CS5100 HOMEWORK 1

Sudharshan Subramaniam Janakiraman

07/05/2020

THEORY

1. The Truck has to route from the starting point to its destination and deliver its package at the intended location. The truck can start initially anywhere but it has to go to the package location, load the package into the truck and unload it after reaching the destination. The truck can move forward , left and right. With these we can develop high level actions.Let
Route(t, l) → take the truck to the location. **Pickup(p,t,i)** → Pickup the package from the initial Location I and load it into the truck. **Deliver(p, t, d)** → Deliver the package that is loaded inside the truck after reaching the destination coordinates. **Load(p, t)** → Load the package to the truck. **Unload(p, t)** → Unload the Package from the Truck. **Forward(t)** → Move the truck in the forward direction. **Right(t)** → Take a right turn. **Left(t)** → Take a left turn. **At(p,l)** → Package at location l, **At(p,t)** → package inside the truck

Hierarchy of High Level Actions for the truck

Refinement(Pickup(p, t, i),
PRECOND: Truck(t) ⋀ At(p, i))
STEPS: [Route(t, i), Load(p, t)])
Refinement(Deliver(p, t, d),
PRECOND:Truck(t) ⋀ At(t, d) ^ At(p, t)
STEPS: [Unload(p, t)])
Refinement(Navigate(t, l),
PRECOND:Truck(t)
STEPS: [Forward(t), Route(t, l)])
Refinement(Route(t, l),
PRECOND:Truck(t)
STEPS: [Left(t), Route(t, l)])
Refinement(Route(t, l),
PRECOND:Truck(t)
STEPS: [Right(t), Route(t, l)])
Refinement(Route(t, l),
PRECOND:Truck(t) ^ At(t,l)
STEPS: [Pickup(p,t,l) or Deliver(p,t,l)])

2. The conditional effects give rise to different meanings for the fields. The DURATION field has constraints. This relates to the effects of the action. The effects of the action should be taken into account.

   Ex: let duration be the time allocated for planning a wedding. Since we have an non deterministic environment we will only have partial details of the world to plan which affects the time allowed for performing the action

   The USE field has constraints that do not care about the effects of the action. It will be solely oriented on the actions alone with no relation to the consequences of the action.

   Ex. let's say the USE field allows you to take a MOVE action in a non deterministic route planning problem. The MOVE action of USE filed moves based on its constraints and it doesn't takes into consideration of what is expected of the action

   The CONSUME field is similar to DURATION field where it gives us the efficiency of the action taken i.e., The resources to be consumed by an action out of all the available resources. This affects the consequences of the action. The CONSUME field has resource constraint which will depend upon the inputs of nondeterministic world

   Ex : let us assume 10 kg of food is allocated for a family to eat in a day cor lunch and dinner combined. If the family eat 5kg for lunch then remaining 5kg will be available for dinner but if they consume 6kg for lunch only 4kg will be available for dinner implying the consequences of the previous action affect the resource consumed by the next action

   Therefore USE field is a different field which can be related only to the action whereas CONSUMER and DURATION field can be combined together to study the consequences of the action

3.

a. Naive Bayes is a recursive conditional probability algorithm that helps us in determining whether an event will occur or not given a certain list of conditions called the prior along with the likelihood of happening of the event and evidence. The formula for naive bayes classification algorithm is

$P(A \mid B) = P(B \mid A) * P(A) / P(B)$ ⇒ The probability that an event A occurs given that B has already happened can be found out by taking the product of Prior probability(Occurring of B if A happens) and Likelihood of A happening (P(A)) and normalizing the obtained probability by the Obtained evidence.

This naive bayes algorithm is used in AI for classification purposes. It can be used for classification of data into multiple classes for the given condition
Ex: Classifying whether it is suitable to play a game outdoor or not based on the available data of wind , temperature, previous games played in such conditions, previous games not played in such condition.

