(7th March)

# Arithmetic Progression Graphs

Cover the brute force approach before iterating. $G = (V, E)$ where we label with weights each $v \in V$ and $e \in E$. ie. $L_1 : V \to Z+$, where $Z+$ are just positive integers. We say $L_1$ are the vertex labels, $L_2$ are the edge labels.

- Each vertex labels are the sum of it's incident edge labels. So $L_1(v) = \sum_{u \in N(v)} L_2(uv)$.
- These vertex labels if looped (sorted) would have the shape $\{a, a+d, ..., a+(n-1)d\}$ where $n = |V|$

"An APG with parameters $a$ and $d$".

So the question we have is "given a description of a graph" does there exist a labeling such that the vertices form a APG.

## Properties

With $n = |V|$ and $m = |E|$ and arithmetic progression parameters positive;

- $a \geq$ the minimum vertex degree in $G$. (vertex of graph with least incident edges).
- Sum of edge labels is half of the sum of the vertex labels.
- Given $a, d$ parameters of an APG, $s_v = na + (0 + 1 + ... + (n-1)d) = na + \frac{n(n-1)}{2}d$
- $2na + n(n-1)d$ is divisible by 4 (multiplying a known even $s_v$ by 2)
- $s_e = (2na + n(n-1)d)/4 \geq m$

All of those should make sense... So now we know $s_e$, the problem is to tweak numbers $a$ and $d$ whilst dealing with paritionings.

## Solution 1 - Combinatorial

Check all combinations of edge labels that sum up to $s_e = (2na + n(n-1)d)/4$. This is the same as checking all integer paritions of $s_e$ with $m = |E|$ parts.

## Solution 2 - Solving Systems of Equations

(28min)

## Solution 3 - Augmenting Paths Approach

Improving walk technique... (43min)

**Not hugely important to take everything in here**

Mostly to get familiar with the general idea of integer programming so perhaps we create seperate section for just that.