2018-17119 박수현

Explanation of HW3 code

Part 1 Homography

H = compute_h(p1, p2)

Given that this function must return a 3 x 3 homography matrix, I set h to be the flatten-out 9 x 1 matrix and define a least squares problem $||Ah - 0||^2$. 2N x 9 matrix A is constructed using the coordinates of each correspondence in p1 and p2, and singular value decomposition $\mathrm{A} = \mathrm{U}\Sigma\mathrm{V}^{\mathrm{T}}$ is performed. Since I have proven in Question 2 of the theory part of the homework that the *last column of* V is the solution $\hat{h}$, I retrieve the *last row of* V^T obtained by the SVD. Then the output `H` matrix is reshaped into a 3 x 3 form.

H = compute_h_norm(p1, p2)

A normalization step can improve the numerical stability of the solution, to be more stable in presence of noise and to prevent divergence from the correct result.

I follow the data normalization technique explained by Rafael Garcia[1], where the method was originally proposed by Hartley and Zisserman[2].

First, the coordinates in each image are translated so that the centroid of the points is set as the origin of coordinates.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \text{and} \quad \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

Next, the coordinates are scaled so that the average distance from a point to the origin is $\sqrt{2}$. The initial average distance from every point to the origin of coordinates is calculated as follows:

$$\bar{d} = \frac{\sum_{i=1}^{n} \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}}{n}$$

Now the scaling factor can be computed as $s = \frac{\sqrt{2}}{\bar{d}}$. As a result, the translation and scaling can be performed by means of the transformation matrix T.

$$\mathbf{T} = \begin{pmatrix} \frac{\sqrt{2}}{\bar{d}} & 0 & -\left(\frac{\sqrt{2}}{\bar{d}}\bar{x}\right) \\ 0 & \frac{\sqrt{2}}{\bar{d}} & -\left(\frac{\sqrt{2}}{\bar{d}}\bar{y}\right) \\ 0 & 0 & 1 \end{pmatrix}$$

I derive two translation matrices `T1` and `T2`, each for `p1` and `p2`. Through matrix multiplication, `p1` and `p2` are normalized, which will be passed to `compute_H()` and get the initial homography matrix. The standard homography matrix can be obtained through the following denormalization equation:

$$\mathrm{H} = (T1)^{-1}H(T2)$$

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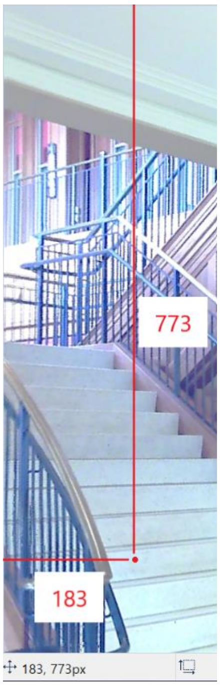
[1] Garcia, Rafael, "A proposal to estimate the motion of an underwater vehicle through visual mosaicking", 2002. pp. 133-134. https://www.researchgate.net/publication/265623150_A_proposal_to_estimate_the_motion_of_an_underwater_vehicle_through_visual_mosaicking

[2] R. Hartley and A. Zisserman, "Multiple View Geometry in Computer Vision," Cambridge University Press, 2000.

Part 2 Mosaicing

`p_in, p_ref = set_cor_mosaic()`

To retrieve coordinates of pixels, I used Windows Paint. One interesting point to note is that Paint gives image coordinates in a similar way matrix indices are done. For example, in the right image, the image coordinate (183, 773) is equivalent to the NumPy index (773, 183). This finding of the coordinate mechanism gains importance as several NumPy operations are performed.



Criterion for selecting correspondences

I selected correspondences according to the following list of criteria:

- ✓ Is located at the overlapping region of the two images
- ✓ Is a corner point of an object, which can be best distinguished from gradients of the surrounding area
- ✓ has an image intensity that sufficiently differs from the surrounding area

Number of correspondences used

I used **9 correspondences**, which is more than the minimum number of correspondences required (4). The correspondences include points of the television, yellow sign near to the camera, pillars of the entrance, table, chair, and some air conditioning vent on the ceiling.

