Comparison of Various Mascon and Spherical Harmonic Solutions from Different GRACE Providers and Their Quality Assessment with Respect to Nepal Region

Lab Rotation-A (Physical Geodesy)

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M.Sc Remote Sensing and Geoinformatics

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February 1, 2025

original **Taskdescription**, *signed by the task creator*

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Topic of the task

The system earth is subject to various temporal changes. Those associated with mass shifts can be studied by monitoring the Earth's gravitational field. Satellite-based geodetic sensors open up the possibility of detecting both temporal and spatial variations in mass. Monthly solutions for the time-varying gravitational potential of the Earth are generated from this data, the long-wave components of which are determined very precisely.

For the necessary calculation steps, the candidate is provided with Matlab m-files, which are to be adapted to the task. The results obtained are to be presented graphically. Special emphasis is put on a thorough discussion and interpretation of the results obtained.

Task Creator: H. Kutterer
……………………………………….
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Supervisor: Dr.-Ing. Kurt Seitz

Start of the Lab Rotation: November 2024

DECLARATION

I, hereby declare that the content of this report is my original work, and I have not utilized any external resources without proper acknowledgment within the report.

Karlsruhe, Germany, 28 Jan. 2025
Signed

____

Name

Acknowledgement

Firstly, I would like to express my genuine gratitude to my (Supervisor) Examiner, Dr.-Ing. Kurt Seitz. Without him, this journey of the Lab Rotation in Physical Geodesy would not have been possible. From my early days at university during his lectures to reaching out to Prof. Nico Sneeuw at the Geodetic Institute of Stuttgart (GIS), University of Stuttgart through video calls and mail, on my behalf before I began my Lab Rotation at GIS, he has been a remarkable mentor and supporter for me. Moreover, during the process of generating results, he guided me both theoretically and practically.

Secondly, I extend my heartfelt thanks to Prof. Dr.-Ing. Nico Sneeuw for his insightful guidance, motivating feedback, and support in shaping a coherent research story with the data I had. His directions on how to proceed with the research were invaluable. I am deeply grateful to him for providing me with the opportunity to undertake a lab rotation at GIS, University of Stuttgart.

Thirdly, I would like to express my gratitude to my local supervisor, Dr. Yixiati Dilixiati, for his valuable input and feedback on my results during our weekly meetings at GIS.

Additionally, I would like to acknowledge Dr.-Ing. Michael Mayer for his valuable suggestions and counseling on administrative matters as an excellent student advisor.

I also wish to thank the following data providers for granting open access to their data:

- Mascon Solutions - GSFC, CSR, JPL
- Spherical Harmonic Solutions - ITSG, CSR, JPL
- River basins data – hydrosheds.org
- Shapefile data - gadm.org

I would like to remember my grandmother (*Hajuraa*) for her continuous selfless love and blessings. My acknowledgements would not be complete without expressing my heartfelt thanks to my parents and my brother for giving me the opportunity to pursue my dream of obtaining research-level education in Germany.

Lastly, I want to thank all the wonderful individuals who, directly or indirectly, contributed to this work and supported me in reaching this stage of my academic journey.

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List of Abbreviations

GRACE Gravity Recovery and Climate Experiment

SAR Synthetic Aperture Radar

GPS Global Positioning System

GNSS Global Navigation Satellite System

DEM Digital Elevation Model

WGS World Geodetic System

UTM Universal Transverse Mercator

NCEP National Centers for Environmental Prediction

RTM Regional Tomography Model

VM Virtual Model

Abstract

Firstly, I would like to express my genuine gratitude to my (Supervisor) Examiner, Dr.-Ing. Kurt Seitz. Without him, this journey of the Lab Rotation in Physical Geodesy would not have been possible. From my early days at university during his lectures to reaching out to Prof. Nico Sneeuw at the Geodetic Institute of Stuttgart (GIS), University of Stuttgart through video calls and mails, on my behalf before I began my Lab Rotation at GIS, he has been a remarkable mentor and supporter for me. Moreover, during the process of generating results, he guided me both theoretically and practically.

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1 Introduction

1.1 Background

The study of Earth's water cycle has evolved significantly over the past century, with various techniques developed to monitor and understand this complex system. Early methods relied heavily on ground-based measurements and hydrologic modeling, which began in the 1850s with Mulvany's work on peak discharge calculations and Darcy's groundwater flow experiments (Singh, 2018). While these approaches provided valuable insights, they were limited by their localized nature and inability to capture global-scale processes.

The advent of satellite technology in the 1960s marked a significant leap forward, offering a new perspective on Earth's water cycle across geopolitical boundaries (Lawford and Unninayar, 2017). By the 1970s, quantitative radiometric measurements from satellites enabled the estimation of variables such as precipitation and soil moisture (Lawford and Unninayar, 2017). However, these electromagnetic-based measurements had limitations in penetrating the Earth's surface and accurately quantifying subsurface water storage changes.

