Karlsruher Institut für Technologie

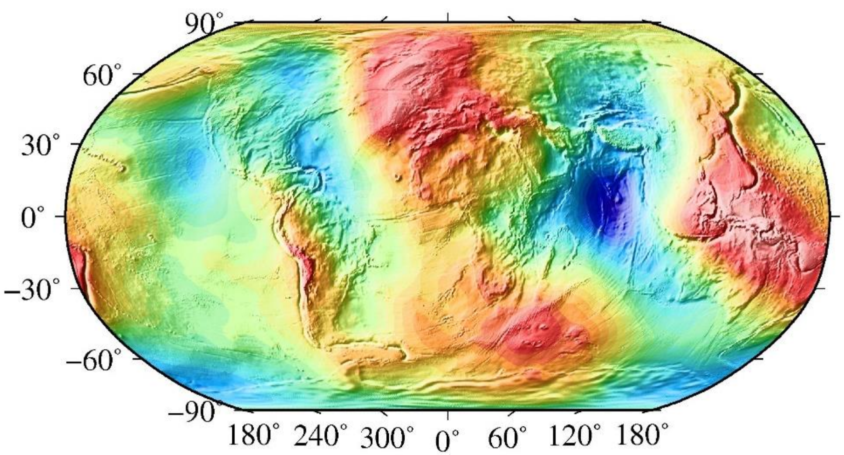
M.Sc. **Environmental Geodesy – Geodetic Earth Observation**
Mass Variations

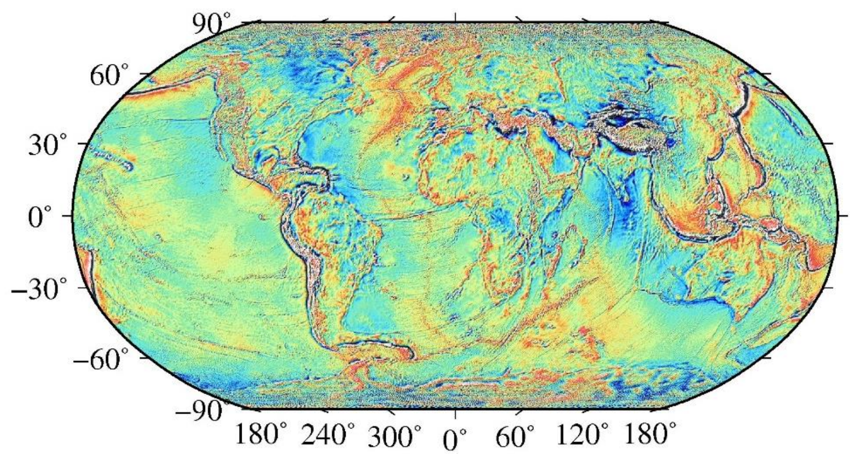
Isostatic reductions to gravity anomalies

Kurt Seitz

Winter-Semester

GEODETIC INSTITUTE – GEODETIC EARTH SYSTEM SCIENCE



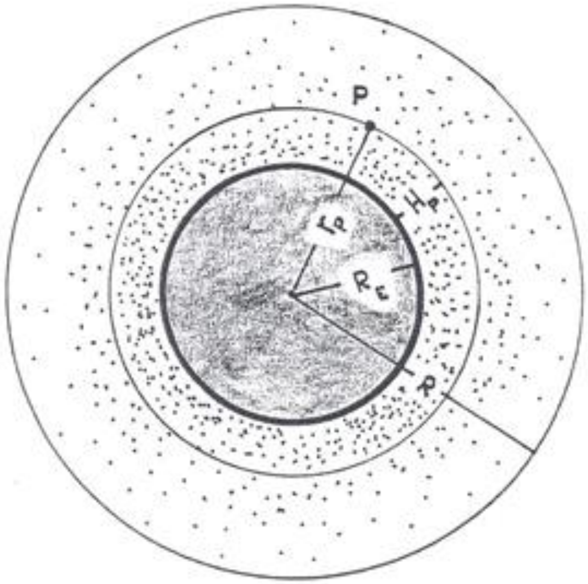


Reductions to gravity

Reductions

Atmospheric reduction

In geoid determination, the mass of the Earth's atmosphere is added to the mass of the solid earth. A correction must therefore be made to the measured gravity values, which shifts the atmospheric mass under the geoid.



This correction is always positive due to the increased mass below the point level.

From the model, the mass of the entire Earth's atmosphere is shifted as point mass into the center of the earth and a height-dependent correction term is attached to the observations. This creates a small indirect effect.

It would also be possible to eliminate the masses of the Earth's atmosphere by projection into infinity, but this would have a very large indirect effect.

Reductions

Atmospheric reduction
The following table shows the corrections for some elevation values.
They are always positive and must be added to the measured severity value.

The values are set according to the approximation formula

$$\delta g_A \approx 0{,}874 - 9{,}9 \cdot 10^{-5} \cdot H + 3{,}5625 \cdot 10^{-9} \cdot H^2$$

units H in [m] and δg_A in [mgal].

H [km]	[mgal]
0	0,874
1	0,779
2	0,688
3	0,606
4	0,535
5	0,468
6	0,410
7	0,356
8	0,310
9	0,272

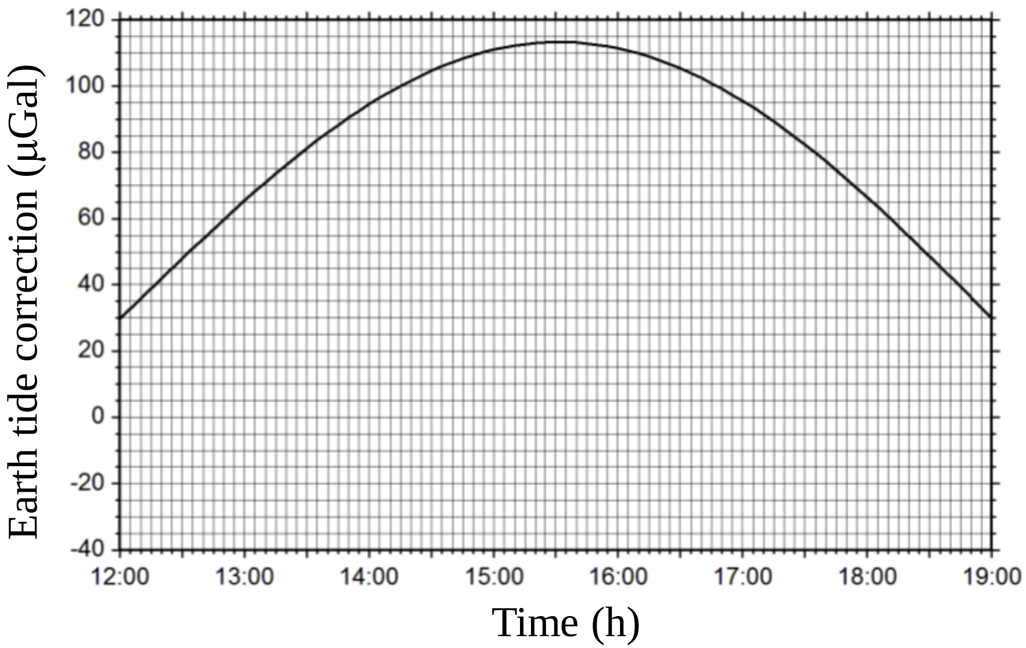
H [km]	[mgal]
10	0,231
20	0,047
30	0,010
40	0,002
50	0,001
60	0,000

Reductions

Earth tides

- Are caused by
  - Sun
  - Moon
  - Planets
- They can be modelled analytically
  - Earth tidal correction by means of tidal potential development according to Cartwright, Tayler and Edden (1973) with 505 single waves

$$\boxed{\delta g(\varphi,\lambda,h,t) = \sum_{i=1}^{505} \delta_i \, A_{i(starr)} \cos(\omega_i \, t + \Phi_{i(starr)} + \Delta\Phi_i)}$$



Reductions

Free air reduction

The free air reduction represents a pure vertical distance reduction, which is $\boxed{\delta g_F = \frac{\partial g}{\partial h} \cdot H}$

It reduces the measured gravity value ***g*** to a different height level.

Applications:

Reduce gravitational measurement to marking.

Geoid calculation: The reduction of the gravity value measured at the Earth's surface to the reference surface (geoid) takes place after the masses between the Earth's surface and geoid have been eliminated by means of the topographical reduction, by applying the **Free Air reduction**.

