CSE-200 Final Presentation

Red Black Tree

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Department of CSE
Bangladesh University of Engineering and Technology

We Need to Store and Search Data

- Everything is **tree-structured**

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- **Insert** data into the structure

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We Need to Store and Search Data

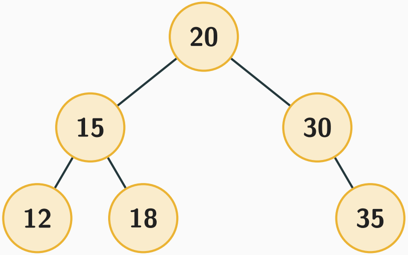
- Everything is **tree-structured**
- **Insert** data into the structure
- **Delete** data efficiently
- **Search** for data quickly

Good way to do all of this?

Use a BST!

The BST Rule

How does BST decide where to put a node?



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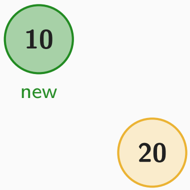

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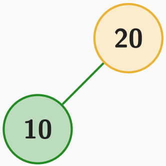

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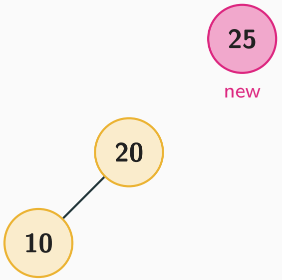

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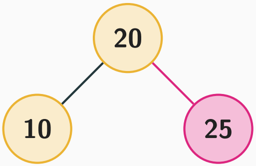

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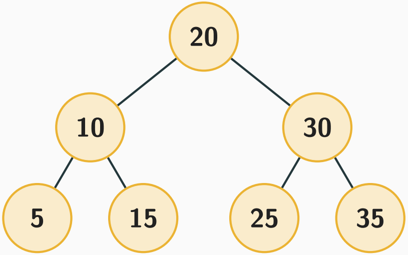

Good technique!

Insert the roll numbers in a class sequentially

1, 2, 3, 4 …10

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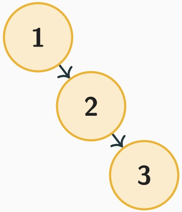

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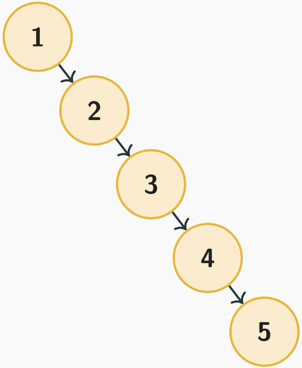

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- The tree just keeps **growing** right...

...and on

Insert the roll numbers in a class sequentially

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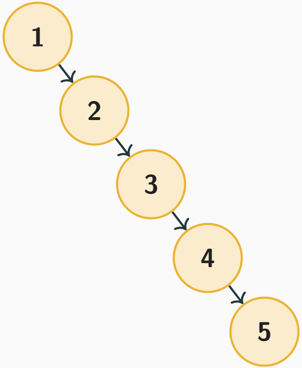

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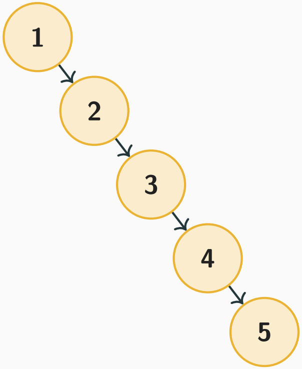
Still works!

...and on

But, What's the Problem?

- **Height becomes n**



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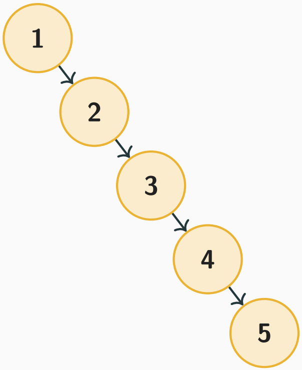

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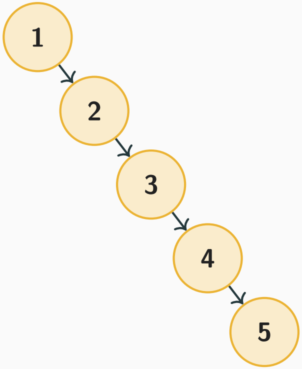

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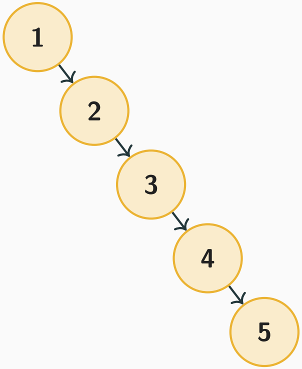

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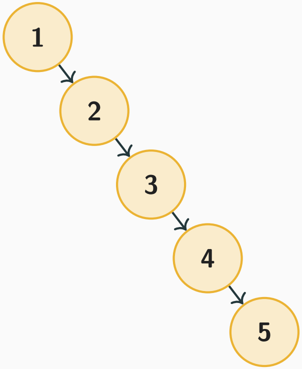

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- Insertion takes $O(n)$
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Time complexity becomes $O(n)$

The Solution?

Use a BST that **promises** to keep its height **logarithmic**

no matter how and what element you insert.

The Solution?

Examples of Self-Balancing Trees:

- AVL Tree

The Solution?

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Examples of Self-Balancing Trees:

- AVL Tree
- **Red-Black Tree**
- Splay Tree
- B-Tree

Let's look at Red-**Black** Trees

What is Red-Black Tree

A Red-Black Tree rebalances itself by coloring nodes **red** and **black**, ensuring no two **red** nodes are **adjacent** and all **paths** have the same **black-height**, which keeps its height **logarithmic**.

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Height becomes log(n) here!

How does RBT do it:Properties

Five points to remember

How does RBT do it:Properties

- **Property 1:** Every node is either red or black

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Hence, the name Red Black Tree

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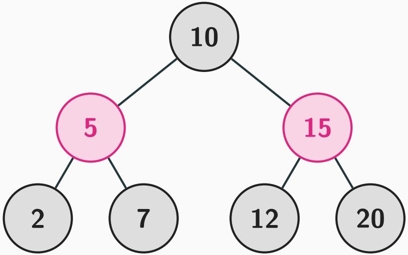

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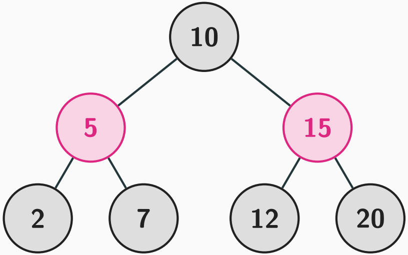