Recognizing the need for a more comprehensive approach to monitoring Earth's water cycle, NASA and the German Aerospace Center (DLR) collaborated to develop the Gravity Recovery and Climate Experiment (GRACE) mission, launched in March 17, 2002 (NASA Jet Propulsion Laboratory, 2016; European Space Agency (ESA)). GRACE introduced a revolutionary method for studying Earth's water cycle by measuring minute changes in the planet's gravitational field caused by water mass redistribution over land, atmosphere, and oceans (NASA Jet Propulsion Laboratory, 2016).

Unlike previous satellite missions, GRACE employed a unique twin-satellite configuration with a microwave ranging system to detect these gravitational variations (NASA Jet Propulsion Laboratory, 2016; European Space Agency (ESA)). This innovative approach allowed scientists to track changes in ice sheets, groundwater, ocean currents, and other components of the water cycle with unprecedented accuracy and global coverage (European Space Agency (ESA)).

GRACE's primary objective was to provide high-resolution monthly global maps of Earth's gravity field and surface mass changes (NASA Jet Propulsion Laboratory). This capability addressed the limitations of previous techniques by offering direct observations of water storage variations, including those occurring beneath the Earth's surface (NASA Jet Propulsion Laboratory, 2006). The mission's success in balancing the water cycle budget for entire continents, as demonstrated in

South America, highlighted its transformative impact on hydrological studies (NASA Jet Propulsion Laboratory, 2006).

The GRACE mission, followed by GRACE-FO in 2018, has significantly advanced our understanding of Earth's changing freshwater landscape, with profound implications for climate change research and water resource management (Tapley et al., 2019). By providing a truly global perspective on water cycle dynamics, GRACE has enabled scientists to study long-term trends and decadal variability in water storage, offering crucial insights for policy-making, resource planning and management (NASA Jet Propulsion Laboratory, 2016).

Two primary approaches have emerged for processing GRACE data: spherical harmonic (SH) solutions and mass concentration (mascon) solutions. Each method has its strengths and limitations, particularly when applied to regions with complex topography like Nepal. Spherical harmonic solutions represent the gravity field as a series of coefficients, offering global coverage but often requiring additional post-processing to reduce noise and signal leakage. Mascon solutions, on the other hand, estimate mass changes directly for predefined regions on Earth's surface, potentially offering better noise reduction and spatial localization of total water storage (TWS) signals (GRACE JPL NASA).

1.2 Statements of Problems

Analyzing and interpreting GRACE and GRACE-FO time series over the Nepal region is extremely challenging due to following reasons:

i. The limited size of Nepal region compared to GRACE's inherent spatial resolution of approximately 300-400 km makes precise satellite gravimetric measurements challenging.

ii. The existing fundamental vertical aggregation of mass signals, i.e. the fact that satellite gravimetry cannot separate groundwater storage change from surface water change, glacier melt and tectonic signals without external information.

iii. The presence of the high noise levels in the gravimetric data, such as striping patterns, bias; necessitates robust filtering, which induces leakage effects. Hence, for instance strong gravitational signals from ground water abstraction in the Ganges plain, Tibetan Plateau will likely leak into the Nepal signal.

1.3 Objectives

i. Specific Objectives

a. To compare various mascon solutions and spherical harmonic solutions from different GRACE data providers and their quality assessment with respect to reference CSR mascon solution, in terms of interpreting total water storage for Nepal region.

ii. General Objectives

a. To evaluate the strengths and limitations of mascon and spherical harmonic solutions in capturing total water storage dynamics in region with complex topography.

b. To assess the impact of different GRACE data processing methodologies on the interpretation of hydrological processes in mountainous areas like Nepal.

c. To investigate the potential advantages of mascon solutions over spherical harmonic solutions in reducing noise levels and bias, such as North-South striping patterns.

d. To provide insights that could inform the selection of appropriate GRACE data products for hydrological studies in topographically complex Nepal region or similar other regions.

1.4 Scope and Limitations of the Study

i. Scope

a. To grasp the satellite gravimetric signal content relative to the noise level of GRACE products over a small Nepal region.

b. Illustration to download, process and compare different mascons solutions and SH solutions from different GRACE providers.

c. To perform exploratory data analysis, both statistical and visual, on time series and/or regional maps from mascons and SH solutions

d. Evaluation of the performance of different GRACE solutions in capturing seasonality and long-term trends in TWS for Nepal region.

d. To obtain a spectrum of perspectives for the stability (or rather instability) of the basic GRACE products for small areas like Nepal from the exploratory data analysis.

ii. Limitations

a. Grasped satellite gravimetric signal content relative to the noise level of GRACE products over Nepal region does not apply corrections for bias (external noises), and induced leakage effects as a result of strong filtering.

b. Smaller size of Nepal region, in comparison to the inherent spatial resolution of GRACE.

c. Coarse spatial resolution of GRACE data (approximately 300-400 km), which may limit the ability to capture fine-scale hydrological processes in Nepal's diverse terrain.

d. Monthly temporal resolution of GRACE data, which may not capture rapid hydrological changes or short-term events.

e. TWS is not disaggregated into individual water storage components under this research (which requires additional data or modeling).

f. GRACE products are not validated against ground-based insitu observations for comparison.

g. Potential uncertainties in GRACE-derived TWS estimates (which is not the part of research here), which may vary between different solutions as a result of different post-processing strategies for Mascons and SH solutions.

h. Seggregation of anthropogenic and climate-driven changes in TWS is not done under this research.

