Quantification of Energy Transformation in Kinetic and Thermal Energy

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Abstract

Keywords-energy conservation, kinetic energy, thermal energy, rotational energy, heat capacity

Energy is never created or destroyed but rather transforms into different types as a dynamical system evolves in time. In this paper, we aim to quantify all the major transformations of potential energy to kinetic, rotational, and thermal energy and carefully demonstrate the first law of thermodynamics, or so called conservation of energy. A total of 356.1g mass connected to a string wrapped around a plastic spindle which turns a rotor through a pot of honey. Then the weight was dropped to the ground and induced the axis to shred and produce a certain amount of heat in the honey against its viscous drag. The heat capacity of the heated subject(honey) and the moment of inertia of the spindle were experimentally found. Using the data and calculations, experimental and theoretical heat gain in the honey per drop were compared. We were able to observe that the experiment successfully represented the first law of thermodynamics and approved that this experiment is apt to be used in classroom applications such as labs or demonstrations.

I. Introduction

Although energy conservation is the most familiar concept of introductory mechanics courses, students hardly get to test it in the lab because the instability of energy makes it so hard to make a simple but controlled environment for lab exercises. In this paper, we aim to analyze the previously tried methods and to accumulate in a guide for lab instruction for student understandings in future.

Thanks to the hard work of many previous experimenters who worked on this topic, the apparatus and procedure were ready to maximize the conservation. The very first model measured the heat gain from sliding lead weights in a plastic cylinder by manually rotating the cylinder a certain number of times. Then realizing the compromised accuracy of the man-powered system, the next experimenter limited the movement of the cylinder by putting it on the wheel with a handle steering it. It seemed promising when the steering wheel system yielded the desired result, but they soon realized that the radiation from the human body was canceling out the heat loss this time. In 2020, the latest experimenter decided to stick to the classic with Joule's experiment in 1849 [2], dropping weights from a height to make the attached spindle rotate and heat up a certain amount of honey. With the help from the insulated box system and automated motion with gravity, the latest version of the lab was within an accuracy of the one-third of the desired result.[5]

After recreating the previous experiment, we found that the data were corrupted by the mass of the falling object, the movement of honey in the preparation of trials, the limitation of temperature measuring method, and the rotational energy of the spindle. Since the previous

maximum falling mass of 100g was not heavy enough to produce significant heat gain, we enabled the system to hold twice more mass by changing the direction of the motion. There was another way to work with bigger falling mass if the drag force of honey shredded by the axis were amplified by simply increasing the amount of honey, but it also counteracted to make it harder to heat up the bigger volume with limited kinetic energy from the mass. Thus we kept the amount of honey constant. Also, we separated the unrelated kinetic energy from the honey by making a new axis that does not rotate in one direction which we wind to prepare for the fall, but clamps on and rotates the axis in the direction the mass falls and the string unwinds. After a few experiments, we realized that the rotational energy of the spindle was not negligible, and thus calculated the original rotational energy with an empty bottle.
We also experimentally found that our specific heat of honey was 3J/g°C. Note that the instructor should expect some degree of inaccuracy for classroom applications because the specific heat of honey will vary in every bottle. It is also assumed that the relationship between millivolts measured and the corresponding Celsius degree of heat for the type E thermocouple follows the best fit provided by the online calculator used in the undocumented gap between two interpolated data points, so it may have some imperfections.[1]

