

# Artificial Intelligence

## Classical Planning: Planning via Search

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# Recap: Classical Planning

Classical planning seeks a path from the initial state to a goal through a **finite, deterministic, fully-observable** search space. We describe a classical planning task using PDDL (STRIPS) syntax:

## PDDL domain description:

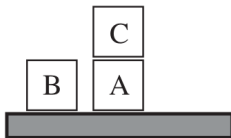
- States (predicates)
- Action scheme  $a$ 
  - Parameters
  - Preconditions  $Precond(a)$
  - Effects  $Effect(a)$ :  $Add(a)$ ,  $Del(a)$

## PDDL problem description:

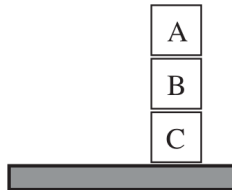
- Initial state  $I$
- Goal  $g$

### Example. [Blocks world domain]

- Predicates:  $onTable(x)$ ,  $on(x, y)$ ,  $clear(x)$
- Action  $moveToTable(x, y)$ 
  - Preconditions:  $clear(x) \wedge on(x, y)$
  - Effects:  $clear(y) \wedge onTable(x) \wedge \neg on(x, y)$
- Action  $moveToBlock1(x, y, z)$ 
  - Preconditions:  $clear(x) \wedge clear(z) \wedge on(x, y)$
  - Effects:  $clear(y) \wedge on(x, z) \wedge \neg clear(z) \wedge \neg on(x, y)$
- Action  $moveToBlock2(x, y)$ 
  - Preconditions:  $clear(x) \wedge clear(y) \wedge onTable(x)$
  - Effects:  $on(x, y) \wedge \neg clear(y) \wedge \neg onTable(x)$



Start State



Goal State

# Planning Algorithms: An Overview

**Question.** How to build a planner?

- Planning is a problem that is closely linked to topics that we have studied.
- **Search** is a prevalent problem solving paradigm:
  - State space
  - Plans are paths from initial state to goals
- **Logic** is a description language to specify planning tasks:
  - Proposition logic
  - First-order logic

# Planning Algorithms: An Overview

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Building a planner relies on our understanding of the task's relations with **search** and **logic**. There are therefore **two main paradigms**:

- Search-based **(to be covered in this lecture)**
  - Forward (Progression) planning
  - Backward (Regression) planning
- Logic-based **(to be covered in the next lecture)**
  - Propositional logic-based planning: SATPlan
  - Logic programming-based planning: Prolog planner

# Search-based Planning Algorithm

**Planning v.s. Search:** The main difference is on **state representation**.

- **Search:** A state (typically the atomic representation) is used as a single entity by the search algorithm.
- **Planning:** A state (structured representation) has structural information which is used by the planning algorithm.

In general, planning can be an **application** of search algorithms.

	<b>Search</b>	<b>Planning</b>
<b>States</b>	data structures	Logic sentences
<b>Actions</b>	transitions	Preconditions/effects
<b>Goals</b>	data structures	Logical sentence
<b>Plan</b>	sequence of states	Sequence of actions

# Search-based Planning Algorithm

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A planning system does the following:

- ① **Action selection** based on the internal structures of action and goal representations.
- ② **Subgoaling** through regression.
- ③ **Heuristics** through relaxing preconditions.

In this lecture we will discuss all the above.

# Algorithm 1: Forward (Progression) Search

**Forward (progression) planning** starts at the initial state, iteratively applying actions in the forward direction, hoping to reach a goal.

## Definition

An action scheme  $a$  implicitly defines the following functions:

- For state  $s$ , the **applicable set** at state  $s$  is:

$$\text{Applicable}(s) = \{a \mid a \text{ is an action, } s \models \text{Precond}(a)\}$$

- For state  $s$  and action  $a$ , the **progressed state** is the resulting state after applying  $a$  at state  $s$ :

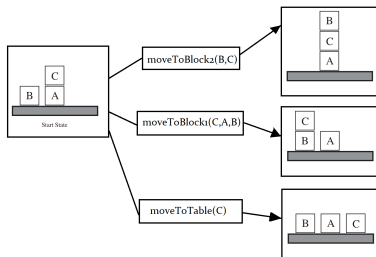
$$\text{Progress}(s, a) = (s \setminus \text{Del}(a)) \cup \text{Add}(a).$$

**Note.** Here we use set notation ( $\setminus$ ,  $\cup$ ) on conjunctions of literals.



E.g.[blocks world] Suppose  $s$  is  $onTable(B) \wedge on(C,A) \wedge onTable(A) \wedge clear(C) \wedge clear(B)$ .

