Jillian Paulin and Shana Lee
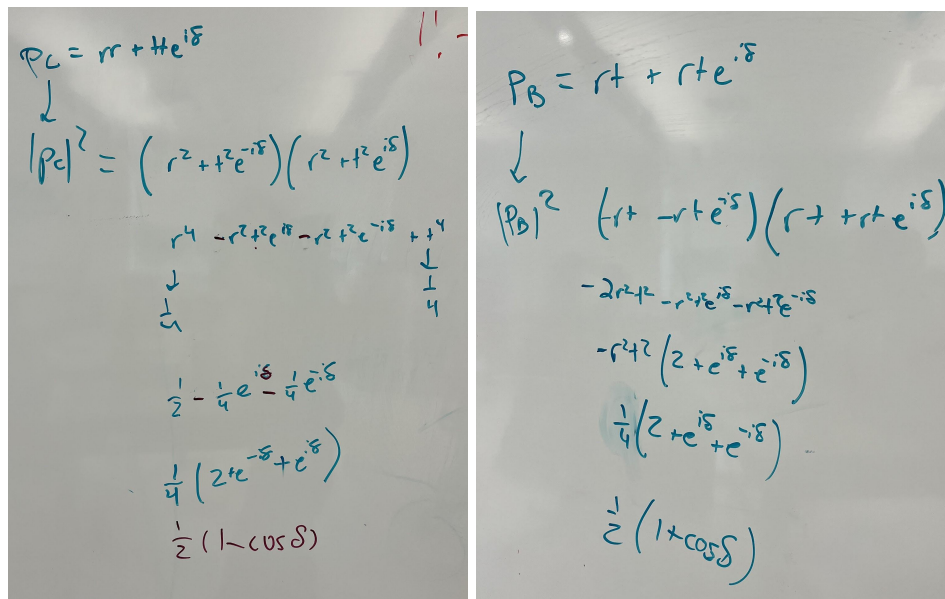Quantum Mechanics- Lab 4: The Quantum Eraser
4/1/2022

Introduction

The purpose of this lab is to explore the impact of something called the "quantum eraser". Essentially we will send light through an optical setup in which the paths are indistinguishable, in which case interference will be observed. When we change the optical setup such that the paths are distinguishable, the interference disappears. After placing an additional optical element after the distinguishable setup, interference will once again be observed, "erasing" all memory of the distinguishability of the two previous paths.

Theory

<u>Question 1</u>: Show that $P_B + P_C = 1$ (by hand)

Thus, the sum of P_C and P_B is $\frac{1}{2}(1 - cos(\delta)) + \frac{1}{2}(1 + cos(\delta)) = \frac{1}{2} + \frac{1}{2} = 1$.

<u>Question 2</u>: Find the probability $P'_B = \left| p'_B \right|^2$.

First, recall that $p'_B = rtq + rtqe^{i\delta}$ and that $t = 1/\sqrt{2}$, $q = 1/\sqrt{2}$, and $r = i/\sqrt{2}$. Also note that $\delta = 2\pi\Delta L/\lambda$ where λ is the wavelength of the light, and ΔL is the difference in length between the two arms. Multiplying p'_B by its complex conjugate, we get the following:

$$-\left(r + g + r t g\, e^{-i\delta}\right)\left(r t g + r t g\, e^{i\delta}\right)$$

$$-\ r^2 t^2 g^2\left(1 + e^{-i\delta}\right)\left(1 + e^{i\delta}\right)$$
$$\underset{\downarrow\downarrow}{}$$

$$\cdots \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\ \left(|x|^2 + e^{i\delta} + e^{-i\delta}\right)$$

$$\frac{1}{8}\left(2 + e^{i\delta} + e^{-i\delta}\right) = \frac{1}{8}\left(2 + 2\cos\delta\right)$$

$$= \frac{1}{4}\left(1 + \cos\delta\right)$$

So the probability that the photon is detected in B is $\frac{1}{4}\left(1 + cos(\delta)\right)$.

<u>Question 3</u>: Find the matrix M_2 for the mirrors of the interferometer.

