Assignment 2: Network Models --- Project Management

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This study focused on conducting a critical path analysis for a software development project to present its cost and timeline to the clients. By analyzing a defined list of tasks required for a successful product rollout, the team identifies key dependencies that determine the minimum project duration. Using linear programming, the analysis provides the best-case, expected, and worst-case estimates for completion time and cost, enabling the team and client to optimize resource allocation, mitigate risks, and track project progress effectively.

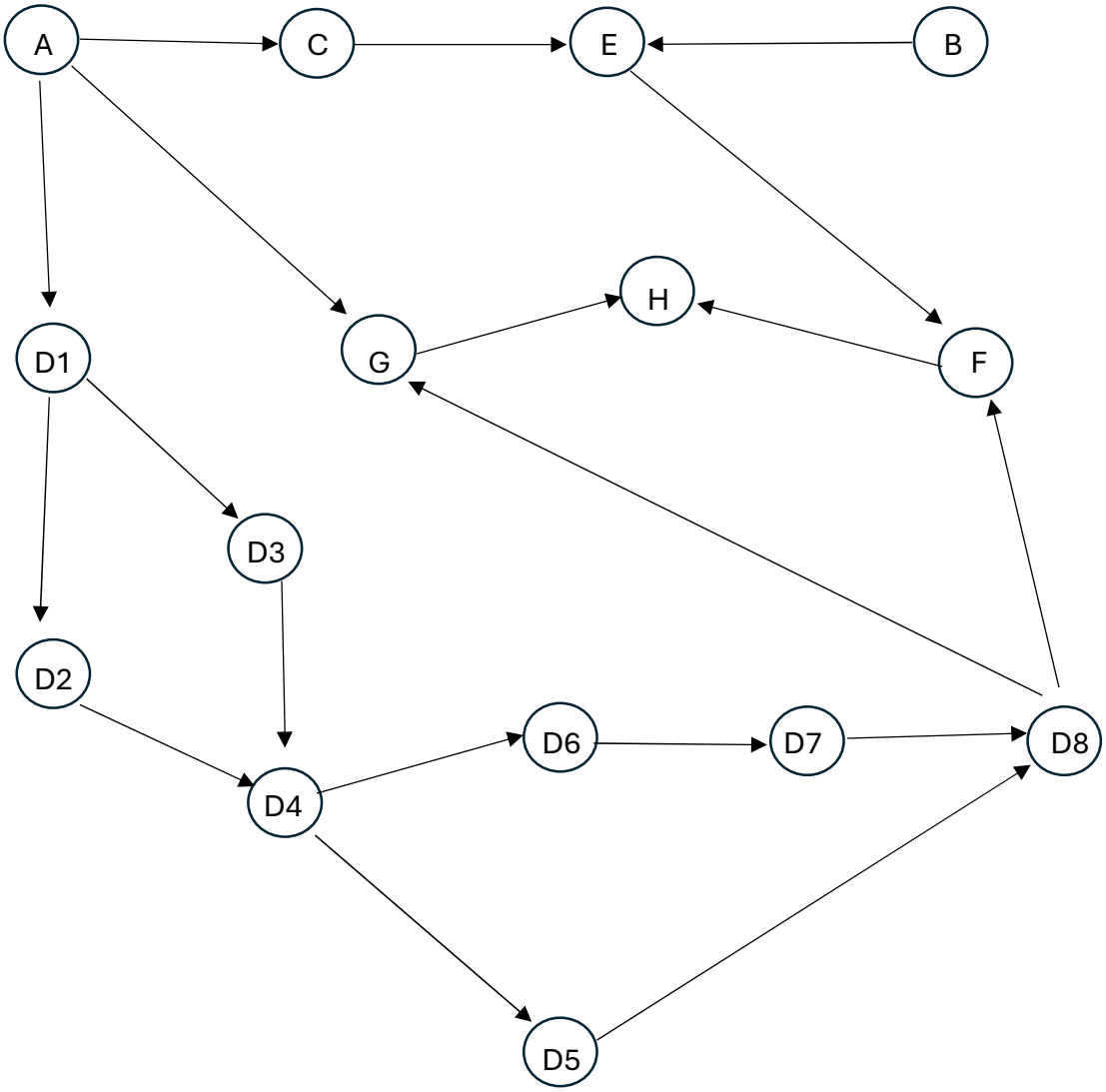
The goal of this problem is to determine the optimal sequence of tasks required to complete the project efficiently. To establish the problem setup, the team estimated the duration of each task under best-case, expected, and worst-case scenarios, along with an hourly rate for project contributors—values generated using ChatGPT. Time and cost are critical factors, as the critical path is determined by the duration and cost of each task. However, there is inherent uncertainty in these estimates, as the projected hours are not based on actual data, and the hourly rate has been generalized at $100 per hour for each contributor. A full directed graph illustrating the network of tasks and associated costs can be found in Appendix 1.

