COMPSCI 761: ADVANCED TOPICS IN ARTIFICIAL INTELLIGENCE

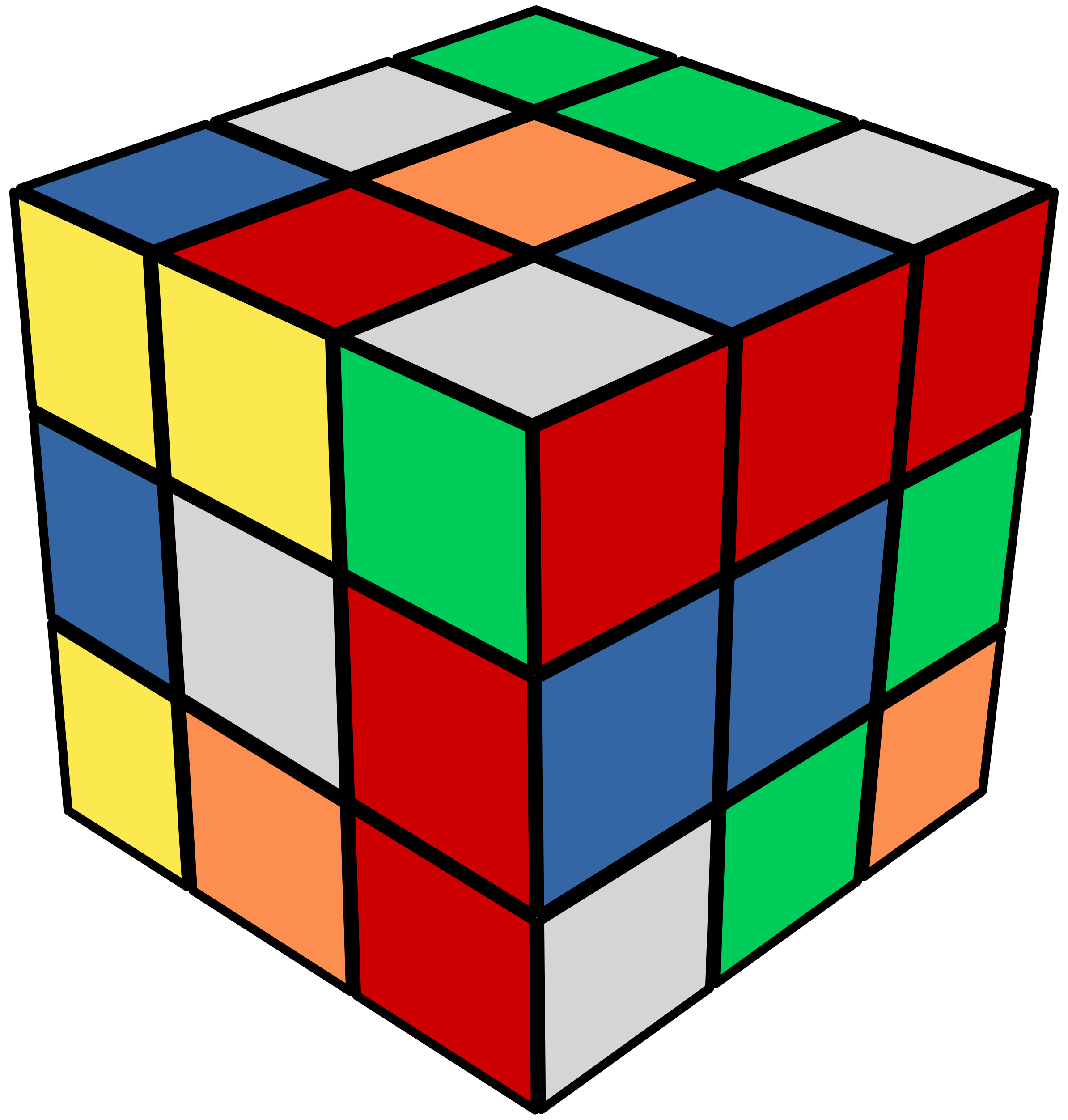
CONSTRAINT SATISFACTION PROBLEM I

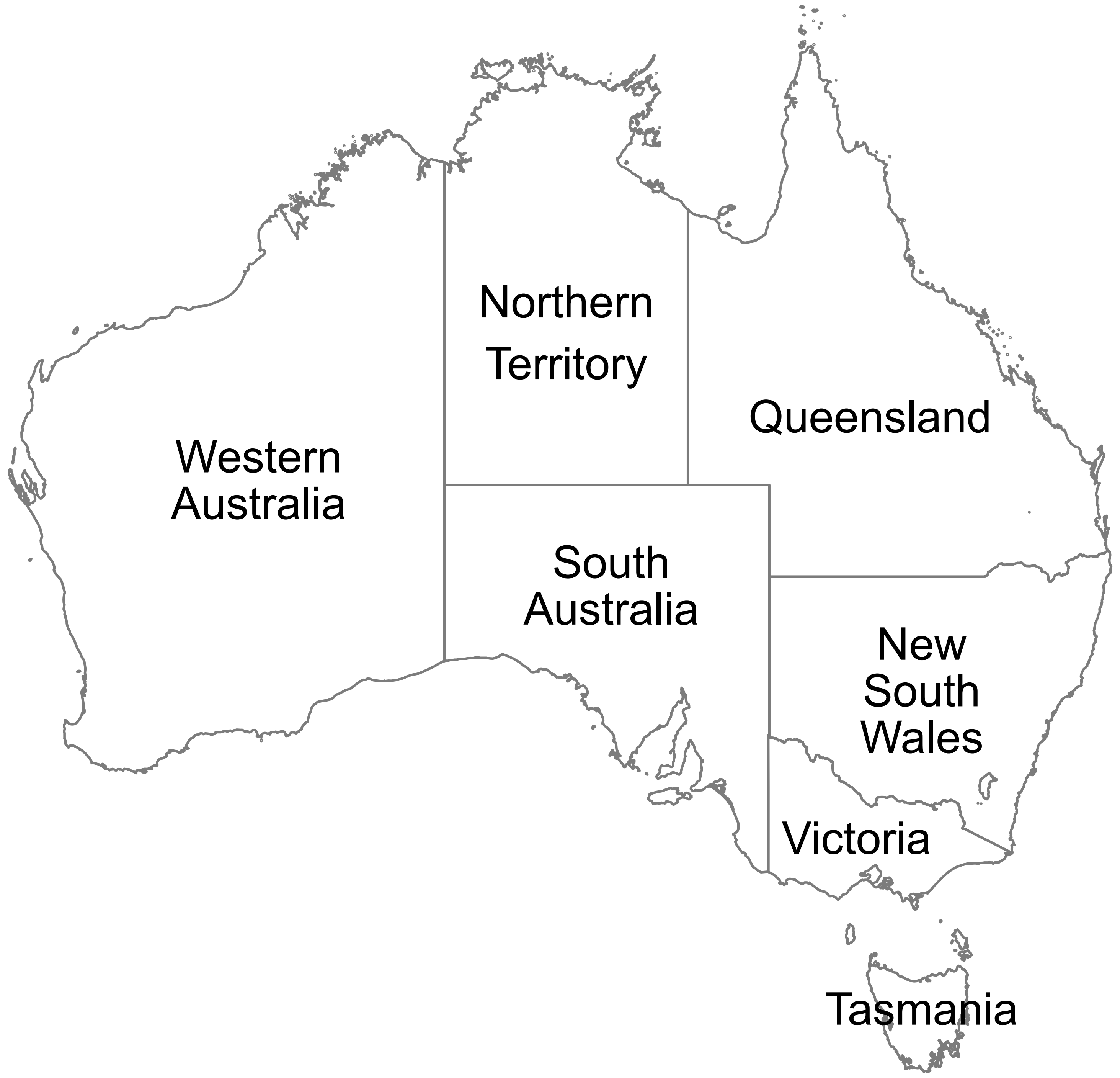
