Physics 106b: Lecture 1 — 7 January, 2014

Principle of Relativity

The Principle of Relativity states:

> The laws of physics are the same in all inertial frames.

In Special Relativity *inertial frames* are a special set of frames in which the laws of physics are particularly simple, with any two frames in the set moving at a constant relative velocity. In General Relativity, an inertial frame is any *free floating* frame, and includes frames freely falling under gravity.

The principle is the same as the Galileo-Newton Principle of Relativity. As the laws of electromagnetism became understood, it appeared that these were not consistent with the principle, since Maxwell's equations predict a wave propagating with a speed c that can be determined by static measurements. With the Newtonian understanding of space and time, if a speed is c in one inertial frame, then it would be a different speed in another inertial frame. Thus either there is a *special* frame (the "ether") in which Maxwell's equations are true, and the Principle of Relativity fails for electromagnetism, or the Principle of Relativity holds for electromagnetism, in which case the Newtonian notion of space-time must fail. Einstein proposed the latter. To emphasize this a second postulate of Special Relativity is often added:

> Yes, really!

(meaning all laws of physics, including E&M), or more commonly

> The speed of light is the same in all inertial frames.

A consequence, as we will see, is that time is no longer absolute. For example, spatially separated events that are simultaneous in one inertial frame need not be simultaneous in a different inertial frame. One way to proceed is to see how the description of space and time transform between different inertial frames. This introduces the *Lorentz transformation*. In the next lecture we will proceed more geometrically.

Lorentz Transformation

How are space coordinates (x, y, z) and time t in one inertial frame S of reference related to space coordinates (x', y', z') and time t' in a second inertial frame S'? First we establish what we mean by coordinates.

Coordinates

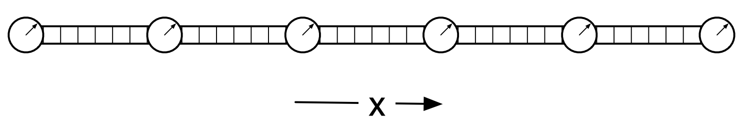

Figure 1: Lattice of rulers and clocks for one dimension

In my inertial frame S I imagine setting up space-time coordinates from a lattice of rulers and synchronized clocks. I show this in Fig. 1 for just one space dimension. The clocks can be synchronized in a number of ways, just as we usually do. I could purchase a large number of identical synchronized clocks from China, and transport them to the various sites. I might worry about time dilation effects if I move them too rapidly, but these can always be made negligible by transporting the clocks slowly enough. Or I could send a radio signal from a central site, and include the correction for the travel time at the speed of light. Or for a pair of clocks I could send a flash of light from a point midway between, and set the clocks to some agreed time when the flashes arrive.

We use this lattice of rulers and clocks to measure the coordinates (x, y, z) and time t in the frame S of a definite point in space-time. We call a precise space-time point an *event*. As the name implies, it is conceptually useful to think of a definite physical event as defining a space-time point, e.g. an atom emits a flash of light, or I clap my hand[1]. It important to think about physical processes involving things happening at different space-time points in terms of events making up the process.

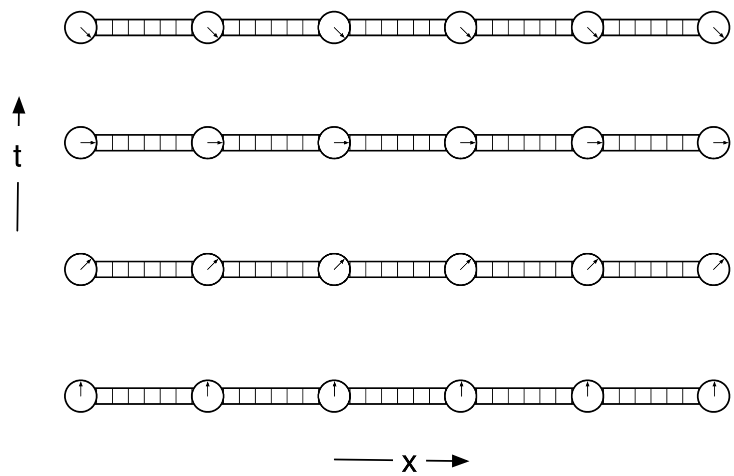

Figure 2: The same lattice of rulers and clocks for 4 successive times

Figure 2 shows the rulers and clocks for four successive times. The coordinate of an event is given by the ruler division and the clock reading at the event. Plotting the coordinates of a sequence of events gives a space-time diagram. Usually, of course, we will just use x, t axes, and not draw clocks and rulers!