We conducted this calculation in Mathematica and received the following. Note that our starting matrix was the Jones matrix for a mirror, which is defined in the first line of our code. We took the tensor product of this Jones matrix with the identity to find the matrix for the mirrors in the interferometer. This is because mirrors do not alter polarization of the incident light. Notice that our output is a 4x4 matrix, just as it was for the beam splitter in the example.

```
In[119]:= mirrorJones = {{0, 1}, {1, 0}}

Out[119]= {{0, 1}, {1, 0}}

In[120]:= M2 = KroneckerProduct[mirrorJones, IdentityMatrix[2]]

Out[120]= {{0, 0, 1, 0}, {0, 0, 0, 1}, {1, 0, 0, 0}, {0, 1, 0, 0}}
```

<u>Question 4</u>: Find the matrix A_2 for the interferometer phase.

We are given that A_2 is the tensor product of A_1 with the identity, as well as the definition of the identity. We defined A_1 and once again found this tensor product in Mathematica using the code shown below:

In[41]:= `A1 = {{1, 0}, {0, Exp[I delta]}}`

Out[41]= $\{\{1, 0\}, \{0, e^{i\,delta}\}\}$

In[42]:= `MatrixForm[KroneckerProduct[A1, IdentityMatrix[2]]]`

Out[42]//MatrixForm=
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\,delta} & 0 \\ 0 & 0 & 0 & e^{i\,delta} \end{pmatrix}$$

Question 5: Find an expression for the full interferometer matrix.

This can be quite a tedious exercise if done by hand. However, we used Mathematica once again. The full interferometer matrix will be the product of the following matrices:

$$Z = B_2 A_2 M_2 W_2(\theta, \, 0) B_2$$

where we have found A_2 and M_2 previously, and B_2 and $W_2(\theta, \, 0)$ are provided to us in the lab. The output is shown below in terms of t, r, and θ as follows:

Out[42]//MatrixForm=
$$\begin{pmatrix} -r\,t + e^{i\,\delta}\,r\,t\,Cos[2\,\theta] & e^{i\,\delta}\,r\,t\,Sin[2\,\theta] & -t^2 + e^{i\,\delta}\,r^2\,Cos[2\,\theta] & e^{i\,\delta}\,r^2\,Sin[2\,\theta] \\ e^{i\,\delta}\,r\,t\,Sin[2\,\theta] & r\,t - e^{i\,\delta}\,r\,t\,Cos[2\,\theta] & e^{i\,\delta}\,r^2\,Sin[2\,\theta] & t^2 - e^{i\,\delta}\,r^2\,Cos[2\,\theta] \\ -r^2 + e^{i\,\delta}\,t^2\,Cos[2\,\theta] & e^{i\,\delta}\,t^2\,Sin[2\,\theta] & -r\,t + e^{i\,\delta}\,r\,t\,Cos[2\,\theta] & e^{i\,\delta}\,r\,t\,Sin[2\,\theta] \\ e^{i\,\delta}\,t^2\,Sin[2\,\theta] & r^2 - e^{i\,\delta}\,t^2\,Cos[2\,\theta] & e^{i\,\delta}\,r\,t\,Sin[2\,\theta] & r\,t - e^{i\,\delta}\,r\,t\,Cos[2\,\theta] \end{pmatrix}$$

Question 6: Verify that when $\theta=\pi/4$, we get the same answer as in the lab.

We do recover this same matrix. Our output from Mathematica is shown below:

In[130]:= `MatrixForm[Simplify[B2.A2.M2.W2.B2]]`

Out[130]//MatrixForm=
$$\begin{pmatrix} -r\,t & e^{i\,d}\,r\,t & -t^2 & e^{i\,d}\,r^2 \\ e^{i\,d}\,r\,t & r\,t & e^{i\,d}\,r^2 & t^2 \\ -r^2 & e^{i\,d}\,t^2 & -r\,t & e^{i\,d}\,r\,t \\ e^{i\,d}\,t^2 & r^2 & e^{i\,d}\,r\,t & r\,t \end{pmatrix}$$

Question 7: Find the vector for $P_x Z_2 | XV >$

We performed this calculation in Mathematica as follows:

In[40]:= **Px.Z.ketXV**

Out[40]= $\left\{\left\{\frac{1}{2} \ddot{\imath} \, e^{\, i \, d}\right\}, \left\{\frac{\ddot{\imath}}{2}\right\}, \{0\}, \{0\}\right\}$

Question 8

In[165]:= **Transpose[ketXH].sand.ketXH.Transpose[ketXH].Conjugate[sand].ketXH + Transpose[ketXV]. sand.ketXV.Transpose[ketXV].Conjugate[sand].ketXV**
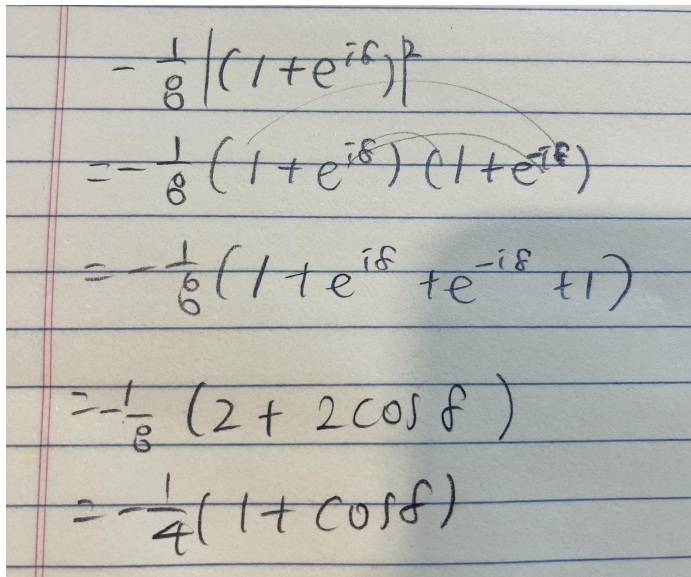
Out[165]= $\left\{\left\{\frac{1}{4}\left(1 - e^{\, i \, d}\right)\left(1 - e^{-i \, \text{Conjugate}[d]}\right) + \frac{1}{4}\left(-1 + e^{\, i \, d}\right)\left(-1 + e^{-i \, \text{Conjugate}[d]}\right)\right\}\right\}$

The variable d just means delta. Since we know the angle can't be imaginary, treat Conjugate d as d. Then these two terms cancel each other. Distinguishability is the max at theta=0.

Question 9

In[49]:= **Simplify[MatrixPower[{{1/Sqrt[2], 1/Sqrt[2], 0, 0}}.ExyZ.ketXV, 2]]**

Out[49]= $\left\{\left\{-\frac{1}{8}\left(1 + e^{\, i \, d}\right)^2\right\}\right\}$

$$-\frac{1}{8}\left|\left(1 + e^{i\delta}\right)\right|^2$$

$$= -\frac{1}{8}\left(1 + e^{i\delta}\right)\left(1 + e^{-i\delta}\right)$$

$$= -\frac{1}{8}\left(1 + e^{i\delta} + e^{-i\delta} + 1\right)$$

$$= -\frac{1}{8}\left(2 + 2\cos\delta\right)$$

$$= -\frac{1}{4}\left(1 + \cos\delta\right)$$

Planned Procedures:

1. For Part I of this lab, we will first collect AB and AC coincidence counts without blocking either of the arms. We will vary the voltage from 1-4V and observe interference patterns.
2. For Part II of this lab, we will repeat our first data collection, but this time, with one arm blocked, thus creating distinguishable paths.
3. Rotating the HWP to change the visibility
- Variable: HWP rotation by theta 0 to 45 degrees, 5 degrees increment
4. With an eraser, arms unblocked
5. With an eraser, arms blocked
6. HWP 0 arms blocked unblocked
7. HWP 45 arm b u
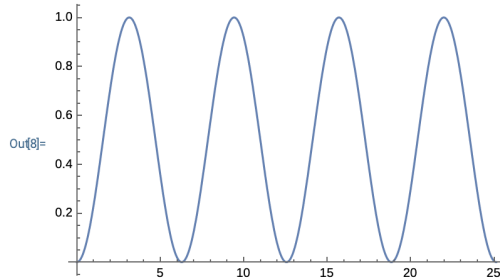8. HWP 10 arm b u