Anna Trofimova, July 2022

TODAY

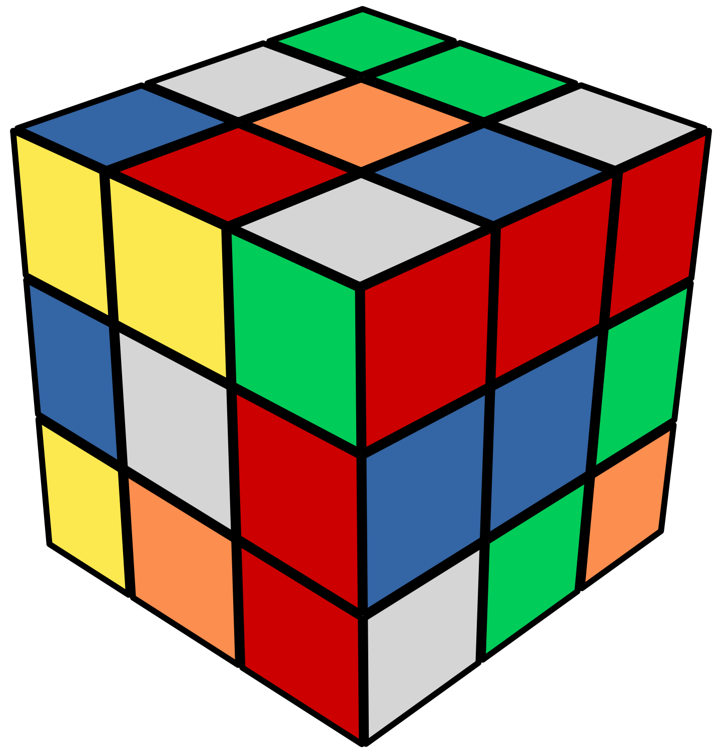
- Atomic Representation vs Factored representation
- CSP Definition
- Examples of CSP