The Naive bayes algorithm uses 2 assumptions that the features are independent and contributes equally to the output.

b. Let the Probability that a person buys a PC or not be P(Yes) and P(No). From the given data we can infer that

$$P(Yes) = 5 / 11 \Rightarrow 0.4545$$
$$P(No) = 6 / 11 \Rightarrow 0.5455$$

Let the Probability that the person has bought a PC before or not be P(P`Y) and P (P`N)

	Yes	No	
Prior Yes (P`Y)	4	1	P(P`Y) : 5/ 11 ⇒ 0.4545
Prior No (P`N)	1	5	P (P`N) = 6 /11 ⇒ 0.5455
Total	5	6	
	P(Yes): 5/11 ⇒ 0.4545	P(No): 6/11 ⇒ 0.5455	

The probability that the person has a purchased a PC before given that the person has purchased now and not purchased is given by is given by

$$P(P`Y \mid Yes) = 4 / 5 \Rightarrow 0.8$$
$$P(P`Y \mid No) = 1 / 6 \Rightarrow 0.167$$

Similarly the probability has not purchased a PC before given that the person has purchased a PC now or not is given by

$$P(P`N \mid Yes) = 1 / 5 \Rightarrow 0.2$$
$$P(P`N \mid No) = 5 / 6 \Rightarrow 0.833$$

Let the Probability that the person is in high school or college be P(H) and P (C)

	Yes	No	
High School (H)	1	3	P(H) : 4/ 11 ⇒ 0.3636
College (C)	4	3	P (C) = 7 /11 ⇒ 0.6364

Total	5	6	
	P(Yes): 5/11 ⇒ 0.4545	P(No): 6/11 ⇒ 0.5455	

The probability the person Studies is HighSchool when the person purchased a PC is given by

$$P(H \mid Yes) = 1 / 5 \Rightarrow 0.2$$
$$P(H \mid No) = 3 / 6 \Rightarrow 0.5$$

The probability that the person Studies is College when the person purchased a PC is given by

$$P(C \mid Yes) = 4 / 5 \Rightarrow 0.8$$
$$P(C \mid No) = 3 / 6 \Rightarrow 0.5$$

Let the Probability that the person has low , medium or high income be P(LI) , P (MI), P(CI)

	Yes	No	
High Income (HI)	3	1	P(H) : 4/ 11 ⇒ 0.3636
Medium Income (MI)	1	2	P (M) = 3 /11 ⇒ 0.2727
Low Income(LI)	1	3	P (L) = 4 / 11 ⇒ 0. 3636
Total	5	6	
	P(Yes): 5/11 ⇒ 0.4545	P(No): 6/11 ⇒ 0.5455	

The probability that the person has High Income when the person purchased a PC is given by

$$P(HI \mid Yes) = 3 / 5 \Rightarrow 0.6$$
$$P(HI \mid No) = 1 / 6 \Rightarrow 0.167$$

The probability that person has medium income when the person purchased a PC is given by

$$P(MI \mid Yes) = 1 / 5 \Rightarrow 0.2$$
$$P(MI \mid No) = 2 / 6 \Rightarrow 0.333$$

The probability that the person has low Income when the person purchased a PC is given by

$$P(LI \mid Yes) = 1 / 5 \Rightarrow 0.2$$
$$P(LI \mid No) = 3 / 6 \Rightarrow 0.5$$

Let the Probability that the person is a Young Adult, Middle Age or senior income be P(LI) , P (MI), P(CI)

	Yes	No	
Young Adult (Y)	3	1	P(H) : 4/ 11 ⇒ 0.3636
Middle Age (M)	1	3	P (M) = 4 /11 ⇒ 0.3636
Senior(S)	1	2	P (L) = 3 / 11 ⇒ 0. 2727
Total	5	6	
	P(Yes): 5/11 ⇒ 0.4545	P(No): 6/11 ⇒ 0.5455	

The probability that the person Young Adult when the person purchased a PC is given by

P(Y | Yes) = 3 / 5 ⇒ 0.6

P(Y | No) = 1 / 6 ⇒ 0.167

The probability that the person is of middle age when the person purchased a PC is given by

P(MI | Yes) = 1 / 5 ⇒ 0.2

P(MI | No) = 3 / 6 ⇒ 0.5

The probability that the person is a senior when the person purchased a PC is given by

P(S | Yes) = 1 / 5 ⇒ 0.2

P(S | No) = 2 / 6 ⇒ 0.333

Given Problem: To find the probability that a person will buy a laptop who is a senior with a high income and hold a high school education and has a prior purchase of PC

I.e., P (Yes | S, HI, H, P`Y) = ?

