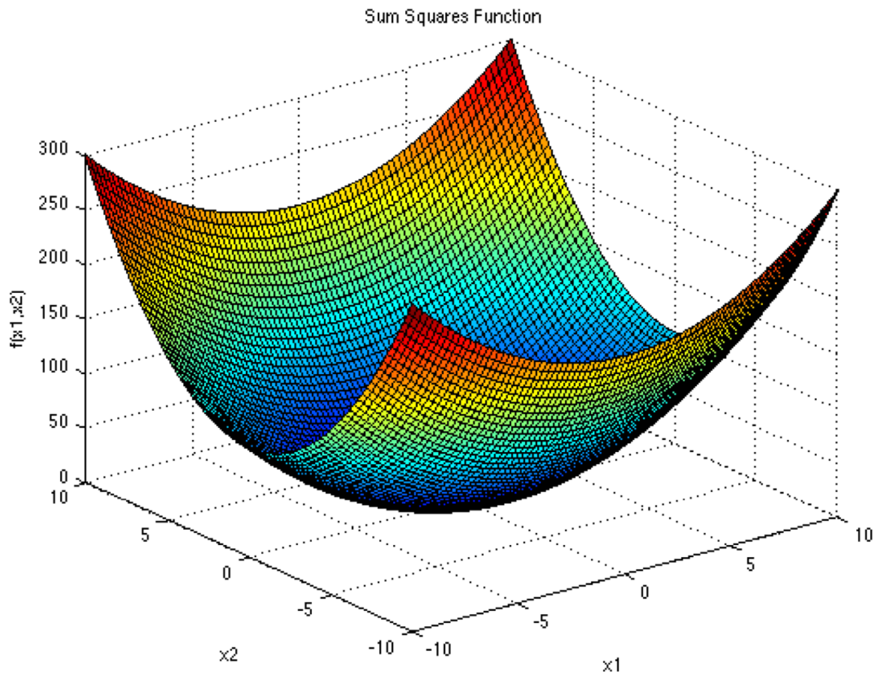
SUM SQUARES FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d} i x_i^2$$

Description:

Dimensions: d

The Sum Squares function, also referred to as the Axis Parallel Hyper-Ellipsoid function, has no minimum except the global one. It is continuous, convex and unimodal. It is shown here in its tw dimensional form.

Input Domain:

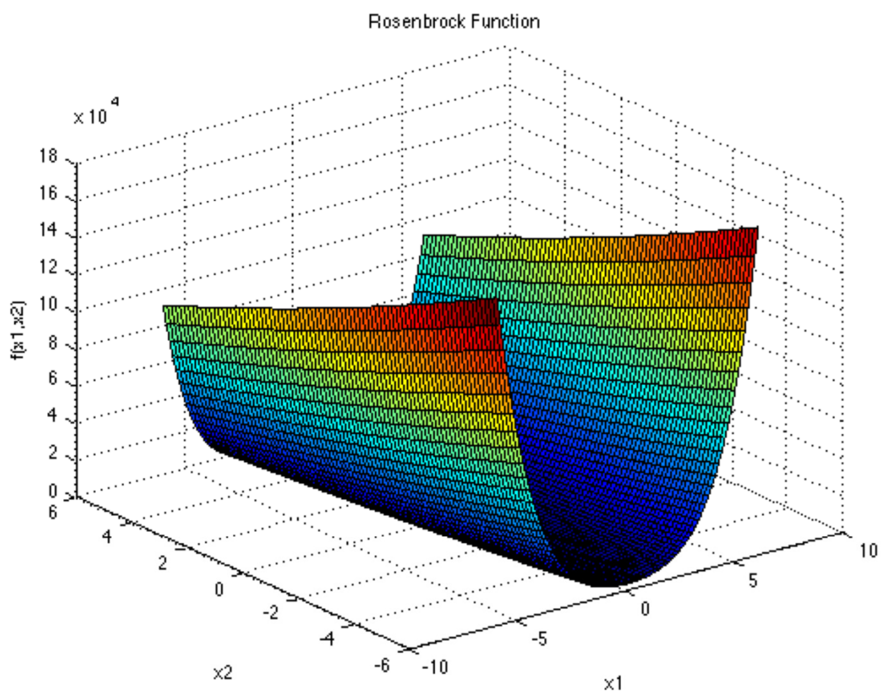
The function is usually evaluated on the hypercube $x_i \in [-10, 10]$, for all i = 1, …, d, although this restricted to the hypercube $x_i \in [-5.12, 5.12]$, for all i = 1, …, d.