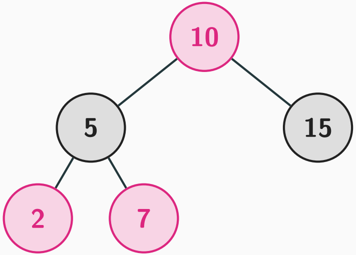

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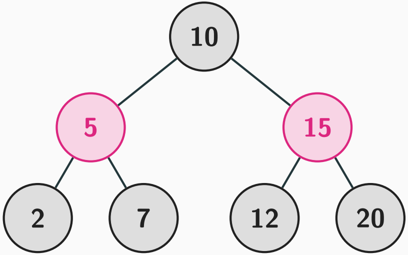
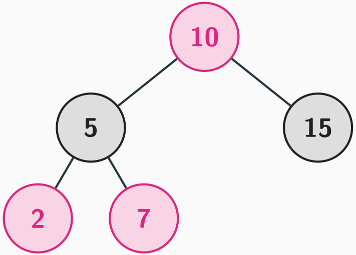


Correct



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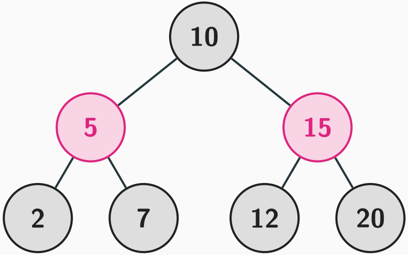

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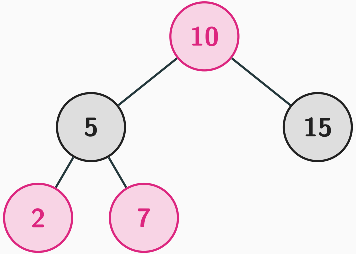

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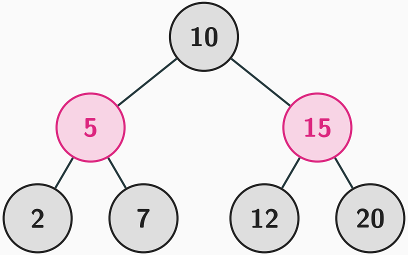

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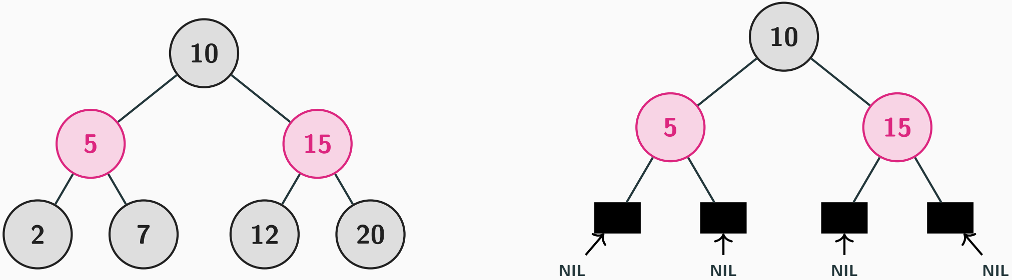

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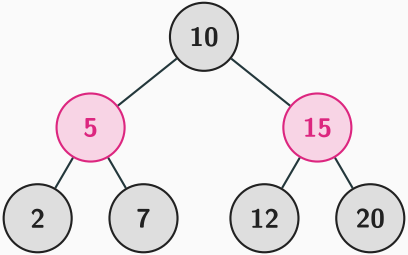

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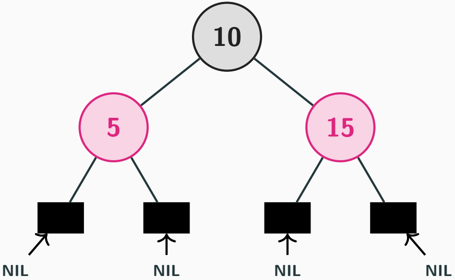

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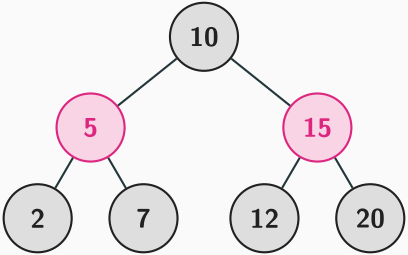

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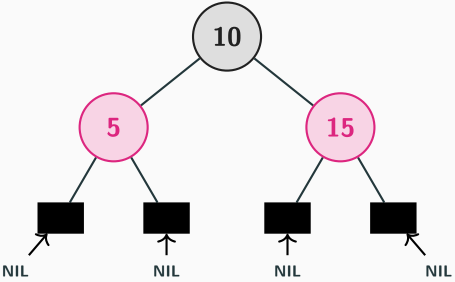

NIL nodes (counted as Black)

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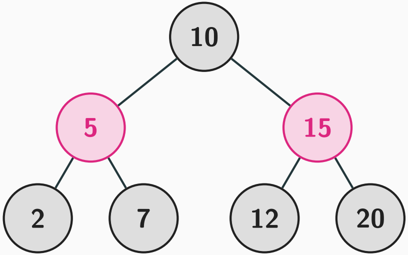

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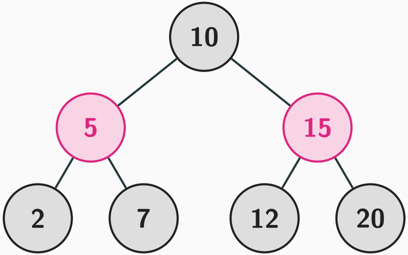

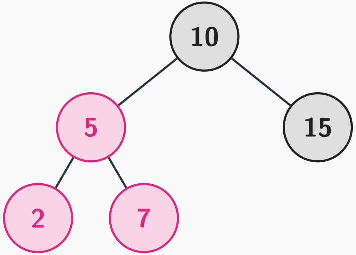

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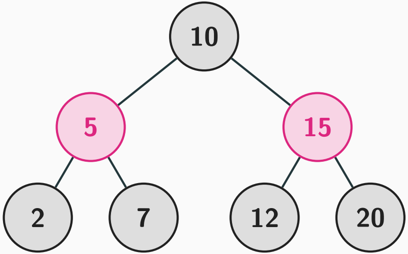

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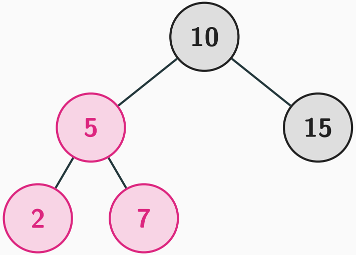

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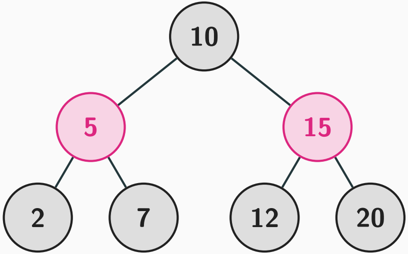

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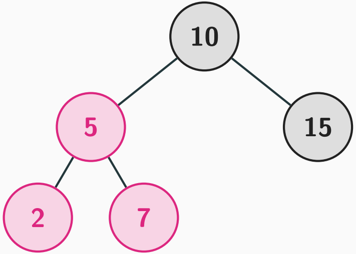

