(1st March)

Finishing up Permutations with Unranking

To unrank P_j for permutation of length j Let $P_{j-1} = \lfloor P_j/j \rfloor$ and remainder $R_{j-1} = P_j \bmod j$. That is $P_j = P_{j-1} + R_{j-1}$.

Compute $\pi_{j-1} = unrank(P_{j-1})$... If P_{j-1} even then $\pi_j = \pi_{j-1}$ with "j" inserted at the position $R_j - 1$ from the right. If P_{j-1} odd then $\pi_j = \pi_{j-1}$ with "j" inserted at the position $R_j - 1$ from the left.

Example Unrank(19)

Table makes more sense, but cbf'd writing one out sorry...working down... we see;

$$j = 4, Unrank(19), P_{j-1} = 4, R_{j-1} = 3, \pi_j = ?, direction = ? \tag{1}$$
$$j = 3, Unrank(4), P_{j-1} = 1, R_{j-1} = 1, \pi_j = ?, direction = ? \tag{2}$$
$$j = 2, Unrank(1), P_{j-1} = 0, R_{j-1} = 1, \pi_j = ?, direction = ? \tag{3}$$
$$j = 1, Unrank(0), P_{j-1} = 0, R_{j-1} = 0, \pi_j = ?, direction = ? \tag{4}$$

And then working back up from our starting permutation of "1". Note direction is derived on **current** P_{j-1} and position is based on **current** R_{j-1}.

$$j = 4, Unrank(19), P_{j-1} = 4, R_{j-1} = 3, \pi_j = 4231, \leftarrow \tag{5}$$
$$j = 3, Unrank(4), P_{j-1} = 1, R_{j-1} = 1, \pi_j = 231, \rightarrow \tag{6}$$
$$j = 2, Unrank(1), P_{j-1} = 0, R_{j-1} = 1, \pi_j = 21, \leftarrow \tag{7}$$
$$j = 1, Unrank(0), P_{j-1} = 0, R_{j-1} = 0, \pi_j = 1 \tag{8}$$

Subset

So emphasis is on lack of order importance, ie; $\{1,2\} = \{2,1\}$. We dive right into binomial coefficients $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n_k}{k!}$.

Some facts (*based of really cool binary string mapping of possible subsets*);

- $\sum_{k=0}^{n} \binom{n}{k} = 2^n$, basically the number of subsets possible for a set of objects size n, is 2^n.
- $\binom{n}{k} = \binom{n}{n-k}$
- $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Example $\binom{5}{3}$

$|\{(123), (124), (125), (134), (135), (145), (234), (235), (245), (345)\}|$
$= 10 = \frac{5!}{3!(5-3)!}$

Co-lexicographic Ordering (Colex)

So how we just listed the above, we opted for a lexicographic ordering for sake of nicety above... Right to left (while keeping confusing for representation by still keeping the set order left to right).

$$
\begin{gathered}
(123)\\
(124)\\
(134)\\
(234)\\
(125)\\
(135)\\
(235)\\
(145)\\
(245)\\
(345)
\end{gathered}
\tag{9}
$$

Okay that took a while to get right mentally...