II. Apparatus

i. Apparatus 1

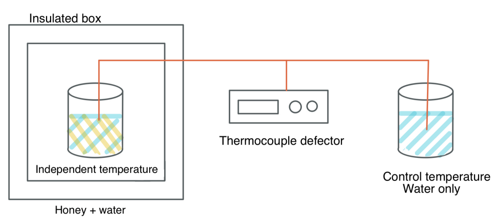


Figure 1: Apparatus 1 for specific heat capacity of honey Experiment

ii. Apparatus 2

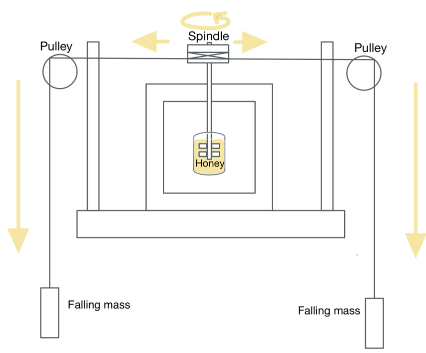


Figure 2: Apparatus 2 for Moment of inertia of the spindle & Main experiment

III. Fiducial Experiments

- Specific Heat Capacity of Honey
- Moment of Inertia of the Spindle

i. Specific Heat Capacity of Honey

First, we observed that the honey was crystallized over time after mixing it looking like in the Figure below. Then we decided to use the new honey if the next experiment is conducted 24 hours after the previous one. Moreover, such instability of physical properties made us question the specific heat of this certain honey. Regarding that honey is a natural substance, it is possible that the physical properties, especially the heat capacity, may vary from different types of honey produced by different bees and flowers. Then we decided to experimentally find out the heat capacity of the specific honey we are using (PICS brand honey in Price Chopper)

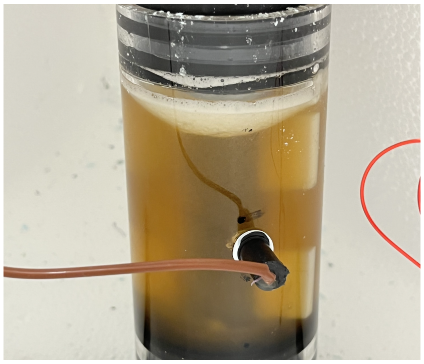


Figure 3: Crystalized honey after few mixings

To measure the specific heat of honey, a beaker of honey was inserted in the oven for at least a day to make sure to be heated over 60 degrees Celsius. Two beakers of room temperature water were weighted and placed one in an insulated styrofoam box and one outside the box. Each end of the thermocouple was inserted in the water beakers and made sure to be touching the water itself, not the surface of the beaker or exposed to the air. Then we poured the heated honey in the beaker of water in the insulated box, closed the box, and measured the temperature drop of the mixture for the next 30 minutes. Then we measured the mass of the mixture and determined the amount of honey mixed with the water. Specific heat of a substance can be calculated as:

$$Q = cm\Delta T \tag{1}$$

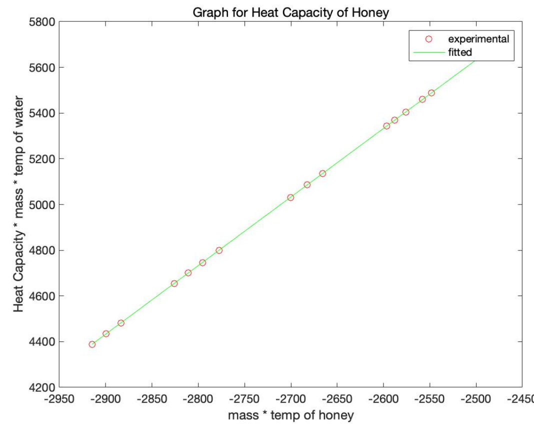
where Q is the heat energy gained or lost, c specific heat of energy, m mass of the substance, and ΔT temperature difference. Since the amount of heat energy lost from the honey will be gained by the water in this mixture, we can incorporate this conservation phenomenon into a formula as

$$c_w m_w \Delta T_w = c_h m_h \Delta T_h \tag{2}$$

where the subscript of w means water and h means honey. Everything except heat capacity of honey was either measured or known, so we can rearrange this formula as:

$$C_h = \frac{c_w m_w \Delta T_w}{m_h \Delta T_h} \tag{3}$$

To see this direct relationship in (3), $m_h \Delta T_h$ vs. $c_w m_w \Delta T_w$ was plotted and fitted to a straight line of which the slope represents the heat capacity of honey.