1.5 Outline of the Project

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This study aims to comprehensively compare various mascon and SH solutions from different GRACE data providers (Mascon Solutions: GSFC, CSR, JPL; SH Solutions: ITSG, CSR, JPL) over a 20-year period (April 2002 to April 2022), focusing on their performance in interpreting TWS changes in the Nepal region. By analyzing mean, variability, time series characteristics, linear trends, and seasonal patterns, we seek to evaluate the strengths and limitations of each solution type when applied to a hydrologically complex area.

Our analysis includes a multi-faceted approach, incorporating time series decomposition, and spatial mapping of linear trends and seasonal averages. Additionally, we employ statistical techniques such as scatter density plots, regression analysis, and evaluation metrics (R, RMSE, Bias, and MAE) to quantify the agreement between different datasets and a reference CSR Mascon solution. This comprehensive assessment aims to provide insights into the relative performance of mascon and SH solutions in capturing TWS dynamics in Nepal, potentially informing future water resource management strategies and enhancing our understanding of hydrological processes in mountainous regions (Croteau et al., 2021; Janák et al., 2020; GRACE JPL NASA).

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2 Theoretical Aspects

Several theoretical aspects were reviewed before diving into the generation of the results for the comparision of the various mascon solutions and SH solutions via scientific papers, reports, presentations, news.

2.1 GRACE Mission, Common Terms, and Data Products

2.1.1 GRACE concepts and its Principles

2.2 Spherical Harmonic Synthesis (SHS)

Spherical harmonics are given in Equation (1). Spherical Harmonic Synthesis is the forward computation of a function on the sphere from a series of surface spherical harmonics (**?**). The spherical harmonic coefficients derived from GRACE level-2 data is used to express the gravity field of earth.

$$V(P) = \frac{GM}{r} \sum_{n=0}^{\infty} \left(\frac{R}{r}\right)^n \sum_{m=0}^{n} (\bar{C}_{nm}\ cosm\lambda + \bar{S}_{nm}\ sinm\lambda)\bar{P}_{nm}(sin\varphi) \tag{1}$$

where,
GM : Geocentric gravitational constant
R : Radius of reference sphere.
$\bar{C}_{nm}, \bar{S}_{nm}$: Fully normalised spherical harmonic coefficients of degree n and order m
$\bar{P}_{nm}$: Fully normalised Legendre functions of degree n and order m.

2.3 Variance propagation

The degree variance quantifies the amplitudes of the spherical harmonic coefficients, which vary across different degrees. Summing up of squared coefficients express the overall energy in the signal as shown in Equation (2). The plot depicts that there is a higher degree variance at lower degrees, and the value drops as the degree (n) increases. This shows that there is a higher concentration of signal at lower degrees when compared to higher degrees.

$$C_n^2(\bar{C}_{nm}, \bar{S}_{nm}) = \sum_{m=0}^{n} (C_{nm}^2 + S_{nm}^2) \tag{2}$$

This helps to understand the distribution of signal intensity through the frequency range. Degree variance for the coefficients obtained for this study has been analysed, which shows that the signal has higher energy in the long wavelength part and it drops in shorter wavelength part of the signal. The plots for degree variance for all the months are similar with marginal variations, so an example plot for January, 2022 is shown in Figure 1. The degree variance is non dependent on altitude of P.

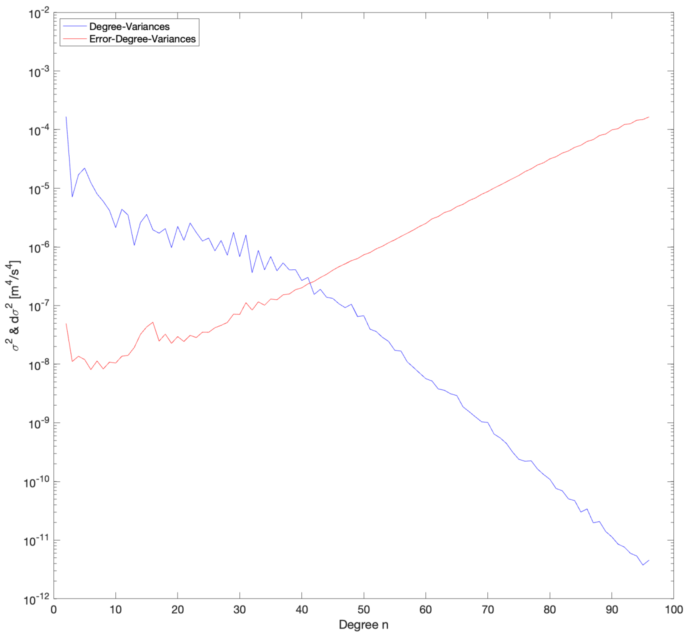

Figure 1: Degree variance and Error degree variance

2.4 Mass discretization using Tesseroids

Tesseroids are three dimensional shapes of constant height bounded by longitudinal planes (λ_1, λ_2), latitudinal planes (φ_1, φ_2), and concentric spherical planes (r_1, r_2) as shown in Figure 2.

