Computer Science 750 (2022)

Assignment 1

This assignment is worth 30 marks representing 30% of your total course grade.

Due date: 28 August 2022 submitted via Canvas before 23.55

Notes

- You solutions have to be typed.
- To obtain full credit, your script must clearly explain *why* your answers are correct.
- You can use without proof any result proved in class. You need to state correctly the result and show how it applies to the specific problem you solve.
- Research is encouraged. You must reference any source you rely on. Avoid quotes.
- **Your submissions must adhere to the Academic Integrity standards. Copying from any source, even translated, is academic misconduct.**

Questions

1. Question 1 [Program analysis]

   Goldbach's conjecture states that every even integer greater than 2 is the sum of two prime numbers.

   a) Construct an algorithm $\mathbf{A}(N)$ that on every even integer $N > 2$ decides whether N is the sum of two primes. **[2 marks]**

   b) Does $\mathbf{A}(N)$ halt for every N? Justify with proof your answer. **[3 marks]**

   c) Assume $\mathbf{A}(N)$ returns 1 if N is the sum of two prime numbers and 0 otherwise. Does the algorithm:

   1. $G = 1$.
   2. $N = 1$.
   3. If $\mathbf{A}(N) = 1, N = N + 1$, go to 2.
   4. $G = 0$.
   5. Stop.

   halt? Justify with proof your answer. **[2 marks]**

   Does it solve the Goldbach conjecture? Justify with proof your answer. **[2 marks]**

2. Question 2 [Research quation]

   Write 800-1000 words discussing the claim that

   Any function can be computable. Use can use the reference: John Baez, Computing the Uncomputable,
   https://johncarlosbaez.wordpress.com/2016/04/02/computing-the-uncomputable/.

   (a) State and explain the main result. **[2 marks]**

   (b) Explain in your own words the proof. **[4 marks]**

   (c) Does the main result contradict the Halting Theorem? Justify with proof your answer. **[3 marks]**

3. Question 3 [Mathematical modeling]

(d) Define the property "the infinite binary sequence contains a monochromatic arithmetic progression". **[4 marks]**

(e) Does every infinite binary sequence contain a monochromatic arithmetic progression? Justify with proof your answer. **[4 marks]**

(f) Answer questions a) and b) for ternary sequences. Justify with proof your answers. **[4 marks]**

These here seem rather trivial
given an infinite binary sequence s = s1,s2,s3,...

t > 0, k > 1 there exists some si, si+t, si+2t, …, si+(k-1)t substring such that this substring is either in {0^k} or {1^k).

Proof:
Give me some infinite binary string… look at the first two letters:

case 00, 11:
clearly a monochromatic substring
case 10, 01:

Way too obvious no?