Units

Our conventional unit of time (second) is defined as the time for 9192631770 oscillations of radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom. Our conventional unit of length (meter) is defined as $1/(2.99792458 \times 10^8)$ of the distance traveled by electromagnetic radiation in one of these seconds. The speed of light is therefore, by definition of our units, $c = 2.99792458 \times 10^8$ meters/second. In setting up the mathematical description of the physics it makes little sense to include these numbers arrived at by historical accident. We will instead, from now on, pick some convenient unit for time, e.g. the time for 1 oscillation of the Cs radiation, and then choose as the units for distances (x, y, x) the distance traveled by the electromagnetic radiation (in vacuum) in this time. With these choices the speed of light $c = 1$, and the symbol c will not appear in any of the expressions. To return to conventional units, factors of c are inserted to give correct dimensions.

[1] Obviously the "spread" of the physical event must be small compared to the space-time resolution desired.

Space-time diagrams

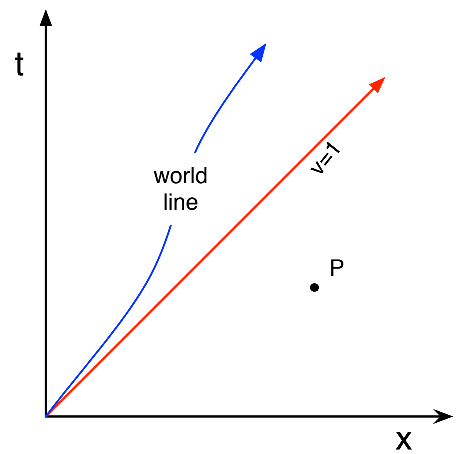

Figure 3: Space-time diagram for one space dimension, showing an event $\mathcal{P}$, the worldline of a particle (blue), and the worldline of a photon (red).

An event is represented by a point in a four dimensional space, three space dimensions and time. To represent this pictorially we often use a graph with time as the ordinate, and one (or sometimes two) space coordinates as abscissa(s). An example is shown in Fig. 3 for x and t. The plot shows an event $\mathcal{P}$, and the worldline of a particle moving in the x-direction — the line joining all the events at which the particle is present — and of a pulse of light or a photon which moves at the speed of light. For the relativistic choice of units, the slope of the photon worldline is unity.

Standard configuration

We consider a "standard configuration" for our two frames of reference S with coordinates t, x, y, z and S' with coordinates (t', x', y', z'): the coordinate axes are aligned, and the origins of the axes coincide at $t = t' = 0$. The frame S' moves at speed v along the x-axis relative to S (and so the frame S moves at the speed v along the negative x' axis relative to S'). Thus the event "the origins coincide" has the coordinates $x = y = z = t = 0$ in S and $x' = y' = z' = t' = 0$ in S'. If an event $\mathcal{P}$ has coordinates t, x, y, z in S, what are the coordinates in S'? Since you have done this before, and one derivation is worked out in Hand and Finch §12.1-4, I will not go through the details of the argument here – you can also review the slides to Lecture 1. The main points of the argument are:

- The coordinates transverse to the direction of relative motion are unchanged $y = y'$, $z = z'$. We now just look at x and t.
- The coordinate origin $x' = 0$ in S' (i.e. the t' axis) is described by the coordinates $x = vt$ in S by definition of the two frames.
- By considering a simple thought-experiment, the time $t' = 0$ in S' (i.e. the x' axis) is given by coordinates $x = t/v$ in S. (In m-sec units this would read $x = c^2 t/v$.) The S' axes are shown in the S space-time diagram in Fig. 4.
- Linearity of the transformation in the coordinates.

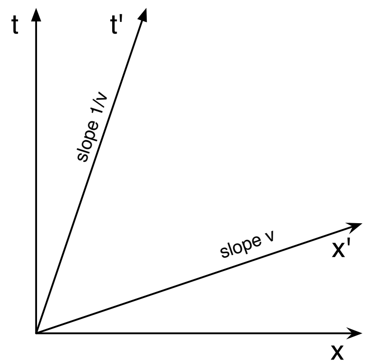

Figure 4: x' axis ($t' = 0$) and t' axis ($x' = 0$) in S frame space-time diagram.

- The transformation $S \to S'$ is the same as the transformation $S' \to S$ with $v \to -v$.