Newton's Integral can be applied on the tesseroids to derive gravitational potential values V of a solid volume Ω.

$$V(r,\varphi,\lambda) = G\rho \iiint\limits_{\Omega} \frac{1}{l} d\Omega = G\rho \int\limits_{\lambda_1}^{\lambda_2} \int\limits_{\varphi_1}^{\varphi_2} \int\limits_{r_1}^{r_2} \frac{r'^2 cos\varphi'}{l} dr' d\varphi' d\lambda' \tag{3}$$

$$l(P,Q) = \sqrt{r^2 + r'^2 - 2rr'cos\psi} \tag{4}$$

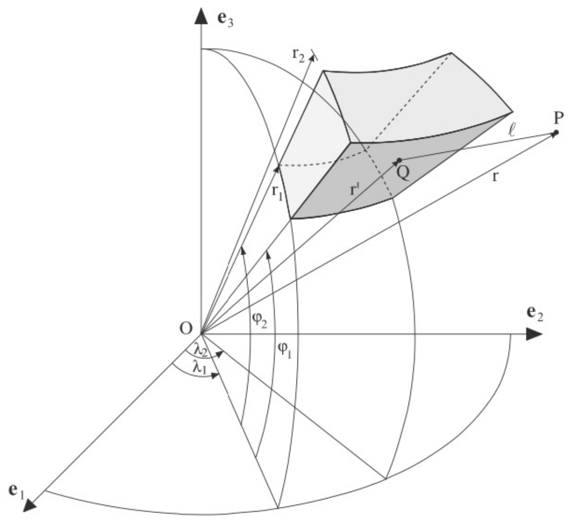

Figure 2: Geometry of a spherical tesseroid (**?**)

$$cos\psi = sin\varphi\, sin\varphi' + cos\varphi\, cos\varphi'\, cos(\lambda' - \lambda) \tag{5}$$

G : Newton's gravitational constant

ρ : density function at running integration point (Q).

l : Euclidean distance between the computation point $P(r, \varphi, \lambda)$ and the running integration point $Q(r', \varphi', \lambda')$

ψ : Angle between the points P and Q.

Newton' integral as mentioned in Equation (3) cannot be solved analytically in the case of tesseroids due presence of elliptical integrals. Taylor series expansion can be used for best possible solution. The Taylor point $(r_0, \varphi_0, \lambda_0)$ for the series expansion is set at the geometrical centre of each tesseroid for maximum efficiency in calculation of potentials (**?**).

$$r_0 = \frac{(r_1 + r_2)}{2}; \quad \varphi_0 = \frac{(\varphi_1 + \varphi_2)}{2}; \quad \lambda_0 = \frac{(\lambda_1 + \lambda_2)}{2} \tag{6}$$

The integrand in equation (3) which represents the kernel $K(r', \varphi', \lambda')$, upon taylor expansion forms Equation (8). The coefficients K_{ijk} denotes the partial derivatives of integration kernel.

$$K(r', \varphi', \lambda') = \sum_{i,j,k} K_{ijk}(r' - r_0)^i(\varphi' - \varphi_0)^j(\lambda' - \lambda_0)^k \tag{7}$$

From above equations, gravitational potential of tesseroid with constant density can be formulated as:

$$v(r,\varphi,\lambda) = G\rho\Delta r\Delta\varphi\Delta\lambda \left[K_{000} + \frac{1}{24}(K_{200}\Delta r^2 + K_{020}\Delta\varphi^2 + K_{002}\Delta\lambda^2) + \mathcal{O}(\Delta^4)\right] \tag{8}$$

with even powers of coefficient K as the odd terms cancel out due to the position of Taylor point being at the geometric centre of the tesseroid (**?**). The Taylor expansion terms in the order of four and above are neglected, which is represented by $\mathcal{O}(\Delta^4)$. Here,

$$\Delta r = r_2 - r_1; \quad \Delta\varphi = \varphi_2 - \varphi_1; \quad \Delta\lambda = \lambda_2 - \lambda_1 \ . \tag{9}$$

2.5 Inverse Modeling and Parameter regularization

Gravity forward modeling is a computation technique in which the gravity field of the Earth can be temporally simulated in relation to the mass distribution within the Earth by using a set of parameters.

3 Data

The level-2 data used in this study is the monthly solutions from GRACE-FO mission. It includes fully normalized spherical harmonic coefficients that assist in the computation of gravitational potential of the Earth, generated by release 6.1 of GeoForschungsZentrum (GFZ) (**?**) compiled by ICGEM.