Data & Analysis:

Part I

- $P_C = \frac{1}{2}(1 - cos(\delta))$

- $P_B = \frac{1}{2}(1 + cos(\delta))$

AB and AC Counts, With Arm Unblocked
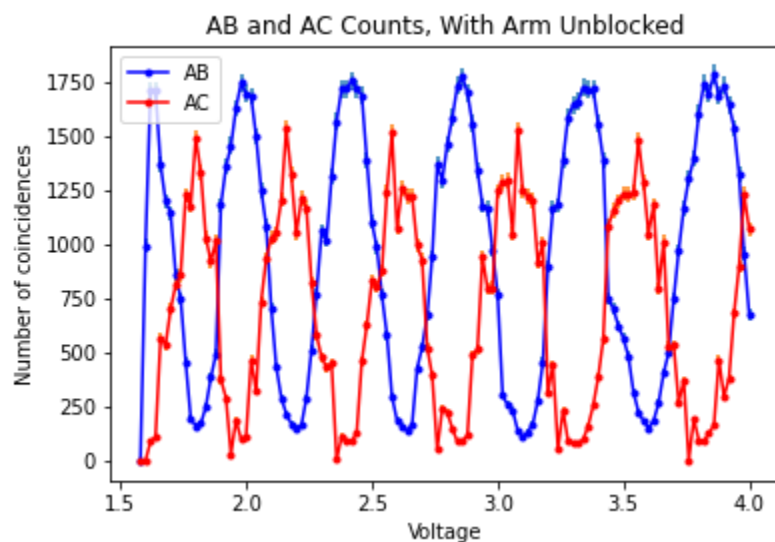
In[1]:= `Pc = 1 / 2 (1 - Cos[d])`

Out[1]= $\frac{1}{2}$ (1 - Cos[d])

In[8]:= `Plot[Pc, {d, 0, 8 π}]`

Out[8]=

In[4]:= `Pb = 1 / 2 (1 + Cos[d])`

Out[4]= $\frac{1}{2}$ (1 + Cos[d])

In[9]:= `Plot[Pb, {d, 0, 8 π}]`

Out[9]=
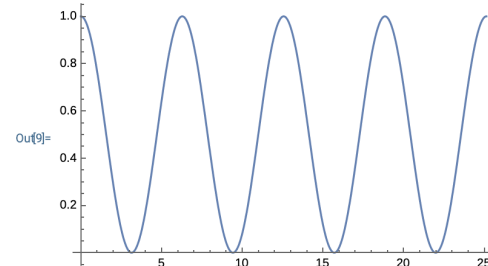
It matches our expectations. We did not plot them because we did not know how exact formula explaining the relationship between applied voltage and path length differences but we at least know that they are directly proportional.


AB and AC Counts, With Arm Blocked
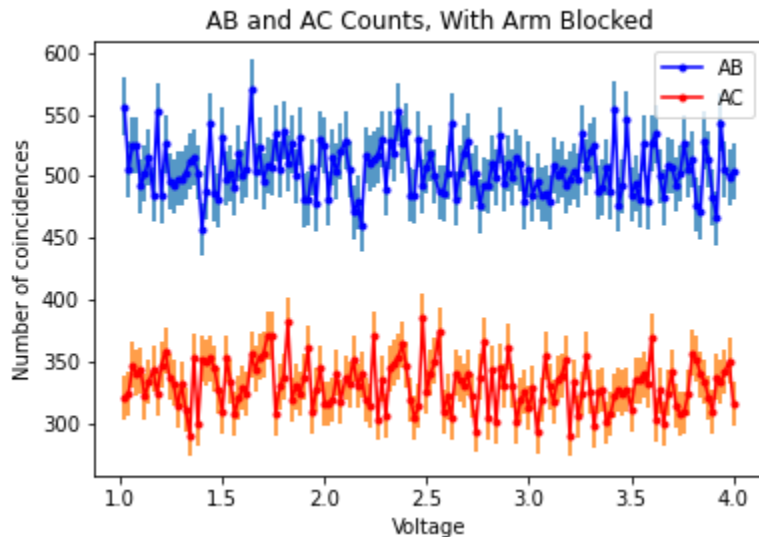
Part II:
- Eq 19: N = N$_0$P(δ)
- Eq 17: V =cos2θ   -> D=1-V
- δ = 2πΔL/λ (phase length differences) -> will be controlled by the
- θ=HWP rotation angle
- θ=pi/4 means the HWP is rotating the wave by 90degrees. That means distinguishability is the biggest at θ=pi/4.