`igs_warp, igs_merge = warp_image(igs_in, igs_ref, H)`

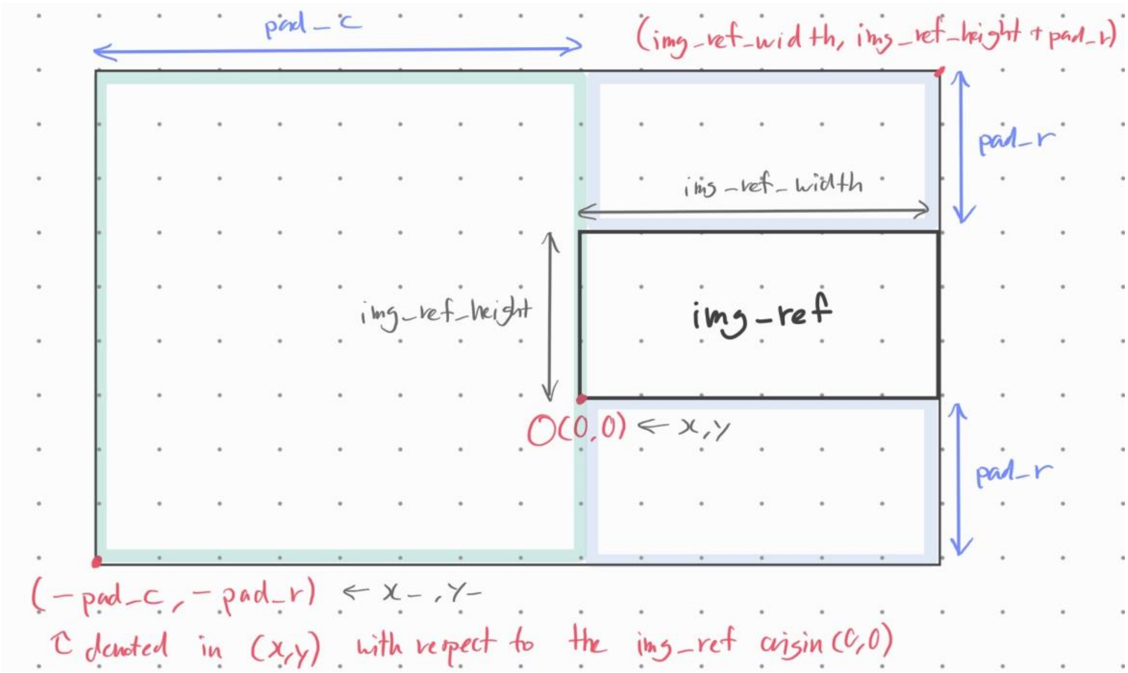
Determining the size of `igs_merge` and defining the coordinate system

The figure below shows how I formed the `igs_merge` canvas and denoted the origin of the image.

`igs_merge` is formed through zero padding with the padding size of `((pad_r, pad_r), (pad_c, 0), (0, 0))`. The optimal `igs_merge` size that makes both `igs_warp` and `igs_ref` best fit to the borders was determined after some calibration attempts. The resulting padding size is calculated from `pad_r, pad_c = 461, 1640`.

