

# Computation of the routing paths across the internet

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## 1 Commercial relationships

The internet is an interconnected collection of about 75,000 networks, called Autonomous Systems (ASes). Two ASes are neighbors if they have one link in each direction between them, allowing them to exchange traffic. Every pair of neighbor ASes engages in a commercial relationship. For simplicity, just two types of relationships are considered: (1) in a *provider-customer* relationship, the customer pays the provider to transit its traffic to and from the rest of the internet; (2) in a *peer-peer* relationship, the two peers exchange traffic between themselves and their customers without charges. The customer-provider relationships create an hierarchy in the Internet. At the bottom of the hierarchy are ASes without customers, called *stubs*. All other ASes are *Internet Service Providers* (ISP), the business of which is to transit traffic. At the top of the hierarchy are ASes without providers, called *Tier-1s*. The majority of ASes are stubs, which include: (1) content provider and content distribution networks, such as those of Google, Facebook, Amazon, Microsoft, Apple, Netflix, Akamai, and Cloudflare, each of which typically owns more than one AS; (2) access networks, university networks, and enterprise networks. There are only a dozen or so Tier-1s, including AT&T, Lumen Technologies, Verizon Enterprise Solutions, Deutsche Telekom Global Carrier, Telxius, NTT communications, and Tata Communications. The Center for Applied Internet Data Analysis (CAIDA) provides inferred topologies of the Internet annotated with the type of commercial relationship between neighbor ASes ([www.caida.org](http://www.caida.org)).

## 2 Routing paths

Routing paths across the internet are mostly determined by two factors. The first is the *destination-based* nature of data-packet forwarding. Each AS uses only information about the destination of data-packets to decide to which neighbor AS they should be forwarded. As a consequence, internet paths are not necessarily optimal. Rather, internet paths to a common destination form

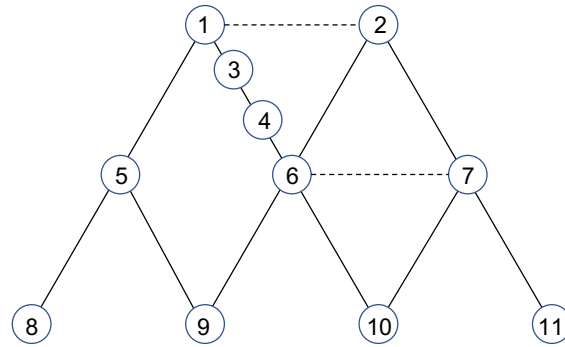


Figure 1: An internet. Solid lines join providers to customers, with providers drawn above customers. Dashed lines join peers.

an equilibrium such that no AS sees an improvement in the delivery of its data-packets to the common destination by unilaterally changing its forwarding AS neighbor. The second factor shaping internet routing paths is the commercial relationships between neighbor ASes. An AS-path is *valid* if, and only if, all intermediate ASes along the path are paid to transit traffic, either by their upstream or their downstream AS neighbors along the path, or by both. Among the valid AS-paths, a *customer (peer, provider) path* is one that starts with a link from an AS to a customer (peer, provider) AS. Given a choice among valid AS-paths from an origin to a destination AS, a customer path is preferred to a peer path and the latter is preferred to a provider path. And given a choice among AS-paths of the same type, customer, peer, or provider, a shorter AS-path is preferred. An internet is *commercially connected* if a valid path from every AS to every other.

In the internet of Figure 1, a provider is joined to a customer by a solid line, with the provider drawn above the customer, while two peers are joined by a dashed line. AS 1 is a provider of both AS 3 and AS 5, and a peer of AS 2. Each solid line represents two links in opposite directions.

- The shortest AS-path from AS 5 to AS 6 is  $5 - 9 - 6$ . However, that AS-path is not valid because, both being providers of AS 9, neither AS 5 nor AS 6 pays AS 9 to transit traffic.
- There are five AS-paths from AS 5 to AS 6, but only two of them are valid. They are  $5 - 1 - 3 - 4 - 6$  and  $5 - 1 - 2 - 6$ , both of which are provider paths. Data-packets from AS 5 to AS 6 are delivered along the first of these two AS-paths, although it is longer than the second. This is because AS 1 prefers customer path  $1 - 3 - 4 - 6$  over peer path  $1 - 2 - 6$ .
- Both  $1 - 5 - 9$  and  $1 - 3 - 4 - 6 - 9$  are customer paths from AS 1 to AS 9. The former path is preferred because it is shorter.
- The internet is commercially connected.

For more information on inter-AS routing, the interested student can start from the CAIDA site and the following references: Gao and Rexford [2001], Gill et al. [2013], Mazloum et al. [2014], Kastanakis et al. [2023].