Equation 19 of the lab manual gives the number of counts as a function of the interference phase δ. We need to find values of V for various degrees of distinguishability.

To do this, we varied the angle of the half wave plate by increments of 5 degrees from 0 to 45 degrees.

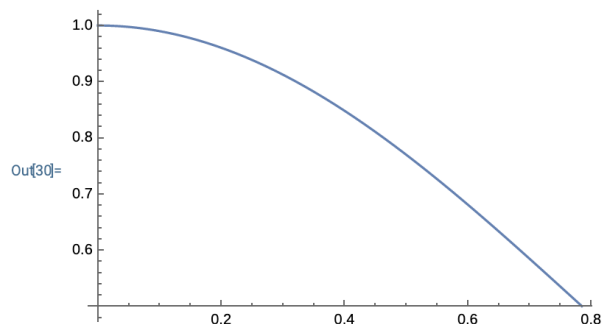Theoretical, we let N0 to be 1 just to see the proportional graph.

AB and AC Counts, Varying HWP Angle
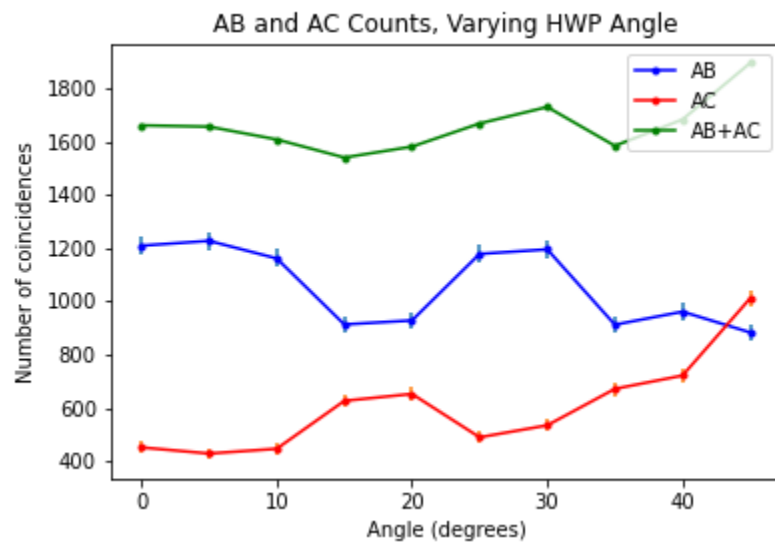
When path length is the same

In[29]:= `Pdelt = 1 / 2 (1 + V * Cos[0])`

Out[29]= $\frac{1}{2}\left(1 + \frac{\frac{1}{2}(-1 + Cos[2\,theta]) + \frac{1}{2}(1 + Cos[2\,theta])}{\frac{1}{2}(1 - Cos[2\,theta]) + \frac{1}{2}(1 + Cos[2\,theta])}\right)$
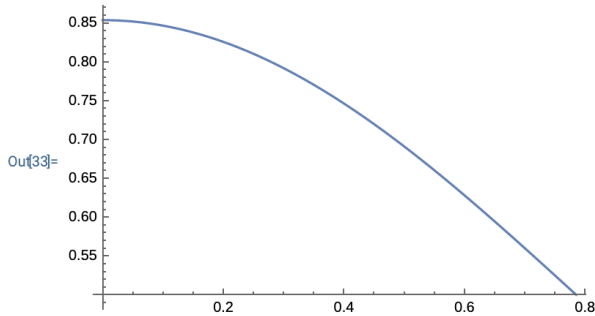
In[30]:= `Plot[Pdelt, {theta, 0, π / 4}]`

