y}{\gamma(1 + u'_x v)}, \quad u_z = \frac{u'_z}{\gamma(1 + u'_x v)}
$$