Physics 106a — Classical Mechanics

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California Institute of Technology

Fall Term, 2013

Lecture 1

Course Website

http://www.pma.caltech.edu/∼mcc/Ph106a/

Please read the course policies

Me

- Michael Cross
- E-mail: mcc@caltech.edu (questions, comments, arrange meeting…)
- Office: 128 Bridge
- Office hours: Thursdays 1:00 - 2:00
- Class ombudsperson

TAs

Office hours: Thursday 7-9 pm in 422 Downs

- Chan Youn Park, splendid at caltech.edu
- Ke (Kevin) Ye, kye at caltech.edu
- Petr Kravchuk, pkravchuk at caltech.edu

Chan will grade assignments 1 and 4, Ke will grade assignments 2 and 5, and Petr will grade assignments 3 and 6. Each will hold office hours on the day before the homework they will grade is due (i.e. Chan will hold office hours on October 10, 31 etc., Ke on October 17, etc. Petr on October 24, etc.)

Texts

- Recommended
  - Analytical Mechanics by *Hand and Finch*: main text
  - Classical Mechanics by *John Taylor*: a nice book, with more review of the basics than Hand and Finch, but slightly less advanced than the level of the class; it will need supplementing with other reading in places
- Other
  - Classical Mechanics by *Goldstein, Poole, and Safko*: an alternative discussion that is less readable than Hand and Finch, but sometimes more precise; more advanced overall
- Reference
  - Classical Dynamics of Particles and Systems by *Thornton and Marion*: not as advanced
  - Mechanics by *Landau and Lifshitz*: classic but terse

Topics for Fall Term

1. Review of Newtonian mechanics
2. Variational approach
3. Lagrangian mechanics
4. Constrained dynamics
5. Equilibria and oscillations
6. Central forces
7. Hamiltonian dynamics
8. Formal methods
9. Rotations and rotating coordinate systems
10. Dynamics of rigid bodies
11. Small vibrations and normal modes
12. Continuum mechanics

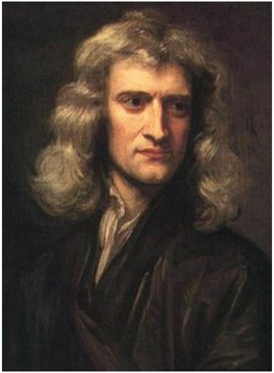
Assignments

Due 4pm Fridays in 149 Bridge; available on website $\sim$ 1 week before

Classical Mechanics is a subject where practice on different kinds of problems is essential for a good understanding

- Work other examples (e.g. from Hand and Finch) not just the assigned ones
- Don't always rely on a collaborative group — make sure you can do the problems on your own

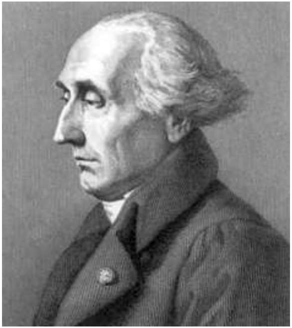
Mechanics Formulations



Isaac Newton [1643 - 1727]

$$\vec{F} = m\vec{a}$$

Vectorial mechanics



comte de Lagrange [1736 - 1813]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0$$

Lagrangian, analytical mechanics

Review of Newtonian Mechanics

In the absence of forces, a particle moves in a straight line with constant speed v.

Equivalently: particle moves with a constant *velocity* $\vec{v}$ where $\vec{v} = v\hat{v}$, with $\hat{v}$ a unit vector giving the direction of the straight line motion).

For a particle of mass m, the *acceleration* $\vec{a}$ for an imposed force $\vec{F}$ is given by

$$\vec{F} = m\vec{a}$$

An equivalent formulation is in terms of the *momentum* $\vec{p} = m\vec{v}$

$$\vec{F} = \frac{d\vec{p}}{dt} \equiv \dot{\vec{p}}$$

If object 1 exerts a force $\vec{F}_{12}$ on object 2, then object 2 exerts a reaction force $\vec{F}_{21}$ on object 1 given by

$$\vec{F}_{21} = -\vec{F}_{12}$$

or "action and reaction are equal and opposite".

Other Concepts

- The notions of the framework of space and time; the Newtonian concepts, particularly the idea of absolute time, need to be modified to more subtle ones in special relativity.
- An observer system or *reference frame* (e.g. a set of rulers and clocks) for quantifying distances and time intervals.
- Inertial frames: the special set of reference frames ("nonaccelerating frames") for which the laws of motion hold. Different inertial frames may be in relative motion, but only with a constant relative velocity $\vec{V}$.
- Vectors, such as $\vec{a}$, $\vec{v}$, $\vec{p}$, which take advantage of physical principle of the rotational symmetry of space to write the equations in a form independent of coordinate basis.
- A system of coordinates (Cartesian, polar ...) to evaluate the consequence of the laws of motion.

Galilean Invariance

An important *invariance* or *symmetry* is that Newton's laws of motion are unchanged by transforming to a different inertial frame.