We can use the Naive bayes formula to compute the above probability

$P(C_k \mid x_1, x_2, \ldots, x_n)$ = (1 / n) * [Product for i ranging from 1 to n($P(x_i \mid C_k)$)]*$P(C_k)$
Where n = sum for all k [$P(C_k)P(x \mid C_k)$]

Let X = {Senior, High, High School, Prior Yes}

$$P(Yes \mid X) = P(X \mid yes) * P(Yes) / P(X) \quad \rightarrow 1$$

$$P(No \mid X) = P(X \mid No) * P(No) / P(X) \quad \rightarrow II$$

$$P(X \mid Yes) = P(S \mid Yes) * P(HI \mid Yes) * P(H \mid Yes) * P(P`Y \mid yes)$$

$$P(X \mid Yes) = 0.2 * 0.6 * 0.2 * 0.8 = 0.0192$$

Similarly

$$P(X \mid No) = P(S \mid No) * P(HI \mid No) * P(H \mid No) * P(P`Y \mid No)$$

$$P(X \mid No) = 0.333 * 0.167 * 0.5 * 0.1667 = 0.00464$$

$$P(X) = P(X \mid Yes) * P(Yes) + P(X \mid No) * P(No)$$

$$P(X) = 0.0192 * 0.4545 + 0.00464 * 0.5455 = 0.01125$$

Substituting all the values in I and II we get

$$P(Yes \mid X) = 0.0192 * 0.4545 / 0.01125 = 0.775$$

$$P(No \mid X) = 0.00464 * 0.5455 / 0.01125 = 0.225$$

There is **77.5** percentage probability the person **will buy** a PC in the given condition which is very much higher than **22.5** percentage probability that he **will not buy** so the classifier will provide **Yes** as the output

4.

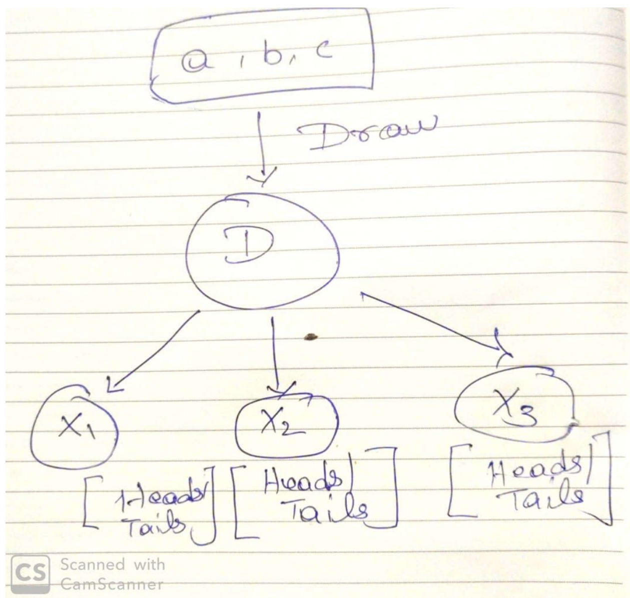
	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 13.3 A full joint distribution for the Toothache, Cavity, Catch world.