- Action  $moveToBlock1(C,A,B)$ 
  - $Precond(moveToBlock1(C,A,B)) = clear(C) \wedge clear(B) \wedge on(C,A)$
  - $moveToBlock1(C,A,B) \in Applicable(s)$
- Action  $moveToBlock2(B,C)$ 
  - $Precond(moveToBlock2(B,C)) = clear(B) \wedge clear(C) \wedge onTable(B)$
  - $moveToBlock2(B,C) \in Applicable(s)$ .
- Action  $moveToTable(C,A)$ 
  - $Precond(moveToTable(C,A)) = clear(C) \wedge on(C,A)$ .
  - $moveToTable(C) \in Applicable(s)$ .
- Action  $moveToBlock2(A,B)$ 
  - $Precond(moveToBlock2(A,B)) = clear(A) \wedge clear(B) \wedge onTable(A)$ .
  - $s \not\models moveToBlock2(A,B)$ .



### Example. [blocks world]

Suppose the state  $s$  is

$$onTable(B) \wedge on(C,A) \wedge onTable(A) \wedge clear(C) \wedge clear(B).$$

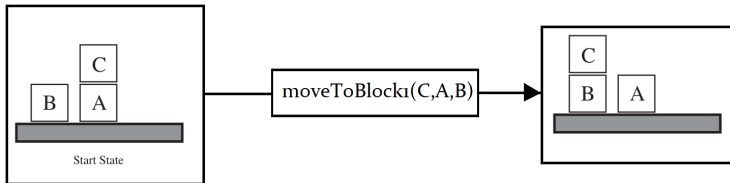
For action  $a = moveToBlock1(C,A,B)$ :

- $Del(moveToBlock1(C,A,B)) = \{clear(B), on(C,A)\},$
- $Add(moveToBlock1(C,A,B)) = \{clear(A), on(C,B)\}.$  So

$$Progress(s, moveToBlock1(C,A,B))$$

$$=(s \setminus Del(moveToBlock1(C,A,B))) \cup Add(moveToBlock1(C,A,B))$$

$$=(onTable(B) \wedge onTable(A) \wedge clear(C) \wedge clear(A) \wedge on(C,B))$$



### Example. [blocks world]

Suppose  $s$  is  $onTable(B) \wedge on(C,A) \wedge onTable(A) \wedge clear(C) \wedge clear(B)$ .

For action  $a = moveToBlock2(B,C)$ :

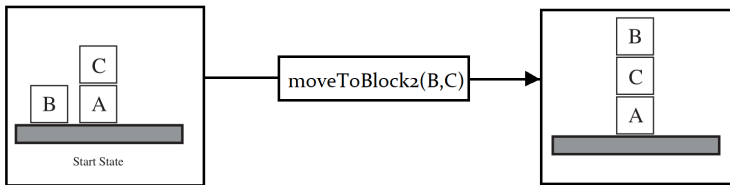
- $Del(moveToBlock2(B,C)) = \{clear(C), onTable(B)\},$
- $Add(moveToBlock2(B,C)) = \{on(B,C)\}.$

