

(1st March)

## Finishing up Permutations with Unranking

To unrank  $P_j$  for permutation of length  $j$  Let  $P_{j-1} = \lfloor P_j/j \rfloor$  and remainder  $R_{j-1} = P_j \bmod j$ . That is  $P_j = P_{j-1} + R_{j-1}$ .

Compute  $\pi_{j-1} = \text{unrank}(P_{j-1})...$  If  $P_{j-1}$  even then  $\pi_j = \pi_{j-1}$  with "j" inserted at the position  $R_j - 1$  from the right. If  $P_{j-1}$  odd then  $\pi_j = \pi_{j-1}$  with "j" inserted at the position  $R_j - 1$  from the left.

### Example Unrank(19)

Table makes more sense, but cbf'd writing one out sorry...working down... we see;

$$j = 4, \text{Unrank}(19), P_{j-1} = 4, R_{j-1} = 3, \pi_j = ?, \text{direction} = ? \quad (1)$$

$$j = 3, \text{Unrank}(4), P_{j-1} = 1, R_{j-1} = 1, \pi_j = ?, \text{direction} = ? \quad (2)$$

$$j = 2, \text{Unrank}(1), P_{j-1} = 0, R_{j-1} = 1, \pi_j = ?, \text{direction} = ? \quad (3)$$

$$j = 1, \text{Unrank}(0), P_{j-1} = 0, R_{j-1} = 0, \pi_j = ?, \text{direction} = ? \quad (4)$$

And then working back up from our starting permutation of "1". Note direction is derived on **current**  $P_{j-1}$  and position is based on **current**  $R_{j-1}$ .

$$j = 4, \text{Unrank}(19), P_{j-1} = 4, R_{j-1} = 3, \pi_j = 4231, \leftarrow \quad (5)$$

$$j = 3, \text{Unrank}(4), P_{j-1} = 1, R_{j-1} = 1, \pi_j = 231, \rightarrow \quad (6)$$

$$j = 2, \text{Unrank}(1), P_{j-1} = 0, R_{j-1} = 1, \pi_j = 21, \leftarrow \quad (7)$$

$$j = 1, \text{Unrank}(0), P_{j-1} = 0, R_{j-1} = 0, \pi_j = 1 \quad (8)$$

## Subset

So emphasis is on lack of order importance, ie;  $\{1, 2\} = \{2, 1\}$ . We dive right into binomial coefficients  $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n_k}{k!}$ .

Some facts (*based of really cool binary string mapping of possible subsets*);

- $\sum_{k=0}^n \binom{n}{k} = 2^n$ , basically the number of subsets possible for a set of objects size  $n$ , is  $2^n$ .
- $\binom{n}{k} = \binom{n}{n-k}$
- $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

### Example $\binom{5}{3}$

$$\begin{aligned} & |\{(123), (124), (125), (134), (135), (145), (234), (235), (245), (345)\}| \\ &= 10 = \frac{5!}{3!(5-3)!} \end{aligned}$$

### Co-lexicographic Ordering (Calex)

So how we just listed the above, we opted for a lexicographic ordering for sake of nicety above... Right to left (while keeping confusing for representation by still keeping the set order left to right).

(123)

(124)

(134)

(234)

(125)

(135)

(235)

(145)

(245)

(345)

(9)

Okay that took a while to get right mentally...