P(toothache) = sum of all the probabilities that is under the column of toothache

P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

5. Let D be the random variable which can take the value a, b, c as we draw randomly from the bag The bayes net like like the below diagram where the random variable D acts as the root which can assume values a , b , c and X1, X2, X3 are child of D



The CPT for D is given by following table (a, b, c has equal chances)

D	P(D)
a	1/3
b	1/3
c	1/3

The CPT for Xi is given by the following table

	a	b	c
P(Heads)	0.2	0.6	0.8

P(Tails)	0.8	0.4	0.2

Outcome1	P(X1)	Outcome2	P(X2)	Outcome3	P(X3)	P(D)
H	0.2	H	0.2	H	0.2	0.008
H	0.2	H	0.2	T	0.8	0.032
H	0.2	T	0.8	H	0.2	0.032
H	0.2	T	0.8	T	0.8	0.128
T	0.8	H	0.2	H	0.2	0.032
T	0.8	H	0.2	T	0.8	0.128
T	0.8	T	0.8	H	0.2	0.128
T	0.8	T	0.8	T	0.8	0.512

P(D = a | 2 heads and 1 Tails) = 0.032 + 0.032 + 0.032 = 0.096

Outcome1	P(X1)	Outcome2	P(X2)	Outcome3	P(X3)	P(D)
H	0.6	H	0.6	H	0.6	0.216
H	0.6	H	0.6	T	0.4	0.144
H	0.6	T	0.4	H	0.6	0.144
H	0.6	T	0.4	T	0.4	0.096
T	0.4	H	0.6	H	0.6	0.144
T	0.4	H	0.6	T	0.4	0.096
T	0.4	T	0.4	H	0.6	0.096
T	0.4	T	0.4	T	0.4	0.064

P(D = b | 2 heads and 1 Tails) = 0.144 + 0.144 + 0.144 = 0.432

Outcome1	P(X1)	Outcome2	P(X2)	Outcome3	P(X3)	P(D)
H	0.8	H	0.8	H	0.8	0.512
H	0.8	H	0.8	T	0.2	0.128
H	0.8	T	0.2	H	0.8	0.128
H	0.8	T	0.2	T	0.2	0.032
T	0.2	H	0.8	H	0.8	0.128
T	0.2	H	0.8	T	0.2	0.032
T	0.2	T	0.2	H	0.8	0.032
T	0.2	T	0.2	T	0.2	0.008

P(D = c | 2 heads and 1 Tails) = 0.128 + 0.128 + 0.128= 0.384

P(D = b | 2 heads and 1 Tails) produces the largest probability value out of the the 3 coins so if we receive 2 heads and 1 tail as the outcome then there is high possibility that the coin is b

6.

a. According to conditional probability

P(A , B) = P(A)P(B) when a and b are independent ⇒ P(A | B) = P(A)* P(B) / P(B) ⇒ P(A)
Here B is considered the evidence and A is the event. If A and B are Independent then the evidence of B is not required. And if the evidence is not given then it can be assumed A is independent of B

Similarly since the evidence is not given in the following question then earthquake and Burglary are to be considered independent
P (Burglary, Earthquake) = P(Burglary) * P(Earthquake) = 0.001 * 0.002 = 0.000002

Topologically we can say Earthquake and Burglary are connected to Alarm which is considered as the evidence . If the evidence is Absent then Alarm is not present then Earthquake and Burglary are isolated from one another declaring them as independent events

b. Inorder to check conditional independence we have to prove that P(B,E |A) = P(B|A) * P(E |A)

Where B → Burglary, E → Earthquake, A → Alarm

LHS : **P (B,E | A) = P (A | B, E) P(B, E) / P(A)** → Bayes Law

P (A | B, E) = 0.95 → 1

P(B,E) = 0.001 * 0.002 (P(B) * P(E)) → 2

P(A) = P (A | B, E) P(B, E) + P (A | B, -E) P(B,- E)
+P (A | -B, E) P(-B, E)+ P (A |- B,- E) P(-B,- E)

P(A) = (0.95 * 0.001 * 0.002) + (0.94 * 0.001 * 0.998) + (0.29 * 0.999 * 0.002)
+ (0.001 * 0.999 * 0.998)

P(A) = 0.00252 → 3

Substituting 1 2 3 in **P (B,E | A)** equation we get

P (B,E | A) = 0.95 * 0.001 * 0.002 / 0.00252 = 0.0008

RHS : P(B|A) * P(E |A)

P(B|A) = P(A | B) * P (B) / P(A)