Out[30]=

When path length difference is 1/8 of the wavelength

In[32]:= **Pdelt = 1 / 2 (1 + V * Cos[Pi / 4])**

Out[32]= $\frac{1}{2}\left(1 + \frac{\frac{1}{2}(-1 + \text{Cos}[2\,\text{theta}]) + \frac{1}{2}(1 + \text{Cos}[2\,\text{theta}])}{\sqrt{2}\left(\frac{1}{2}(1 - \text{Cos}[2\,\text{theta}]) + \frac{1}{2}(1 + \text{Cos}[2\,\text{theta}])\right)}\right)$

In[33]:= **Plot[Pdelt, {theta, 0, π/4}]**

Out[33]=

When path length difference is ¼ of the wavelength

In[26]:= **Pdelt = 1 / 2 (1 + V * Cos[Pi / 2])**

Out[26]= $\frac{1}{2}$

In[28]:= **Plot[Pdelt, {theta, 0, π/4}]**

Out[28]=
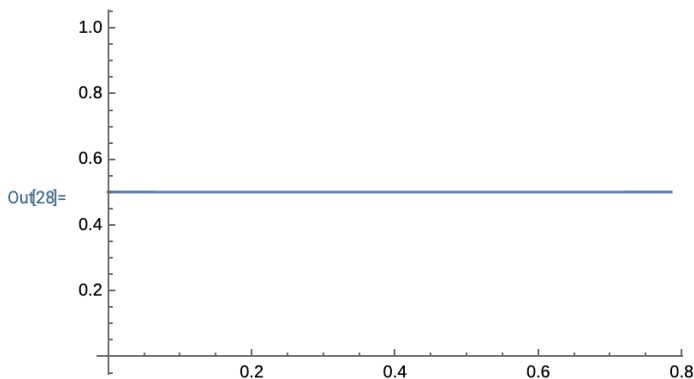
We assume this is the graph that represents our experiment. Sum of those two graphs (green line) adds up to be nearly constant. We assume that we were putting amount of Voltage suitable to make the path length difference to be ¼ of the wavelength. We can also calculate that N0 will be twice as the sum according to the probability calculation.

Part III: Erasure
 -   Eq 26: $|\langle XD|ExyZ|XV\rangle|2 = 1/4(1 + \cos\delta)$
Since we do not have a data for <XD|, we will compare
  -   $|\langle XH|ExyZ|XV\rangle|^2$ and $|\langle XV|ExyZ|XV\rangle|^2$ And $|\langle X10|ExyZ|XV\rangle|^2$ where 10 means V rotated 10 degrees. If <X10| looks closer to our expected probability of

1/4(1 + cos δ), we can assume that $|\langle XD|ExyZ|XV\rangle|2$ = 1/4(1 + cos δ) will be true.

The following is our data for AB and AC coincidence counts with the eraser added, when the half wave plate is rotated to 10 degrees:
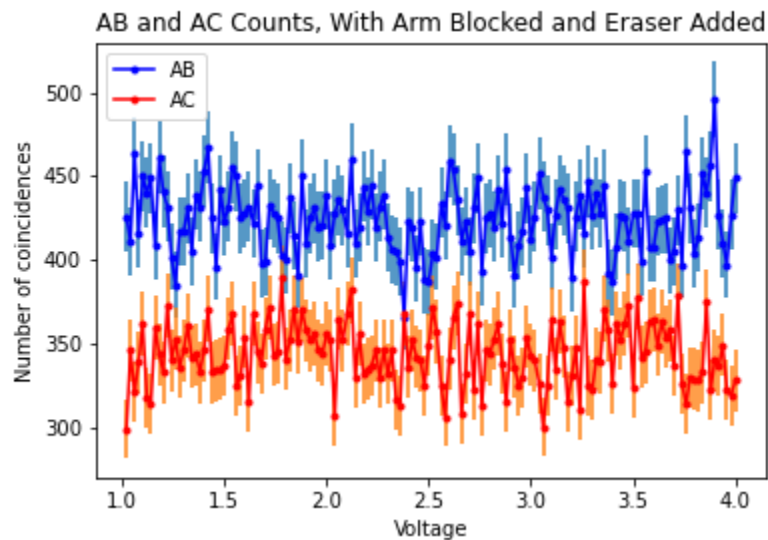
AB and AC Counts, With Arm Blocked and Eraser Added