Used value for the gradient: Lack of knowledge of the actual vertical gravity gradient. This is replaced by the normal gravity gradient

So that the following applies to the free air reduction:

$$\boxed{\delta g_F = \frac{\partial g}{\partial h} \cdot H \approx \delta \gamma_F = \frac{\partial \gamma}{\partial h} \cdot H = -0.3086\,mgal/m \cdot H[m] = -0.3086 \cdot 10^{-5} \cdot s^{-2} \cdot H[m]}$$

Reductions

Used value for the gradient: Lack of knowledge of the actual vertical gravity gradient.
This is replaced by the **normal gravity gradient**
So that the following applies to the free air reduction:

$$\delta g_F = \frac{\partial g}{\partial h} \cdot H \approx \delta \gamma_F = \frac{\partial \gamma}{\partial h} \cdot H = -0.3086\, mgal / m \cdot H[m] = -0.3086 \cdot 10^{-5} \cdot s^{-2} \cdot H[m]$$

The normal gravity at the ellipsoidal height h[km] reads as follows:

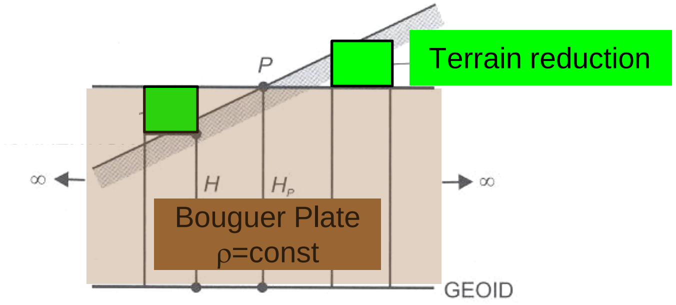
$$\gamma = \gamma_0 - \left(3.0877 \cdot 10^{-3} - 4.3 \cdot 10^{-6} \cdot \sin^2 \varphi\right) \cdot h + 0.72 \cdot 10^{-6} \cdot h^2$$

Reductions

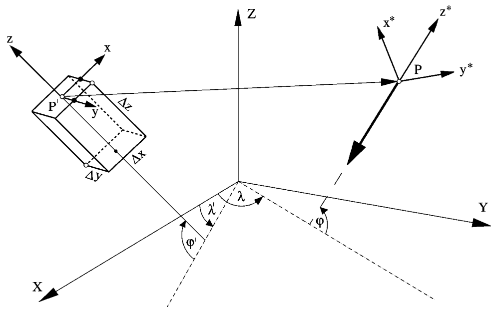
Topographic reduction $\delta g_{topo} = \delta g_{BP} + \delta g_{terrain}$

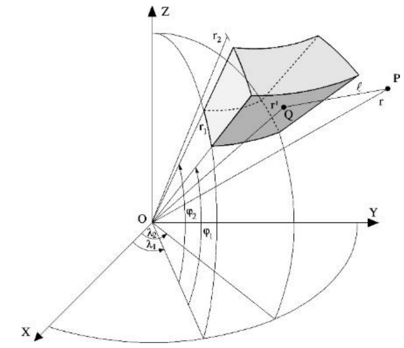
The topographical reduction consists of the

- ❑ Plate reduction (Bouguer's plate reduction) δg_{BP}
- ❑ and the terrain reduction $\delta g_{terrain}$



- ❑ Mass discretizatins are neccessary
  - ❑ Prism
  - ❑ Tesseroid
  - ❑ Planar (Bougure-)plate $\delta g_{BP} = 2 \cdot \pi \cdot G \cdot \rho \cdot H_P$





The normal gravity potential

The normal gravity on the ellipsoid can be calculated with the formula of Somigliana depending on the geographical latitude φ

$$\gamma_0 = \frac{a \cdot \gamma_a \cdot cos^2 \varphi + b \cdot \gamma_b \cdot sin^2 \varphi}{\sqrt{a^2 \cdot cos^2 \varphi + b^2 \cdot sin^2 \varphi}}$$

or specify in a series development:

$$\gamma_0 = \gamma_a \big(1 + 0.0052790414 \cdot sin^2 \varphi + 0.0000232718 \cdot sin^4 \varphi + 0.0000001262 \cdot sin^6 \varphi + 0.0000000007 \cdot sin^8 \varphi\big)$$

The normal gravity at the ellipsoidal height h[km] reads as follows:

$$\gamma = \gamma_0 - \left(3.0877 \cdot 10^{-3} - 4.3 \cdot 10^{-6} \cdot sin^2 \varphi\right) \cdot h + 0.72 \cdot 10^{-6} \cdot h^2$$

Gravity anomalies

Free air anomaly

$$\Delta g_F = g_P - \delta g_F - \gamma_0$$

Bouguer anomaly

$$\Delta g_B = g_P - \delta g_F - \delta g_{BP} - \delta g_{terrain} - \gamma_0$$

$$\Delta g_B = g_P - \delta g_F - \delta g_{topo} - \gamma_0$$

Faye anomaly

$$\Delta g_{Fa} = g_P - \delta g_F - \delta g_{terrain} - \gamma_0$$

Isostatic anomaly

$$\Delta g_I = g_P - \delta g_F - \delta g_{BP} - \delta g_{terrain} - \delta g_I - \gamma_0$$

Isostatic Reductions

Isostatic Concepts

Isostatic Concepts

- **Isostasy**: State of hydrostatic equilibrium of the Earth's crust or lithosphere. Mass surpluses due to the topography in the continental area or mass deficits in the oceanic area are each balanced out by compensation masses. The phenomenon of isostasy was first discovered in 1749 by P. Bouguer, who noticed that the mass attraction of the Andes was significantly smaller than expected. This observation was confirmed as part of the colonial survey in India (1800-1870): The impact of the masses of the Himalayas on the vertical deviation was significantly smaller than calculated from the visible masses.
- 1855, J.H. Pratt, deflection of the vertical in the southern Himalayas:
  - Calculated from topographical masses: 28″
  - Measured: 5″
- These deviations can be explained by the fact that the visible mass excesses in the Earth's interior are compensated for. The aim of the isostatic reduction is to take the gravity of the compensation masses into account.
- From the middle of the 19th century, various isostasy models were introduced to describe and explain the phenomenon.

- Torge (1975)
- Heiskanen & Moritz (1967)
- http://www.geodz.com/deu/d/Isostasie