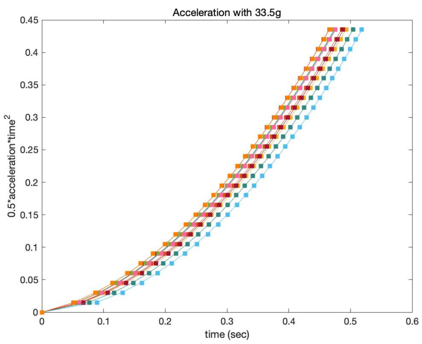


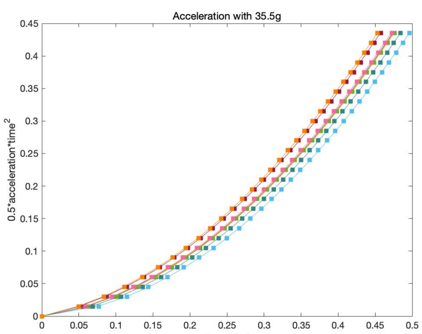
41.3g of 19°C water and 57.6g of 75°C honey were mixed and we used the generally accepted heat capacity of water as 4.184J/g°C. Then we used the temperature of mixture to deduce the temperatures decreased from 75°(honey) and increased from 19°C (water). As a result, we determined the heat capacity of the honey we are using to be 3.000J/g°C, slightly smaller than the accepted heat capacity of finely granulated honey,3.054J/g°C, and bigger than that of the general honey of 2.510J/g°C.

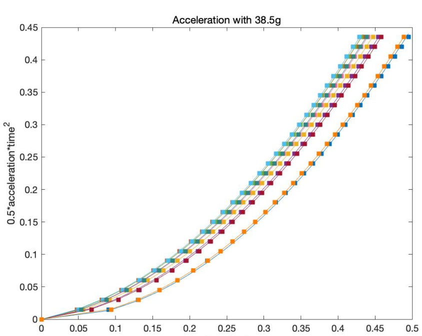
ii. Moment of Inertia of the Spindle

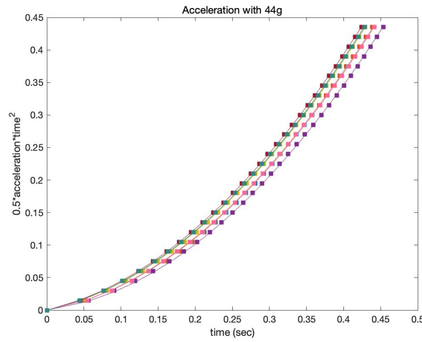
We noticed that a significant amount of energy dissipated to various means, and one of the biggest energy losses was rotational kinetic energy produced by the spindle. We prepared apparatus 1, which is the same as the main experiment except that we did not have honey inside the bottle, to measure the constant rotational energy in a controlled environment. Velocity, acceleration, and height of the falling mass and the diameter of the spindle were measured after falling 5 different hanging masses to calculate the moment of inertia around the axis of rotation.

Since the horizontal motion of the hanging mass blurred the instant acceleration measured from the detector, we first calculated the relative acceleration of two neighboring points with bigger step size of time to minimize the noise. Then we calculated one average acceleration for 30 trials with each of five different masses. Figure 4 below represents time versus distance of the fall for the five falling mass and it is clear that the distance is related to the square of time. Since the graphs looked quiet similar, we agreed that different masses do not change the behavior of the fall









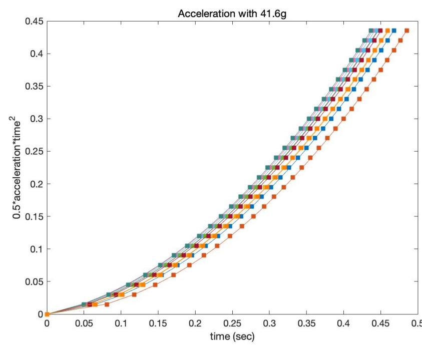


Figure 4: Plots of acceleration over time for five different masses

Now with the acceleration calculated with each masses, we were able to find the moment of inertia of the spindle according the the calculation executed below.