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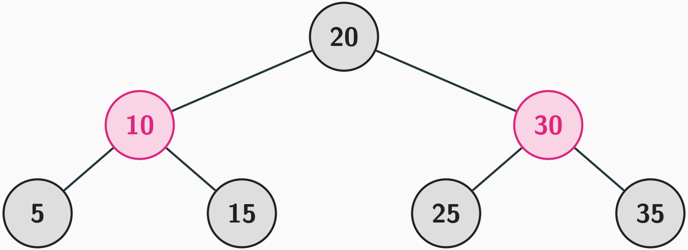

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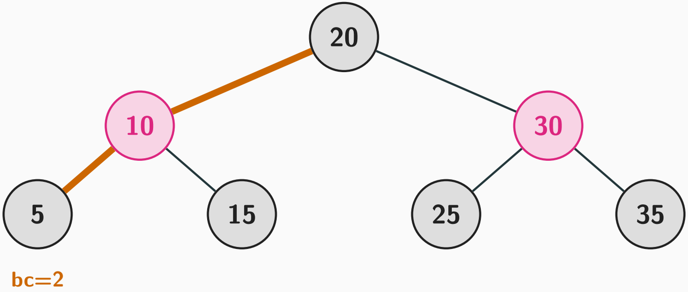

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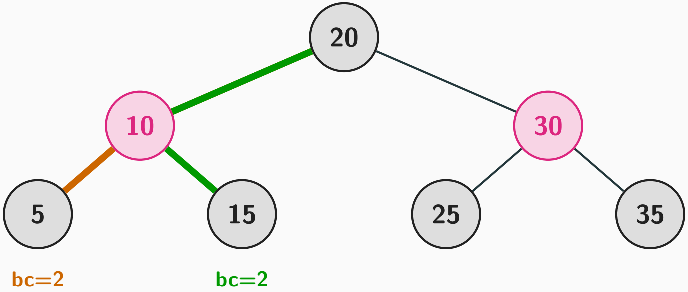

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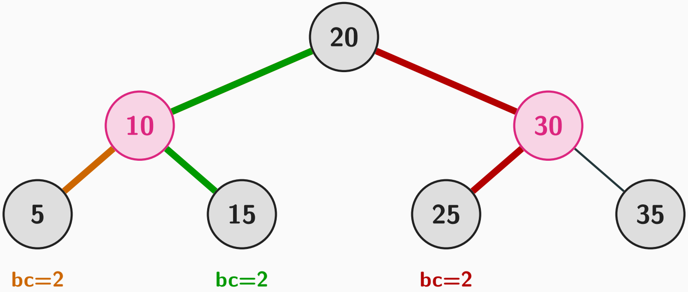

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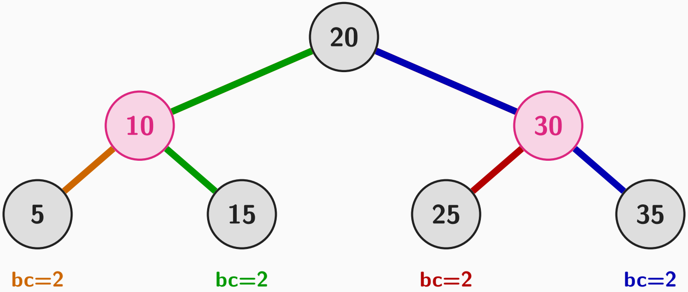

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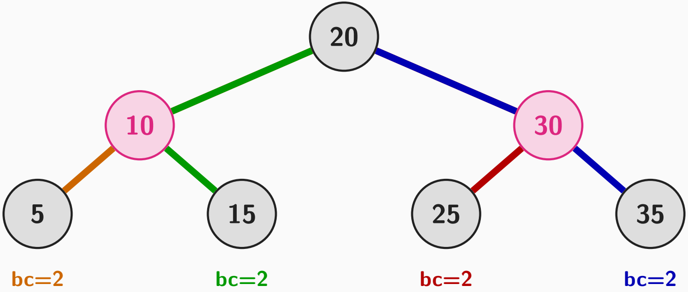

All paths from root have same black count = 2

RBT Operations

Now, How do these points ensure the ”rebalancing” feature of Red Black Tree?

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Let’s see some operations....

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Insert node x in a Red Black Tree

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Pseudocode

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color[x] = RED

y = root[T]

while y ≠ NIL do
  if key[x] > key[y]
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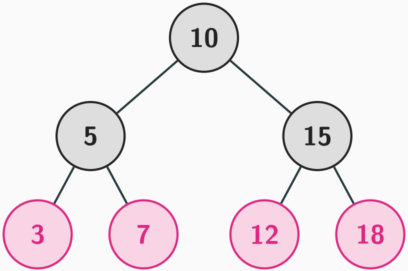
Existing RBT



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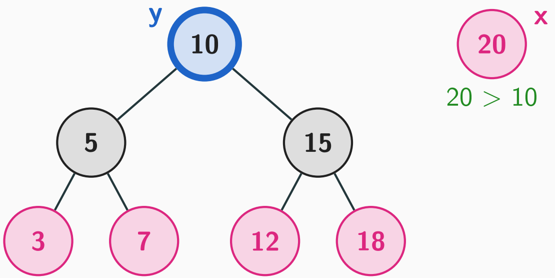

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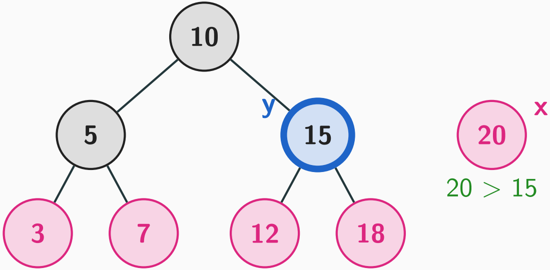

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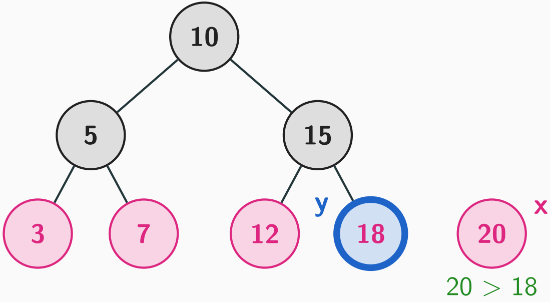

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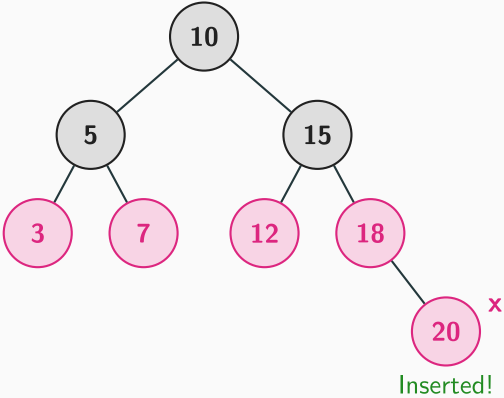
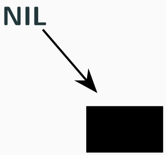
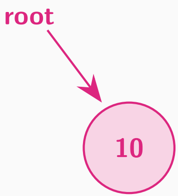