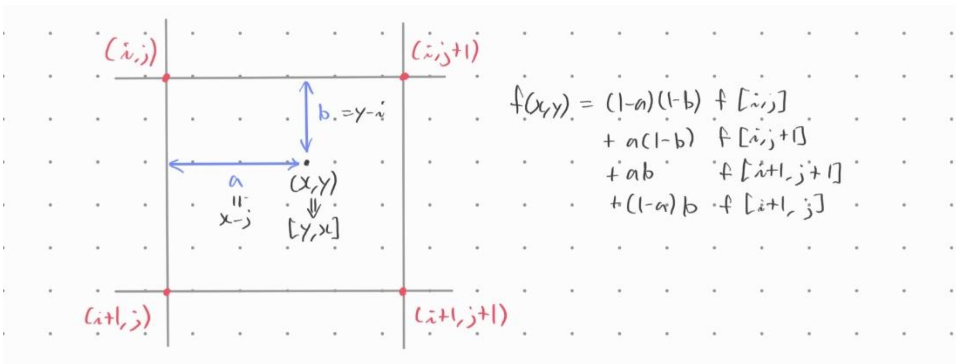
Since the inverse warping method requires pixels from coordinates centered to `img_ref`, I design the algorithm to operate along the origin (0,0) set at the bottom left corner of `img_ref`. That is, from the perspective of `img_ref`, the coordinates of `igs_merge`'s bottom left corner A should be A`(-pad_c, -pad_r)` and the upper right corner B should be B`(img_ref_width, img_ref_height + pad_c)`. Therefore, I looped through coordinates `(x_, y_)` of `igs_merge` from A to B and calculated their corresponding locations `(x, y)` in `igs_in`. Note that in NumPy notations, each locations refer to `igs_merge[y_, x_]` and `igs_merge[y, x]`, respectively.

`(x, y)` is obtained by matrix multiplication of H^{-} and individual homogeneous coordinates of `(x_, y_)`.