$$\Sigma E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgz$$

and ω can be replaced with v/R

$$= \frac{1}{2}mv^2 + \frac{1}{2}I(\frac{v}{R})^2 - mgz$$

Take a derivative of each side, and it should equal to zero because E is constant

$$\frac{dE}{dt} = mva + \frac{I}{R^2}va - mgv = 0$$

and velocity is canceled out

$$0 = ma + \frac{I}{R^2}a - mg$$

And we can derive two different relations.

$$\frac{1}{a} = \frac{1}{g} + \frac{I}{mgR^2} \tag{4}$$

$$\frac{I}{R^2} = \frac{m(g-a)}{a} \tag{5}$$

Since the inverse of mass and the inverse of acceleration are linearly related as represented in (4), we graphed inverse of acceleration to inverse of mass so that we can measure the slope, $I/(g*R^2)$.

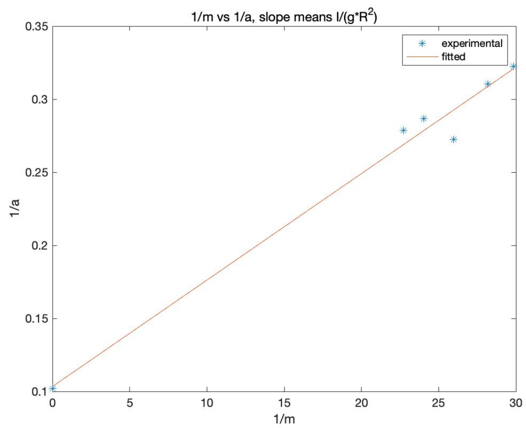


Figure 5: Plot of inverse of acceleration over inverse of mass

We found that I was $8.7551*10^{-5}kg/m^2$, which can be used for further calculations in the energy translation experiment to be followed.

IV. Main Experiment: Translation of Mechanical Energy to Heat Energy

Now that all the main components of the energy conversion were identified, we were able to represent the energy conservation in the translation of potential energy to rotational energy, mechanical energy, and heat energy in the medium contact to the spinning axis. We especially decided to use honey as the heat energy collector because its viscosity acts against the gravitational fall and minimizes the noise from fast falling speed and acceleration.

i. Procedure

A weighed amount of honey was poured into the bottle attached with one end of the thermocouple. The bottle was closed with a lid penetrated with a vertical rotary shaft and placed in an insulated styrofoam box. Two opposite ends of the round spindle were connected to hanging masses and the axis with shredding wings was inserted into the middle hole of the spindle. Then, the motion of fall was directed to the floor by the pulley at the sides of the insulated box. We rotated the spindle so that the string could wind around it and elevate the mass, and let it fall. We measured the velocity, height, and acceleration of the falling motion and temperature change in honey. We repeated this procedure from spindle rotation to measurement of data for 2 sets of 44 drops each, excluding the anomalous trials.

ii. Results and Analysis

By the law of energy conservation, potential energy and the sum of all translated energy including rotational, thermal, and kinetic energy should be equal.

$$potentialE = kineticE + rotationalE + thermalE$$

where E denotes for energy. This can be expressed as

$$mgh = 1/2m_f v^2 + 1/2Iw^2 + m_h c_h \Delta T_h$$

where f denotes for falling and subscript denotes h for honey Now that everything else than the ΔT is identified, we can isolate ΔT on the left-hand side to calculate expected energy gain, resulting in 0.01734 J/drops or 0.7631 Joules per 44 drops. However, it was not true for either of the sets of 44 drops, and only 0.4250 J, 0.4373J of heat energy were gained in the honey after each set. Still, it was noticeable in the Figure below that the honey cooled down after every trial since most of the temperature after the drop in the previous trial was bigger than the temperature before the drop in the next trial. Therefore, the energy gain of each drop was compared with the expected value of 0.01734 J/drops and the accuracy went up to 0.01364 and 0.01843, yielding 21.3% and

