Mass Variations
Exercise Sheet 1

Elaboration of:

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Comment:	

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Mass Variations

Exercise 1: Normal Gravity Potential and Normal Gravity

In this exercise, the normal gravitational acceleration of a normal gravity field has to be calculated.

(1) Normal gravity in geodetic coordinates (φ, h)

The normal gravity on the ellipsoid can be calculated with the formula of Somigliana depending on the geographical latitude φ

$$\gamma_0 = \frac{a \cdot \gamma_a \cdot \cos^2\varphi + b \cdot \gamma_b \cdot \sin^2\varphi}{\sqrt{a^2 \cdot \cos^2\varphi + b^2 \cdot \sin^2\varphi}}$$

or specify in a series development:

$$\gamma_0 = \gamma_a \big(1 + 0.0052790414 \cdot \sin^2\varphi + 0.0000232718 \cdot \sin^4\varphi + 0.0000001262 \cdot \sin^6\varphi + 0.0000000007 \cdot \sin^8\varphi\big)$$

The normal gravity at the ellipsoidal height h[km] reads as follows:

$$\gamma = \gamma_0 - \left(3.0877 \cdot 10^{-3} - 4.3 \cdot 10^{-6} \cdot \sin^2\varphi\right) \cdot h + 0.72 \cdot 10^{-6} \cdot h^2$$

1a) Compute the normal gravity along the ellipsoidal normal at Pi(φ=9°, hi = i·Δh)
With Δh=50m and 0 ≤ i ≤ 20.
Plot the contributions of γ0
Plot the linear term with h
Plot the quadratic term with h2
and plot the sum in one plot
Discuss your results!

Ans:

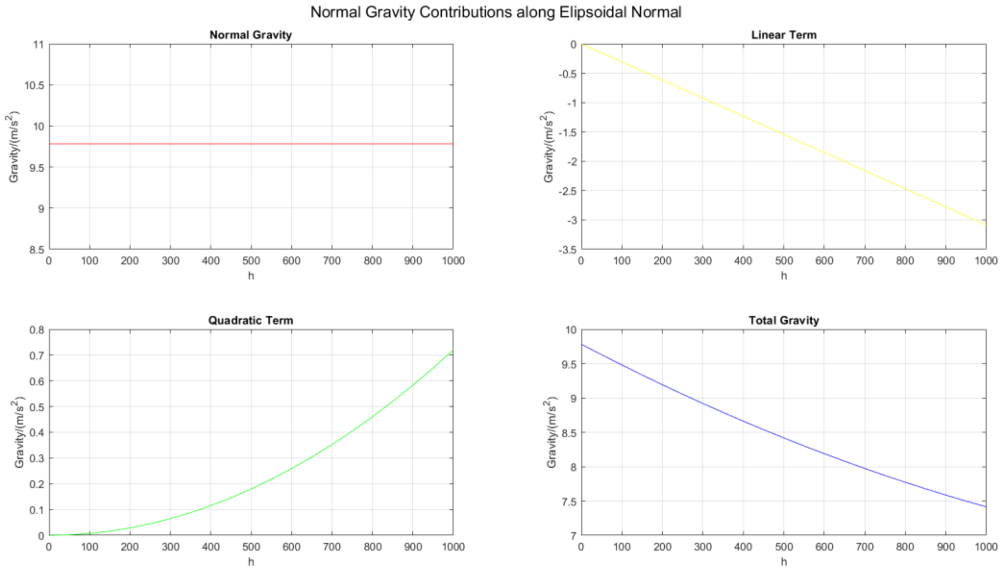

Fig 1: The normal gravity along the ellipsoidal normal points at different heights.

Discussions:

i) contributions of γ_0

With the increasing height, the normal gravity doesn't change. It remains constant at 9.7816 m/s^2. In the Fig 1, it's a line in parallel to the x-axis. It is because on the surface of the reference ellipsoid, the normal gravity values are constant in a Somigliana-Pizzetti normal field.
In the plot, contributions of γ_0 is shown by the red color line in Fig 1.

ii) linear term with h

In the plot, the linear term with h decreases linearly with the height. It is shown by the yellow color line in Fig 1.

iii) quadratic term with h^2

In the plot, the quadratic term increases quadratically with height. It is shown by the green color line in Fig 1.

iv) total gravity

The total gravity is the sum of all these contributions (γ_0 contributions, linear term with h contributions and quadratic term with h^2 contributions) and represents the overall gravity along the ellipsoidal normal.

$$\gamma = \gamma_0 - (3.0877 \cdot 10^{-3} - 4.3 \cdot 10^{-6} \cdot sin^2\varphi) \cdot h + 0.72 \cdot 10^{-6} \cdot h^2 \qquad (1.1)$$

In the plot, the total gravity decreases with height. It is shown by the blue line in Fig 1.