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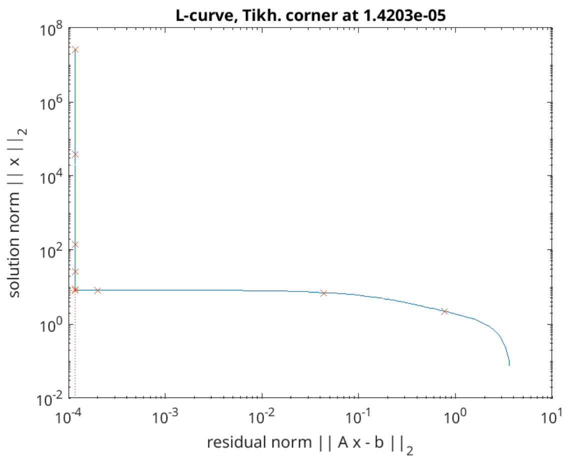
4 Methodology Overview

The residual gravitational potential δV values can be projected in a 1°x1° grid on the earth surface, separately for altitudes of computation point P. Spherical harmonic coefficients can be applied in Equation (1) to perform Spherical Harmonic Synthesis as shown in Figure 3 and 5. The discretized mass elements known as tesseroids possess uniform surface panels $(\Delta\varphi, \Delta\lambda)$ with constant density where the radial component Δr is assumed to create residual gravitational potential δV. Thus the residual potential can be expressed in terms of Δr, as shown in Equation (10), which is used to develop the design matrix A.

$$\delta V(r,\varphi,\lambda) = G\rho\Delta\varphi\Delta\lambda \sum_k \Delta r_k \left[K_{000} + \frac{1}{24}(K_{200}\Delta r^2 + K_{020}\Delta\varphi^2 + K_{002}\Delta\lambda^2) + \mathcal{O}(\Delta^4)\right]_{k'} \quad (10)$$

Potential values derived through synthesis are introduced as observations L in Equation (10) for least squares adjustment. The vector of unknowns represent the radial height change Δr of tesseroid bodies. Here, the observation vector L is of size [m x 1], unknowns vectors x is of size [n x 1] and the design matrix A of size [m x n], such that $m > n$ to ensure a over determined system of equations (**?**). So the observations from synthesis is derived for a grid of 1°x1° spatial resolution at computation point P, while the unknowns Δr at Q is calculated for a spatial grid of 2°x2°. The solution for unknowns can be formulated as:

$$\hat{x} = (A^T P A)^{-1} A^T P L; \qquad P = I\ . \quad (11)$$



(a) (b)

Figure 3: **(a)** shows L-curve for P at an altitude of 260 km for the month of January, 2022. **(b)** shows L-curve for P at an altitude of 500 km for the month of January,2022. L-curve for all months has been included in Appendix 8.1

Table 1: Tikhonov (optimum) corner values for all observations.

Months in 2022	Regularization parameter (260 km)	Regularization parameter (500 km)
January	1.420e-05	2.183e-06
April	8.601e-06	1.118e-06
July	1.017e-05	6.773e-07
October	1.017e-05	2.580e-06