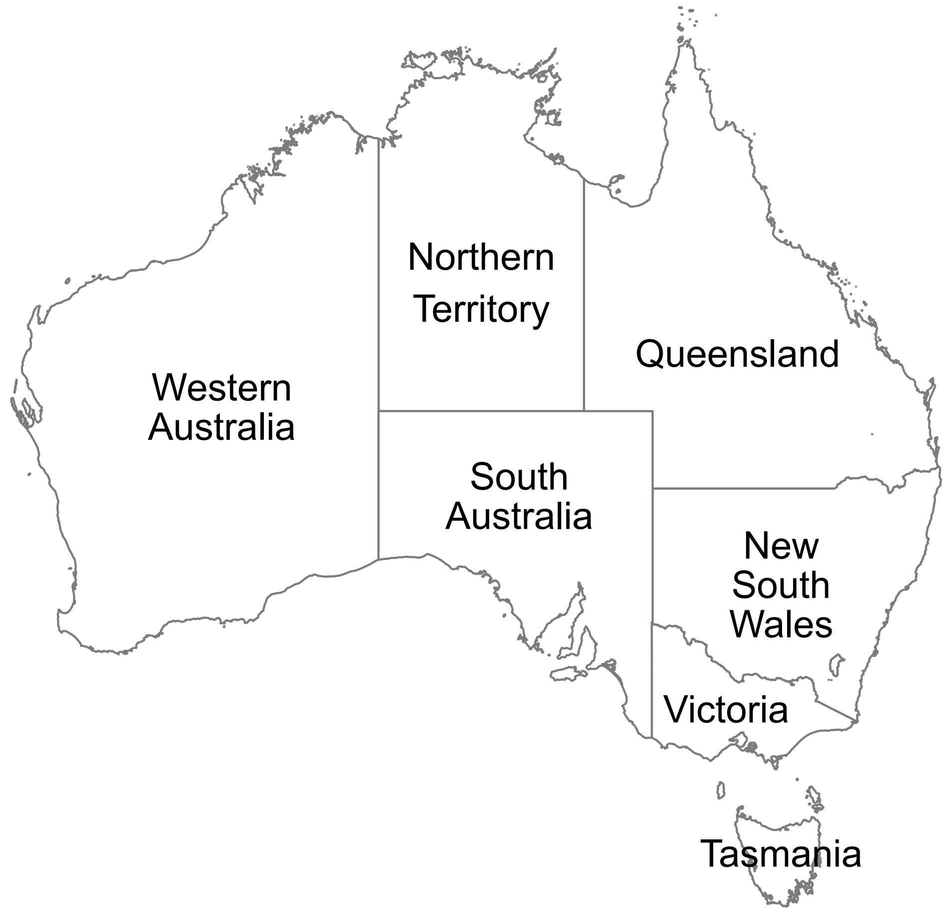
RUBIK'S CUBE VS MAP COLOURING





RECAP: SOME SEARCH PROBLEMS TYPES





Models with atomic representation have no internal structure; the state either does or does not match the goal state.

In Rubik's cube, the tiles alignment is either correct or not.

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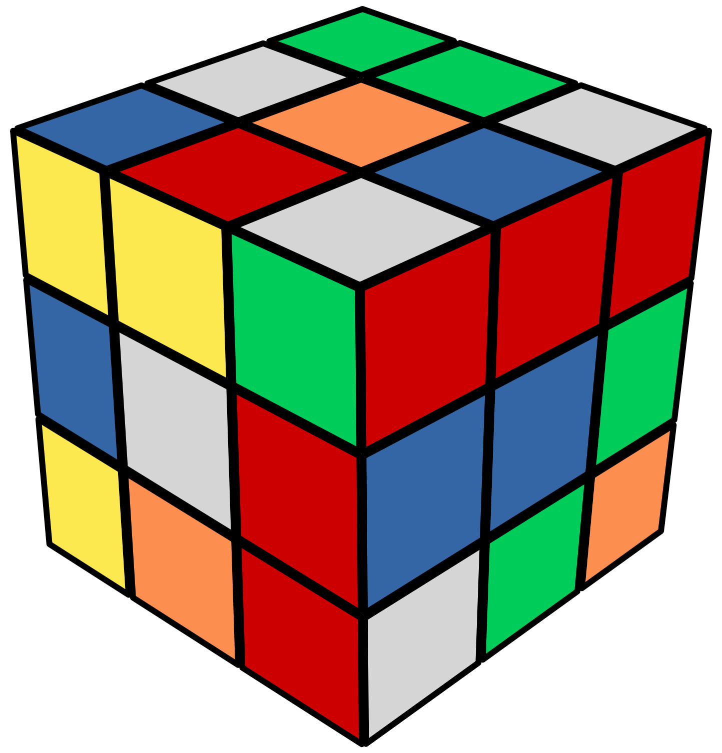

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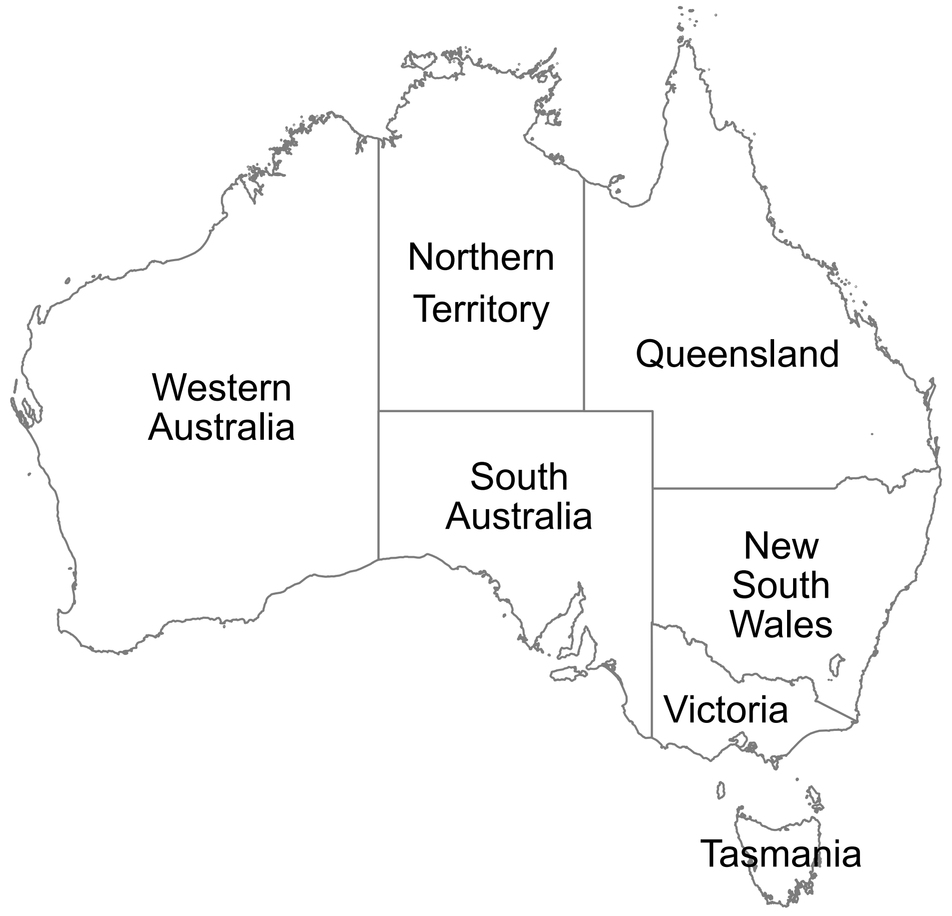


Models with factored representation have a set of variables, each of which has a value.