So

$Progress(s, moveToBlock2(B,C))$

$= (s \setminus Del(moveToBlock2(B,C))) \cup Add(moveToBlock2(B,C))$

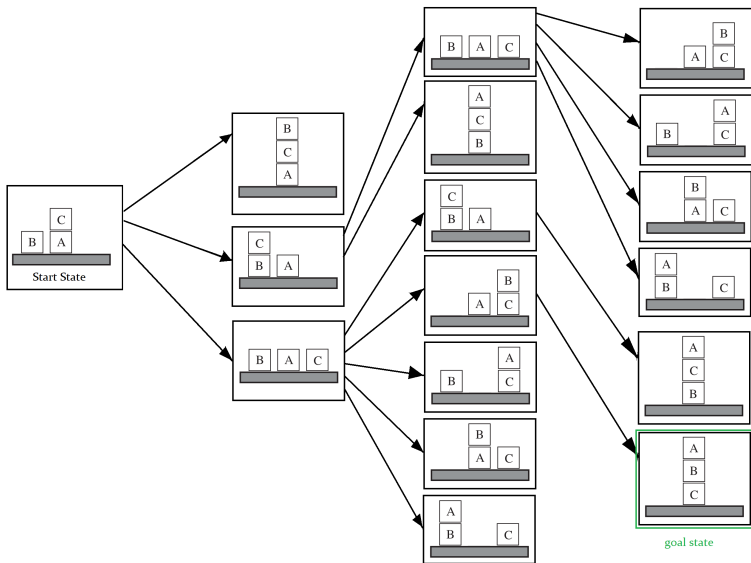
$= (on(C,A) \wedge onTable(A) \wedge clear(B) \wedge on(B,C))$



## Progression planning:

- ① **Initialisation:** Set  $s_0$  as the initial state.
- ② **Repeat:** At iteration  $t > 0$ , suppose we are at state  $s_{t-1}$ .
  - ① Choose an action  $a_t$  such that  $a_t \in \text{Applicable}(s_{t-1})$
  - ② Update state:  $s_t = \text{Progress}(s_{t-1}, a_t)$ .
  - ③ If  $s_t \models \text{Goal}$ , declare that a plan is found and stop.
- ③ **Termination:** If goal is found, return the plan. Otherwise, return *failure*.

## Example. [blocks world]



## Complexity issue with progression planning:

- **Irrelevant actions:** Forward search could explore states and actions that are not relevant to the goal.

**E.g. [buying a book]** Action schemas: *Buy(isbn)* with effect *Own(isbn)*. Goal: *Own(0136042597)*.

- “Blind” forward planning would search over all possible isbn numbers.
  - There are potentially  $10^{10}$  isbn numbers.
- **Large search space:** State space becomes too large for uninformed search.

**E.g. [air cargo]** Suppose there are 10 airports, each airport has 5 planes, and 20 pieces of cargo.

Goal: Move all the cargo at AKL to WLG.

Forward search may search over all possible actions:

- Flying 50 planes, each to 9 possible destinations.
  - 200 packages may be loaded/unloaded to any plane.

## Algorithm 2: Backward (Regression) Search

**Backward (regression) planning** starts at the goal, iteratively applying actions in the backward direction, until we find a sequence of steps that reaches the initial state.

### Definition

Let  $g$  be a goal.

- An action  $a$  is **relevant** to  $g$  if
  - at least one of  $a$ 's effects (either positive or negative) unifies with an element of  $g$ ; and
  - none of  $a$ 's effects (positive or negative) negates an element of  $g$

In this case, we write  $a \in \text{Relevant}(g)$ .

- The **regressed subgoal** from  $g$  over action  $a$  is

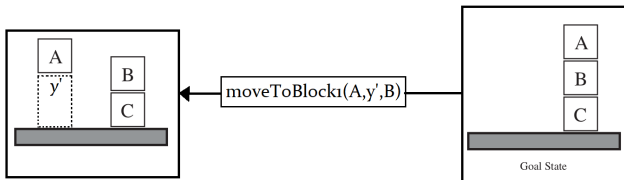
$$\text{Regress}(g, a) = (g \setminus \text{Add}(a)) \cup \text{Precond}(a)$$

**E.g. [blocks world]** Suppose the goal  $g$  is  $\text{on}(A, B) \wedge \text{on}(B, C)$

- An action with effect  $\neg \text{on}(A, B) \wedge \text{on}(B, C)$  is irrelevant.
- An action with effect  $\text{clear}(y) \wedge \text{onTable}(x) \wedge \neg \text{on}(x, y)$  is irrelevant.