Insert 10



What can go wrong

Insert 10



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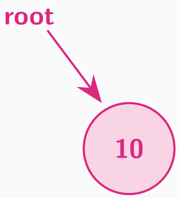

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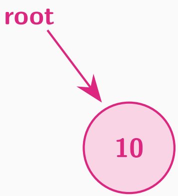
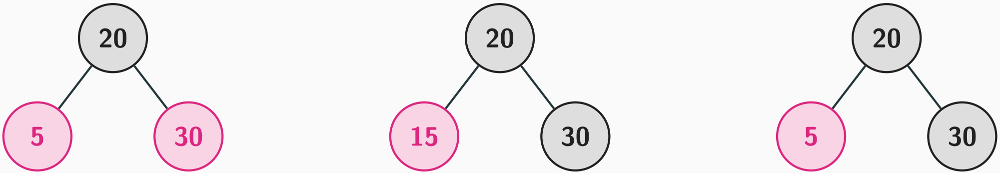


Root can't be RED

Case 1

What can go wrong

Insert 10



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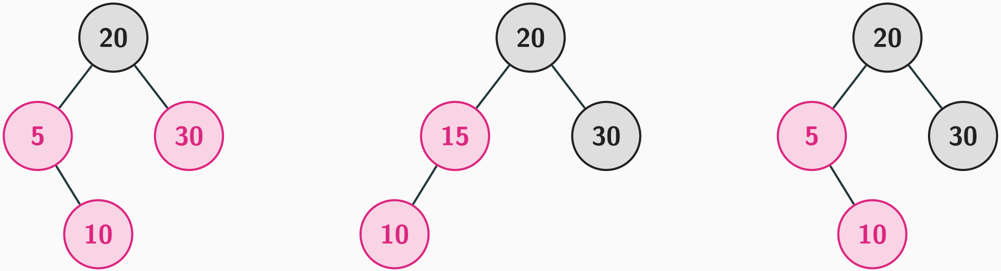

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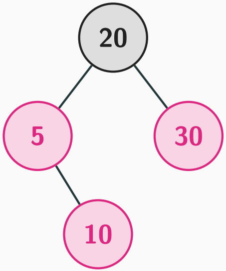
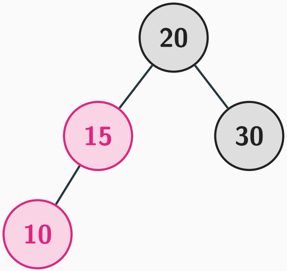
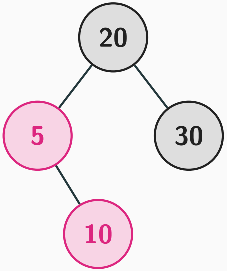






Case 2 **Case 3** **Case 4**

Solutions to Violations

Case 1

Violation

Root is **RED** — Property 2 broken

Fix

Recolor root to **BLACK**

Solutions to Violations

Case 1

Violation

Root is **RED** — Property 2 broken

Fix

Recolor root to **BLACK**

Case 2

Violation

Uncle is **RED** — two reds adjacent

Fix

Recolor parent, uncle **BLACK**; grandparent **RED**

Solutions to Violations

	Violation	Fix
Case 1	Root is **RED** — Property 2 broken	Recolor root to **BLACK**
Case 2	Uncle is **RED** — two reds adjacent	Recolor parent, uncle **BLACK**; grandparent **RED**
Case 3	Uncle is **BLACK** — Right-Right	Left rotate at grandparent, then recolor

Solutions to Violations

	Violation	Fix
Case 1	Root is **RED** — Property 2 broken	Recolor root to **BLACK**
Case 2	Uncle is **RED** — two reds adjacent	Recolor parent, uncle **BLACK**; grandparent **RED**
Case 3	Uncle is **BLACK** — Right-Right	Left rotate at grandparent, then recolor
Case 4	Uncle is **BLACK** — Left-Right	Left rotate at parent, then apply Case 3

Remember This Problem?

Let's try inserting **1, 2, 3, 4, 5** again...

But this time in a Red-Black Tree!

Remember This Problem?

Let's try inserting **1, 2, 3, 4, 5** again...

But this time in a Red-Black Tree!

Watch the magic happen!

Insert 1

- First node is always **root**
- Insert as **RED** (default color)

Insert 1

- First node is always **root**
- Insert as **RED** (default color)

After Insert

Insert 1

- First node is always **root**
- Insert as **RED** (default color)
- But Root cannot be **RED!**
  Property 2 violated - Case : 1

Violation — Root is RED!

Insert 1

- First node is always **root**
- Insert as **RED** (default color)
- But Root cannot be **RED!**
  Property 2 violated - Case : 1
- Recolor root to **BLACK**
- **Fixed!**

After Recolor

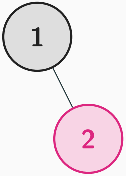
Insert 2

- Right child of 1
- Insert as **RED** (default color)

Insert 2

- Right child of 1
- Insert as **RED** (default color)
- Parent is BLACK — **no violation** ✔

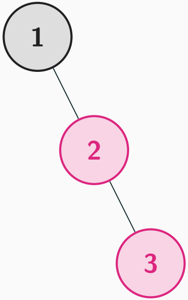
After Insert



Insert 3

- Right child of 2
- Insert as **RED** (default)

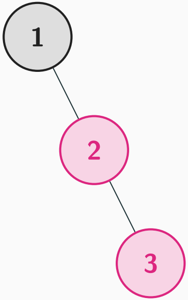
After Insert



Insert 3

- Right child of 2
- Insert as **RED** (default)
- Uncle is **NIL/BLACK**
- **Case: 3**
  Left rotate at node 1

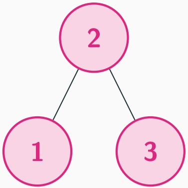
Violation Found — Two RED in a row!



Insert 3

- Right child of 2
- Insert as **RED** (default)
- Uncle is **NIL/BLACK**
- **Case: 3**
  Left rotate at node 1
- Recolor: 2 $\rightarrow$ **BLACK**, children $\rightarrow$ **RED**

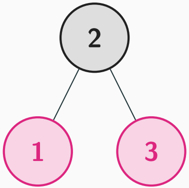
After Left Rotation



Insert 3

- Right child of 2
- Insert as **RED** (default)
- Uncle is **NIL/BLACK**
- **Case: 3**
  Left rotate at node 1
- Recolor: 2 $\rightarrow$ **BLACK**, children $\rightarrow$ **RED**
- **Fixed!**

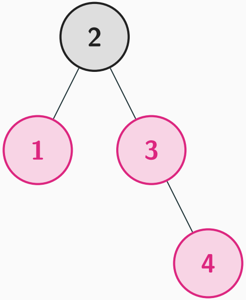
After Recolor



Insert 4

- Right child of 3
- Insert as **RED** (default)

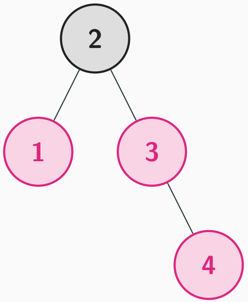
After Insert



Insert 4

- Right child of 3
- Insert as **RED** (default)
- Uncle (node 1) is **RED**
- **CASE 2**
  Uncle is RED — just recolor!

