Assignment 1: Linear Programming Example - The Diet Problem Revisited

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This study focused on a diet optimization problem constrained by the U.S. Food and Drug Administration's (USFDA) daily nutrition intake recommendations, using five personally selected food items. The chosen foods were priced packaged items with listed nutrition facts for convenient calculations. While the challenge was identifying foods that not only reflected my typical diet but also included all seven nutrients, I opted for a ribeye steak, a chicken and apple sausage, kimbap, Greek yogurt, and tofu. See Appendix 1 for their nutrition facts.

The goal of this problem is to determine the number of servings of the five food items to purchase and consume to minimize costs while meeting the nutritional requirements. My weekly diet consists of a ribeye steak for \$6.84 per serving, chicken and apple sausage for \$1.12 per serving, kimbap for \$1.66 per serving, Greek yogurt for \$0.83 per serving, and tofu for \$0.63 per serving. See Appendix 2 for price breakdown. The USFDA recommends a maximum intake of 5,000mg of sodium and minimums of 2000kcal, 50g of protein, 20mcg of vitamin D, 1,300mg of calcium, 18mg of iron, and 4,700mg of potassium. The number of servings must also be nonnegative. The objective is to minimize the weekly cost. See Appendix 3 for problem formulation.

There are five decision variables representing the number of servings for each food item. Since the objective is to minimize the total food cost, the objective function is a minimization problem, summing the cost and serving amount for each food item. The problem is subject to eight constraints: seven nutritional requirements and the non-negativity constraint. Each food's nutritional content is broken down in Appendix 4. For each nutritional requirement, its constraint sums the nutritional values of all foods, multiplied by the respective servings and scaled to reflect weekly intake. The linear programming (LP) problem was formulated in Python and solved using PuLP. The optimal solution is 22.22 servings of Greek yogurt and 15 servings of tofu, resulting in a weekly cost of \$27.97. See Appendix 5 for problem formulation in Python.

To increase variety to the suggested diet plan, I added additional constraints to ensure at least one serving of each food. This resulted in 1 serving each of the steak, sausage, and kimbap, 22.22 servings of Greek yogurt, and 13.02 servings of tofu. The number of servings were adjusted except for the yogurt, raising the weekly cost to \$36.27 from \$27.97 shown in Appendix 6. To further diversify and personalize the diet, I would include carbohydrates, cholesterol and other nutrients to be more mindful of or ensure the inclusion of food groups like vegetables.

To compare the performance of large language models (LLM) in solving similar problems, I experimented with ChatGPT -https://chatgpt.com/. See Appendix 7 for full ChatGPT conversation. For the first prompt, I assigned the AI agent the role of data scientist, provided context for the problem, and instructed to construct a model, write code for the problem, and ask questions for clarification. The agent successfully generated the problem setup and corresponding Python code and asked data-related questions. After a couple failed attempts of instructing online search for nutrition facts and submitting images of them, the third attempt of sending a table containing all the necessary and correct data allowed the agent to complete the code with the provided data. However, when asked to solve it, I encountered a data analysis limit. Regardless, I believe an LLM agent can fully complete the assignment when given correct data, context, and access to subscribed version.

This study demonstrated the application of an optimization technique in an LP problem using Python and an LLM model. While Python is a great diverse tool for solving simple and complex problems, LLMs can assist in model construction and data analysis with accurate data and advanced analytics capabilities.

Appendix 1: Nutrition Facts

Aldi Grass Fed Ribeye Steak

Aidells Chicken & Apple Sausage

Trader Joe's **Kimbap**

Nutrition Facts Serving Size 1 serving (4 oz) Amount Per Serving 390 **Calories** % Daily Values* Total Fat 35.00g Saturated Fat 15.000g 75% Trans Fat 0.000g Cholesterol 80mg 27% Sodium 60mg Total Carbohydrate 0.00g 0% Dietary Fiber 0.0g 0% Sugars 0.00g Includes 0.00g Added Sugars 0% Protein 18.00g Vitamin D 0mcg 0% Calcium 30mg Iron 2.00mg 11% Potassium 260mg 6% * The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