5 Methodology

5.1 Research Design

5.2 Data Collection

5.3 Data Analysis

5.4 Ethical Considerations

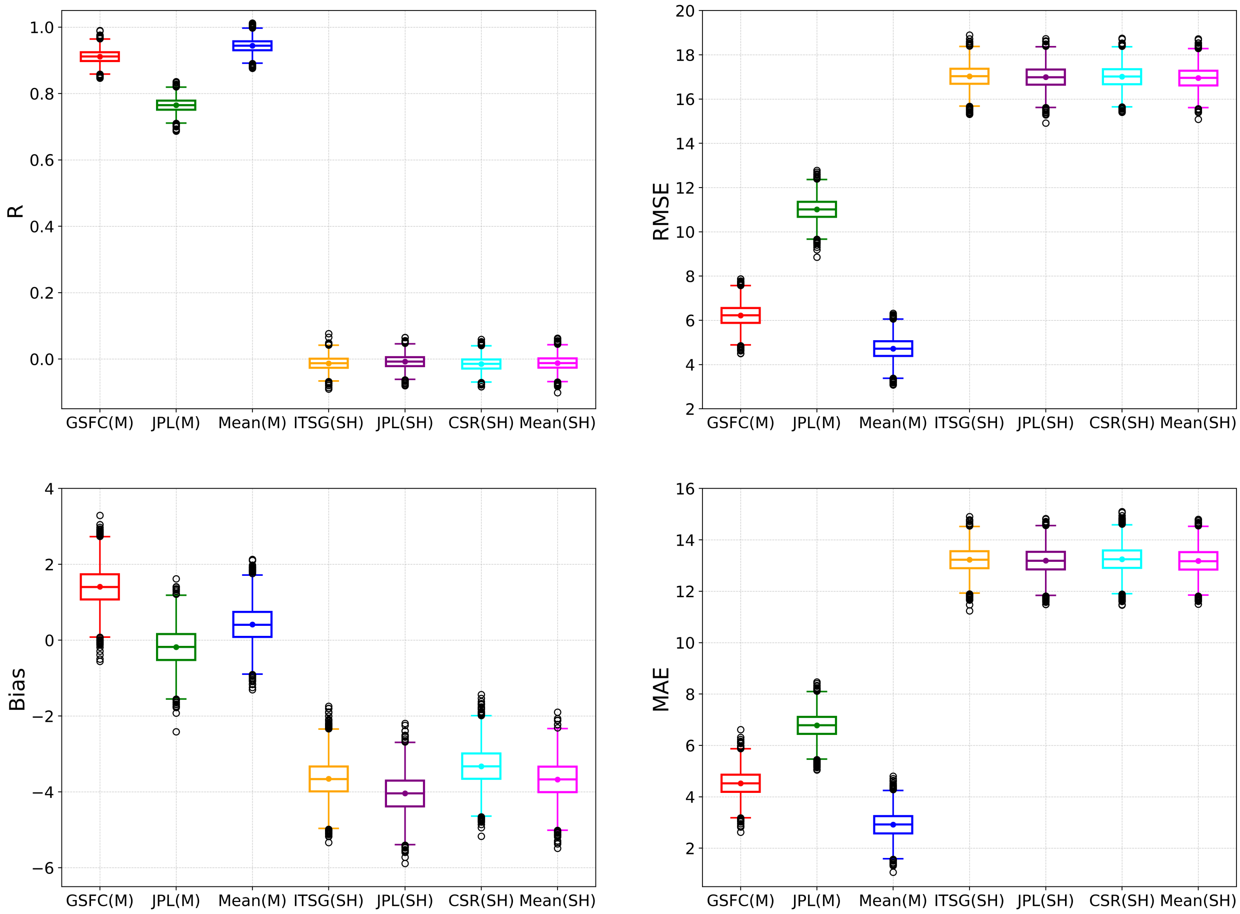


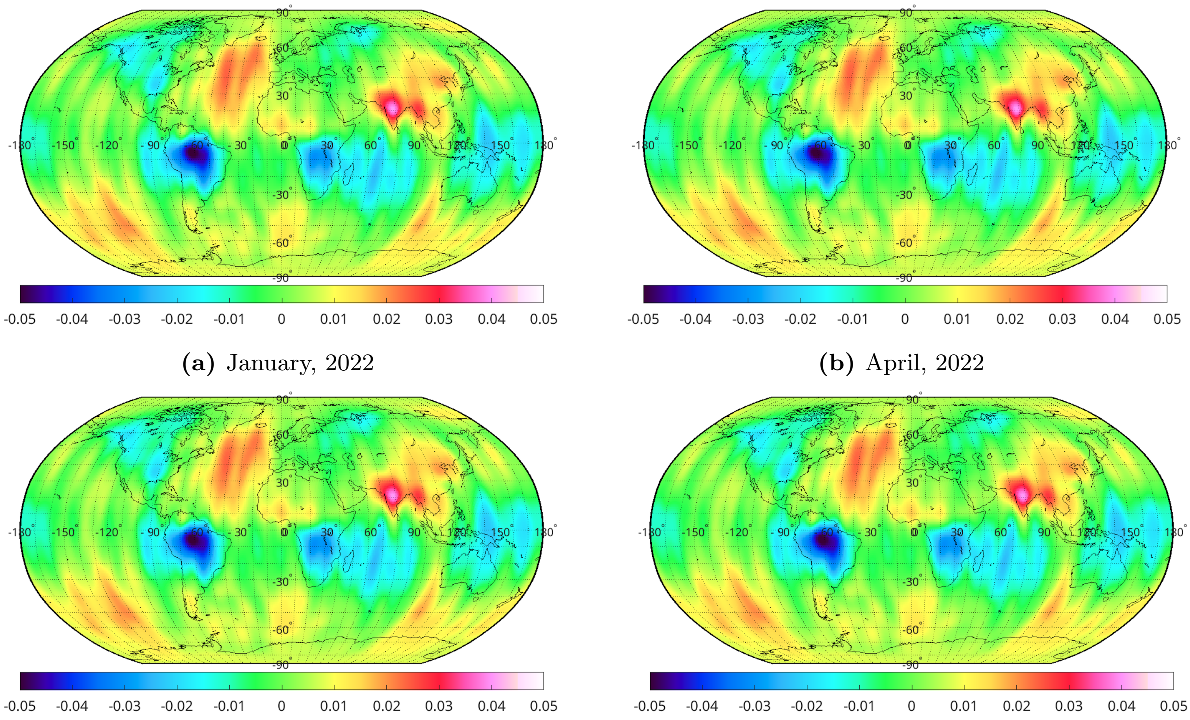
Figure 4: boxplot

6 Results and Discussion

The results obtained from spherical harmonic synthesis of coefficients and derived estimates of TWS through inversion techniques has been included in this section. The estimated heights of tesseroids has been refined using adjustment and regularization techniques. The potential values derived using Equation (10) is compared to the potential of synthesis to analyze its deviation.