The following is our data for AB and AC coincidence counts when the half wave plate is rotated 45 degrees:

AB and AC Counts, With Arm Blocked and Eraser Added
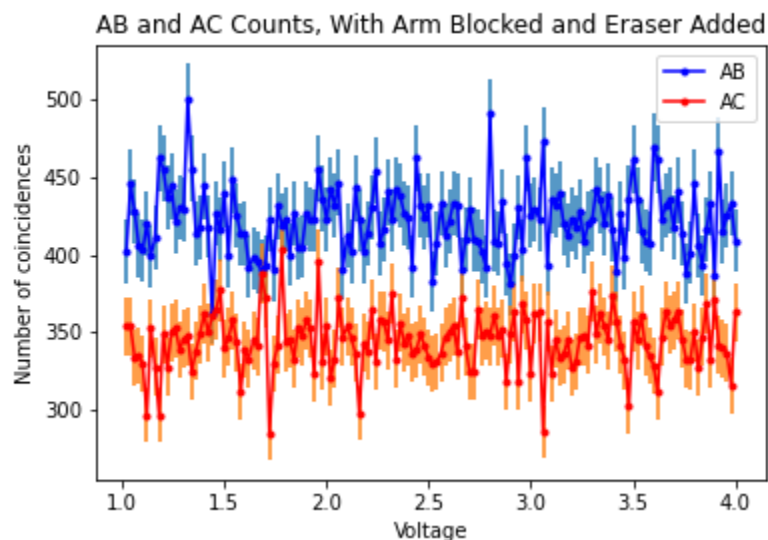
Finally, the following is our data for AB and AC coincidence counts when the half wave plate is not rotated:

AB and AC Counts, With Arm Blocked and Eraser Added
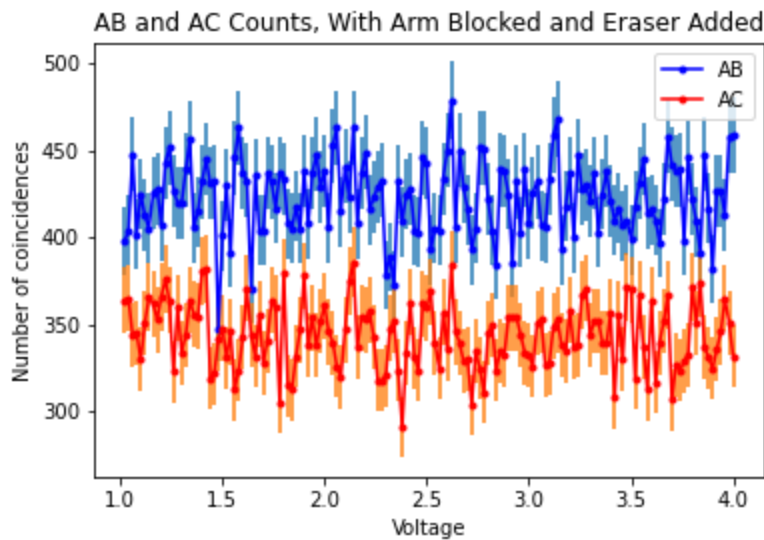
Theoretical probability:
Expectation for $|\langle X10|ExyZ|XV \rangle|^2$

Out[59]= $\left\{\left\{\left(r\,t - e^{i\,\delta}\,r\,t\,Cos[2\,\theta]\right)\left(\frac{1}{2}\,Cos\left[\frac{\pi}{18}\right] + \frac{1}{2}\,Sin\left[\frac{\pi}{18}\right]\right) + e^{i\,\delta}\,r\,t\left(\frac{1}{2}\,Cos\left[\frac{\pi}{18}\right] + \frac{1}{2}\,Sin\left[\frac{\pi}{18}\right]\right)Sin[2\,\theta]\right\}\right\}$

The above data does not show clear interference and is in fact too noisy to make any confident conclusions. This may be because the angle of the HWP used in our experiment was insufficient to have a noticeable effect. We should have used an angle of 22.5 degrees and instead collected data using an angle of 10 degrees. Additionally, it is possible that the alignment of the optical components may have been imperfect.