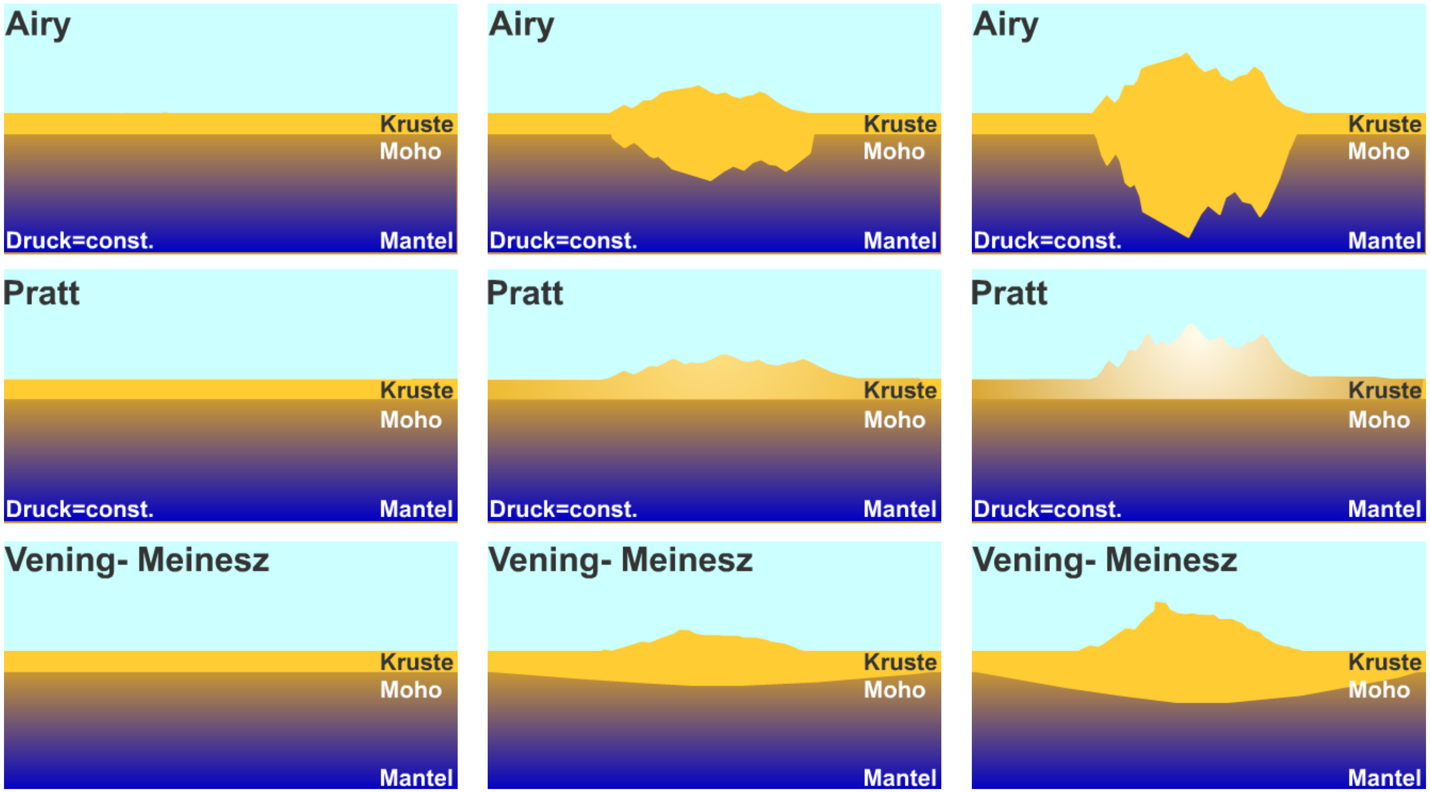
Isostatic Concepts

Determination of isostatic masses

- Classic approach: Isostatic balancing masses are calculated from the loading of the topographic masses (Heiskanen and Moritz, 1967)
- Principle of mass equality in relation to a compensation depth to be determined
- Classic models assume a constant density distribution of the topographic masses
- Models must be transferred to the 3-layer approach of the RWI method
    - RWI = Rock Water Ice
  - Model of Airy-Heiskanen
  - Model of Pratt-Hayford
  - combined Airy-Heiskanen / Pratt-Hayford Model

Isostatic Concepts

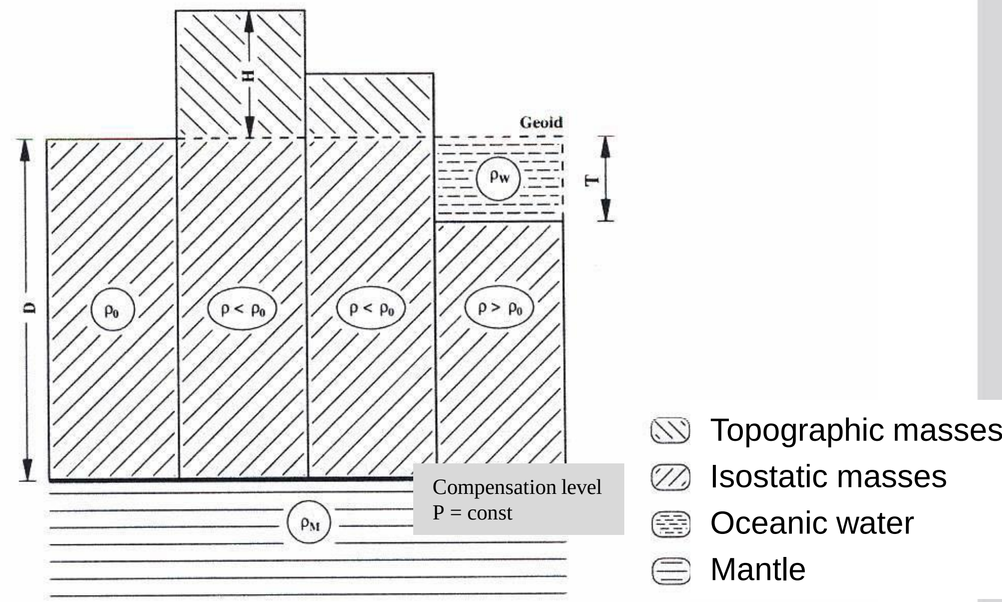
- **Mountain growth**
  - Airy-Heiskanen model: local compensation - "floating iceberg"
  - Pratt-Hayford model: local compensation - "rising yeast dough"
  - Vening- Meinesz model: regional compensation -elastic-plastic plate



- http://www.geophysik.uni-kiel.de/~sabine/DieErde/Werkzeuge/Geophysik/M1-Grav/5Isostasie/Isostasie.htm

Pratt-Hayford-Model (planar)

- Topographic masses
  - Density ρ_{TM}
- Mantle
  - Density ρ_M
- Isostatic masses
  - Normal colum with density ρ_0
  - Isostatic mass-column has density ρ
- Compensation level in the depht D
  - Assumption: constant preasure
    - Pressure balance
    - p_0 = Pressure of the normal column
  - Specification of standard values
    - D = 100 km
    - ρ_0 = 2670 kg/m³
- Wanted: individual density ρ in each column
- **Modeling the isostatic effect**
  - **Density contrast $\Delta\rho = \rho - \rho_0$**
  - **Thickness of the column is constant = D**



- Planar modeling
- Pressure balance:

$$p_0 = p_1$$
$$D \cdot \rho_0 \cdot g = (H + D) \cdot \rho_1 \cdot g$$

- Density of the isostatic masses:

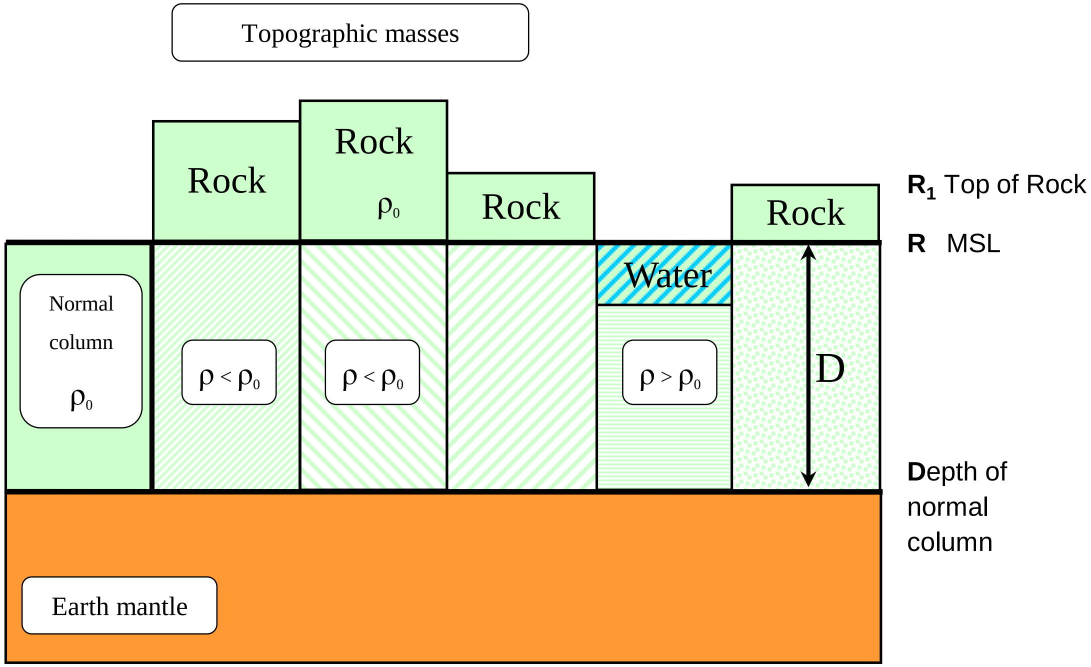
$$\rho_1 = \frac{D}{(H+D)} \cdot \rho_0$$

- Density contrast:

$$\boxed{\Delta\rho = \rho_0 - \rho_1 = \frac{h}{(H+D)} \cdot \rho_0}$$

Pratt-Hayford-Model (spherical)

- Unknown: individual density ρ in each column
- Constant compensation surface at depth D
- Constant density of the normal column ρ_0
  - Specification of standard values
    - D = 100 km
    - $\rho_0 = 2670\ kg/m^3$