6.1 Assessment of SHS results

this is nice



(a) January, 2022

(b) April, 2022

(c) July, 2022

(d) October, 2022

Figure 5: δV (in m^2/s^2) calculated using forward computation with computation point P at 260 km altitude.

SHS results are described in the next section ...

Table 2: Statistics for inversion results for TWS values (mm)

Computation point at altitude of 260 km					
Months in 2022	Minimum	Maximum	Mean	RMS	Std. Dev.
January	-280.189	315.375	5.512	65.537	12.092
April	-324.638	420.048	-3.904	79.007	16.514
July	-334.347	397.598	1.679	65.684	14.205
October	-364.401	336.873	-7.065	81.753	17.012
Computation point at altitude of 500 km					
Months in 2022	Minimum	Maximum	Mean	RMS	Std. Dev.
January	-270.296	303.833	5.427	63.612	11.687
April	-314.842	404.001	-3.846	76.562	15.968
July	-321.763	383.310	1.660	63.627	13.738
October	-350.411	326.312	-6.971	79.275	16.495

7 Conclusion

GRACE mission is the pioneer mission that allowed reliable quantification of TWS in a global scale. The quality of user data provided by the gravity mission has shown a significant increase in the recent years since the launch of GRACE, especially in regard to the temporal and spatial resolution. Compared to RL01, the current release RL06.1, a significant reduction in data errors has been achieved. This report delves into the estimation of terrestrial water storage using an inverse tesseroid approach, where the masses that generate changes in gravitational potential is considered to be water masses. These masses are discretized as tesseroids in uniform grids of 2°x2° and the changes in height of each tesseroid is assumed to be the responsible parameter that causes changes in potential. This demanded Taylor series expansion due to the presence of elliptical integrals. Geometrical centre of tesseroid was chosen as the Taylor point for peak effectiveness. The computed potential values from synthesis of spherical harmonic coefficients are introduced as observation equation to do a least squares adjustment where the height of tesseroids are considered as unknowns, which is associated to TWS. The generated results for four months, representing each season of the year, are analyzed. The analysis is done for computation altitude of 260 km and 500 km, which reveals the importance of choosing computation point altitude depending on the scope of study. At higher altitude the effect small scale features on earth on gravitational potential is diminished and the same effect is visible in case of TWS results.

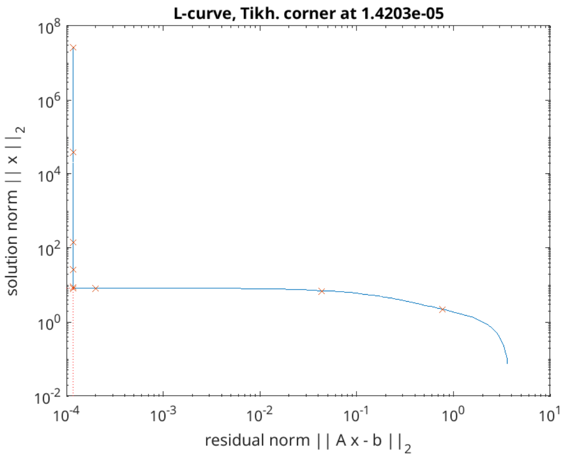
A more precise depiction of TWS can be achieved by eliminating the striping effect observed in the results. Various post processing techniques can help in reducing this correlation errors. A usage of 'correlated error filter' has been analysed in (**?**). Successor mission for GRACE is expected to adopt a mission architecture that could considerably reduce undesirable errors such as the correlation errors (**?**).

8 Acknowledgement

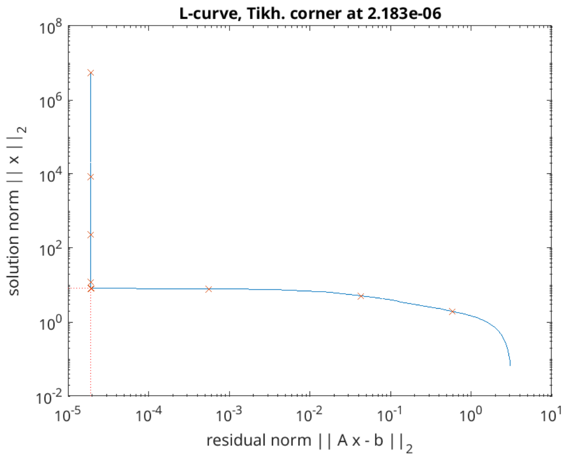
The author wishes to extend his gratitude to Prof. Dr.-Ing. Hansjörg Kutterer , for the Lab Rotation opportunity. Gratitude is also owed to the supervisor, Prof. Dr. Kurt Seitz for his valuable guidance and support throughout the course of Lab rotation. His constructive feedback has been helpful in shaping the outcome of this study. The author acknowledges the support from Geodätisches Institut (GIK) for providing necessary resources and facilities. Additionally, the author acknowledges ICGEM (**?**) for the data provided for this study.