Violation Found — Uncle is RED



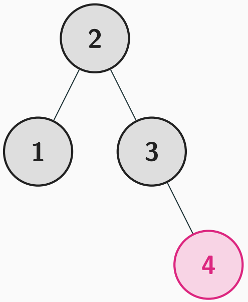
Insert 4

- Right child of 3
- Insert as **RED** (default)
- Uncle (node 1) is **RED**
- **CASE 2**
  Uncle is RED — just recolor!
- Recolor: parent & uncle $\rightarrow$ **BLACK**
- Grandparent stays **BLACK**

Insert 4

- Right child of 3
- Insert as **RED** (default)
- Uncle (node 1) is **RED**
- **CASE 2**
  Uncle is RED — just recolor!
- Recolor: parent & uncle $\rightarrow$ **BLACK**
- Grandparent stays **BLACK**
- **Fixed!**

After Recolor



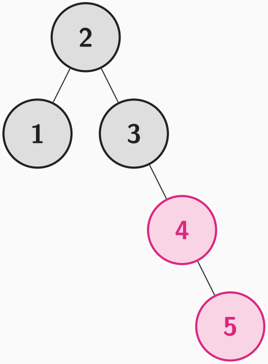
Insert 5

- Right child of 4
- Insert as **RED** (default)

Insert 5

- Right child of 4
- Insert as **RED** (default)

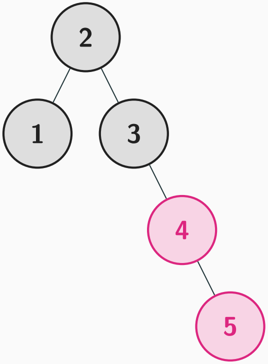
After Insert



Insert 5

- Right child of 4
- Insert as **RED** (default)
- Uncle (node 1) is **BLACK**
- **CASE 3**
  Uncle is **BLACK** — rotate & recolor!

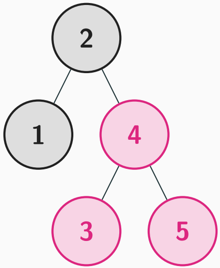
Violation — Two RED in a row!



Insert 5

- Right child of 4
- Insert as **RED** (default)
- Uncle (node 1) is **BLACK**
- **CASE 3**
  Uncle is **BLACK** — rotate & recolor!
- Left rotate at node 3 $\rightarrow$ 4 moves up

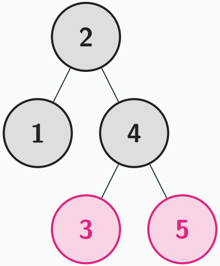
After Left Rotation



Insert 5

- Right child of 4
- Insert as **RED** (default)
- Uncle (node 1) is **BLACK**
- **CASE 3**
  Uncle is **BLACK** — rotate & recolor!
- Left rotate at node 3 → 4 moves up
- Recolor: 4 → **BLACK**, children → **RED**
- **We're done!**

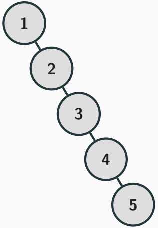
After Recolor



BST vs. Red-Black Tree

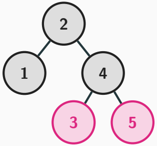
Inserting $\{1, 2, 3, 4, 5\}$ in order

Regular BST



Height = 5 ▪ $O(n)$

Red-Black Tree



Height = 3 ▪ $O(\log n)$

Insertion is the easy half

Now, What happens when we delete a node?

Deletion

Deletion is even more... interesting!

Deletion is even more... interesting!

Deleting a **RED** node

Deleting a **BLACK** node

Deletion is even more... interesting!

Deleting a RED node

- No problem!

Deleting a BLACK node

Deletion is even more... interesting!

Deleting a RED node

- No problem!
- Just remove it

Deleting a BLACK node

Deletion is even more... interesting!

Deleting a RED node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a BLACK node

Deletion is even more… interesting!

Deleting a RED node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a BLACK node

- Oh boy…

Deletion is even more... interesting!

Deleting a RED node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a BLACK node

- Oh boy...
- Black height changes!

Deletion is even more... interesting!

Deleting a RED node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a BLACK node

- Oh boy...
- Black height changes!
- **Need "double black" fix**

Deletion is even more... interesting!

Deleting a RED node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a BLACK node

- Oh boy...
- Black height changes!
- **Need "double black" fix**
- Complex cases

Deletion is even more... interesting!

Deleting a RED node

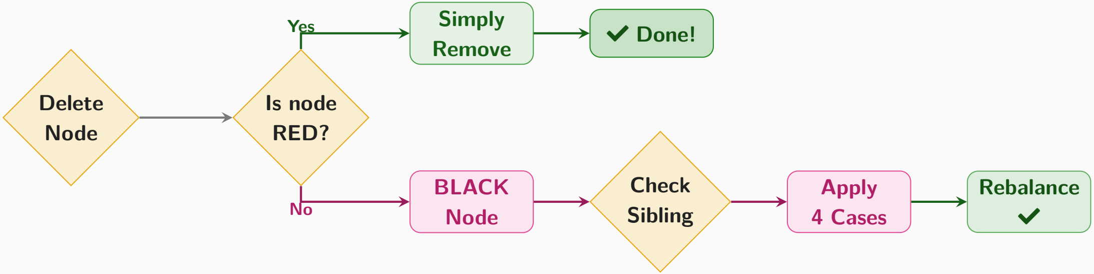
- No problem!
- Just remove it
- **Properties still hold**

Deleting a BLACK node

- Oh boy...
- Black height changes!
- **Need "double black" fix**
- Complex cases

Let's see both cases...