However, we can still compare what we have here.

Expectation for $\langle XV|ExyZ|XV \rangle$

In[47]:= **Simplify[Transpose[ketXV].ExyZ.ketXV, 2]**

⋯ Simplify : 2 is not a well–formed assumption.

Out[47]= $\left\{\left\{\frac{1}{4}\,i\,\left(1 + e^{i\,d}\right)\right\}\right\}$
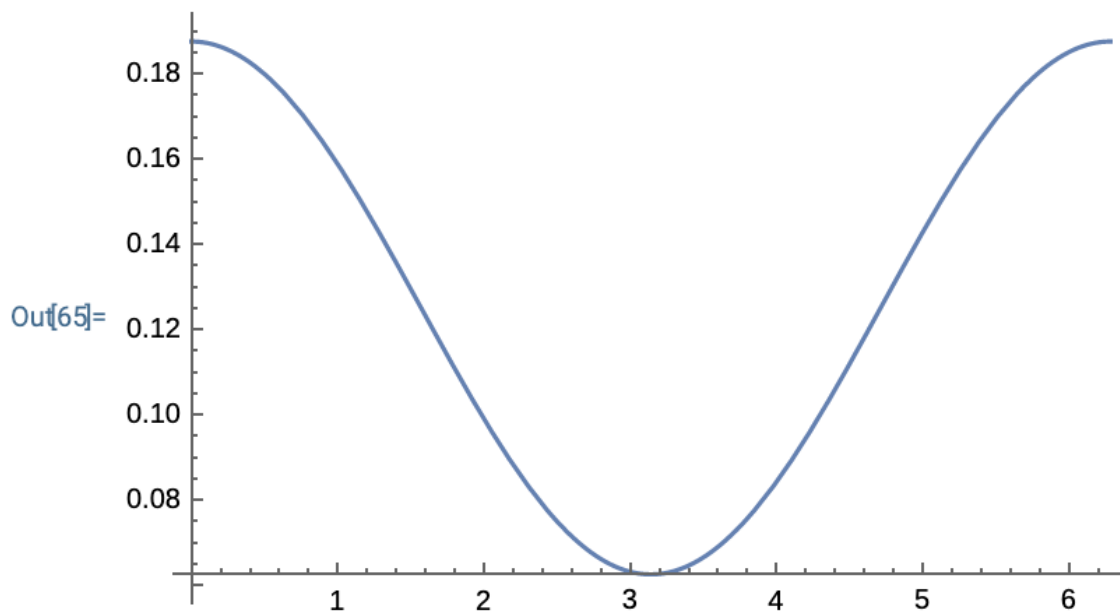
Expectation for $\langle XH|ExyZ|XV \rangle$

In[50]:= `Simplify[Transpose[ketXH].ExyZ.ketXV]`

Out[50]= $\left\{\left\{\frac{1}{4} i \left(1 + e^{i d}\right)\right\}\right\}$

Probability amplitude comes from absolute square so, it simplifies to 1/16*(2+cos(d=angle))

In[65]:= $\text{Plot}\left[\frac{1}{16} (2 + \text{Cos}[d]), \{d, 0, 2 \text{Pi}\}\right]$

Out[65]=

Seeing that we made a noticeable change in those graphs that is not following either of our expectations for H and V, we can assume that it may follow the expectation for 22.5(45 rotation) in an adequate setting.

Conclusions

The results of the experiment are drastically different between when paths are blocked and unblocked. Although we should have observed interference once again after adding the quantum eraser, we did not, as this half wave plate was oriented at the incorrect angle for this to occur. We expect to produce more accurate results if we were to align the optical equipments in correct angles and collect a new dataset.