P(A | B) = P (A | B, E) P(E) + P (A | B, -E) P(- E) = (0.95 * 0.002) + (0.94 * 0.998)

P(A | B) = 0.94

P(B) = 0.001

P(A) = 0.00252

P(B|A) = 0.94 * 0.001 / 0.00252 = 0.373

P(E|A) = P(A | E) * P (E) / P(A)

P(A | E) = P (A | B, E) P(B) + P (A |- B,-E) P(-B) = (0.95 * 0.001) + (0.29 * 0.999)

P(A | E) = 0.29

P(E) = 0.002

P(A) = 0.00252

P(E|A) = 0.29 * 0.002 / 0.00252 = 0.23

P(B|A) * P(E |A) = 0.373 * 0.23 = 0.086

LHS is not same as RHS

P(B,E |A) not equal to P(B|A) * P(E |A)
E and B are not conditionally independent if A is given

7. .

a. For 2 step transition the transition probability matrix is P * P

P = [[0.6 0.4]
[0.4 0.6]]

⇒ P * P = [[0.6 0.4] [0.4 0.6]] * [[0.6 0.4] [0.4 0.6]] = [[0.36+0.16 0.24+0.24] [0.24+0.24 0.36+0.24]]

P * P = [[0.52 0.48]
[0.48 0.52]]

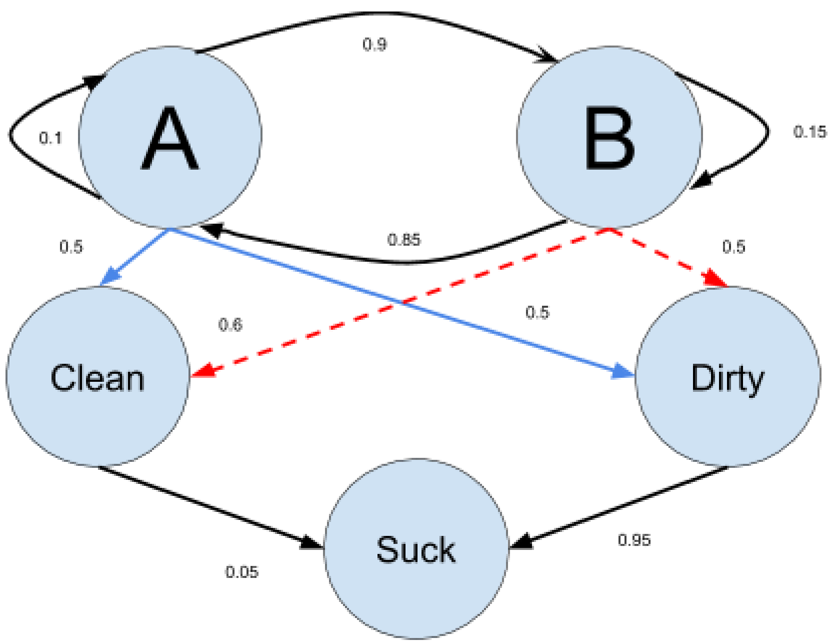
b. From 5.30 to 8.30 pm there exists 3 hrs of time which implies the possibility of maximum 3 transitions implying the the requirement of 3 step transition probability matrix

P * P * P = [[0.52 0.48] [0.48 0.52]] * [[0.6 0.4] [0.4 0.6]] = [[0.312+0.192 0.208+0.288] [0.288+0.208 0..192+0.312]]

P * P * P = [[0.504 0.496]
[0.496 0.504]]

The probability that it will be in Mode I at 8:30 pm on the same day is 0.504

8. .



HMM are very much used in robotics. In the real world, The sensors collect information which are not complete. The fully observable states are less compared to the actual states but the fully observable states are obtained from the hidden states.

Let us consider the Vaccum world problem where there exists 2 states A and B and a vacuum cleaning agent which is present to clean the states

The agent will never know if it is in state A or B . it sensor can only detect wall and sense if a state is dirty or clean . This make the dirt/ clean to be the fully observable states of the vacuum agent in the vacuum world. The states A and B become the hidden state

A and B correspond to the behavioural state of the model as they are not sensed directly but can be obtained with enough information.