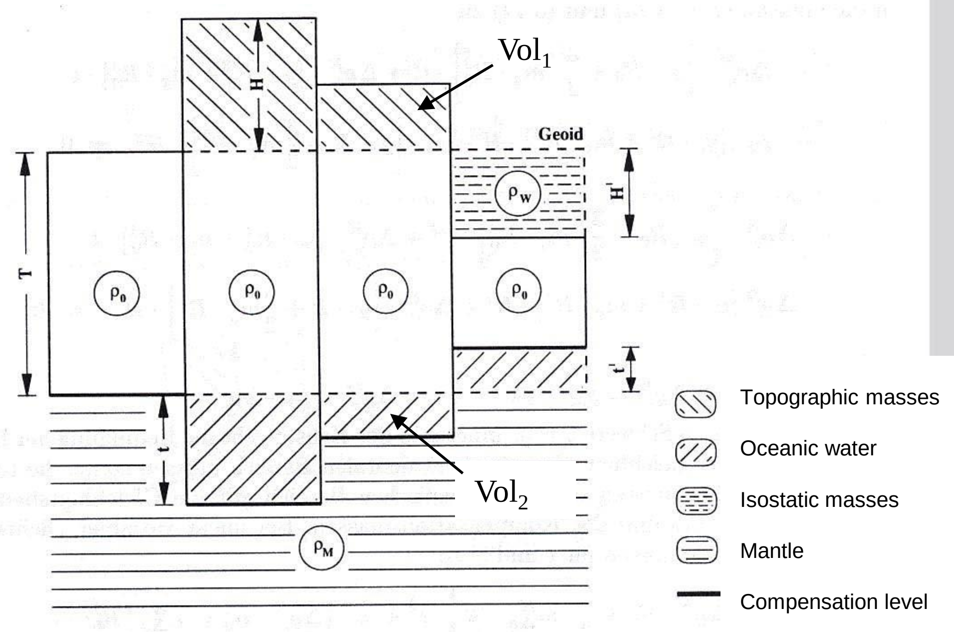
- **Spherical modeling**
- Mass equality between
  - Topographic masses
  - Isostatic masses
- Individual density ρ in each column
- Modeling the isostatic effect
  - Density contrast $\Delta\rho = \rho - \rho_0$

$$\rho_0 \cdot \left(R^3 - (R-D)^3\right) = \rho \cdot \left(R^3 - (R-D)^3\right) + \rho_0 \cdot \left(R_1^3 - R^3\right)$$

$$\boxed{\rho = \rho_0 \cdot \frac{2R^3 - (R-D)^3 - R_1^3}{R^3 - (R-D)^3}}$$

Airy-Heiskanen-Model (planar)

- Topographic masses
  - Density ρ_{TM}; Volume Vol_1
- Mantle
  - Density ρ_M
- Isostatic masses
  - Assumption: swimming balance
  - Dipping the crust into plastic material
  - Normal column has density ρ_0
  - Normal column has thickness T
    - ~Mohorovičić-Diskontinuity
  - Isostatic compensation masses
    - have the density contrast $\Delta\rho = \rho_M - \rho_0$
    - and the volume Vol_2
  - Specification of standard values
    - T = 25…30 km
    - $\rho_0 = 2670$ kg/m³
    - **$\Delta\rho = \rho_M - \rho_0$** = 600 kg/m³
- Wanted: root depth **t** of each column
- **Modeling the isostatic effect**
  - **Constant density contrast $\Delta\rho = \rho_M - \rho_0$**
  - **Variable root depth t**



- Planar modeling
- Swimming balance:
  - Load of floating material $\rho_0 \cdot g \cdot Vol_1 = \rho_0 \cdot g \cdot A \cdot H$
  - Buoyancy force of the displaced material
$$(\rho_M - \rho_0) \cdot g \cdot Vol_2 = (\rho_M - \rho_0) \cdot g \cdot A \cdot t$$
- Root depth: $\boxed{t = \frac{\rho_0 \cdot H}{\rho - \rho_0} = \frac{\rho_0}{\Delta\rho} \cdot H}$

Airy-Heiskanen-Model (spherical)

- Unknown: immersion depth t'
- Free parameters: T and $\Delta\rho$
  - Specification of standard values

    T thickness of the normal column

    T = 25 km

    ρ_R thicknes of the crust (Rock)

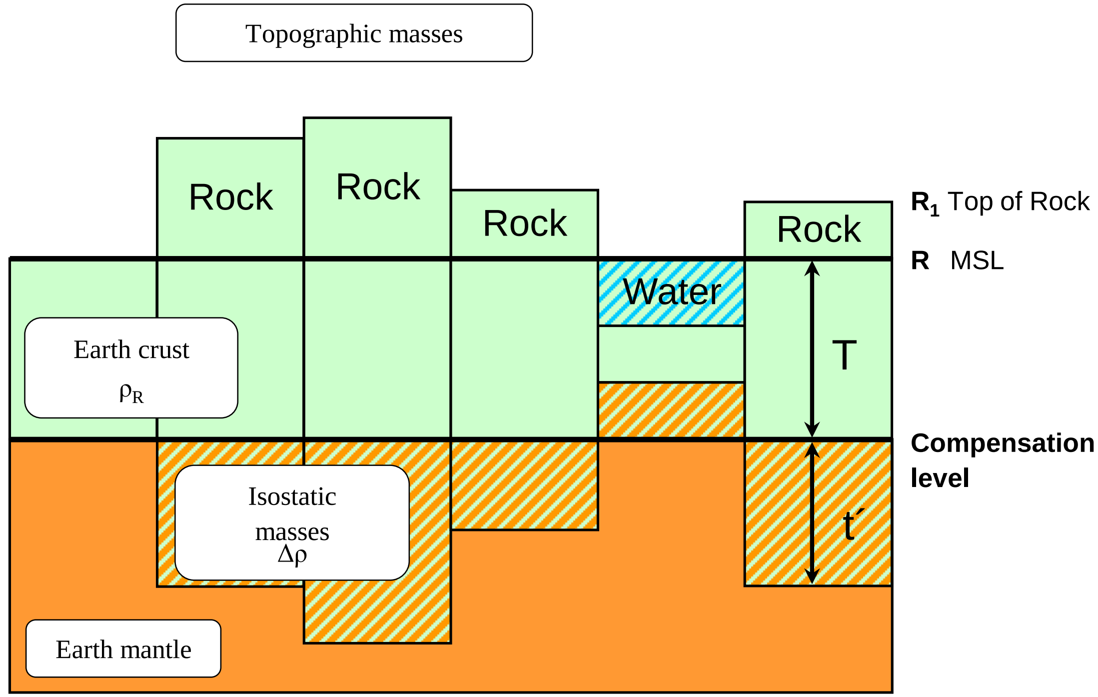
    ρ_R = 2670 kg/m³

    ρ_M Density of the mantle

    ρ_M = 3270 kg/m³

    $\Delta\rho$ Density contrast

    $\Delta\rho = \rho_M - \rho_K$ = 600 kg/m³
- Mass equality between
  - Topographic masses
  - Isostatic masses



$$t' = R - T - \sqrt[3]{(R-T)^3 - \frac{\rho_R}{\Delta\rho}\cdot\left(R_1^3 - R^3\right)}$$

Airy-Heiskanen-Modell with RWI-Method

- Unknown: immersion depth t'
- Free parameters: T and $\Delta\rho$
  - Specification of standard values

    T thickness of the normal column

    T = 25 km

    ρ_R density of the crust (Rock)

