(March 9th)

Set Paritions

We want to divide $\{1, 2, ..., n\}$ into disjoint union of subsets (blocks).

The number of set paritions of n items is called the "Bell numbers", B_n.

The number of set paritions of n items with k blocks is called the "Sterling Number (of the second kind)" $S(n, k)$.

We're setting up the basis for a DP problem..

$$S(n, k) = S(n-1, k-1) + k \cdot S(n-1, k)$$
$$B_n = \sum_{j=1}^{n} S(n, j)$$
$$S(n, 1) = 1$$
$$S(n, n) = 1$$

Enumeration

Enumerate via enumerating all restricted growth functions (of length n). Any vector $(v_1, v_2, ..., v_n)$ satisfying $v_1 = 1$ and $v_i \leq max(v_1, v_2, ..., v_{i-1}) + 1$ (in lexicographic ordering).

Example; $n = 3$

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Theorem

There is a bijection between restricted growth functions of length n and set partitions of n items.