Every state on the map is a variable that has a colour value.

CONSTRAINT SATISFACTION PROBLEMS (CSPS)

- Constraint Satisfaction Problems are defined by a set of variables X_i, each with a domain D_i of possible values, and a set of constraints C that specify allowable combinations of values.

- The aim is to find an assignment of the variables X_i from the domains D_i in such a way that none of the constraints C are violated.
  - i.e. all of the constraints C are satisfied

EXAMPLE: MAP-COLOURING

Variables: WA, NT, Q, NSW, V, SA, T

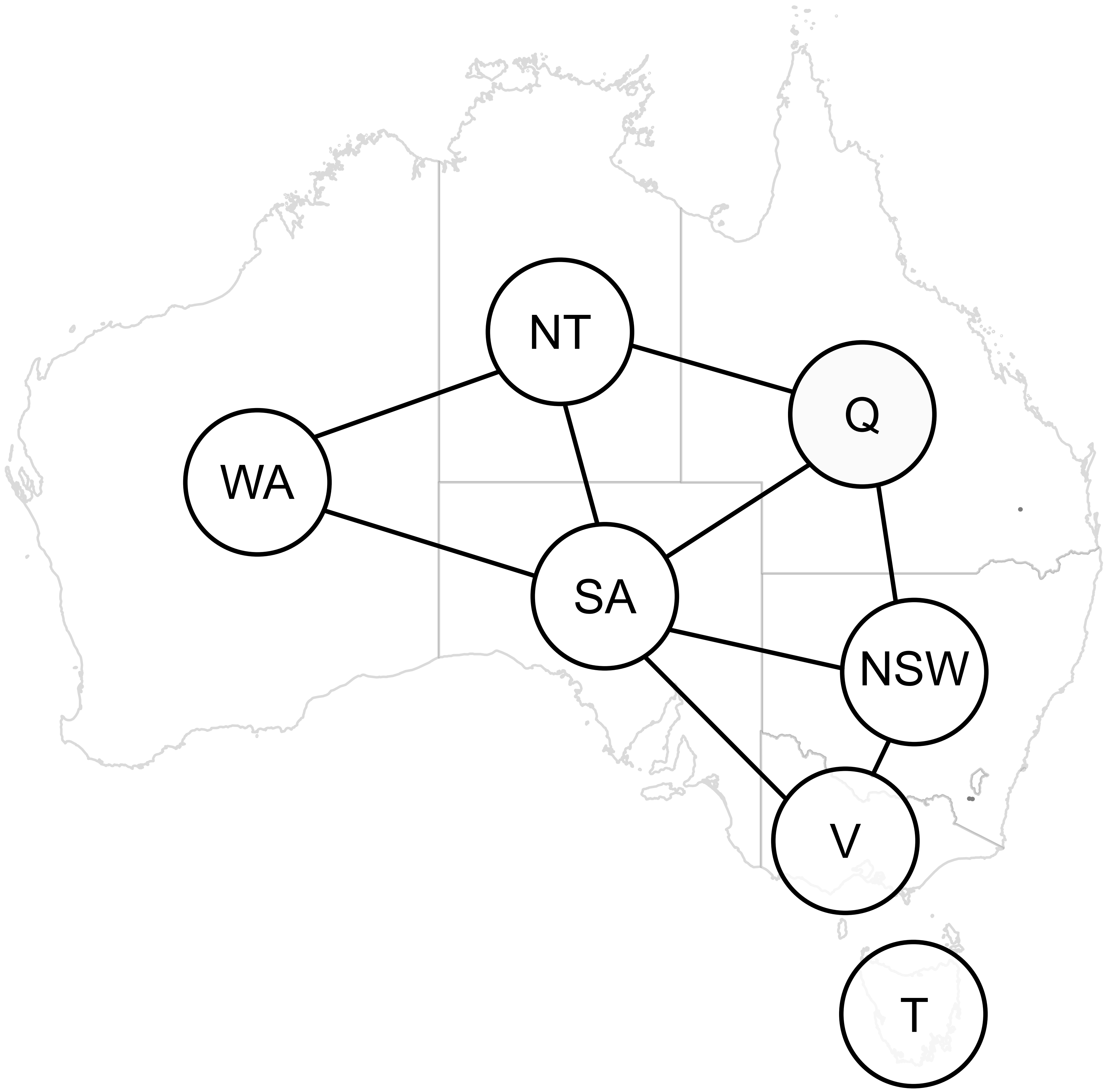