From these arguments you can construct the following results for the $S \to S'$ transformation

$$t' = \gamma(t - vx) \tag{1a}$$
$$x' = \gamma(x - vt) \tag{1b}$$
$$y' = y \tag{1c}$$
$$z' = z \tag{1d}$$

with $\gamma = 1/\sqrt{1 - v^2}$. The $S' \to S$ transformation is given by $v \to -v$

$$t = \gamma(t' + vx') \tag{2a}$$
$$x = \gamma(x' + vt') \tag{2b}$$
$$y = y' \tag{2c}$$
$$z = z' \tag{2d}$$

From these equations you can derive the important invariant

$$t^2 - x^2 - y^2 - z^2 = t'^2 - x'^2 - y'^2 - z'^2. \tag{3}$$

To use the Lorentz transformation:

- describe the physical phenomenon of interest in terms of events;
- relate the coordinates of the events using these equations.

The key to resolving many apparent paradoxes is that spatially separated events that are simultaneous (occur at the same time) in one frame of reference may not be simultaneous in other frames.

Clocks and rulers

Let us look at the Lorentz transformation in terms of the clocks and rulers notionally used to set up the coordinates. This introduces the ideas of *time dilation* and *length contraction*.

I set up clocks and rulers to define coordinates in my S frame of reference. Let's suppose that you are in the inertial frame S'. You would do exactly the same thing, and set up a lattice of clocks and rulers defining

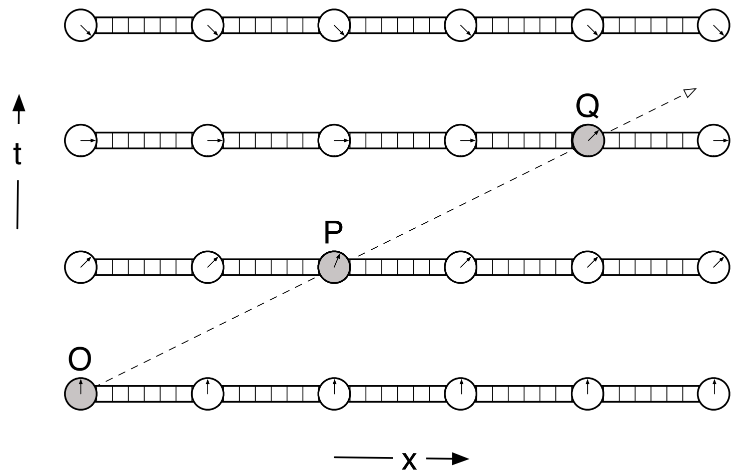

Figure 5: Clock at the origin of the S' frame (grey) moving through the S frame. The S frame clock at the position of the grey clock is not shown, but would, of course read the same time as the other clocks in the lattice.

the coordinates (x', y', z') and time t' of an event. We have agreed to origins for our space coordinates that coincide at some time (an event!), and set our clocks to zero at that event, so that $x = y = z = t = 0$ corresponds to $x' = y' = z' = t' = 0$. You and I agree we have set up equally good ways of measuring space and time, since we did it in the same way, and of course the mechanics of the clocks are the same in our two inertial frames by the principle of relativity. However we will not necessarily agree on the coordinates assigned to events.

For example consider two events: $\mathcal{O}$ the coincidence of the origins that has coordinates $x = y = z = t = 0$ and $x' = y' = z' = t' = 0$ (we agree on this one), and $\mathcal{P}$ that occurs at a later time, and at the spatial origin of in your frame $x' = y' = z' = 0$. Of course the spatial coordinates in my frame are different, just as they are in Galilean physics, since your origin is moving in my frame. But in addition the times are different, i.e. the two clocks present at the event $\mathcal{P}$, one of your clocks and one of mine, give *different* readings. How can that be? How can one clock read earlier than the other, since there is a "symmetry" between the frames — you are moving relative to me, and I am moving relative to you? Which one reads the shorter time? The answer is that there is no symmetry for the events described: you have a single clock present at both events; I have different, but synchronized, clocks that read the times of $\mathcal{O}$ and $\mathcal{P}$. Your clock is said to read the *proper time* interval between the two events. It is a *shorter* time than I read from my clocks. This is shown in the S frame for three events $\mathcal{O}$, $\mathcal{P}$, $\mathcal{Q}$ (the clock at the S' origin at three times) in Fig. 5. This is the phenomenon of time dilation. Quantitatively, the event $\mathcal{P}$ has the coordinates $t', x' = 0$, and using Eq. (2a) gives $t = \gamma t'$. Remember $\gamma > 1$, so that $t > t'$.

Another example is the measurement of the length of a moving object. To measure the length of an S' unit-length ruler in frame S we find the coordinate x at $t = 0$ for coordinate $x' = 1$. Remember $x = 0, t = 0$ implies $x' = 0, t' = 0$ so that we are determining the coordinates in S of the two ends of the S' ruler at the same time $t = 0$ — which is how we measure a length! Using Eq. (1b) gives $x = 1/\gamma$: we measure the moving ruler to be length contracted.

Michael Cross, January 6, 2014