Computer Science 750 (2022)

Assignment 1

This assignment is worth 30 marks representing 30% of your total course grade.

Due date: 28 August 2022 submitted via Canvas before 23.55

Notes

- You solutions have to be typed.
- To obtain full credit, your script must clearly explain why your answers are correct.
- You can use without proof any result proved in class. You need to state correctly the result and show how it applies to the specific problem you solve.
- Research is encouraged. You must reference any source you rely on. Avoid quotes.
- Your submissions must adhere to the Academic Integrity standards. Copying from any source, even translated, is academic misconduct.

Questions

1. Question 1 [Program analysis]

Goldbach's conjecture states that every even integer greater than 2 is the sum of two prime numbers.

a) Construct an algorithm $\mathbf{A}(N)$ that on every even integer N>2 decides whether N is the sum of two primes.

[2 marks]

b) Does $\mathbf{A}(N)$ halt for every N? Justify with proof your answer.

[3 marks]

c) Assume A(N) returns 1 if N is the sum of two prime numbers and 0 otherwise. Does the algorithm:

1. G = 1.

2. N = 1

3. If $\mathbf{A}(N) = 1, N = N + 1$, go to 2.

4. G = 0.

5. Stop.

halt? Justify with proof your answer.

[2 marks]

Does it solve the Goldbach conjecture? Justify with proof your answer,

[2 marks]

[2 marks]

2. Question 2 [Research quation]

Write 800-1000 words discussing the claim that

Any function can be computable. Use can use the reference: John Baez, Computing the Uncomputable,

https://johncarlosbaez.wordpress.com/2016/04/02/computing-the-uncomputable/.

(a) State and explain the main result,

(b) Explain in your own words the proof.(c) Does the main result contradict the Halting Theorem? Justify with proof your answer.[3 marks]

3. Question 3 [Mathematical modeling]

(d) Define the property "the infinite binary sequence contains a monochromatic arithmetic progression".

[4 marks]

(e) Does every infinite binary sequence contain a monochromatic arithmetic progression? Justify with proof your answer.

[4 marks]

(f) Answer questions a) and b) for ternary sequences. Justify with proof your answers.

[4 marks]

These here seem rather trivial given an infinite binary sequence s = s1, s2, s3,...

t > 0, k > 1 there exists some si, si+t, si+2t, ..., si+(k-1)t substring such that this substring is either in $\{0^k\}$ or $\{1^k\}$.

Proof:

Give me some infinite binary string... look at the first two letters:

case 00, 11: clearly a monochromatic substring case 10, 01:

Way too obvious no?