Under such a *Galilean* transformation

$$
\begin{aligned}
t &\to t' \quad \text{with} \quad t' = t \\
\vec{r} &\to \vec{r}\,' \quad \text{with} \quad \vec{r}\,' = \vec{r} - \vec{V}t \\
\vec{v} &\to \vec{v}\,' \quad \text{with} \quad \vec{v}\,' = \vec{v} - \vec{V} \\
\vec{a} &\to \vec{a}\,' \quad \text{with} \quad \vec{a}\,' = \vec{a}
\end{aligned}
$$

and it is assumed that the force is the same in the two frames.

Note that Newton's laws are *not* true in non-inertial (accelerating frames) such as on the surface of the Earth! We will discuss consequences of this (e.g. hurricanes) in later lectures.

Comments

Newton's laws:

- profoundly important in establishing quantitative science;
- shape how we think about the world around us (pushes,pulls, dynamics...)

However the concepts such as force turn out not to be the best way to formulate physics in more extreme conditions (and the laws have to be modified):

- small scales: quantum mechanics (use potential, but not force)
- large scales: general relativity (curvature of space time)
- high speeds: special relativity (instantaneous forces only make sense for point collisions)

The Lagrangian/Hamiltonian formulation we will discuss next lecture is more suitable for generalizing to these extreme conditions

Newton's Laws for a Composite Object

For a set of many particles i (not necessarily of the same mass) N2 reads

$$\dot{\vec{p}}_i = \sum_j \vec{F}_{ji} + \vec{F}_i^{(e)}$$

with $\vec{F}_{ji}$ the interparticle forces and $\vec{F}_i^{(e)}$ the forces from external sources. Summing over all particles i gives

$$\dot{\vec{P}} = \sum_{ij} \vec{F}_{ji} + \sum_i \vec{F}_i^{(e)}$$

with $\vec{P} = \sum_i \vec{p}_i$ the *total momentum*.

If N3 applies the first sum vanishes, and we get Newton's law for the total momentum in terms of the total external force

$$\dot{\vec{P}} = \vec{F}^{(e)} \quad \text{with} \quad \vec{F}^{(e)} = \sum_i \vec{F}_i^{(e)}$$

Conservation of Momentum

For zero external force

$$\dot{\vec{P}} = 0$$

and the total momentum is conserved.

We can also write the total momentum as

$$\vec{P} = M\dot{\vec{R}} \quad \text{with} \quad M = \sum_i m_i, \quad \vec{R} = \frac{\sum_i m_i \vec{r}_i}{M}$$

introducing the the *total mass* M and the *center of mass* coordinate $\vec{R}$.

N2 for the composite object can be written

$$M\frac{d^2\vec{R}}{dt^2} = \vec{F}^{(e)}$$

Angular Momentum

For a set of particles with vector positions $\vec{r}_i$ measured from origin O, the *angular momentum* about O is defined as

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

Conservation of Angular Momentum

Taking the time derivative

$$\dot{\vec{L}} = \sum_i \dot{\vec{r}}_i \times \vec{p}_i + \sum_i \vec{r}_i \times \dot{\vec{p}}_i$$

The first term is zero, since $\vec{p}_i$ is parallel to $\dot{\vec{r}}_i$. Newton's second law gives

$$\dot{\vec{L}} = \sum_i \vec{r}_i \times \vec{F}_i^{(e)} + \sum_{ij} \vec{r}_i \times \vec{F}_{ji}$$

The terms in the last sum can be paired up, e.g.

$$\vec{r}_1 \times \vec{F}_{21} + \vec{r}_2 \times \vec{F}_{12} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{21} \quad \text{using Newton's third law}$$

For the special case of *central forces* this vanishes so that

$$\dot{\vec{L}} = \sum_i \vec{r}_i \times \vec{F}_i^{(e)} \equiv \vec{N}^{(e)} \quad \text{total torque}$$

If the total torque is zero *angular momentum is conserved.*

Work and Energy

Define the *work* done by the external force $\vec{F}$ acting on a particle in going from point 1 to point 2 by

$$W_{12} = \int_1^2 \vec{F} \cdot \vec{ds}.$$

Use Newton's 2nd law to write $\vec{F} = m d\vec{v}/dt$, and replace the line integral by an integral over time using $\vec{ds} = \vec{v} dt$ to find

$$W_{12} = m \int_{t_1}^{t_2} \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt = \frac{m}{2} \int_{t_1}^{t_2} \frac{d}{dt}(v^2)\, dt = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2.$$

We call the quantity $\frac{1}{2} m v^2$ the *kinetic energy*.

Conservative Forces

If W_{12} is *independent* of the path taken from 1 to 2 the force is *conservative*

Equivalently

$$\oint \vec{F} \cdot \vec{ds} = 0 \quad \text{or} \quad \vec{F} = -\vec{\nabla} V(\vec{r}),$$

with $V(\vec{r})$ the *potential energy*

For a conservative force

$$\tfrac{1}{2} m v_1^2 + V(\vec{r}_1) = \tfrac{1}{2} m v_2^2 + V(\vec{r}_2)$$

and the *total energy* (kinetic plus potential) is conserved in the dynamics

Not all familiar forces are conservative: electrostatic and gravitational forces are; magnetic forces and friction are not.

Summary

- Newton's laws of motion
- Conservation laws
  - Momentum
  - Angular momentum
  - Energy
- Symmetries
  - Rotational $\Rightarrow$ vectors
  - All inertial frames equivalent $\Rightarrow$ Galilean symmetry