Serving Size	1 link (90 g)
Amount Per Serving Calories	170
	% Daily Values*
Total Fat 12.00g	15%
Saturated Fat 3.500g	18%
Trans Fat 0.000g	
Cholesterol 75mg	25%
Sodium 660mg	29%
Total Carbohydrate 4.0	00g 1 %
Dietary Fiber 1.0g	4%
Sugars 3.00g	
Protein 13.00g	
Vitamin D -	
Calcium -	
Iron -	
Potassium -	

Serving Size 3	pieces (77 g)
Amount Per Serving Calories	140
q	% Daily Values*
Total Fat 4.00g	5%
Saturated Fat 5.000g	25%
Trans Fat 0.000g	
Cholesterol Omg	0%
Sodium 320mg	14%
Total Carbohydrate 23.0	0g 8 %
Dietary Fiber 1.0g	4%
Sugars 2.00g	
Includes 2.00g Added Sug	gars 4%
Protein 4.00g	
Vitamin D 0mcg	0%
Calcium 50mg	4%
Iron 0.50mg	3%
Potassium 70mg	1%

Soonyeowon Firm Tofu

Q

Serving Size 3/4 Cup (170g) Servings Per Container 5	
Servings Fer Container 5	
Amount Per Serving	
Calories 150	0/ P.1 V.L
Total Fat 6g	% Daily Value
Saturated Fat 3.5g	18%
Trans Fat Og	
Cholesterol 25mg	8%
Sodium 65mg	3%
Total Carbohydrate 7g	3%
Dietary Fiber Og	0%
Total Sugars 5g	
Added Sugars Og	0%
Protein 16g	32%
Vitamin D 0.9mcg 4%	• Calcium 170mg 15%
Iron Omg 0%	Potassium 200mg 4%

Serving Size	3 oz (85 g)
Amount Per Serving Calories	80
	% Daily Values*
Total Fat 4.50g	6%
Saturated Fat 1.500g	8%
Trans Fat 0.000g	
Polyunsaturated Fat 0.000	Og .
Monounsaturated Fat 0.00	00g
Cholesterol Omg	0%
Sodium 5mg	0%
Total Carbohydrate 1.00	Og 0 %
Dietary Fiber 1.0g	4%
Sugars 0.00g	
Includes 0.00g Added St	ugars 0%
Protein 10.00g	
Vitamin D 0mcg	0%
Calcium 150mg	12%
Iron 1.19mg	7%
Potassium 140mg	3%

Appendix 2: Price Breakdown

	Total Price	Servings per package	Price per Serving
Ribeye Steak	\$13.69	2	\$6.84
Chicken & Apple Sausage	\$16.79	15	\$1.12
Kimbap	\$4.99	3	\$1.66
Greek Yogurt	\$4.15	5	\$0.83
Tofu	\$3.79	6	\$0.63

Appendix 3: Linear Programming Standard Form

minimize
$$Z = 6.84x_1 + 1.12x_2 + 1.66x_3 + 0.83x_4 + 0.63x_5$$

subject to:

$$7(60x_1 + 660x_2 + 320x_3 + 65x_4 + 5x_5) \le 7 * 5000$$

$$7(390x_1 + 170x_2 + 140x_3 + 150x_4 + 80x_5) \ge 7 * 2000$$

$$7(18x_1 + 13x_2 + 4x_3 + 16x_4 + 10x_5) \ge 7 * 50$$

$$7(0.9x_4) \ge 7 * 20$$

$$7(30x_1 + 50x_3 + 170x_4 + 150x_5) \ge 7 * 13000$$

$$7(2x_1 + 0.5x_3 + 1.19x_5) \ge 7 * 18$$

$$7(260x_1 + 70x_3 + 200x_4 + 140x_5) \ge 7 * 4700$$

$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

Appendix 4: Nutrition Breakdown

	Ribeye Steak	Chicken & Apple Sausage	Kimbap	Greek Yogurt	Tofu
Sodium	60	660	320	65	5
Energy	390	170	140	150	80
Protein	18 g	13	4	16	10
Vitamin D	0	0	0	0.9	0
Calcium	30	0	50	170	150
Iron	2	0	0.5	0	1.19
Potassium	260	0	70	200	140