### 3 Algorithms

You will be asked to design and implement algorithms that compute internet paths, and, using them, extract statistics concerning their types and lengths. The input to all algorithms includes an internet given as a file. Each line of the file describes a link; it consists of a tail AS, a head AS, and the commercial relationship the tail has with the head. AS identifiers are integers between 0 and  $2^{16} - 1 = 65535$ . Numbers 1, 2, and 3 represent a link from provider to customer, from peer to peer, and from customer to provider, respectively. For example, in the list below, the first two lines imply that AS 4323 is a provider of AS 12122 and the last two lines imply that AS 29017 and AS 34309 are peers. It is recommended that the input internet be represented in main memory

|       |       |   |
|-------|-------|---|
| 4323  | 12122 | 1 |
| 12122 | 4323  | 3 |
| 7018  | 17228 | 1 |
| 17228 | 7018  | 3 |
| 29017 | 34309 | 2 |
| 34309 | 29017 | 2 |

with an adjacency list and that a total number of  $2^{16} = 65536$  ASes be represented, even though some of them will not be part of the internet, having neither incoming nor outgoing links.

The following is requested.

- Design algorithm  $\text{COMMERCIAL}(Net, t)$  that receives as input internet  $Net$  and a destination  $t$  in that internet. The algorithm returns the type of AS-path from every AS to  $t$ . What is the worst-case complexity of the algorithm? The algorithm should allow an interactive mode for testing, where a user enters an origin AS and a destination AS and the algorithm outputs the type of AS-path from the origin to the destination.
- Design algorithm  $\text{COMMERCIALCYCLE}(Net)$  that receives as input internet  $Net$  and returns whether or not  $Net$  contains at least one cycle where every AS is a customer of the next around the cycle.
- Design algorithm  $\text{COMMERCIALCONNECTED}(Net)$  that receives as input internet  $Net$ , known to have no cycles where every AS is a customer of the next around the cycle, and returns

whether or not  $Net$  is commercially connected. There is an algorithm for this task that is much more efficient than running  $COMMERCIAL(Net, t)$  over all destinations. What is its worst-case complexity? Can you design  $COMMERCIAL(Net)$  regardless of whether or not  $Net$  contains a cycle where every AS is a customer of the next around the cycle? (This is a more advanced question; do not start here).

- Using  $COMMERCIALCYCLE(Net)$ ,  $COMMERCIALCONNECTED(Net)$ , and  $COMMERCIAL(Net, t)$ , design algorithm  $COMMERCIALALL(Net)$  that returns statistics for the types of AS-paths from every origin AS to every destination AS. The statistics should be presented as a Probability Distribution Function (PDF).
- Design algorithm  $COMMERCIALLENGTHS(Net, t)$  that receives as input internet  $Net$  and a destination  $t$  in that internet and computes the lengths of valid AS-paths from every AS to  $t$ . Are the computed lengths optimal over all valid AS-paths? Are they optimal over all AS-paths of the same type? The algorithm should have an interactive mode.
- Using  $COMMERCIALLENGTHS(Net, t)$ , design algorithm  $COMMERCIALLENGTHSALL(Net)$  that returns statistics of AS-path lengths from every AS to every other. The statistics should be presented as a Complementary Cumulative Distribution Function (CCDF).
- Design algorithm  $SHORTESTALL(Net)$  that receives as input internet  $Net$  and returns statistics of distances from every AS to every other.

Before designing the algorithms, you should try to build a model of the problem. A good model will allow a precise and concise statement of the problem, will identify suitable its elementary operations, and will expose its computational structure.

## 4 Report and evaluation

Students work on the project in groups of two. Each group should design, implement, and test the algorithms requested. Each group will write a report with no more than five pages summarizing its findings. The report must include the following:

- A clear description of all the algorithms designed. Only the most primitive and specialized algorithms, such as  $COMMERCIAL(Net, t)$ , will require pseudo-code. In this case, choose expressive names for the variables, structure and comment the pseudo-code.
- A discussion of the complexity of the algorithms.

- The statistics obtained by algorithms `COMMERCIALALL(Net)`, `COMMERCIALLENGTHSALL(Net)`, and `SHORTESTALL(Net)` on two input internets that will be given later on, representing fragments of the internet from 2009 and 2024.
- Overall conclusions about the subject of this project.

The following aspects are considered in the evaluation:

- I will start by reading your report, which is the means for you to communicate your ideas with others. Organize your report in sections; present high-level, but precise descriptions of your algorithms, highlighting their most subtle steps, if any; draw concise, but unambiguous conclusions.
- If the report is readable, I will run some tests on your programs.
- I will have a discussion with each group, at which time we test your programs and debate your choices. The discussion occurs at the end of the course.
- A mark will be given individually to each member of the group.

I will keep the option of asking for some small, extra requirements on the project. Thus, please stay in tune. You are welcome to share ideas with your colleagues, but the design and implementation of the algorithms and the text of the report must absolutely be your own.

The code and the report should be sent in a .zip file to my email address with subject `para.<group number>.zip` where `<group number>` is your group number. The deadline is Friday, October 11, at 23:59. The discussions will be in the week starting Monday, October 21.

## Bibliography

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- R. Mazloum, M.-O. Buob, J. Augè, B. Baynat, D. Rossi, and T. Friedman. Violation of interdomain routing assumptions. In *Proceedings of the 15th International Conference on Passive and Active Measurement*, pages 173–182, 2014.