    ρ_R = 2670 kg/m³

    ρ_W Density of the water

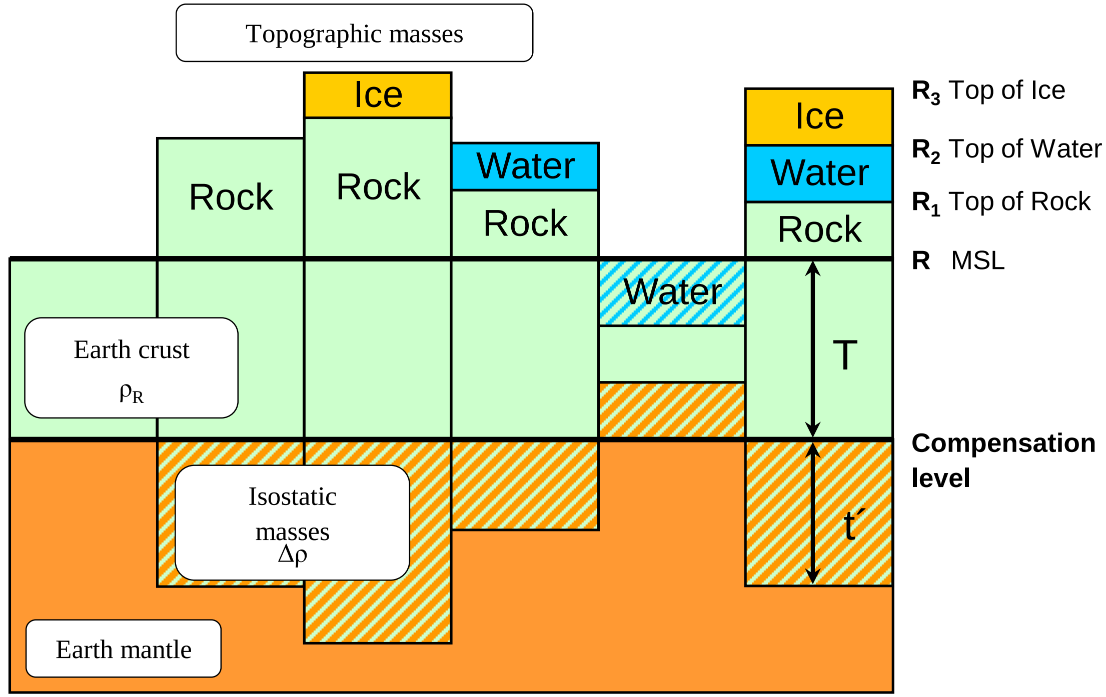
    ρ_I Density of the ice

    ρ_M Density of the mantle

    ρ_M = 3270 kg/m³

    $\Delta\rho$ Density contrast

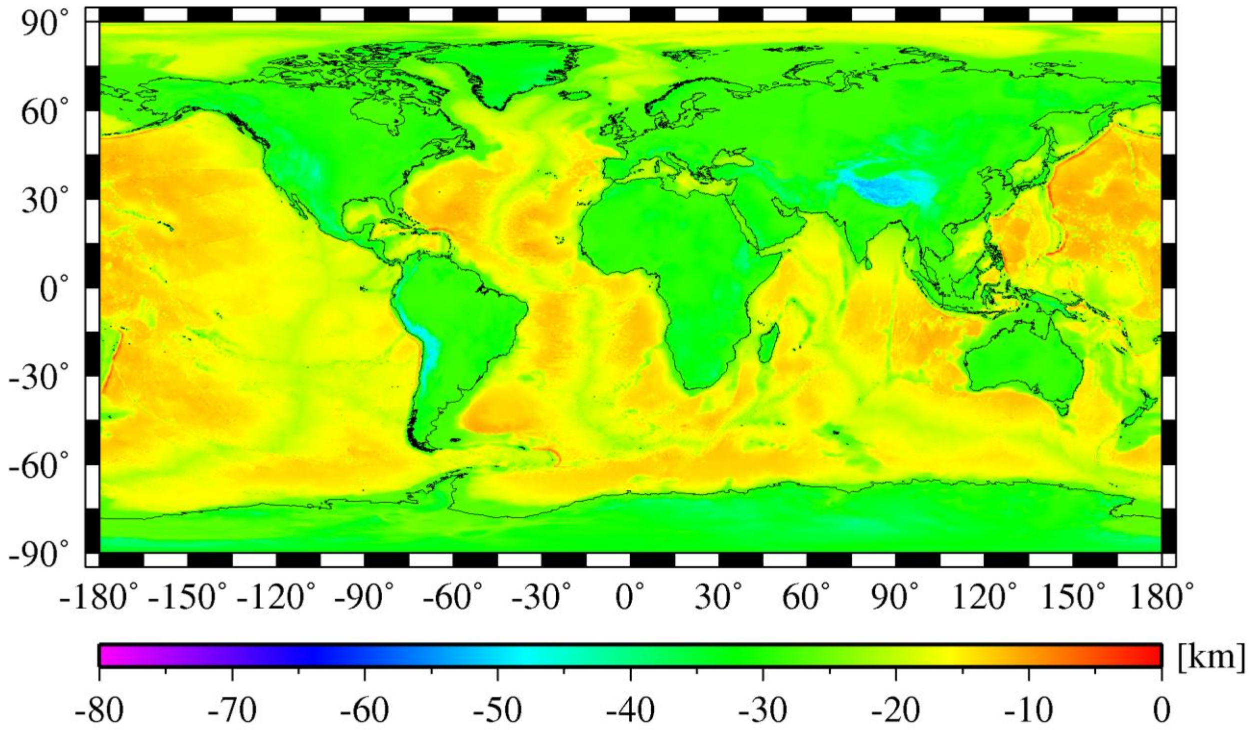
    $\Delta\rho = \rho_M - \rho_K$ = 600 kg/m³
- Mass equality between
  - Topographic masses
  - Isostatic masses



$$t' = R - T - \sqrt[3]{(R-T)^3 - \frac{\rho_R}{\Delta\rho}\cdot\left(R_1^3 - R^3\right) - \frac{\rho_W}{\Delta\rho}\cdot\left(R_2^3 - R_1^3\right) - \frac{\rho_I}{\Delta\rho}\cdot\left(R_3^3 - R_2^3\right)}$$

Airy-Heiskanen-Modell with RWI-Method

- Moho depths t' from the Airy-Heiskanen - model



min	=	-58.681
max	=	-0.545
mean	=	-22.999
rms	=	23.888
std	=	7.442
range	=	58.136

[km]

- Model fails in deep sea trenches (Moho depth less than seabed!)
- Isostatic masses not only result from topographic loading but also reflect density heterogeneities in the Earth's interior

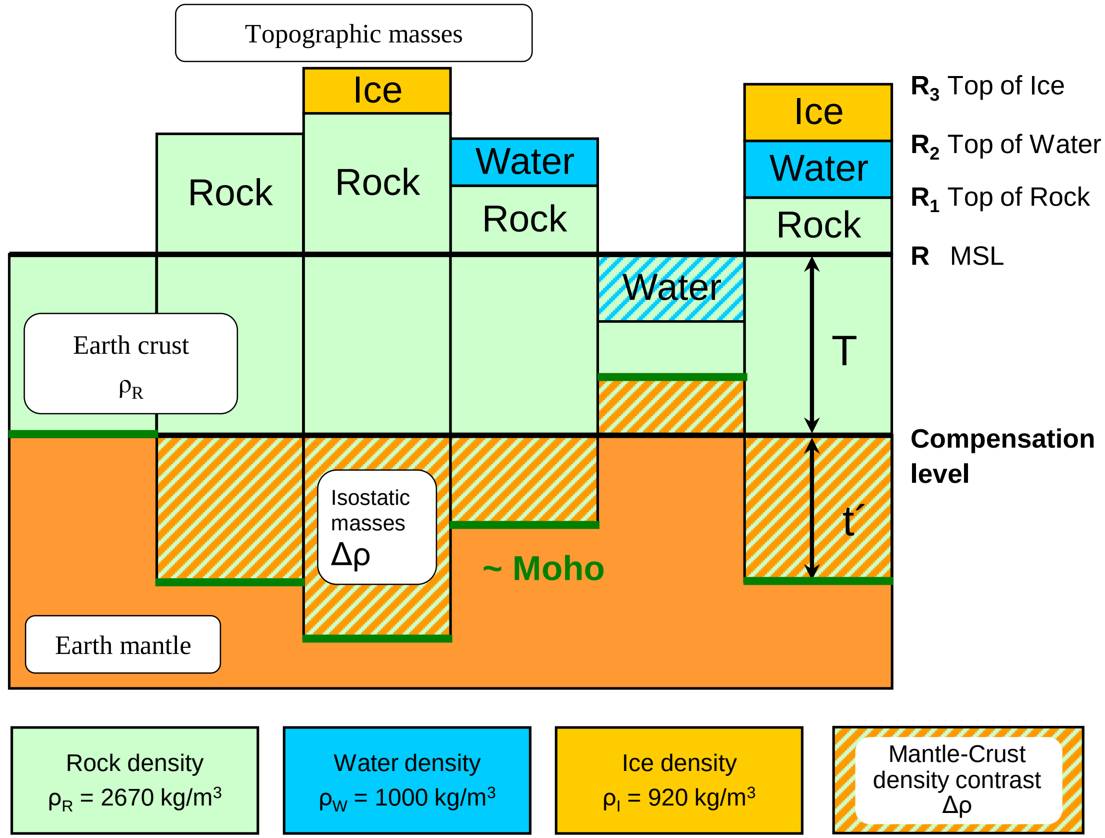
⇨ Extension of the isostatic model to include additional physical information