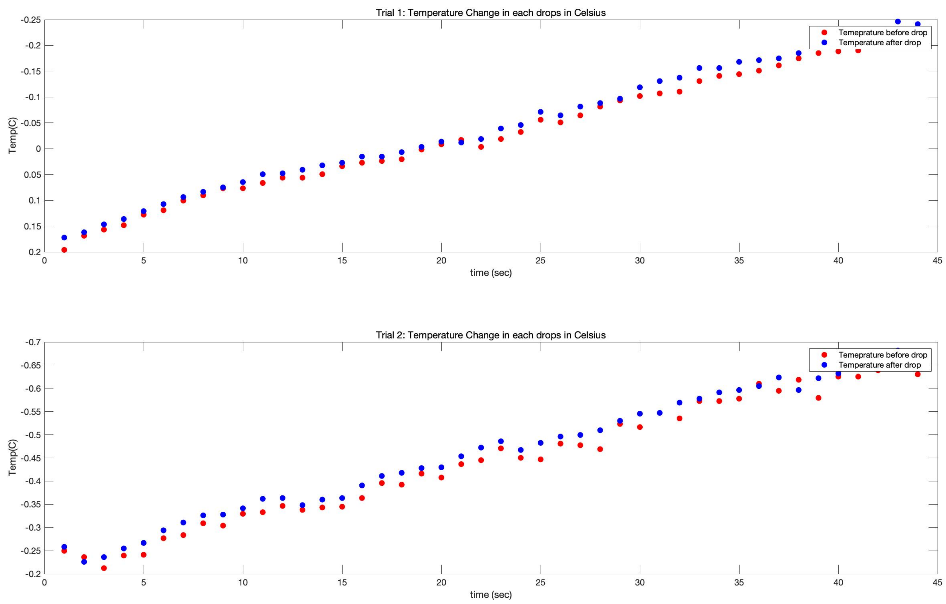


Figure 5: Plot of Temperature Before and After Each Drop

-6.3% relative error from the expected value. Since the mean of two energy gains per drop was 7.51%, we were able to conclude that our experimental data matched the theoretical expectation.

iii. Error Analysis

Although the results are closer to the expectations than any of the previous experiments, some limitations, and room to improve exist, especially in the temperature measurements.
The heat energy transferred out of the honey bottle is concerning because the control thermocouple was placed in that same area that was receiving the heat. Although the average heat loss was smaller than 0.001mV, this small number is almost 5% of the temperature difference we are observing, which could account for a quarter of the error we were experiencing. Thus, it is suggested to try putting the control thermocouple in a separate insulated box in the same environment.
Also, the heat distribution of honey may not be even throughout the volume because the contact area of the rotating wings is limited. It is suggested to have more than one thermocouples in many different points both in vertical and horizontal axes to thoroughly observe the temperature spread. It may sound possible to wait for them to even out, but this is not plausible because the honey bottle was not insulated enough to retain all heat until it evens out. Especially the speed of leakage in the previous paragraph foreshadows that the gain of the accuracy of even distribution will be overshadowed by the heat loss.

V. Conclusion

By mixing the honey with a motion of axis with wings initiated by gravitational fall of a certain object, we were able to see the translation of mechanical energy to thermal energy in a quantifiable way. However, we realized that the mechanical energy of falling mass was not only producing the thermal energy but also the rotational energy with I, a moment of inertia around the axis of rotation, as $8.755 * 10^{-5} kg * m^2$ and other secondary energy we dismissed as minimal. Also, regarding the speciality of physical properties of honey produced in different farms and states of honey, the specific heat capacity of the honey used in the experiment was experimentally determined as 3.000J/g°C. With the heat capacity and rotational energy found, theoretical thermal energy produced by the gravitational fall was calculated to be 0.7631 Joules per one set of 44 drops. However, this did not match with either sets of our data as they were 0.4250 J and 0.4373J, so we divided the expected value per set into 0.01734 Joules per drop to compared with energy gain after each individual drops. Our mean of the experimental data was 0.01604, which agrees with the theoretical expectation with a 7.51% error. We expect this error to be improved if the accuracy of heat measurement and insulation of bottle are ameliorated. Thus we were able to conclude that this translation of kinetic energy to heat energy followed the law of conservation of energy. Thanks to the simplicity of the experiment itself, it may be followed as an educational material for teaching energy conservation and translation.

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