Deletion Decision Flowchart



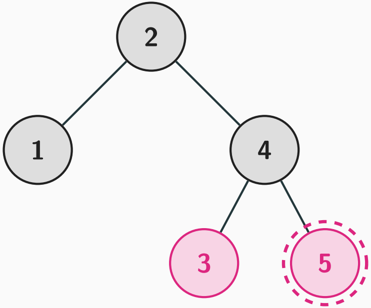
Top path (RED node) = straightforward

Bottom path (BLACK node) = complex

Case 1: Deleting a RED Node

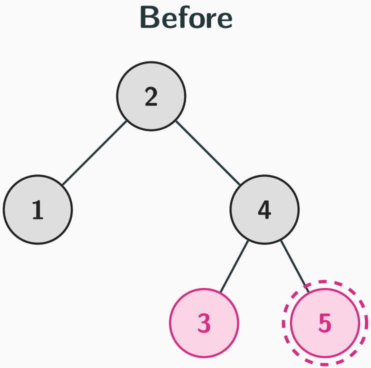
Delete node **5** from the tree

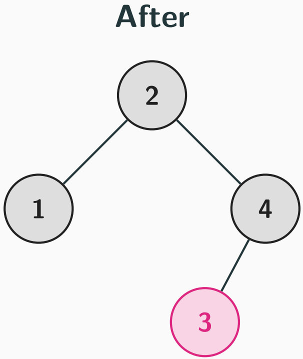
Before



Case 1: Deleting a RED Node

Delete node **5** from the tree





Case 1: Deleting a RED Node

Is it done?

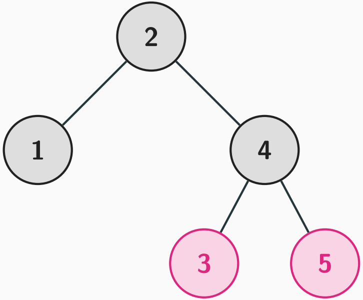
Is it done?

Let's check the black height.

Case 1: Black-Height Stays the Same

Every path still has **bc = 2** black nodes after removing 5

Before (with node 5)



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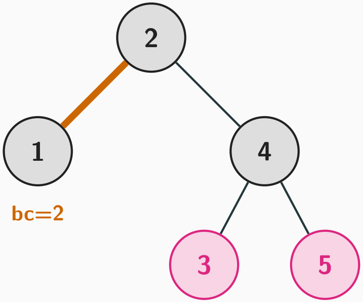

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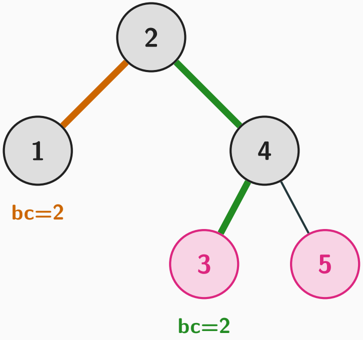

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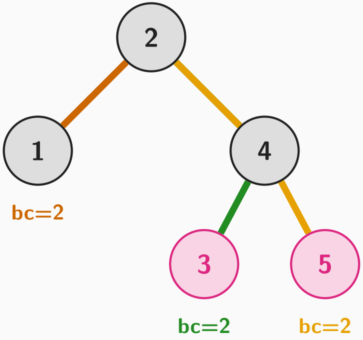

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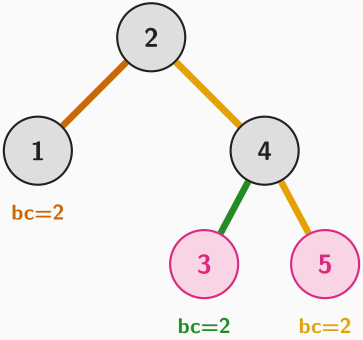
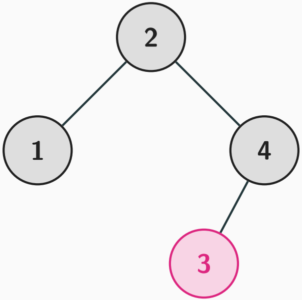
Before (with node 5)



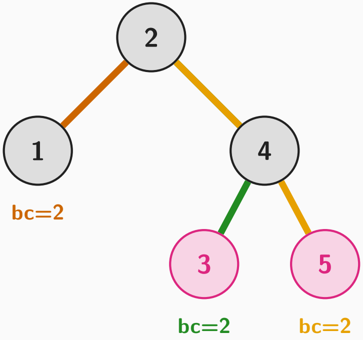
After (node 5 removed)



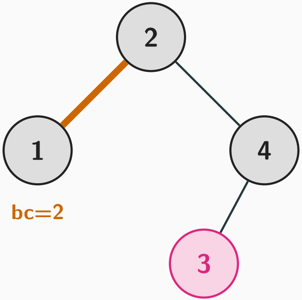
Case 1: Black-Height Stays the Same

Every path still has **bc = 2** black nodes after removing 5

Before (with node 5)

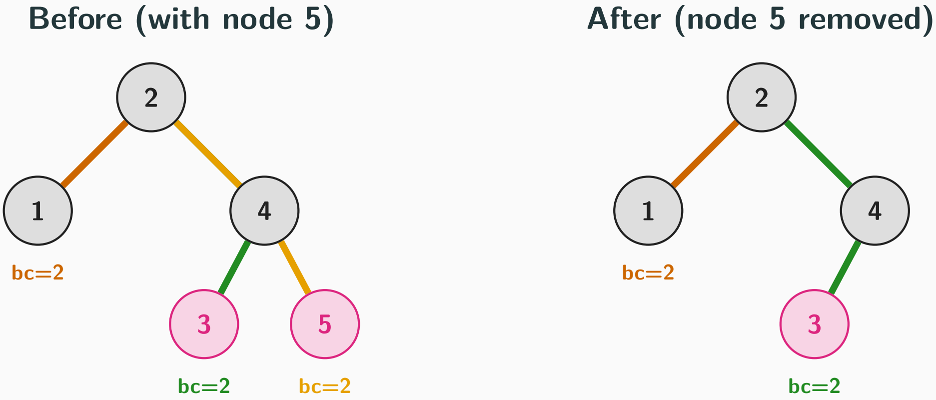


After (node 5 removed)



Case 1: Black-Height Stays the Same

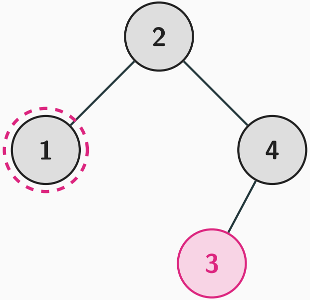
Every path still has $\mathbf{bc = 2}$ black nodes after removing 5



Case 2: Deleting a BLACK Node

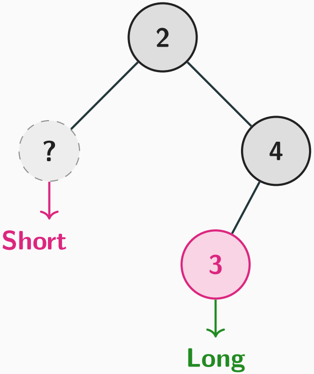
Delete node **1** from the tree

Before deletion:



Case 2: After Deleting the BLACK Node

The node is gone - but now we have a **problem**



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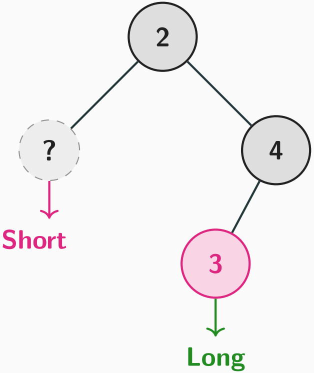
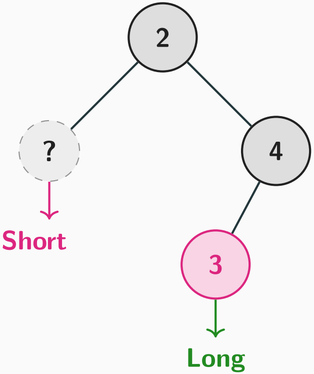


- Left path is now **shorter**

Case 2: After Deleting the BLACK Node

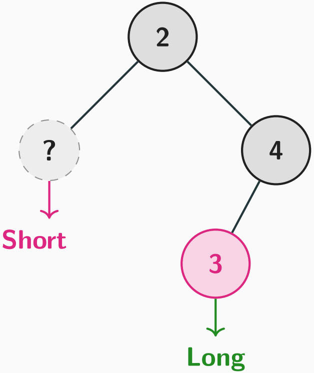
The node is gone - but now we have a **problem**



- Left path is now **shorter**
- Black-height **violated!**

Case 2: After Deleting the BLACK Node

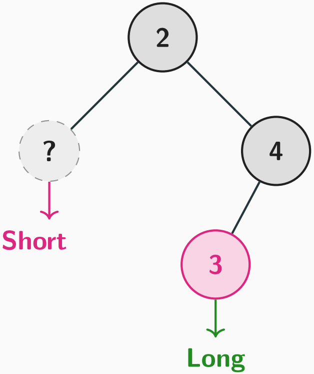
The node is gone - but now we have a **problem**



- Left path is now **shorter**
- Black-height **violated!**
- We call this a **"Double-Black"** node

Case 2: After Deleting the BLACK Node

The node is gone - but now we have a **problem**



- Left path is now **shorter**
- Black-height **violated!**
- We call this a

  "Double-Black" node

IMBALANCED - must fix!

Case 2: The Fix-up

- **Rotate:** Right at 4, then left at 2

Case 2: The Fix-up

- **Rotate:** Right at 4, then left at 2
- **Recolor:** Node 3 $\rightarrow$ Black

Case 2: The Fix-up

- **Rotate:** Right at 4, then left at 2
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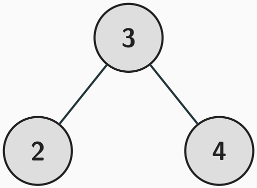

Case 2: The Fix-up

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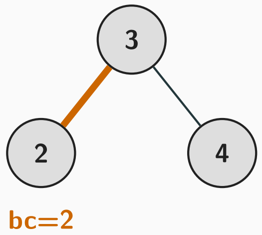

Case 2: The Fix-up

- **Rotate:** Right at 4,
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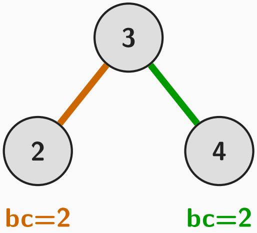
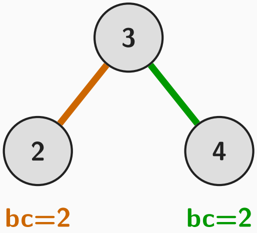
- **Recolor:** Node 3 $\rightarrow$ Black



- **Rotate:** Right at 4, then left at 2
- **Recolor:** Node 3 $\rightarrow$ Black
- **Tree is balanced!**



All paths: $\mathbf{bc = 2}$ - Black-height restored!

Fixing Double-Black: 4 Cases

When we have a **Double-Black** node,

the fix depends on the **sibling's color and children**.

P = Parent **S** = Sibling **L** / **R** = S's children

DB = Double-Black node

Fixing Double-Black: 4 Cases

When we have a **Double-Black** node,

the fix depends on the **sibling's color and children**.

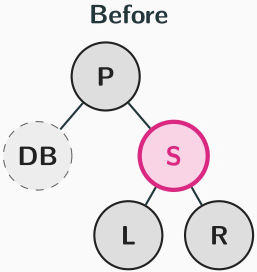
P = Parent **S** = Sibling **L** / **R** = S's children

DB = Double-Black node

4 cases - let's go through them one by one!

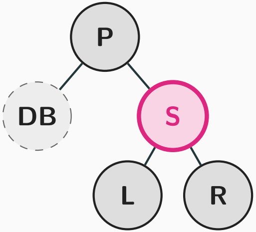
Fix Case 1 of 4: Sibling is RED

The Sibling S is RED



The Sibling S is RED

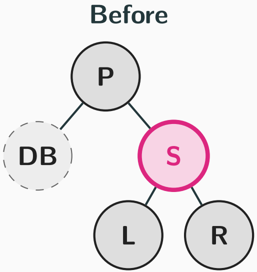
Before



- **Rotate** P to the left

Fix Case 1 of 4: Sibling is RED

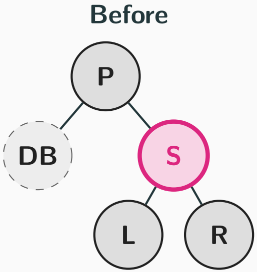
The Sibling S is RED



- **Rotate** P to the left
- **Recolor:** S $\rightarrow$ Black, P $\rightarrow$ Red

Fix Case 1 of 4: Sibling is RED

The Sibling S is RED



- **Rotate** P to the left
- **Recolor:** S $\rightarrow$ Black, P $\rightarrow$ Red

Fix Case 1 of 4: Sibling is RED

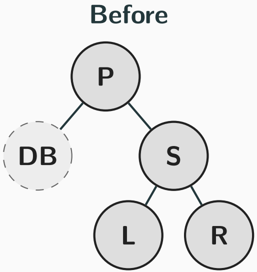
The Sibling S is RED

Now apply Case 2, 3, or 4 to DB

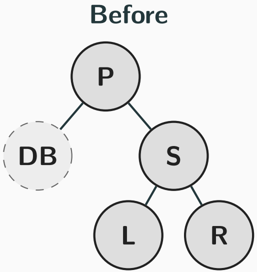
- **Rotate** P to the left
- **Recolor:** S $\rightarrow$ Black, P $\rightarrow$ Red

Fix Case 2 of 4: Sibling & Children All BLACK

S and both children are BLACK



S and both children are BLACK

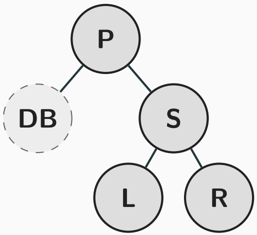


- **Recolor** S $\rightarrow$ Red

Fix Case 2 of 4: Sibling & Children All BLACK

S and both children are BLACK

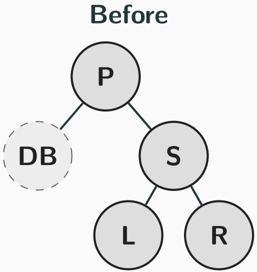
Before



- **Recolor** S $\rightarrow$ Red
- Push the Double-Black **up to P**

Fix Case 2 of 4: Sibling & Children All BLACK

S and both children are BLACK



- **Recolor** S $\rightarrow$ Red
- Push the Double-Black **up to P**

Fix Case 2 of 4: Sibling & Children All BLACK

S and both children are BLACK

DB pushed to P — continue fixing

- **Recolor** S → Red
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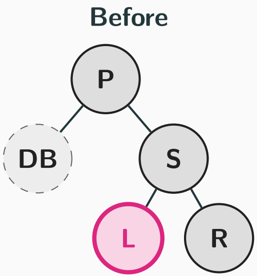
Fix Case 3 of 4: Sibling's Left Child is RED