Airy-Heiskanen-Modell with RWI-Method

- **Expansion of the Airy-Heiskanen-Model**
- Immersion depth t‘

  by fixing a Moho-Model
- Model for Moho
  - Crust2.0 Model (Bassin et al. 2000)
  - Globally poor resolution
    - 2° x 2°
    - Meanwhile 1° x 1° (Crust1.0)
- Free parameters
  - T
  - Density contrast between

    mantle and crust: Δρ

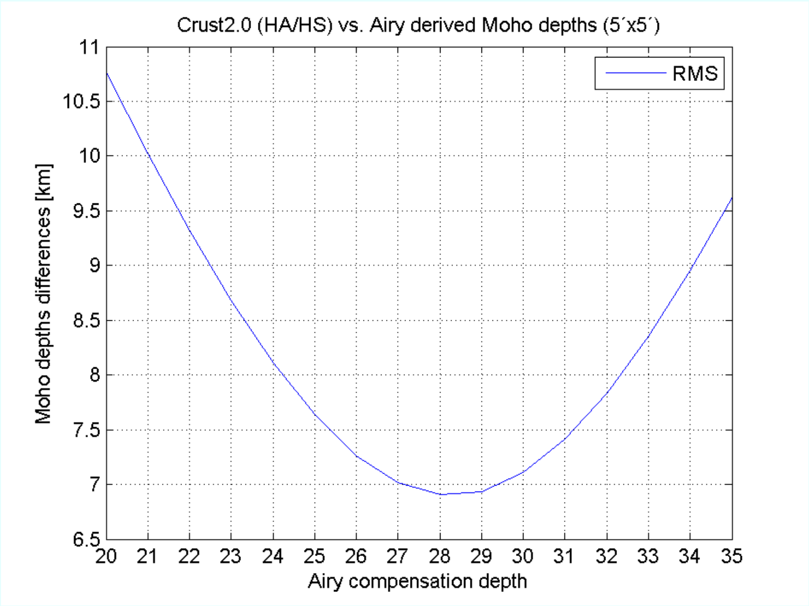


$$t' = R - T - \sqrt[3]{(R-T)^3 - \frac{\rho_R}{\Delta\rho}\cdot(R_1^3 - R^3) - \frac{\rho_W}{\Delta\rho}\cdot(R_2^3 - R_1^3) - \frac{\rho_I}{\Delta\rho}\cdot(R_3^3 - R_2^3)}$$

Determination of suitable model parameters

Until now:

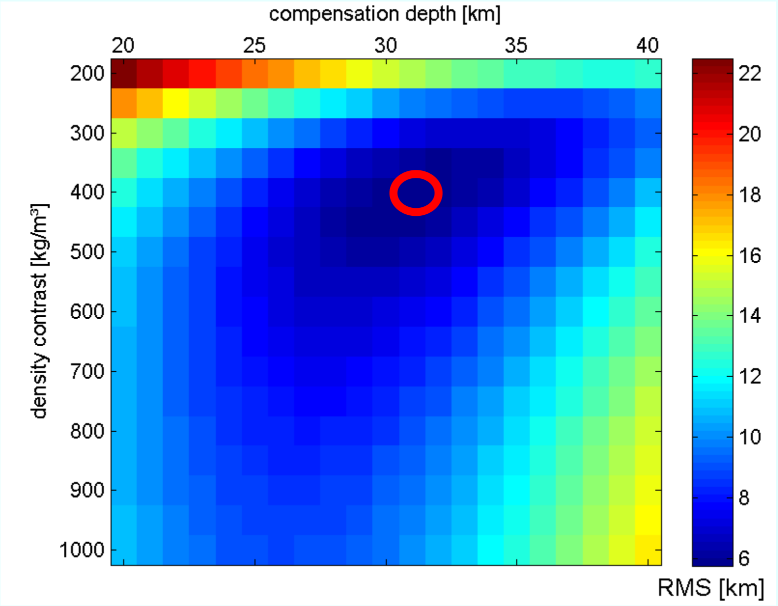
Determination of a global value for the compensation depth T at fixed Δρ = 600 kg/m³



→ **T = 28 km, Δρ = 600 kg/m³** Global

Extension of the model:

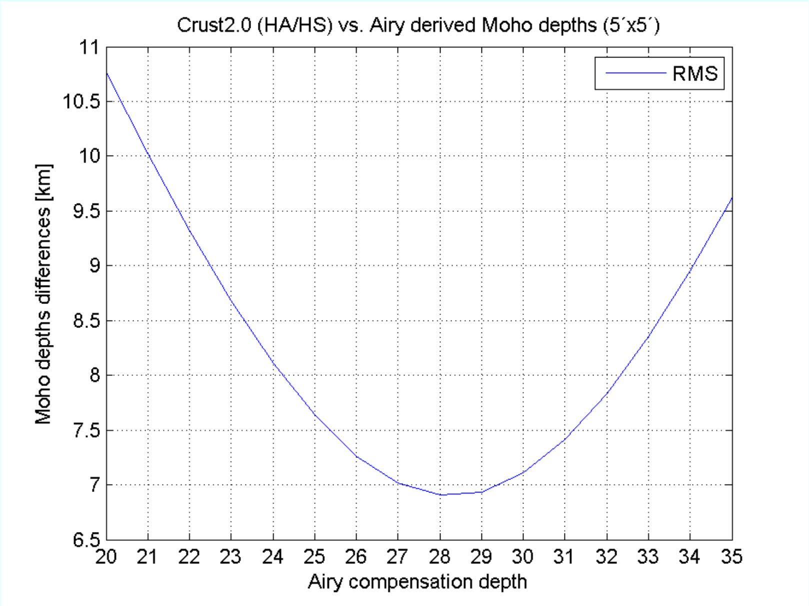
Simultaneous determination of values for T and Δρ (Global, Ocean, Land)

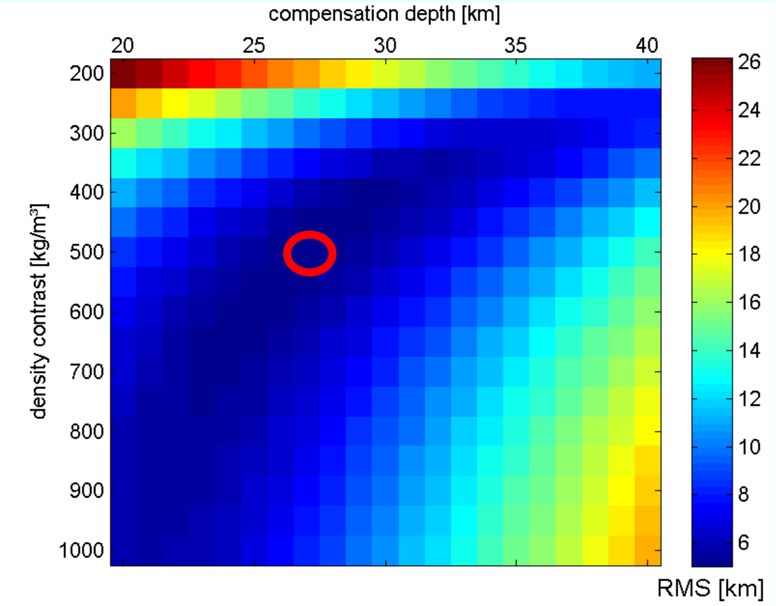


→ **T = 31 km, Δρ = 400 kg/m³** Global

Determination of suitable model parameters

Until now:	Extension of the model:
Determination of a global value for the compensation depth T at fixed Δρ = 600 kg/m³	Simultaneous determination of values for T and Δρ (Global, Ocean, Land)