**Example. [blocks world]** Suppose the goal  $g$  is  $on(A, B) \wedge on(B, C)$ .  
Action  $a$  is  $moveToBlock1(x, y, z)$ :

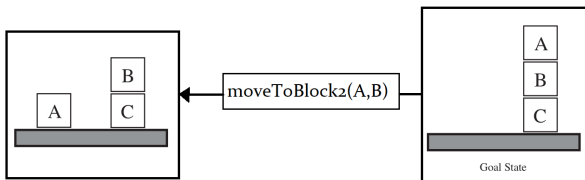
- **Effect:**  $clear(y) \wedge on(x, z) \wedge \neg clear(z) \wedge \neg on(x, y)$ .
- **Unification:**  $on(A, B)$  unifies with  $on(x, z)$ . So  $a \in Relevant(g)$ .
- **After unification:**  $moveToBlock1(A, y', B)$ 
  - Preconditions:  $clear(A) \wedge clear(B) \wedge on(A, y')$
  - Effects:  $clear(y') \wedge on(A, B) \wedge \neg clear(B) \wedge \neg on(A, y')$
- **Regression:**  $Regress(g, a) = on(B, C) \wedge clear(A) \wedge clear(B) \wedge on(A, y')$





**Example. [blocks world]** Suppose the goal  $g$  is  $on(A, B) \wedge on(B, C)$ .  
Action  $a$  is  $moveToBlock2(x, y)$ :

- **Effect:**  $on(x, y) \wedge \neg clear(y) \wedge \neg onTable(x)$ .
- **Unification:**  $on(A, B)$  unifies with  $on(x, y)$ . So  $a \in Relevant(g)$ .
- **After unification:**  $moveToBlock2(A, B)$ 
  - Preconditions:  $clear(A) \wedge clear(B) \wedge onTable(A)$
  - Effects:  $on(A, B) \wedge \neg clear(B) \wedge \neg onTable(A)$
- **Regression:**  
 $Regress(g, a) = on(B, C) \wedge clear(A) \wedge clear(B) \wedge OnTable(A)$



## Regression planning:

- ① **Initialisation:** Set  $g_0$  as the goal.
- ② **Repeat:** At iteration  $t > 0$ , suppose we are at subgoal  $g_{t-1}$ .
  - Choose an action:  $a_t \in \text{Relevant}(g_{t-1})$
  - **Standardise action:** Unify elements of  $\text{Effect}(a_t)$  with elements of  $g_{t-1}$ . Substitute the unbounded variables with new names.
  - Update goal:  $g_t = \text{Regress}(g_{t-1}, a_t)$ .
  - If  $I \models g_t$  (where  $I$  is the initial state), declare that a plan is found.
- ③ **Termination:** If a plan is found, return it; otherwise, return *failure*.

**Advantage of regression planning:** Generally reduced branching factor. Suitable for cases when there is a large number of ground actions.

### Example. [buying a book]

- Action:

*Action(Buy(i), PRECOND : ISBN(i), EFFECT : Own(i))*

**Note.** There are potentially  $10^{10}$  ways to instantiate *Buy(i)*.

- Problem:
  - *Goal(Own(0136042597)),*
  - *Init(ISBN(0000000000), ISBN(0000000001), ..., ISBN(9999999999)).*

Applying regression planning:

- ① Standardise action:

*Action(Buy(0136042597), PRECOND : ISBN(0136042597),  
EFFECT : Own(0136042597))*

- ② Update goal: *Goal(ISBN(0136042597))*
- ③ Plan found: *Buy(0136042597).*

### **Disadvantages of regression planning:**

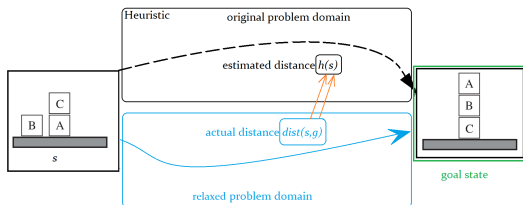
- Plans may still contain redundant steps.
- Plan may not achieve the goal.
- Planning may not terminate for some problems (when DFS is adopted).
- Issues with handling subgoals that have unbounded variables.