Appendix 5: Original Problem

Code:

```
import pulp
from pulp import LpVariable, LpProblem, LpMaximize, LpStatus, value, LpMinimize
# Problem (Initial)
# define variables
x1 = LpVariable("Ribeye", 0, None) # MFR>=0
x2 = LpVariable("Sausage", 0, None) # EOC>=0
x3 = LpVariable("Kimbap", 0, None) # ER>=0
x4 = LpVariable("Yogurt", 0, None) # SC>=0
x5 = LpVariable("Tofu", 0, None) # WM>=0
# defines the problem
prob = LpProblem("problem", LpMinimize)
# define constraints
prob += 7*(60*x1 + 660*x2 + 320*x3 + 65*x4 + 5*x5) <= 7*5000 \# sodium constraint
prob += 7*(390*x1 + 170*x2 + 140*x3 + 150*x4 + 80*x5) >= 7*2000 # energy constraint
prob += 7*(18*x1 + 13*x2 + 4*x3 + 16*x4 + 10*x5) >= 7*50 # protein constraint
prob += 7*(0*x1 + 0*x2 + 0*x3 + 0.9*x4 + 0*x5) >= 7*20 \% vitamin D constraint
prob += 7*(30*x1 + 0*x2 + 50*x3 + 170*x4 + 150*x5) >= 7*1300 # calcium constraint
prob += 7*(2*x1 + 0*x2 + 0.5*x3 + 0*x4 + 1.19*x5) >= 7*18 # iron constraint
prob += 7*(260*x1 + 0*x2 + 70*x3 + 200*x4 + 140*x5) >= 7*4700 # potassium constraint
# define objective function
prob += 6.84*x1 + 1.12*x2 + 1.66*x3 + 0.83*x4 + 0.63*x5
# solve the problem
status = prob.solve()
print(f"Problem")
print(f"status={LpStatus[status]}")
# print the results
for variable in prob.variables():
    print(f"{variable.name} = {variable.varValue}")
print(f"Objective = {value(prob.objective)}")
print(f"")
```

Output:

```
Problem
status=Optimal
Kimbap = 0.0
Ribeye = 0.0
Sausage = 0.0
Tofu = 15.12605
Yogurt = 22.22222
Objective = 27.97385576
```

Appendix 6: Revised Problem

Code:

```
import pulp
from pulp import LpVariable, LpProblem, LpMaximize, LpStatus, value, LpMinimize
# Problem (Initial)
# define variables
x1 = LpVariable("Ribeye", 1, None) # MFR>=0
x2 = LpVariable("Sausage", 1, None) # EOC>=0
x3 = LpVariable("Kimbap", 1, None) # ER>=0
x4 = LpVariable("Yogurt", 1, None) # SC>=0
x5 = LpVariable("Tofu", 1, None) # WM>=0
# defines the problem
prob = LpProblem("problem", LpMinimize)
# define constraints
prob += 7*(390*x1 + 170*x2 + 140*x3 + 150*x4 + 80*x5) >= 7*2000 \# energy constraint
prob += 7*(18*x1 + 13*x2 + 4*x3 + 16*x4 + 10*x5) >= 7*50 # protein constraint
prob += 7*(0*x1 + 0*x2 + 0*x3 + 0.9*x4 + 0*x5) >= 7*20 # vitamin D constraint
prob += 7*(30*x1 + 0*x2 + 50*x3 + 170*x4 + 150*x5) >= 7*1300 \# calcium constraint
prob += 7*(2*x1 + 0*x2 + 0.5*x3 + 0*x4 + 1.19*x5) >= 7*18 # iron constraint
prob += 7*(260*x1 + 0*x2 + 70*x3 + 200*x4 + 140*x5) >= 7*4700 # potassium constraint
# define objective function
prob += 6.84*x1 + 1.12*x2 + 1.66*x3 + 0.83*x4 + 0.63*x5
# solve the problem
status = prob.solve()
print(f"Problem")
print(f"status={LpStatus[status]}")
# print the results
for variable in prob.variables():
   print(f"{variable.name} = {variable.varValue}")
print(f"Objective = {value(prob.objective)}")
print(f"")
```

Output:

```
Problem
status=Optimal
Kimbap = 1.0
Ribeye = 1.0
Sausage = 1.0
Tofu = 13.02521
Yogurt = 22.22222
Objective = 36.270326559999994
```