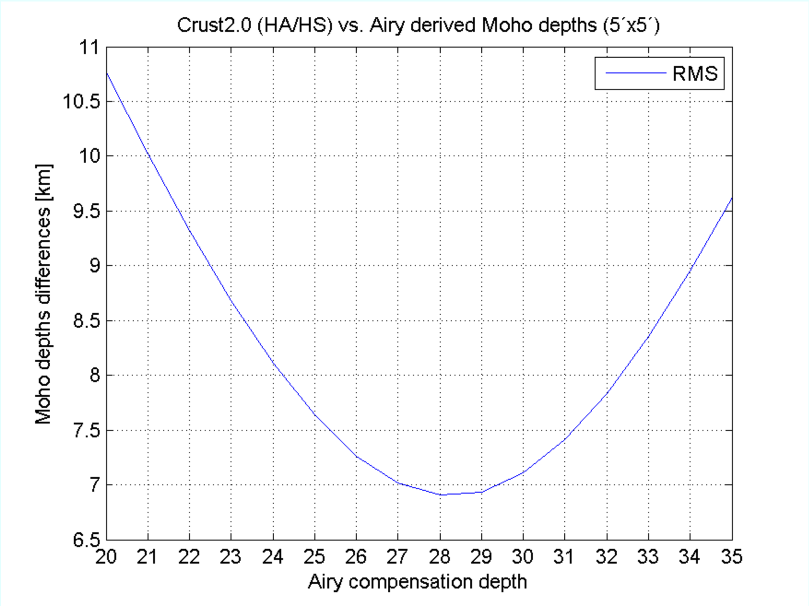
→ T = 28 km, Δρ = 600 kg/m³ Global

→ T = 31 km, Δρ = 400 kg/m³ Global

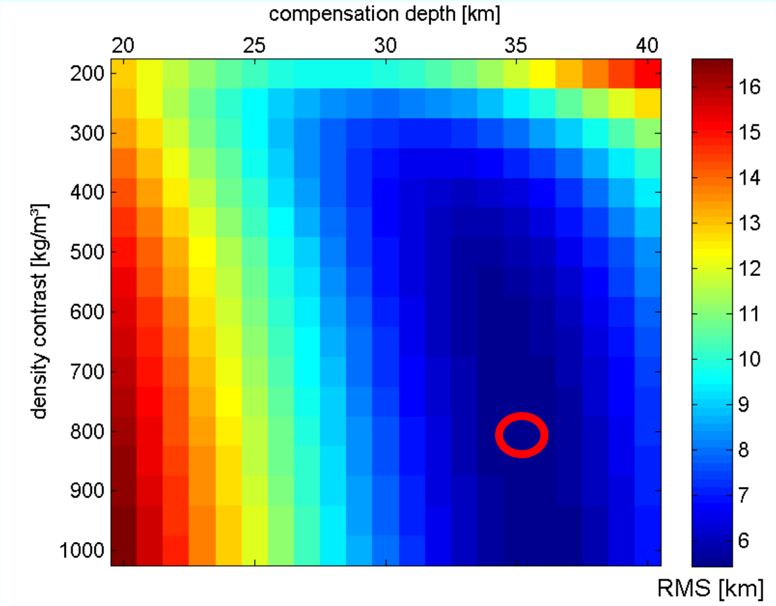
→ T = 27 km, Δρ = 500 kg/m³ Ocean

Determination of suitable model parameters

Until now:	Extension of the model:
Determination of a global value for the compensation depth T at fixed Δρ = 600 kg/m³	Simultaneous determination of values for T and Δρ (Global, Ocean, Land)



→ T = 28 km, Δρ = 600 kg/m³ Global



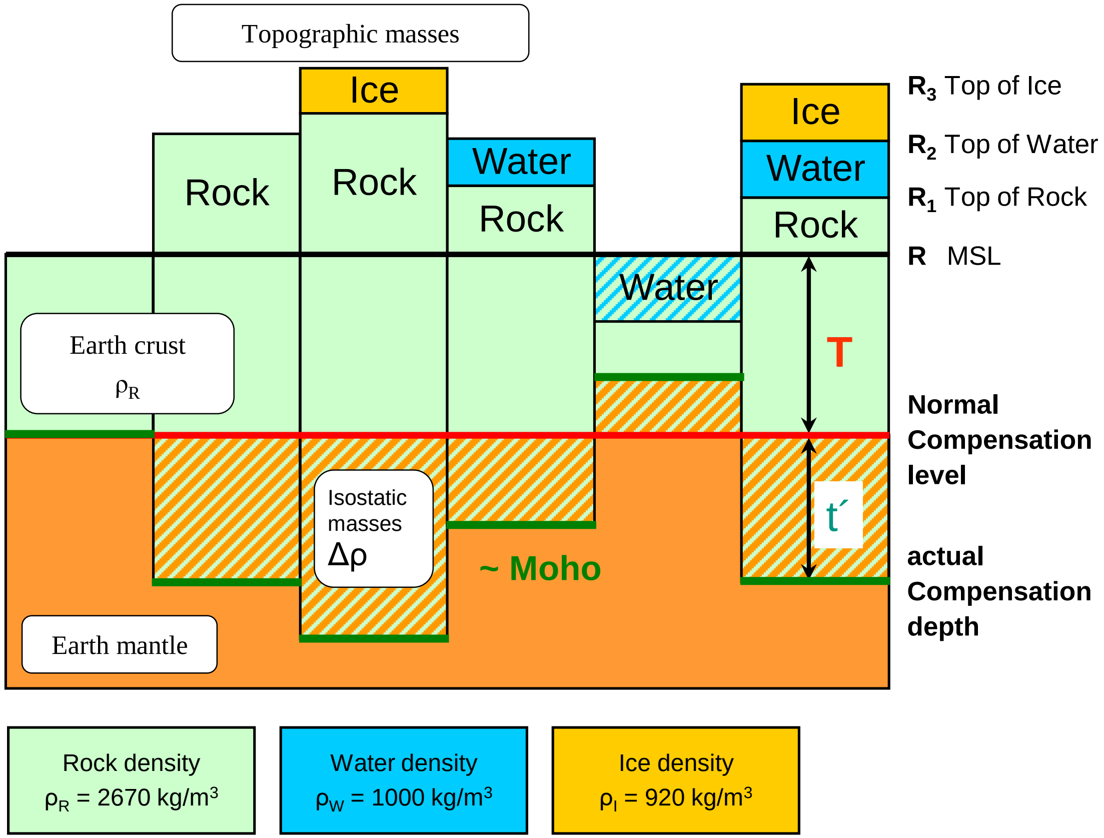
→ T = 31 km, Δρ = 400 kg/m³ Global

→ T = 27 km, Δρ = 500 kg/m³ Ocean

→ T = 35 km, Δρ = 800 kg/m³ Land

Modified Airy-Heiskanen-Model with RWI-Method

- Parameter T = 31 km
  fixed from several variationes
- t' introduced from the Moho-Model (c.f. CRUST2.0)



$$\Delta\rho = \frac{\rho_R \cdot \left(R_1^3 - R^3\right) + \rho_W \cdot \left(R_2^3 - R_1^3\right) + \rho_I \cdot \left(R_3^3 - R_2^3\right)}{\left(R - T\right)^3 - \left(R - T - t'\right)^3}$$

- Now determine the density contrast in each column individually
- Combination AH- and PH-Model

Selection of examined isostatic concepts

Airy-Heiskanen (A-H)

- $T = 28$ km, $\Delta\rho = 600$ kg/m³
- $T = 31$ km, $\Delta\rho = 400$ kg/m³
- $T = 27/35$ km, $\Delta\rho = 500/800$ kg/m³

Pratt-Hayford (P-H)

- $D = 100$ km
- $D = 75$ km
- $D = 75/100$ km

modified Airy-Heiskanen (mod. A-H)

- $T = 28$ km
- $\boxed{T = 31 \text{ km}}$
- $T = 27/35$ km

Combination: P-H / A-H

- $D = 100$ km, $T = 35$ km, $\Delta\rho = 800$ kg/m³

Combination : mod. A-H / P-H

- $D = 75$ km, $T = 27$ km

➢ Different isostatic concepts have little effect at satellite altitude

➢ Distinction between ocean/land does not result in any significant gain

➢ modified Airy-Heiskanen model (T = 31 km) delivers the best smoothing results

Seitz K, Grombein T, Kutterer H (2019):
Schwereanomalien in Baden-Württemberg. zfv 144(4):260–275,
DOI: 10.12902/zfv-0267-2019