Global Minimum:

$$f(\mathbf{x}^*) = 0 \text{ , at } \mathbf{x}^* = (0, \dots, 0)$$

Solve for five variables: $x = (x_1, x_2, x_3, x_4, x_5)^T$

ROSENBROCK FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$$

Description:

Dimensions: d

The Rosenbrock function, also referred to as the Valley or Banana function, is a popular test prc gradient-based optimization algorithms. It is shown in the plot above in its two-dimensional form

The function is unimodal, and the global minimum lies in a narrow, parabolic valley. However, ev this valley is easy to find, convergence to the minimum is difficult (Picheny et al., 2012).

Input Domain:

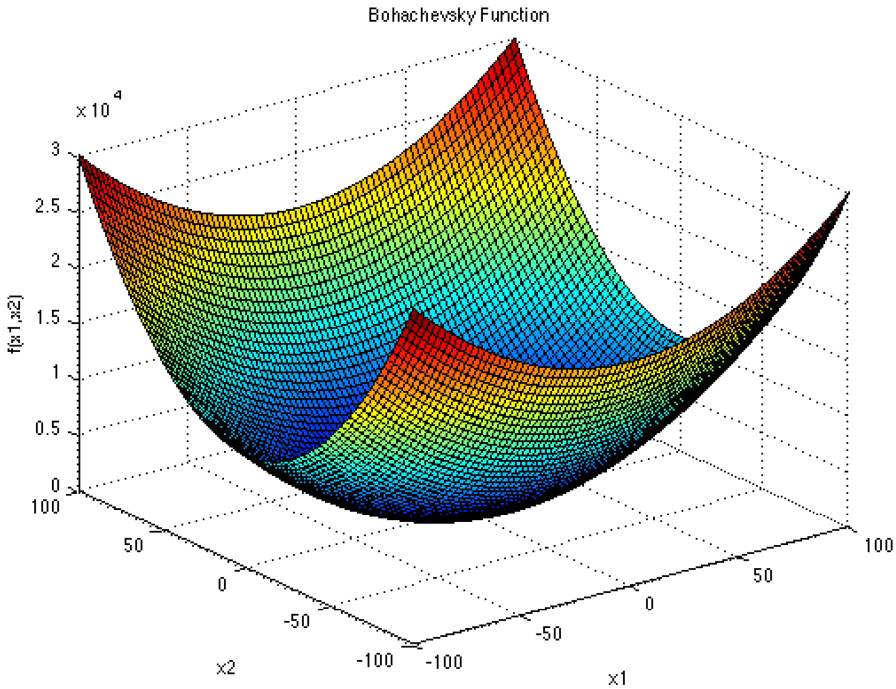
The function is usually evaluated on the hypercube $x_i \in [-5, 10]$, for all i = 1, …, d, although it mє restricted to the hypercube $x_i \in [-2.048, 2.048]$, for all i = 1, …, d.

Global Minimum:

$$f(\mathbf{x}^*) = 0, \text{ at } \mathbf{x}^* = (1, \dots, 1)$$

Solve for three variables: $x = (x_1, x_2, x_3)^T$

BOHACHEVSKY FUNCTIONS



$$f_1(\mathbf{x}) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) - 0.4\cos(4\pi x_2) + 0.7$$
$$f_2(\mathbf{x}) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1)\cos(4\pi x_2) + 0.3$$
$$f_3(\mathbf{x}) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1 + 4\pi x_2) + 0.3$$

Description:

Dimensions: 2

The Bohachevsky functions all have the same similar bowl shape. The one shown above is the

Input Domain:

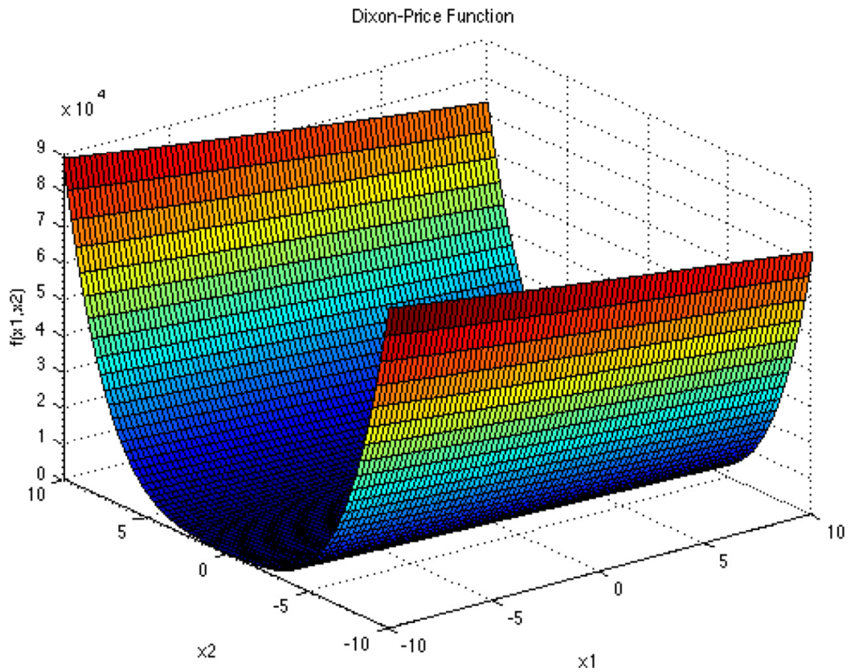
The functions are usually evaluated on the square $x_i \in [-100, 100]$, for all i = 1, 2.