9 Appendices

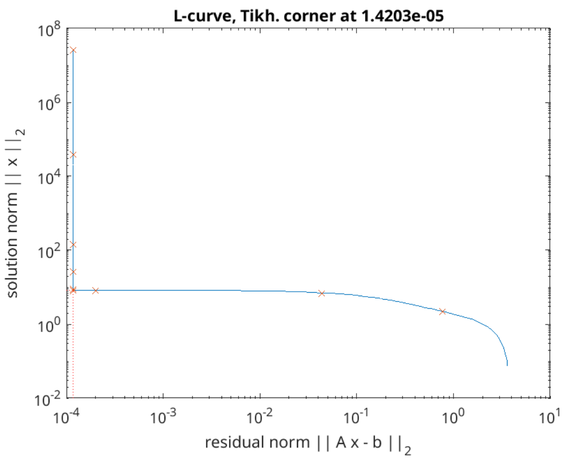
9.1 L-curve of regularization parameter



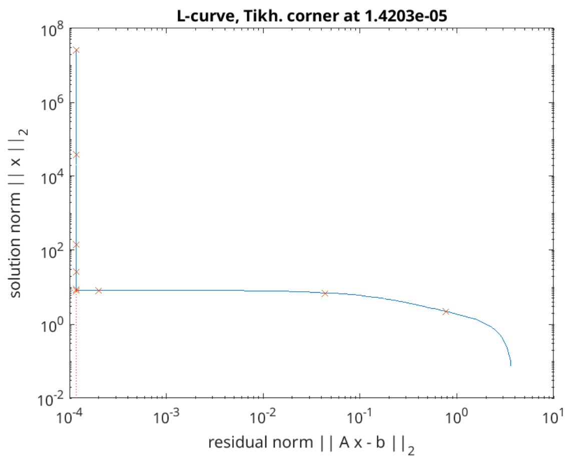
(a) January



(b) April



(c) July



(d) October

Figure 6: L-curves for P at altitude of 260 km

References

M. J. Croteau, T. J. Sabaka, and B. D. Loomis. Grace fast mascons from spherical harmonics and a regularization design trade study. *Journal of Geophysical Research: Solid Earth*, 126:e2021JB022113, 2021. doi: 10.1029/2021JB022113. URL `https://doi.org/10.1029/2021JB022113`.

European Space Agency (ESA). Grace mission objectives. ESA Earth Observation Gateway. URL `https://earth.esa.int/eogateway/missions/grace/objectives`. Accessed: 29 Jan. 2025.

GRACE JPL NASA. Choosing a grace data solution. URL `https://grace.jpl.nasa.gov/data/choosing-a-solution/`. Accessed: 29 Jan. 2025.

J. Janák, A. Novák, and B. Korekáčová. Mascon versus spherical harmonic solutions to global monthly time varying gravity field, egu general assembly g4.1 satellite gravimetry: Data analysis, results and future mission concepts. EGU General Assembly 2020, 2020. URL https://presentations.copernicus.org/EGU2020/EGU2020-21563_presentation.pdf. Accessed: 29 Jan. 2025.

R. Lawford and S. Unninayar. Historical development of the global water cycle as a science framework. Oxford Research Encyclopedia of Environmental Science, March 29 2017. URL https://oxfordre.com/environmentalscience/view/10.1093/acrefore/9780199389414.001.0001/acrefore-9780199389414-e-55. Retrieved 29 Jan. 2025.

NASA Jet Propulsion Laboratory. Grace-fo mission overview. NASA JPL News. URL https://www.jpl.nasa.gov/news/press_kits/grace-fo/mission/. Accessed: 29 Jan. 2025.

NASA Jet Propulsion Laboratory. Nasa satellites find balance in south america's water cycle. NASA JPL News, July 5 2006. URL https://www.jpl.nasa.gov/news/nasa-satellites-find-balance-in-south-americas-water-cycle/. Accessed: 29 Jan. 2025.

NASA Jet Propulsion Laboratory. Applications plan for the gravity recovery and climate experiment (grace) missions: Grace, grace-fo, and future missions. GRACE Website, October 2 2016. URL https://grace.jpl.nasa.gov/internal_resources/114/. Accessed: 29 Jan. 2025.

V. P. Singh. Hydrologic modeling: progress and future directions. *Geoscience Letters*, 5(1):15, 2018. doi: 10.1186/s40562-018-0113-z. URL https://doi.org/10.1186/s40562-018-0113-z.

B. D. Tapley, M. M. Watkins, F. Flechtner, et al. Contributions of grace to understanding climate change. *Nature Climate Change*, 9:358–369, 2019. doi: 10.1038/s41558-019-0456-2. URL https://doi.org/10.1038/s41558-019-0456-2.