Domains: Di = {red, green, blue}

Constraints: adjacent regions must have different colours **e.g. WA≠ NT, etc.**

CONSTRAINT GRAPH

- Constraint graph: nodes are variables, arcs are constraints

- Binary CSP: each constraint relates two variables



EXAMPLE: MAP-COLOURING

Variables: WA, NT, Q, NSW, V, SA, T

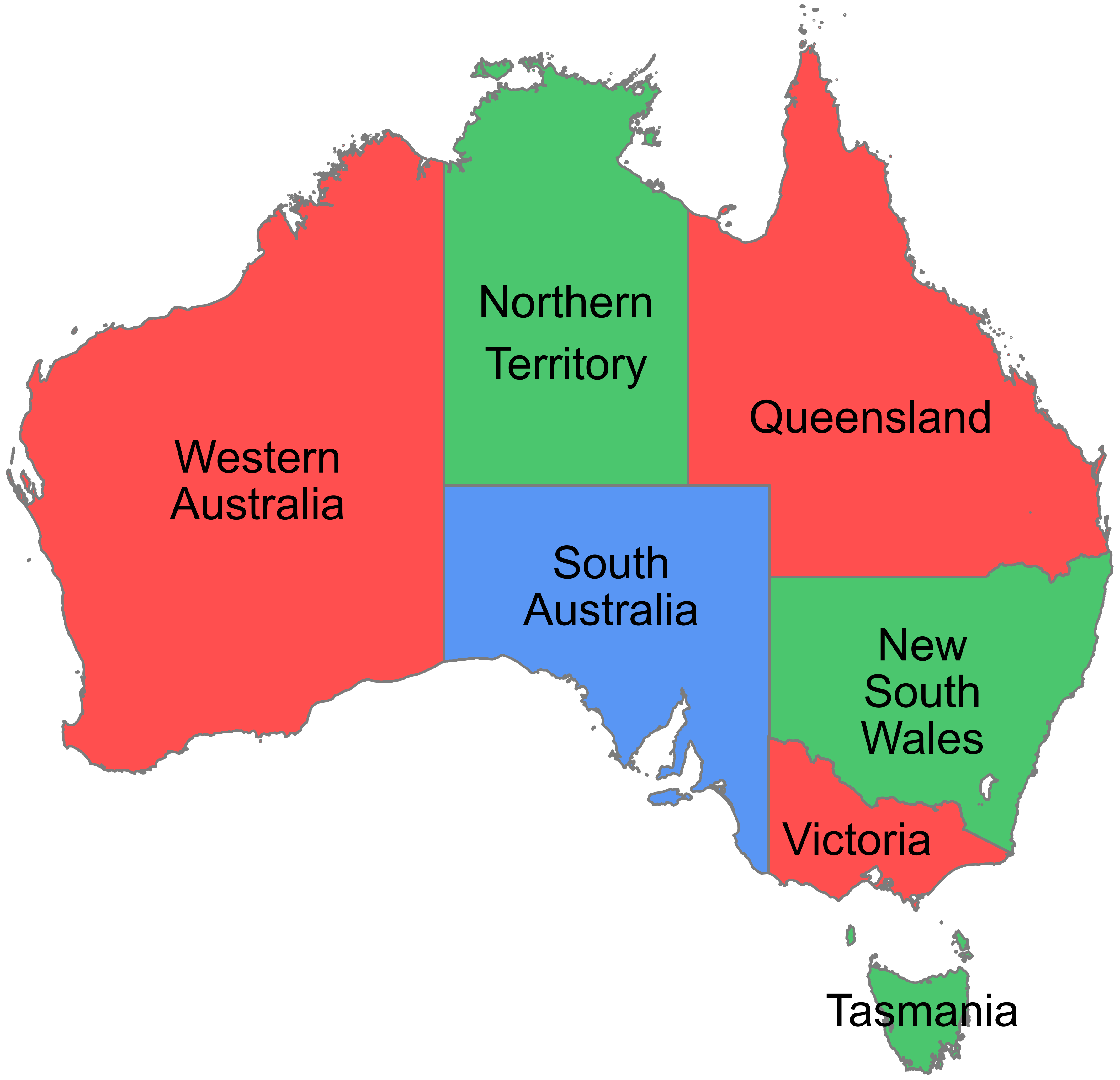
Domains: Di = {red, green, blue}

Constraints: adjacent regions must have different colours

e.g. WA≠ NT, etc.

or (WA,NT) in {(red, green), (red, blue),(green, red), (green, blue), (blue, red),(blue, green)}