Global Minimum:

$$f_j(\mathbf{x}^*) = 0, \text{ at } \mathbf{x}^* = (0, 0), \text{ for all } j = 1, 2, 3$$

Solve for all three functions separately

DIXON-PRICE FUNCTION



$$f(\mathbf{x}) = (x_1 - 1)^2 + \sum_{i=2}^{d} i \left(2x_i^2 - x_{i-1}\right)^2$$

Description:

Dimensions: d

Input Domain:

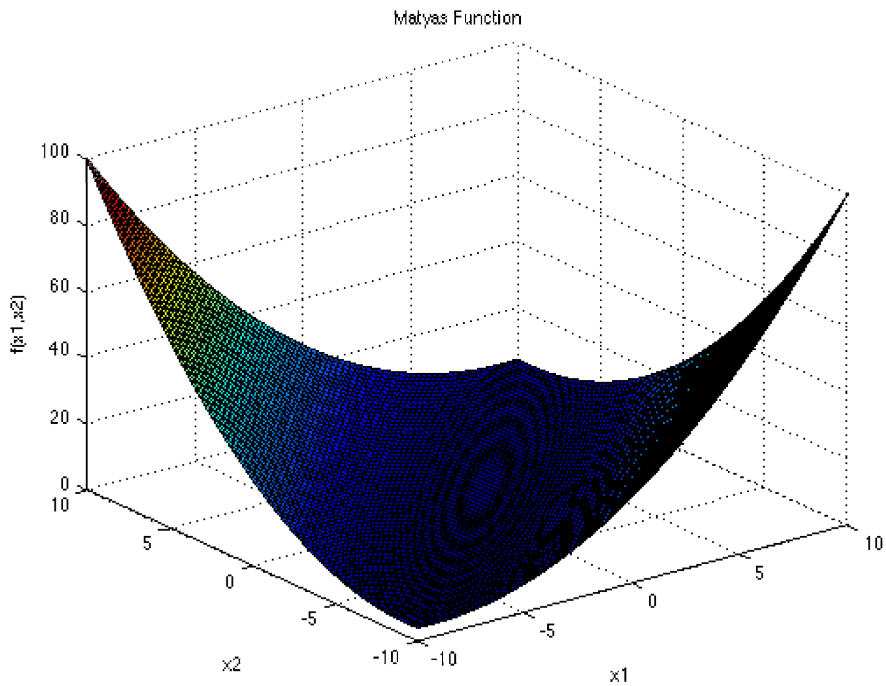
The function is usually evaluated on the hypercube $x_i \in [-10, 10]$, for all i = 1, …, d.

Global Minimum:

Solve for four variables: $x = (x_1, \dots, x_4)^T$

$$f(\mathbf{x}^*) = 0, \text{ at } x_i = 2^{-\frac{2^i-2}{2^i}}, \text{ for } i = 1, \dots, d$$

MATYAS FUNCTION



$$f(\mathbf{x}) = 0.26(x_1^2 + x_2^2) - 0.48x_1x_2$$

Description:

Dimensions: 2

The Matyas function has no local minima except the global one.