HMM relates the fully observable states and the hidden states by probabilities [Note : the probabilities used here are for representation purpose only]

Initially we have no knowledge of where the agent is located in the world or the relation between hidden and observed states.

The probabilities are calculated after multiple observation when the robot executes a task

PROGRAMMING ASSIGNMENT

MDP

FORMULA USED

$$Q^*(s,a) = R(s,a) + \gamma\, \mathbb{E}_{s'}[V^*(s')]$$

$$Q^*(s,a) = R(s,a) + \gamma \sum_{s' \in \mathbb{S}} p(s'|s,a)V^*(s')$$

Since,

$$V^*(S) = \max_a Q^*(s,a)$$

$$V^*(S) = \max_a \left[R(s,a) + \gamma \sum_{s' \in \mathbb{S}} p(s'|s,a)V^*(s') \right]$$

OUTPUT

```
(venv) (base) SUDHARSHANs-MacBook-Air:MDP sudharshan$ python ice.py < easyMDP.txt
Reading mode...MDP
Reading transition probabilities...
D D P R D
R R G P D
U U P D L
U U L L L
Calculating average utility...
Average utility per move: 21.22
```

```
(venv) (base) SUDHARSHANs-MacBook-Air:MDP sudharshan$ python ice.py < slippyMDP.txt
Reading mode...MDP
Reading transition probabilities...
D D P R D
R R G P D
U U P D D
U U L L L
Calculating average utility...
Average utility per move: 7.51
```

```
(venv) (base) SUDHARSHANs-MacBook-Air:MDP sudharshan$ python ice.py < verySlippyMDP.txt
Reading mode...MDP
Reading transition probabilities...
D L P R D
R L G P L
D L P D D
U L U R U
Calculating average utility...
Average utility per move: 14.78
```

```
(venv) (base) SUDHARSHANs-MacBook-Air:MDP sudharshan$ python ice.py < bigMDP.txt
Reading mode...MDP
Reading transition probabilities...
R P R G L P R R G L
P G U P U D U P U U
P P U P P D P D P U
P D U P P D D D L P
R R U L L L L L P G
Calculating average utility...
Average utility per move: 12.78
```

Q LEARNING

FORMULA USED

$$\underbrace{NewQ(s,a)}_{\text{New Q-value for that state and that action}} = \underbrace{Q(s,a)}_{\text{Current Q value}} + \underset{\text{Learning Rate}}{\alpha}[\underbrace{R(s,a)}_{\text{Reward for taking that action at that state}} + \underset{\text{Discount rate}}{\gamma} \underbrace{max\, Q'(s',a')}_{\text{Maximum expected future reward given the new s' and all possible actions at that new state}} - Q(s,a)]$$

OUTPUTS

```
(venv) (base) SUDHARSHANs-MacBook-Air:MDP sudharshan$ python ice.py < easyQ.txt
Reading mode...Q
Reading transition probabilities...
D D P R D
R R G P D
R U P D L
R U L L L
Calculating average utility...
Average utility per move: 21.34
```

```
(venv) (base) SUDHARSHANs-MacBook-Air:MDP sudharshan$ python ice.py < slippyQ.txt
Reading mode...Q
Reading transition probabilities...
D D P R D
R R G P D
U U P D D
U U L L L
Calculating average utility...
Average utility per move: 7.47
```

```
(venv) (base) SUDHARSHANs-MacBook-Air:MDP sudharshan$ python ice.py < verySlippyQ.txt
Reading mode...Q
Reading transition probabilities...
D L P R D
R L G P D
U L P D D
U L L L L
Calculating average utility...
Average utility per move: 11.42
```

```
(venv) (base) SUDHARSHANs-MacBook-Air:MDP sudharshan$ python ice.py < bigQ.txt
Reading mode...Q
Reading transition probabilities...
R P R G L P R R G L
P G L P U D U P U U
P P U P P D P D P U
P D U P P D D D L P
R R U L L L L L P G
Calculating average utility...
Average utility per move: 8.81
```