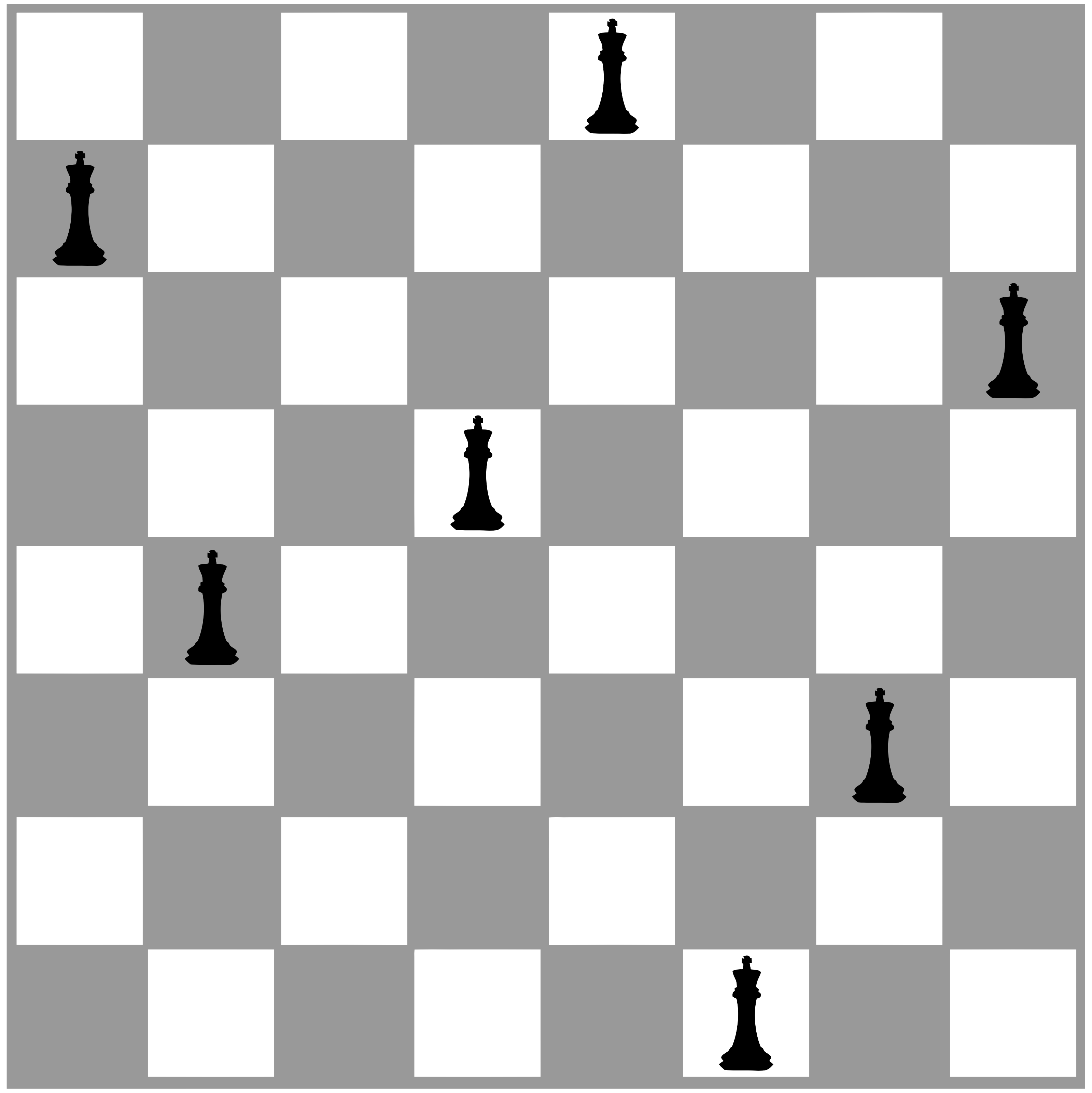
EXAMPLE: MAP-COLOURING



{WA=red, NT=green, Q=red, NSW=green, V=red, SA=blue, T=green}

EXAMPLE: N-QUEENS PUZZLE

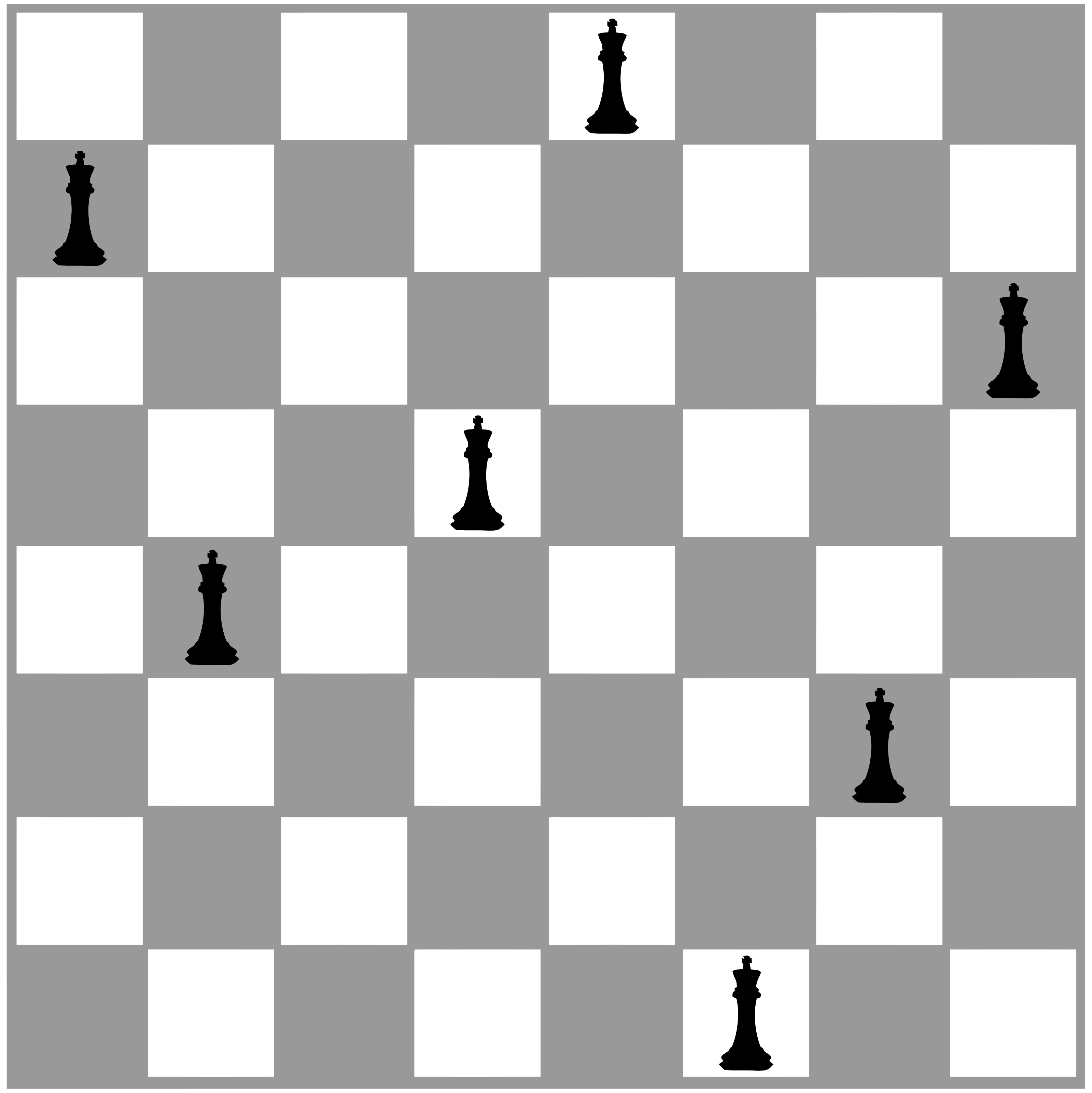
Put n queens on an n-by-n chess board so that no two queens are attacking each other.