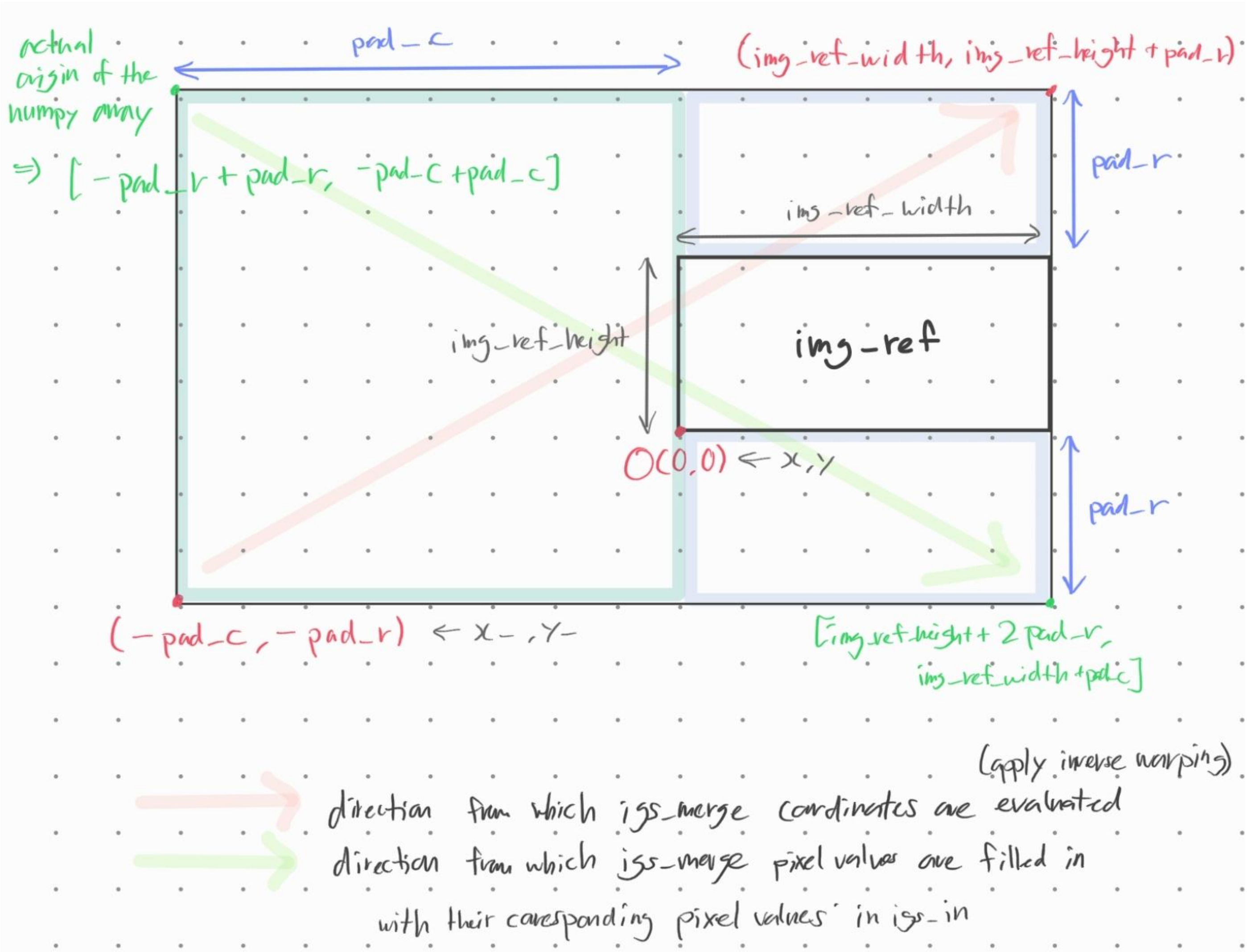
Bilinear interpolation

During inverse warping, it is critical to handle (x, y) coordinates that are float values instead of integers. I use bilinear interpolation to interpolate color value from its neighbors. For more convenient NumPy operations, instead of using image-wise coordinates of (x, y), I use notations of (y, x). Accordingly, I put out the appropriate neighboring coordinates and distance measures as follows:



Coordinate selection and pixel value insertion for `igs_warp` and `igs_merge`

NumPy operations set the starting point (0,0) differently, namely the upper left corner of `igs_merge`. While `igs_merge` coordinates `(x_, y_)` are looped through from bottom left to upper right (see the red text and arrow in the figure below), when inserting pixel values into the NumPy array `igs_merge[y_, x_]`, the indices must be nonnegative. Thus, I add `pad_r` and `pad_c` to `y_` and `x_` respectively so that `igs_merge` will be filled out starting from indices [0, 0]. In this sense, pixel values are inserted to `igs_merge` from the upper left to bottom right (see the green text and arrow).



The operations for `igs_warp` and `igs_merge` are the same until this last coordinate selection and pixel insertion step. The output `igs_warp` is essentially `igs_in` warped to match `igs_ref`'s perspective and dimension. It consists of `igs_in`'s pixel coordinates and RGB values that also exist in `img_ref`. Recalling that coordinates `(y_, x_)` in `igs_merge` are are set with respect to `img_ref` coordinates, and `igs_warp` is the same dimension with `img_ref`, I assign the (interpolated) pixel value to `igs_warp[y_, x_]`. To indicate correct positions within the boundary, `y_` and `x_` are subject to the condition of being nonnegative. The following is a code snippet for this part.

```
if 0 <= y_ < igs_ref_height and 0 <= x_:  # x_ is already smaller than igs_ref_width
    igs_warp[y_, x_] = pixel
igs_merge[y_ + pad_r, x_ + pad_c] = pixel
```

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Part 3 Rectification

`c_in, c_ref = set_cor_rec()`

For `c_in`, I selected the **4 corner points** at which the boundary lines of the Iphone intersect. (left image)

For `c_ref`, I first edited the input image by applying the "Free Transform" function in Photoshop, to get the desired output that the `rectify()` function should produce. This was an attempt to visually justify the resulting points to form a shape that preserves the original scale and dimension of the input image, rather than naively guessing the positions. Just like `c_in`, I retrieved the **4 corner points** that the rectified sample image gives, and adjusted the x-, y- coordinates so that all of the corner angles are equal (90°).

`igs_rec = rectify(igs, p1, p2)`

Image rectification basically follows the same procedure done in `warp_image()`. The difference is that while `warp_image()` needs to cover two images and a canvas of bigger dimension, `rectify()` edits only a single image. This means that the coordinate system doesn't have to be readjusted according to `igs`'s origin, and I can directly loop through the NumPy array coordinates from the upper left to the lower right. Therefore, `igs[y_, x_]` is assigned a bilinear-interpolated pixel value at the position of the inversely-warped correspondence `(x, y)`.