Appendix 7: ChatGPT Conversation

```
you are a data scientist and trying to solve the following problem:
For this assignment, you need to construct a personalized diet using current recommended dietary allowances from the U.S. Food and Drug Administration, updated to account for recent research on sodium intake and health (Mente, 0'Donnell, and Yusuf 2021).
 The constraints for this linear programming problem, should consider seven components of nutrition and their daily values, as shown in the following table:
 Daily Amount and measure
5,000 milligrams (mg)
Energy
Minimum
2,000 Calories (kilocalories, kcal)
Protein
 50 grams (g)
 20 micrograms (mcg)
Calcium
 Minimum
1,300 milligrams (mg)
Iron
 18 milligrams (mg)
  18 milligrams (mg)
   4,700 milligrams (mg)
   Set this up as a standard linear programming problem with decision variables taking any non-negative values. In other words, partial servings are permitted.
   For nutritional constraints, consider setting these to satisfy a weekly diet. That is, multiply each daily requirement by seven (7).
   (Optional) Add an eighth nutritional constraint that is especially relevant to anyone interested in weight control. Define the maximum number of Energy/Calories to be consumed each day, and multiply that daily requirement by seven (7).
   Nutrition labels on packaged foods should contain information about these eight components of nutrition along with other components. Each of the components represents a constraint in the linear programming problem you are developing.
   To adapt the problem to your personal diet, collect nutrition facts from five packaged food items in your household. Use packaged foods that are part of your normal diet and for which you have prices. Also, ensure that the across the set of food items there are positive values for each of the eight components of nutrition. Adjust the price for each food item so that it represents one serving size, as defined on the nutrition facts label. Each food item represents a decision variable in the linear programming problem.
   The goal or objective of this problem is to find the minimum-cost diet (servings of food items) that satisfies the eight nutritional requirements.
   construct a model to solve this problem and the code to go with it. Ask my questions to get more information or to clarify.
   To solve this problem, I can construct a linear programming model and write the code to solve it using Python with libraries such as **PuLP** or **SciPy** for optimization. Here's the general approach to building the model:
   ### **Problem Setup**

1. **Decision Variables**:

- Let \( \( \times 1, \times 2, \) represent the servings of each food item. These will be continuous variables that can take any non-negative values.
   2. **Objective Function**:

- Minimize the total cost: \(\text{Minimize} \) C = p_1 \cdot x_1 + p_2 \cdot x_2 + \ldots + p_5 \cdot x_5 \), where \(\( p_i \) \) is the price per serving of the \(\( i\)\)-th food item.
   3. **Constraints**:
               4. **Optional Constraint**:  
- Maximum energy intake: \( \sum_{i=1}^{5} e_i \cdot x_i 
   5. **Nutritional Values**:
- Extract from nutrition labels for \( s_i, e_i, p_i, \) etc., for all five food items.
```

```
### **Questions for You**
1. Can you provide:
   - The names of five packaged food items you consume.
   - Their nutrition facts (Sodium, Energy, Protein, Vitamin D, Calcium, Iron, Potassium) per serving.
   - Their cost per serving.
2. Do you want to include the optional constraint of maximum energy intake? If yes, what is the daily max?
3. Should we solve this using **PuLP**, **SciPy**, or another optimization library?
### **Code Framework**
Once I have the information, I'll structure the problem and write the code to solve it. Here's an outline:
```python
from pulp import LpProblem, LpVariable, lpSum, LpMinimize
1. Define the problem
problem = LpProblem("Minimum_Cost_Diet", LpMinimize)
2. Define decision variables
x1 = LpVariable("Food1_Servings", lowBound=0)
x2 = LpVariable("Food2_Servings", lowBound=0)

x3 = LpVariable("Food3_Servings", lowBound=0)

x4 = LpVariable("Food4_Servings", lowBound=0)

x5 = LpVariable("Food5_Servings", lowBound=0)
3. Define objective function (minimize cost)
cost = [p1, p2, p3, p4, p5] # Replace with actual values
problem += lpSum([cost[i] * vars[i] for i in range(5)])
4. Define constraints
Sodium (Max)
problem += lpSum([sodium[i] * vars[i] for i in range(5)]) <= 35000</pre>
Energy (Min)
problem += lpSum([energy[i] * vars[i] for i in range(5)]) >= 14000
Protein, Vitamin D, Calcium, Iron, Potassium
Add similar constraints...
5. Solve the problem
problem.solve()
6. Output the results
for v in problem.variables():
 print(f"{v.name} = {v.varValue}")
print(f"Total cost = ${value(problem.objective):.2f}")
Let me know the data so I can refine and implement the full solution for your personalized diet!
```

```
User
User
To answer your questions:

1. The names of five packaged food items are ALDI ribeye steak, Costco Chicken and Apple Sausage, Trader Joes frozen kimbap, Stonyfield organic whole milk greek yogurt, and tofu.