The project is modeled as a linear programming (LP) problem with two decision variables representing the start and end times of each task. The objective is to minimize the total project duration by summing the durations of all tasks. Constraints are defined to enforce task dependencies, ensuring that each task begins only after its predecessors are completed, along with a non-negativity constraint to prevent unrealistic negative start times. The LP problem was formulated and solved using Python and PuLP, yielding optimal project durations of 164 hours (best-case), 246 hours (expected), and 410 hours (worst-case). For the full problem formulation, refer to Appendix 2, and for the complete optimal solution results, see Appendix 3.

The LP solution provides a structured timeline for task completion and identifies the critical path, which determines the longest sequence of dependent tasks that dictate the overall project duration. In all three scenarios (best-case, expected, and worst-case), the critical path followed the sequence: 'DescribeProduct' → 'RequirementsAnalysis' → 'SoftwareDesign' → 'Coding' → 'UnitTesting' → 'SystemTesting' → 'PackageDeliverables' → 'DevelopImplementationPlan' → 'WriteClientProposal'. These nine tasks are essential, meaning any delay in one will directly impact the overall project timeline. Understanding this path allows for better risk management and resource allocation to prevent bottlenecks. Refer to Appendix 4 for the Gantt charts illustrating the project schedules across all scenarios.

This project was targeted at the design of a customer-focused recommendation system for restaurant owners in Marlborough, Massachusetts. The total project cost is primarily driven by labor costs, which depend on the number of contributors and estimated task durations. Using the expected duration of 246 hours and generalized hourly rate of $100 per contributor with 9 total workers, the cost would be $221,400. For the product prototype, which requires 201 hours, the estimated delivery time in a 40-hour work week is 5 weeks. Adding one additional contractor would reduce the delivery time by 9%, shortening it to approximately 4.5 weeks. Given the uncertainty in task durations, a Monte Carlo simulation can be used to model different project timelines. By simulating various duration estimates and hourly cost fluctuations, we can determine probability distributions for project completion time and cost, helping to mitigate risk and improve planning accuracy.

Appendix 1: Directed Graph Diagram



Appendix 2: LP Formulation

Decision Variables

Let:

- S_i be the start time of task i
- E_i be the end time of task i

Parameters

- d_i is the duration of task i
- P_i is the set of predecessor tasks for task i

Objective Function

Minimize the total project duration:

$$\min \sum_{i \in T} E_i$$

where T is the set of all tasks.

Constraints

1. **Task Duration Constraint:**

   Each task's end time is its start time plus its duration:

   $$E_i = S_i + d_i, \quad \forall i \in T$$

2. **Precedence Constraints:**

   A task can only start after all its predecessor tasks have finished:

   $$S_i \geq E_j, \quad \forall j \in P_i, \forall i \in T$$

3. **Non-Negativity Constraints:**

   $$S_i \geq 0, \quad E_i \geq 0, \quad \forall i \in T$$

Appendix 3: LP Solution Python Results

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Critical Path time for Best-case:
DescribeProduct starts at time 0
DevelopMarketingStrategy starts at time 0
WriteClientProposal ends at 164.0 hours in duration

Solution variable values for Best-case:
End_Coding = 84.0
End_DescribeProduct = 8.0
End_DesignBrochure = 18.0
End_DevelopImplementationPlan = 154.0
End_DevelopMarketingStrategy = 12.0
End_DevelopPricingPlan = 146.0
End_PackageDeliverables = 134.0
End_RequirementsAnalysis = 24.0
End_SoftwareDesign = 44.0
End_SurveyPotentialMarket = 34.0
End_SystemDesign = 44.0
End_SystemTesting = 124.0
End_UnitTesting = 100.0
End_WriteClientProposal = 164.0
End_WriteDocumentation = 96.0
Start_Coding = 44.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 8.0
Start_DevelopImplementationPlan = 134.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopPricingPlan = 134.0
Start_PackageDeliverables = 124.0
Start_RequirementsAnalysis = 8.0
Start_SoftwareDesign = 24.0
Start_SurveyPotentialMarket = 18.0
Start_SystemDesign = 24.0
Start_SystemTesting = 100.0
Start_UnitTesting = 84.0
Start_WriteClientProposal = 154.0
Start_WriteDocumentation = 84.0
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Critical Path time for expected case:
DescribeProduct starts at time 0
DevelopMarketingStrategy starts at time 0
WriteClientProposal ends at 246.0 hours in duration