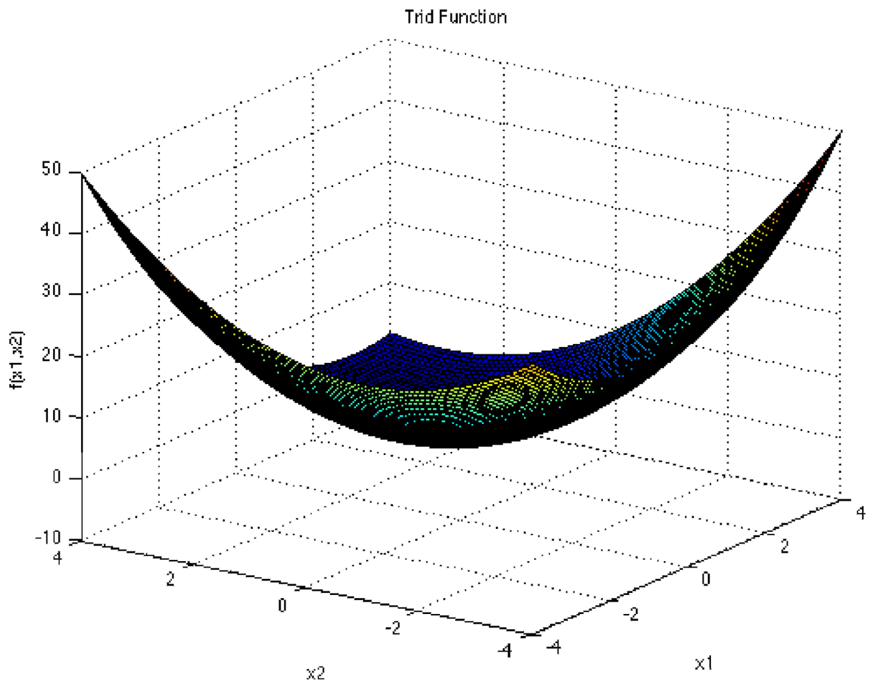
Input Domain:

The function is usually evaluated on the square $x_i \in [-10, 10]$, for all i = 1, 2.

Global Minimum:

$f(\mathbf{x}^*) = 0$, at $\mathbf{x}^* = (0, 0)$

TRID FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d}(x_i - 1)^2 - \sum_{i=2}^{d} x_i x_{i-1}$$

Description:

Dimensions: d

The Trid function has no local minimum except the global one. It is shown here in its two-dimen

Input Domain:

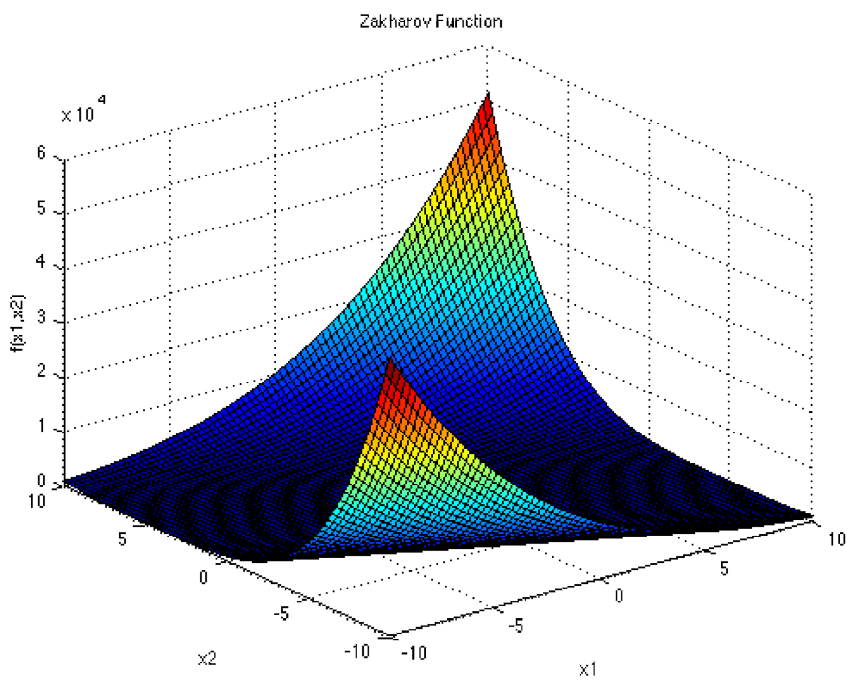
The function is usually evaluated on the hypercube $x_i \in [-d^2, d^2]$, for all i = 1, …, d.

Solve for six variables: $x = (x_1, \ldots, x_6)^T$

Global Minimum:

$$f(\mathbf{x}^*) = -d(d+4)(d-1)/6, \text{ at } x_i = i(d+1-i), \text{ for all } i = 1, 2, \ldots, d$$

ZAKHAROV FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d} x_i^2 + \left(\sum_{i=1}^{d} 0.5 i x_i\right)^2 + \left(\sum_{i=1}^{d} 0.5 i x_i\right)^4$$

Description:

Dimensions: d

The Zakharov function has no local minima except the global one. It is shown here in its two-din form.

Input Domain:

The function is usually evaluated on the hypercube $x_i \in [-5, 10]$, for all i = 1, …, d.

Global Minimum:

Solve for two variables: $x = (x_1,\ x_2)^T$

$$f(\mathbf{x}^*) = 0, \text{ at } \mathbf{x}^* = (0, \dots, 0)$$