2. For their nutrition facts (Sodium, Energy, Protein, Vitamin D, Calcium, Iron, Potassium) per serving, please research online to find their nutrition facts.

3. Their cost per serving for steak is $6.84, sausage is $1.12, kimbap is $1.66, yogurt is $0.83, and tofu is $0.63

4. Do you want to include the optional constraint of maximum energy intake? If yes, what is the daily max? There is no maximum energy intake.

5. Should we solve this using PuLP, ScIPy, or another optimization library? you can use PuLP.
 To construct a personalized, cost-effective diet plan that meets your weekly nutritional requirements, we'll use the five specified packaged food items. Below are the nutrition facts per serving for each item:
 ChatGPT
1. ALDI Ribeye SteakB
- **Serving Size**: 010 oz (283g)00
- **Calories**: 0040 kg (283g)00
- **Protein**: 0145 kg (00
- **Protein**: 0145 kg (00
- **Calcium**: 015 by (-20mg)00
- **Calcium**: 015 by (-25 by (-20mg)00
- **Protein**: 015 by (-4.5 kg)00
- **Potassium**: 015 by (-4.5 kg)00
- **Vitamin D**: 015 by (-4.5 kg)00
- 015 by (-4.5 kg)00
- 015 by (-4.5 kg)00
- **Vitamin D**: 015 by (-4.5 kg)00
- 015 by (-4.5 kg)
2. Costco Chicken and Apple Sausage©
-**Serving Size**: ©1 link (68g)@8
-**Calories**: @12 kcike (68g)@8
-**Protein**: @12 kcike (68g)@8
-**Protein**: @10 specified@8
-**Calcium**: @Not specified@8
-**Calcium**: @Not specified@8
-**Tron**: @Not specified@8
-**Protassium**: @Not specified@8
-**Vitamin D**: @Not specified@8
@cite@turn@search2@8
3. Trader Joe's Frozen Kimbap8

- **Serving Size**: 101 package (170g)100

- **Calories**: 10Not specified100

- **Fortein**: 10Not specified100

- **Sodium**: 10Not specified100

- **Iron**: 10Not specified100

- **Iron**: 10Not specified100

- **Potassium**: 10Not specified100

- **Vitamin D**: 10Not specified100

- **Vitamin D**: 10Not specified100
 4. Stonyfield Organic Whole Milk Greek Yogurt©
- **Serving Size**: EMX cup (170g))000
- **Calories*: EM130 kcale
- **Protein**: EM15gE000
- **Portein**: EM5mgE000
- **Calcium**: EM5mgE000
- **Calcium**: EM5mgF000
- **Portein**: EM5 DV (-195mg))000
- **Portein*: EM5 DV (-0.00))000
- **Portein*: EM5 DV (-0.00))000
- **Portein*: EM5 DV (-0.00))000
 - **Potassium**: @Not specified@@