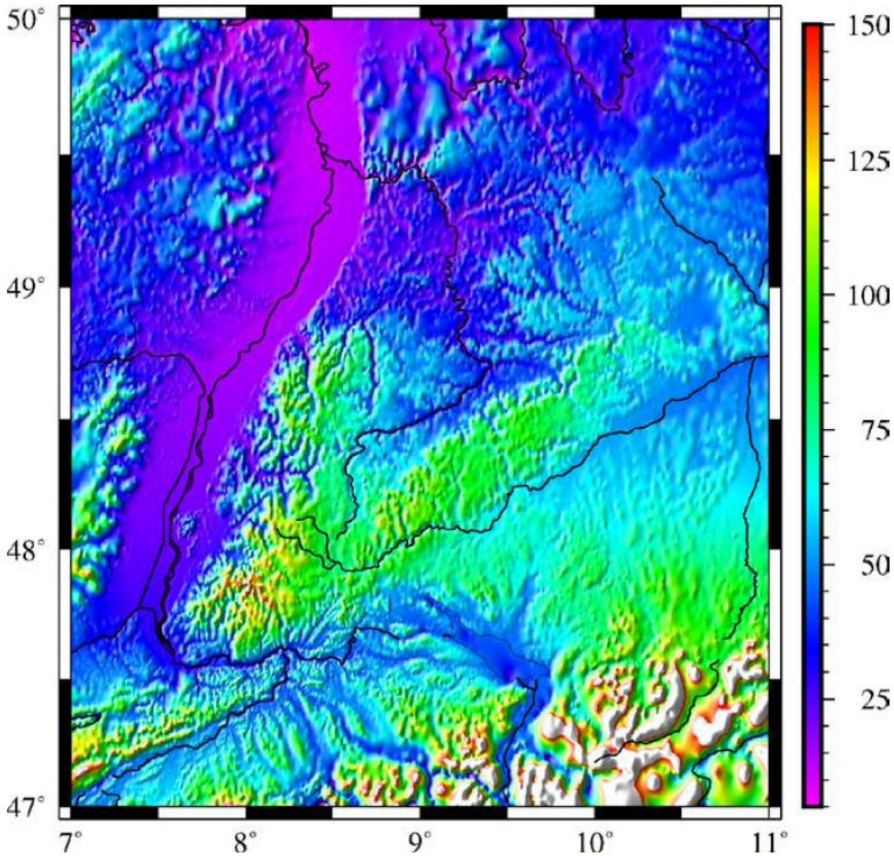


Abb. 9: Topographische Wirkung (mGal)

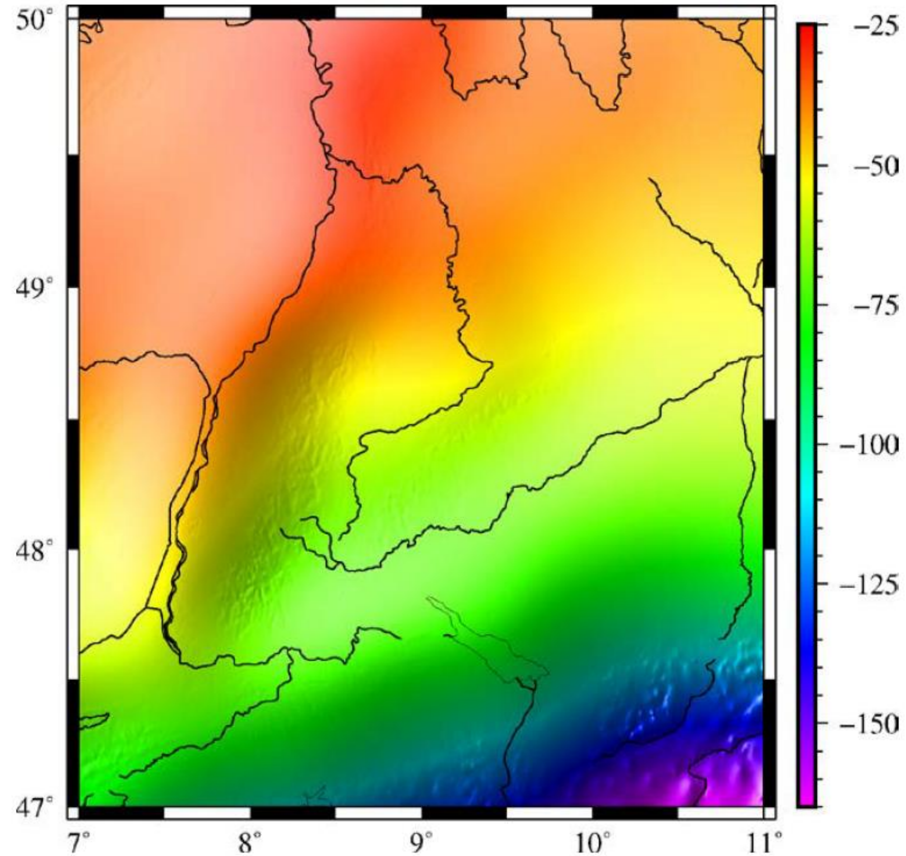


Abb. 10: Isostatische Wirkung (mGal)

Gravity reduction sceem

Karlsruher Institut für Technologie

Measured gravity 9,xxxxmxxX	Gravity anomaly ± 400 mGal
Atmospheric reduction < 0,9 mGal	
Tidal reduction ± 0,1 mGal	
Normal gravity 9,yyyymyyY…	
Free air reduction →	Free air anomaly
Topographic reduction • Terrain reduction • Bouguer reduction →	Bouguer anomaly Faye-anomaly
Isostatic reduction →	Isostatic anomaly

Theory of Stokes

Stokes' theory is that the boundary values are on the geoid and there are no masses outside the boundary.

Reductions are neccessary

$$g_0 = g_P - \delta g_{Gel} - \delta g_{BP} - \delta g_F$$

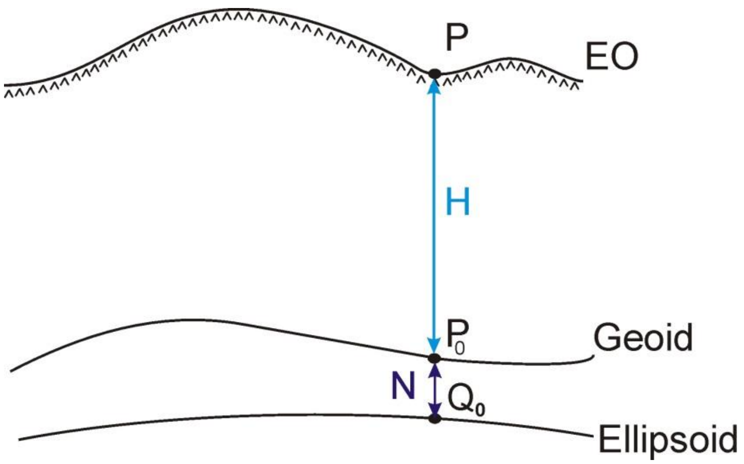
Anomaly: Comparison of reduced gravity in P_0 at the geoid with normal gravity on the ellipsoid in Q_0

$$\Delta g := g_0 - \gamma_0$$

g(P) measured gravity in P

(tidal effects, atmosphere, instruments heights are already corrected)

$\gamma_0(\varphi)$ computable normal gravity value on the ellipsoid



Bougueranomalie:

$$\Delta g_B := g_P - \delta g_{Gel} - \delta g_{BP} - \delta g_F - \gamma_0$$

Summary:

- You know different reductions that are applied to measured gravity values:
  - Atmospheric reduction
  - Tidal reduction
  - Free air reduction
  - Topographic reduction
  - Terrain reduction
  - Bouguer Plates Reduction
  - Isostatic reduction
- You know different isostatic models

References

Bronstein IN, Semendjajew KA, Musiol G, Muehlig H: Taschenbuch der Mathematik. Verlag: Harri Deutsch; Auflage: 5., überarb. u. erw. Aufl. (August 2000)

Heiskanen WA, Moritz H (1967): Physical Geodesy. WH Freeman & Co., San Francisco, USA.

Moritz H (1984): Geodetic Reference System 1980. Bulletin Géodésique (58): 388-398, Springer-Verlag, Berlin, Heidelberg.

Seitz K, Grombein T, Kutterer H (2019): Schwereanomalien in Baden-Württemberg. zfv 144(4):260–275, DOI: 10.12902/zfv-0267-2019

Torge W (2003): Geodäsie. Walter de Gruyter, Berlin, New York, 2. Auflage

Torge W und J Müller (2012): Geodesy. 4th edition, de Gruyter, Berlin, Germany.