Analysis

i. Since the normal gravity γ_0 is a function of longitude (φ) in eq (1.2), which is constant, i.e., φ=9° along the ellipsoidal normal at P_i according to the question, so the plot of the contributions of γ_0 appears constant with the increasing height.

$$\gamma_0 = \frac{a.\gamma_a.cos^2\varphi \ + \ b.\gamma_b.sin^2\varphi}{\sqrt{a^2.cos^2\varphi \ + \ b^2.sin^2\varphi}} \qquad (1.2)$$

ii. The linear term is the function of longitude (φ) and height (h), whereby the linear term is given by eq (1.3)

$$linear\ term = -\left(3.0877 \cdot 10^{-3} - 4.3 \cdot 10^{-6} \cdot sin^2\varphi\right) \cdot h \quad (1.3)$$

Height, furthermore is the function of i, given by

$$hi = i{\cdot}\Delta h \quad (1.4)$$

where, Δh=50m and $0 \leq i \leq 20$.
So the linear term with h decreases linearly with the height in Fig 1.

iii. The quadratic term is the function of the square of height (h^2), whereby the quadratic term is given by eq (1.5)

$$\gamma = 0.72 \cdot 10^{-6} \cdot h^2 \quad (1.5)$$

So the quadratic term with h^2 increases quadratically with height in Fig 1.

iv. The value of the total gravity falls from 9.8 m/s^2 to 7.4 m/s^2 with the increasing height. It is because of the fact that the gravity decreases at longer distance between centers of mass.

1b) Compute the normal gravity along a meridian at Pj (φj = j·Δφ, h = 100m)
With Δφ = 1° and $0 \leq j \leq 90$.
Plot the contributions of γ0
The linear term with φ
The quadratic term with φ2
And the sum in one plot
Discuss your results!

Ans:

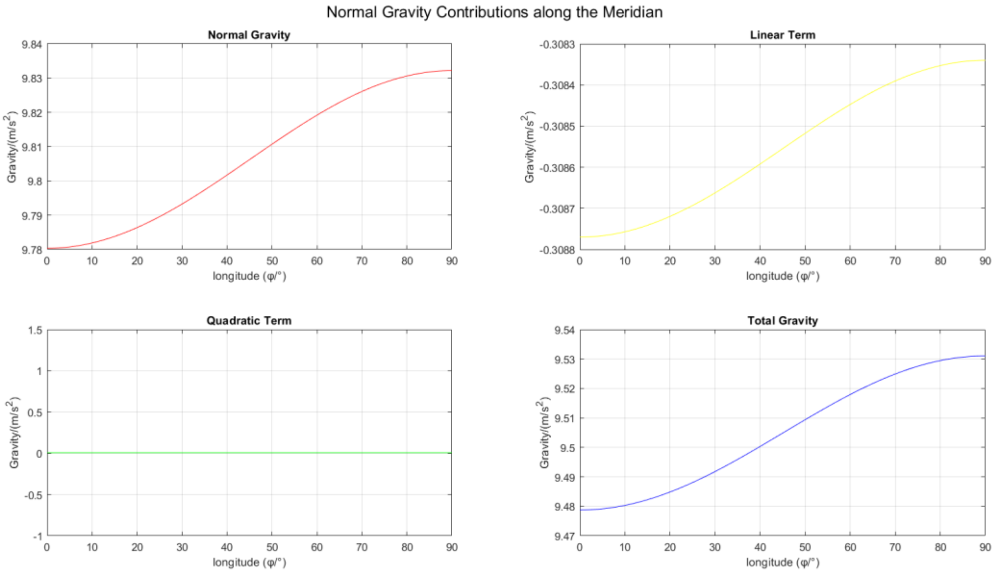

Fig 2: The normal gravity along a meridian at different longitudes.

Analysis and Discussions

i. contributions of γ_0

As mentioned above in (1.2), the normal gravity γ_0 is a function of longitude (φ). The longitude further is a function of j, given by

$$\varphi_j = j{\cdot}\Delta\varphi \quad (1.6)$$

where $\Delta\varphi = 1°$ and $0 \leq j \leq 90$.

The plot of the contributions of γ_0 increases with the trigonometric functions of sine and cosine with the increasing longitude. It starts at 9.7803m/s^2 at the equator, and ends as 9.8322 m/s^2.

In the plot, contributions of γ_0 is shown by the red color line in Fig 2.

ii. linear term with φ

From eq (1.3), the linear term is the function of longitude (φ) and height (h). As the height is kept constant, the linear term with h increases with the longitude. Compared to the normal gravity, the value of linear term is relatively small, increasing from -0.3088 to -0.3083, shown in yellow color line in Fig 2.

iii. quadratic term with h^2

The quadratic term is the function of height (h^2) as given by eq (1.5). So, the quadratic term with h^2 stays the same with increasing longitude and it is a line parallel to the x axis in the given plot shown by the green color line. And its value is 0.0072.

iv. total gravity

The value of the total gravity increases from 9.4788 to 9. 5310 m/s^2 with the increasing longitude. The reason of this increasing trend is the earth is not a standard sphere, but an mathematically approximated ellipsoid. In the equator, the radius of the earth is around 6378137 m, while that at pole is around 6356752m. Furthermore, with the smaller distance, the gravitation force becomes stronger.

Feedback in data processing:

During this exercise, the part of compiling codes is relatively easy. It's divided into two parts. Firstly, write a function which can receive the input data to get the normal gravity values. Then using the method of loop to calculate all the normal gravity values along a meridian and the ellipsoidal normal points at different heights.

We used both Python and Matlab code. Due to the readability in Matlab, not only can we get effective information easily in figures, but we can also check all the values of variables appearing in the calculation. Finally, we choose it as our final answer.

During the compiling, it's also a good chance to recall what was on the slides taught in class. We compare our answer with the figure in slides as a verification.