Variables: $?$
Domains: $?$
Constraints: ?

EXAMPLE: N-QUEENS PUZZLE

Put n queens on an n-by-n chess board so that no two queens are attacking each other.



Variables: $Q1, Q2, \ldots, Qn$
Domains: $\{1, 2, \ldots, n\}$
Constraints:

$Qi \neq Qj$ (cannot be in same row)
$|Qi - Qj| \neq |i - j|$ (or same diagonal)

EXAMPLE: CRYPTARITHMETIC

Variables:
?

Domains:
?

Constraints:
?

```
   S E N D
+  M O R E
----------
 M O N E Y
```

EXAMPLE: CRYPTARITHMETIC

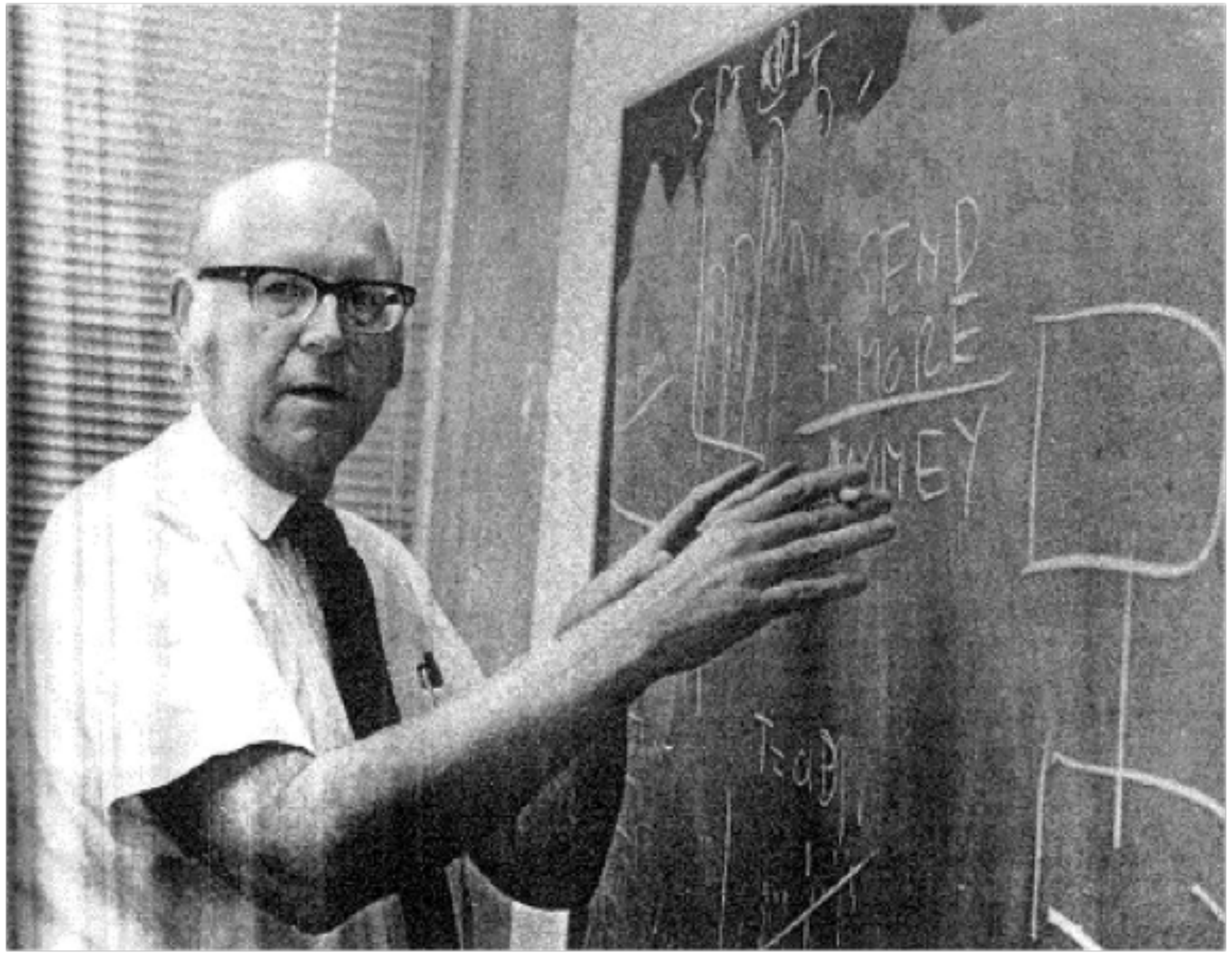
Variables:
D E M N O R S Y

Domains:
{0,1,2,3,4,5,6,7,8,9}

Constraints:
$M \neq 0$, $S \neq 0$ (unary constraints)
Y = D+E or Y = D+E −10, etc.
D ≠ E, D ≠ M, D ≠ N, etc.

$$
\begin{array}{r}
S\ E\ N\ D \\
+\quad M\ O\ R\ E \\
\hline
M\ O\ N\ E\ Y
\end{array}
$$

CRYPTARITHMETIC WITH ALLEN NEWELL



Book: Intended Rational Behavior

CRYPTARITHMETIC WITH ALLEN NEWELL

7.1. Cryptarithmetic

Let us start with cryptarithmetic. This task was first analyzed by Bartlett (1958) and later by Herb Simon and myself (Newell & Simon, 1972). It plays an important role in the emergence of cognitive psychology—at least for me, and perhaps for others. It has been the strongest convincer that humans really do use problem spaces and do search in them, just as the AI theory of heuristic search says.