Solution variable values for expected case:
End_Coding = 126.0
End_DescribeProduct = 12.0
End_DesignBrochure = 27.0
End_DevelopImplementationPlan = 231.0
End_DevelopMarketingStrategy = 18.0
End_DevelopPricingPlan = 219.0
End_PackageDeliverables = 201.0
End_RequirementsAnalysis = 36.0
End_SoftwareDesign = 66.0
End_SurveyPotentialMarket = 51.0
End_SystemDesign = 66.0
End_SystemTesting = 186.0
End_UnitTesting = 150.0
End_WriteClientProposal = 246.0
End_WriteDocumentation = 144.0
Start_Coding = 66.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 12.0
Start_DevelopImplementationPlan = 201.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopPricingPlan = 201.0
Start_PackageDeliverables = 186.0
Start_RequirementsAnalysis = 12.0
Start_SoftwareDesign = 36.0
Start_SurveyPotentialMarket = 27.0
Start_SystemDesign = 36.0
Start_SystemTesting = 150.0
Start_UnitTesting = 126.0
Start_WriteClientProposal = 231.0
Start_WriteDocumentation = 126.0
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Start_DescribeProduct = 0.0
Start_DesignBrochure = 20.0
Start_DevelopImplementationPlan = 335.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopPricingPlan = 335.0
Start_PackageDeliverables = 310.0
Start_RequirementsAnalysis = 20.0
Start_SoftwareDesign = 60.0
Start_SurveyPotentialMarket = 45.0
End_WriteClientProposal = 410.0
End_WriteDocumentation = 240.0
Start_Coding = 110.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 20.0
Start_DevelopImplementationPlan = 335.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopPricingPlan = 335.0
Start_PackageDeliverables = 310.0
Start_RequirementsAnalysis = 20.0
End_WriteClientProposal = 410.0
End_WriteDocumentation = 240.0
Start_Coding = 110.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 20.0
Start_DevelopImplementationPlan = 335.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopPricingPlan = 335.0
End_WriteClientProposal = 410.0
End_WriteDocumentation = 240.0
Start_Coding = 110.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 20.0
Start_Coding = 110.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 20.0
Start_DesignBrochure = 20.0
Start_DevelopImplementationPlan = 335.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopImplementationPlan = 335.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopPricingPlan = 335.0
Start_DevelopPricingPlan = 335.0
Start_PackageDeliverables = 310.0
Start_PackageDeliverables = 310.0
Start_RequirementsAnalysis = 20.0
Start_RequirementsAnalysis = 20.0
Start_SoftwareDesign = 60.0
Start_SurveyPotentialMarket = 45.0
Start_SystemDesign = 60.0
Start_SystemTesting = 250.0
Start_UnitTesting = 210.0
Start_WriteClientProposal = 385.0
Start_WriteDocumentation = 210.0
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Critical Path time for worst case:
DescribeProduct starts at time 0
DevelopMarketingStrategy starts at time 0
WriteClientProposal ends at 410.0 hours in duration

Solution variable values for worst case:
End_Coding = 210.0
End_DescribeProduct = 20.0
End_DesignBrochure = 45.0
End_DevelopImplementationPlan = 385.0
End_DevelopMarketingStrategy = 30.0
End_DevelopPricingPlan = 365.0
End_PackageDeliverables = 335.0
End_RequirementsAnalysis = 60.0
End_SoftwareDesign = 110.0
End_SurveyPotentialMarket = 85.0
End_WriteClientProposal = 410.0
End_WriteDocumentation = 240.0
Start_Coding = 110.0
End_WriteClientProposal = 410.0
End_WriteDocumentation = 240.0
Start_Coding = 110.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 20.0
End_WriteClientProposal = 410.0
End_WriteDocumentation = 240.0
Start_Coding = 110.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 20.0
Start_DevelopImplementationPlan = 335.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopPricingPlan = 335.0
Start_PackageDeliverables = 310.0
Start_RequirementsAnalysis = 20.0
Start_SoftwareDesign = 60.0
Start_SurveyPotentialMarket = 45.0
Start_SystemDesign = 60.0
Start_SystemTesting = 250.0
Start_UnitTesting = 210.0
End_WriteClientProposal = 410.0
End_WriteDocumentation = 240.0
Start_Coding = 110.0
Start_DescribeProduct = 0.0
Start_DesignBrochure = 20.0
Start_DevelopImplementationPlan = 335.0
Start_DevelopMarketingStrategy = 0.0
Start_DevelopPricingPlan = 335.0
Start_PackageDeliverables = 310.0
Start_RequirementsAnalysis = 20.0
Start_SoftwareDesign = 60.0
Start_SurveyPotentialMarket = 45.0
```

Appendix 4: Gantt Charts