Advanced Gravity Field Modelling
Regionale Schwerefeldmodellierung

Exercise Sheet 2

Elaboration of:

Family name, first name	Student number
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If necessary 1. resubmission to	
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Feedback for teachers:

Time required for processing	-
Motivation; 1 (low) … 10 (high)	9
Level of difficulty of the exercise; 1 (low) … 10 (high)	4

Feedback for students:

Estimated time required for processing	
Estimated level of difficulty of the exercise; 1 (low) … 10 (high)	
Evaluation of "clean work"; 1 (scribble) … 10 (very clean)	
Technical correctness; 1 (low) … 10 (high)	
Scientific work; 1 (underdeveloped) … 10 (very pronounced)	
Comment:	

Form of the elaboration

You have to submit a pdf-document, which is created based on this template and therefore contains the title page and feedback, the written elaboration of the exercise sheet and the exercise.

The name (pdf file) must be clearly defined, e.g.
LVLV_BlattNr_ii__nnnvvv.pdf.
LVLV: Name of the course
ii: consecutive number, is incremented within the semester
nnnvvv: first 3 letters of surname, followed by the first 3 letters of surname Letters of the first name.

This elaboration should contain the description of the solution including necessary formulas and graphics. Finally, the results are to be discussed and analysed.

If calculation routines were created during processing, e.g. using MATLAB or Excel, these must also be uploaded to ILIAS.

Resubmission

A maximum of two resubmissions is possible per exercise sheet. Workouts cannot be accepted due to errors in content or for formal reasons. Formal reasons for resubmissions are:
- Late submission of the exercise sheet. If, for important reasons, it is not possible to process an exercise sheet within the scheduled time, this must be communicated to the trainer in good time.
- Incomplete processing of the exercise sheet. All subtasks and questions must be answered.
- Insufficient form of elaboration.

Feedback

Regular feedback is essential to improve teaching. Therefore, please complete the Feedback for Teachers section on the cover page of each exercise sheet.

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Advanced Gravity Field Modelling

Exercise 2: Boundary value problems of potential theory

For all boundary value problems of potential theory, the geometry of the boundary surface S is assumed to be known; the position vectors of the boundary points are therefore given.

What is unknown is the potential V in the space outside the boundary surface S, were the boundary values are given.

Compare the following in a table:
a) Boundary value problems of potential theory: name and boundary values and unknowns
b) Which observables could be used as boundary values in physical geodesy for this? State how these are defined and from which measurements they can be determined. Distinguish between the oceanic and continental areas of the earth's surface.
c) Give the solutions of the 1st to 3rd BVP assuming a spherical boundary surface.
Ans.

Boundary Value Problem	**Boundary Values**	**Unknowns**	**Physical Geodesy Observables**
1. Dirichlet Problem (First BVP)	Potential V on S	Potential V outside S	**Ocean:** Sea surface topography from satellite altimetry **Land:** Geoid heights from GNSS/leveling
2. Neumann Problem (Second BVP)	Normal derivative $\partial V/\partial n$ on S	Potential V outside S	**Ocean:** Gravity anomalies from shipborne/airborne gravimetry **Land:** Gravity anomalies from terrestrial gravimetry
3. Mixed Problem/Poincare) (Third BVP)	Linear combination of V and $\partial V/\partial n$ on S	Potential V outside S	**Ocean:** Combination of sea surface topography and gravity anomalies **Land:** Combination of geoid heights and gravity anomalies

(Dr.-Ing. Kurt Seitz, 2024)

The observables used as boundary values in physical geodesy are defined and determined as follows:

1. Sea surface topography:
i. Definition: The difference between the sea surface and the geoid
ii. Measurement: Satellite altimetry (e.g., Jason, Sentinel-3) measures the sea surface height relative to a reference ellipsoid

2. Geoid heights:
i. Definition: The separation between the geoid and a reference ellipsoid
ii. Measurement: Combination of GNSS positioning and spirit leveling

3. Gravity anomalies:
i. Definition: The difference between observed gravity and normal gravity on the geoid
ii. Measurement:
- Ocean: Shipborne or airborne gravimetry
- Land: Terrestrial gravimetry using absolute or relative gravimeters

c. Give the solutions of the 1st to 3rd BVP assuming a spherical boundary surface
Ans.
Solution for Spherical Boundary:
For a spherical boundary surface, the solutions to the 1st, 2nd, and 3rd boundary value problems (BVPs) in potential theory can be expressed using spherical harmonic expansions. Here are the solutions for each BVP:
1. 1st BVP (Dirichlet):

$$V(r,\vartheta,\lambda)=\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\left(v_{nm}^{C}\cos m\lambda+v_{nm}^{S}\sin m\lambda\right)\cdot P_{nm}(\cos\vartheta) \qquad (1)$$

2. 2nd BVP (Neumann):
In the special case of a spherical boundary surface and the kernel function H, the solution of the 2nd boundary value problem is:

$$V(r,\vartheta,\lambda)=-\frac{R}{4\pi}\iint_{\sigma} f(\vartheta',\lambda')\cdot H(\psi)\cdot d\sigma \qquad (2)$$

3. 3rd BVP (Mixed Problem):
For points on the spherical boundary S (r ⟶ R) the following Stroke's relation applies (do not get the formula for the potential on the spherical boundary surface as per 3rd BVP):

$$S(r=R,\psi)=S(\psi)=\frac{1}{\sin\frac{\psi}{2}}+1-6\sin\frac{\psi}{2}-5\cos\psi-3\cos\psi \qquad (3)$$

References:
Dr.-Ing. Kurt Seitz. (2024). *Advanced Gravity Field Modeling*.