S is Black, S's Left child is RED



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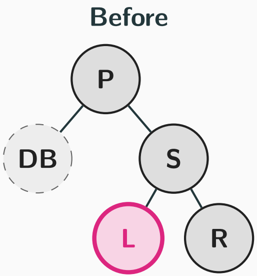

- **Right-rotate** at S, & **Swap colors** of S and L

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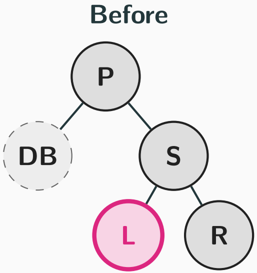

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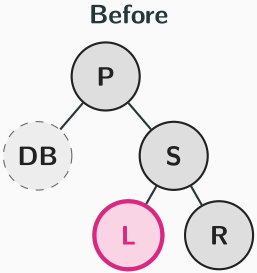

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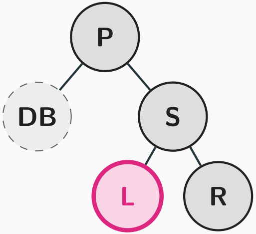

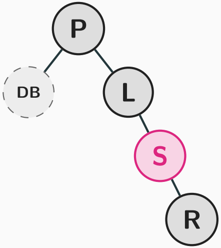

Now proceed with Case 4

- **Right-rotate** at S, & **Swap colors** of S and L

Fix Case 4 of 4: Sibling's Right Child is RED

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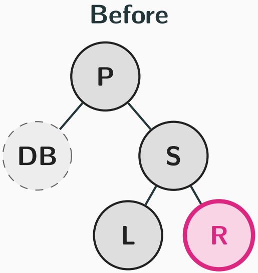

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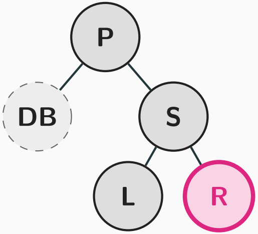

- **Left-rotate** at P

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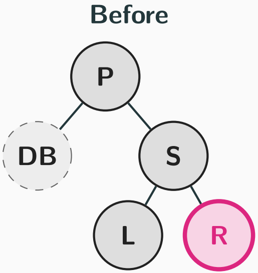

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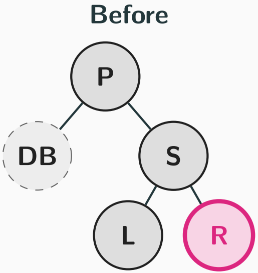

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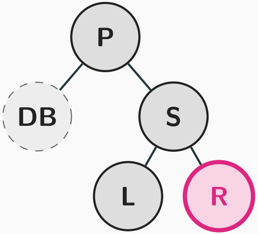

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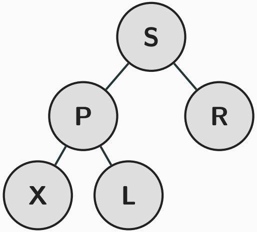

Double-Black fully resolved!

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Summary

The cases form a **chain**:

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$$\textbf{Case 1} \xrightarrow{\text{rotate}} \textbf{Case 2/3} \xrightarrow{\text{transform}} \textbf{Case 4} \xrightarrow{\text{rotate}} \checkmark\ \textbf{Done!}$$

Goal: Eventually reach Case 4 to fully eliminate Double-Black

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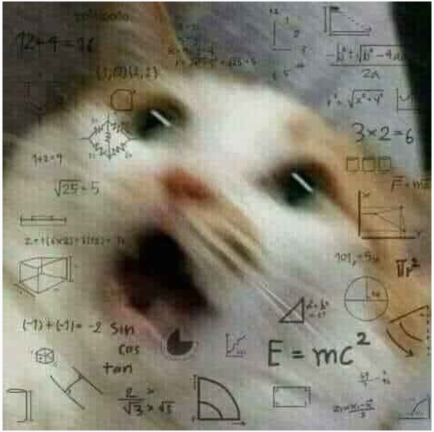
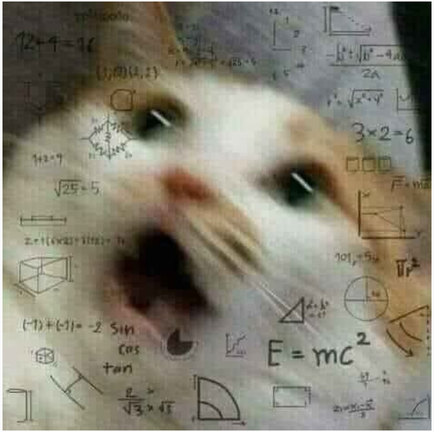
Goal: Eventually reach Case 4 to fully eliminate Double-Black

Case 2 may propagate upward; Cases 1 & 3 always lead to Case 4



Confused?

Too Many Cases?



Confused?

If this felt like **a lot** at once - that's because **it is !**

Even seasoned programmers keep a reference sheet open for this one.

The Reality of Deletion

We know deletion is complex - and that's *okay!*

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We'll skip the gory details for now!

All that work.

Was All That Worth It?

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What did it actually buy us?

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Let's finally see the payoff.

Rotation Complexity: $O(1)$ Operations

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Left Rotation:

- Restructures tree locally

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Why Rotations Work: The Magic Behind Balance

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Key Insight

Rotations are $O(1)$ because they only modify a **constant number of pointers**!

No tree traversal required.

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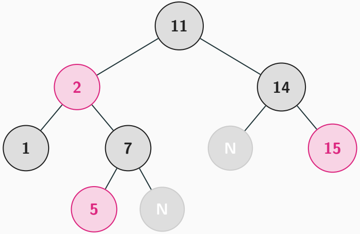

Black height $= 2$, Total height $= 4$

Why Red-Black Trees Are Space Efficient

Space Complexity: $O(n)$ Memory Usage

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RBTs achieve $O(n)$ **space** with **1 bit** per node (color field).

The Evolution: From Red-Black to Modern Trees

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Note: Even Sedgewick evolved the design—Left-Leaning RBTs reduce implementation complexity.

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Next Frontier

Adaptive structures: learning access patterns to optimize tree shape dynamically.

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Words to Live By

"We know exactly how to balance a tree.
We just haven't been outside to see one."

— Unknown

Thanks for listening!