S. Tofu@

- **Serving Size**: @3 oz (85g)@@

- **Calories**: @70 kcal@@

- **Portein*: @85g@@

- **Sodium**: @15mg@@

- **Calcium**: @15mg@@

- **Iron**: @85 DV (-1.44mg)@@

- **Iron**: @85 DV (-1.44mg)@@

- **Potassium**: @Not specified@@

- **Vitamin D**: @Not specified@@
 Costs per Serving:0
- **Ribeye Steak**: @$6.84@0
- **Chicken and Apple Sausage**: @$1.12@0
- **Kimbap**: @$1.66@0
- **Yogurt**: @$0.83@0
- **Tofu**: @$6.63@0
 Note: @Some nutritional values are not specified in the available data. For a comprehensive analysis, please refer to the nutrition labels on the packaging of these products.@@
 Linear Programming Model Setup: D
- **Decision Variables**: D
 - Tilet ((x_1, x_2, x_3, x_4, x_5 \) represent the number of weekly servings of Ribeye Steak, Chicken and Apple Sausage, Kimbap, Yogurt, and Tofu, respectively.
 - **Objective Function**: - EMinimize the total cost: EM
 - **Constraints**:
 Constraints:0
- "*Energy (Calorises)**:0
- DMinimum: \(2,000 \times 7 = 14,000 \) kcal per week.00
- DN[940x_1 + 120x_2 + \text{(Calories in Kimbap)}x_3 + 130x_4 + 70x_5 \gqq 14,000 \] \]00
- "Protein":0
- DMinimum: \(5 0 \times 7 = 350 \) grams per week.00
- DN[45x_1 + 13x_2 + \text{(Protein in Kimbap)}x_3 + 15x_4 + 8x_5 \gq 350 \] \]00
- "Sodium*:0
- "Maximum: \(5,000 \times 7 = 35,000 \) mg per week.00
- DN[150x_1 + \text{(Sodium in Sausage)}x_2 + \text{(Sodium in Kimbap)}x_3 +
 [File]: file-6eJoMrSRhrxsausjxhysV5-image.png
[File]: file-MSZpf6ISL1fkbkZMp7gXmr-image.png
[File]: file-Gn9g4SSCAWSV5DrpKaug-image.png
those are not the correct nutrition facts. please use the following pictures for steak, sausage, and kimbap respectivefully.
 ChatGPT
 Thank you for providing the updated images of the nutrition facts for the steak, sausage, and kimbap. Let me extract the relevant information from the uploaded files.
```

It seems there was an issue locating the uploaded images. Let me re-check and ensure the images are processed correctly. I'll extract the information again.

```
ChatGPT
It looks like the text extraction worked for only one image (the sausage nutrition facts), but the other two images weren't correctly accessed due to file path issues. Could you re-upload the images for the steak and kimbap so I can properly process them?
User this graph to get to all the nutrition facts of all the food to solve the problem: Ribeye Steak Chicken & Apple Sausage Kimbap Greek Yogurt Tofu Sodium 60 660 320 65 5
Energy 390 170 140 150 80
Protein 18 g 13 4 16 10
Vitamin D 0 0 0 0,9 0
Calcium 30 0 50 170 150
Iron 2 0 0 150 0 1.19
 5
80
10
0.9
150
1.19
200
Iron 2
Potassium
Thank you for providing the nutritional data for all the foods. I'll now construct the linear programming model using PuLP to solve for the minimum-cost diet that satisfies the weekly nutritional requirements. Let's proceed with the solution.
It seems the required `PuLP` library is not installed in this environment. You can run the following code locally if you have Python installed. Make sure to install PuLP using:
```bash
pip install pulp
Here's the code for you to use locally:
```python
from pulp import LpProblem, LpVariable, LpMinimize, lpSum
Define the problem
problem = LpProblem("Minimum_Cost_Diet", LpMinimize)
}
weekly_requirements = {
 "Sodium": 5000 * 7,
 "Energy": 2000 * 7,
 "Protein": 50 * 7,
Decision variables (number of servings of each food)
servings = {food: LpVariable(food, lowBound=0, cat="Continuous") for food in foods}
Objective function: Minimize cost problem += lpSum(costs[food] * servings[food] for food in foods), "Total Cost"
Sodium constraint (maximum limit)
Solve the problem
problem.solve()
Results
results = {food: servings[food].varValue for food in foods}
total_cost = sum(costs[food] * results[food] for food in foods)
print("Optimal servings of each food:", results)
print("Total cost of the diet: $", total_cost)
If you need further help with this, let me know!
solve the problem. give me the optimal solution for the decision variables and total cost
ChatGPT
It seems like I can't do more advanced data analysis right now. Please try again later. If you'd like, I can guide you through solving this problem on your own system using Python and PuLP. Let me know if you'd like me to provide the complete code and instructions for running it.
```

#### You've reached your data analysis limit.

Upgrade to ChatGPT Plus or try again tomorrow after 2:22 PM.





Message ChatGPT