# Heuristics for Planning

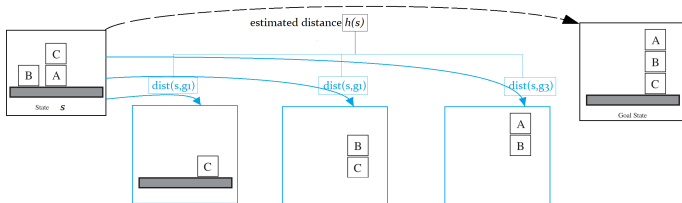
**Task:** Define a heuristic function to **estimate the distance from a state  $s$  to the goal  $g$**  during the search.

**Two approaches:**

① **Relaxed problem:**



② **Subgoal independence:**

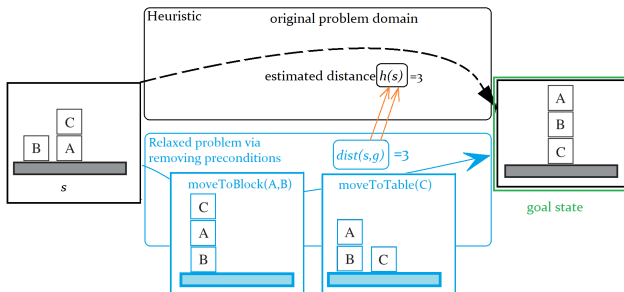


# 1. Relaxed problem:

- 1 Define a **relaxed problem** with the same current state  $s$  and goal  $g$ , but with different definitions of the actions.  
**Note.** The relaxed problem should in principle be **much easier to solve**.
- 2 Solve the relaxed problem and compute the distance  $dist(s, g)$  in the **relaxed problem**.
- 3 Set  $h(s) = dist(s, g)$ . Use this heuristic in the **original** planning problem.

**Possible relaxation:** **Removing preconditions**. This relaxed problem is obtained by removing all preconditions of actions in the original problem.

**Note.** In the relaxed problem any action can be applied at any step.



### Other possible relaxations.

- Removing preconditions from some (but not all) actions.
- Emptying delete list of actions.

E.g.

$Goal(A \wedge B \wedge C)$

$Action(X, EFFECT : A \wedge P)$

$Action(Y, EFFECT : B \wedge C \wedge Q \wedge \neg A)$

$Action(Z, EFFECT : B \wedge \neg C)$

Distance between initial state (empty) to goal is 2 (X, Y) in the relaxed problem.

- Ignoring some literals (thus simplifying the state space).

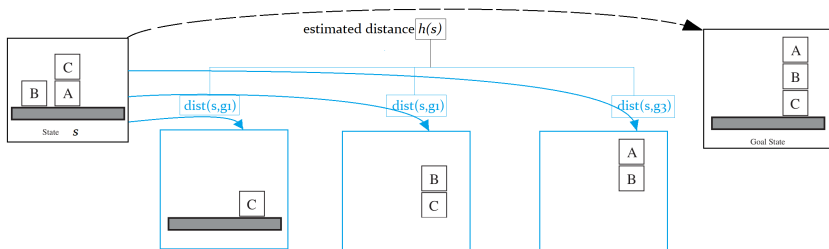
E.g. [blocks world] Removing *onTable* predicate.

## 2. Subgoal independence:

- 1 Divide the goal  $g$  into a number of disjoint subgoals  $g_1 \wedge g_2 \wedge \dots \wedge g_k$ .
- 2 Find plans  $P_1, \dots, P_n$  from  $s$  to  $g_1, g_2, \dots, g_k$  separately.
- 3 Set  $h(s)$  as  $|P_1| + |P_2| + \dots + |P_n|$ .

### Note.

- The idea behind this heuristic is **divide-and-conquer**.
- **Independence condition:** If the effects of any  $P_i$  leave all the preconditions and goals of other  $P_j$ s unchanged, then  $h$  is **admissible**.





# Summary of The Topic

The following are the main knowledge points covered:

- There is a close resemblance between **search** and **planning**.
- The main difference is on representations (of states, actions, goals).
- Search-based planning algorithm 1: **Forward (progression) search**.
  - Starting from the initial state.
  - Choose actions among the **applicable set**.
  - Generate **progressed state**.
- Search-based planning algorithm 2: **Backward (regression) search**.
  - Starting from the goal.
  - Choose actions among the **relevant set**.
  - Generate **regressed subgoal**.
- Heuristics for planning:
  - Relaxed problem: e.g. Removing preconditions
  - Subgoal independence