A cryptarithmetic task is just a small arithmetical puzzle (see Figure 7-1). The words *DONALD*, *GERALD*, and *ROBERT* represent three six-digit numbers. Each letter is to be replaced by a distinct digit (that is, D and T must each be a digit, say $D = 5$ and $T = 0$, but they cannot be the same digit). This replacement must lead to a correct sum, such that $DONALD + GERALD = ROBERT$. Mathematically viewed, the problem is one of satisfying multiple integer constraints involving equality, inequality, and unequality.

Humans can be set to solving cryptarithmetic tasks, and pro-

CRYPTARITHMETIC WITH ALLEN NEWELL

Intendedly Rational Behavior ▪ 365

Assign each letter a unique digit to make a correct sum

$$\begin{array}{r} \text{DONALD} \\ +\ \text{GERALD} \\ \hline \text{ROBERT} \end{array} \qquad D = 5$$

Figure 7-1. The cryptarithmetic task.

tocols can be obtained from transcripts of their verbalizations while they work (Newell & Simon, 1972). Analysis shows that people solve the task by searching in a problem space and that the search can be plotted explicitly.[1] Figure 7-2 shows the behavior of S3 on *DONALD + GERALD = ROBERT*.[2] The plot is called a *problem-behavior graph (PBG)*; it is just a way of spreading out the search so it can be examined (Figure 1-4 showed a PBG for chess). S3

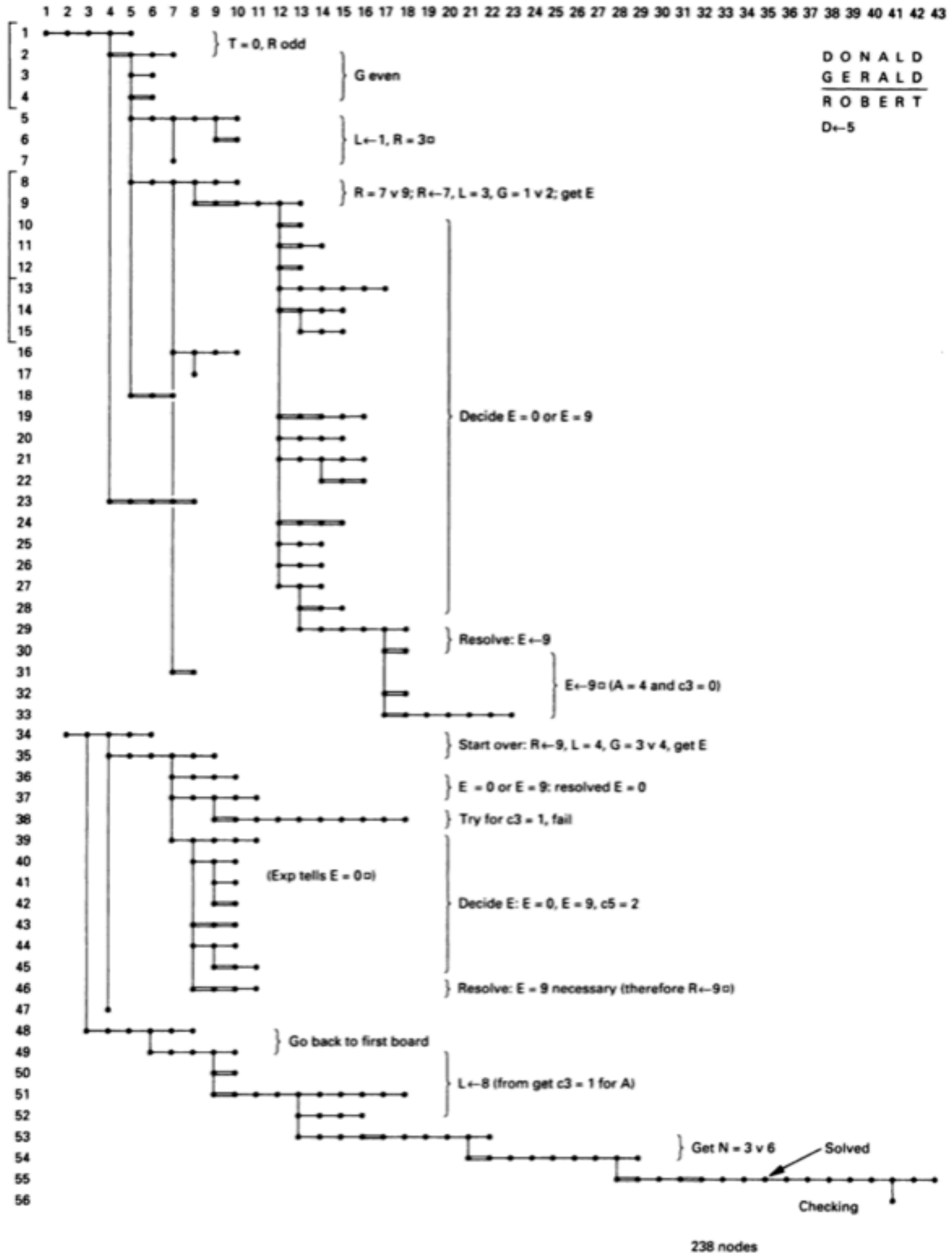


Figure 7-2. Problem-behavior graph of subject S3 on *DONALD + GERALD = ROBERT.*