# Red-Black Trees

Why Even the Inventor Moved On...

Your Name

February 19, 2026

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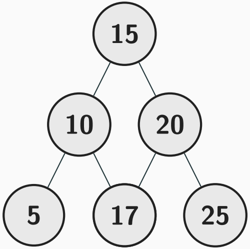

# We All Love to Sort Things!

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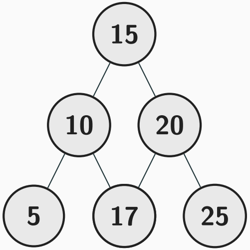
- **TREES!** Specifically Binary Trees




**Binary Trees are Awesome!**

# Why Binary Trees Are Good

## Beautiful Structure

- Everything has its place
- Search: $O(\log n)$
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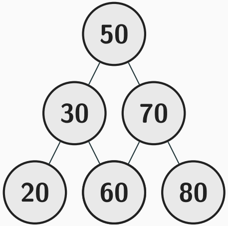


### The Magic

Logarithmic time = **Sports car performance!**

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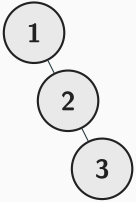

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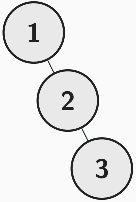


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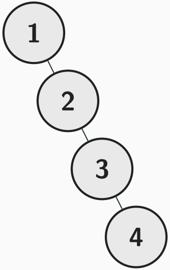

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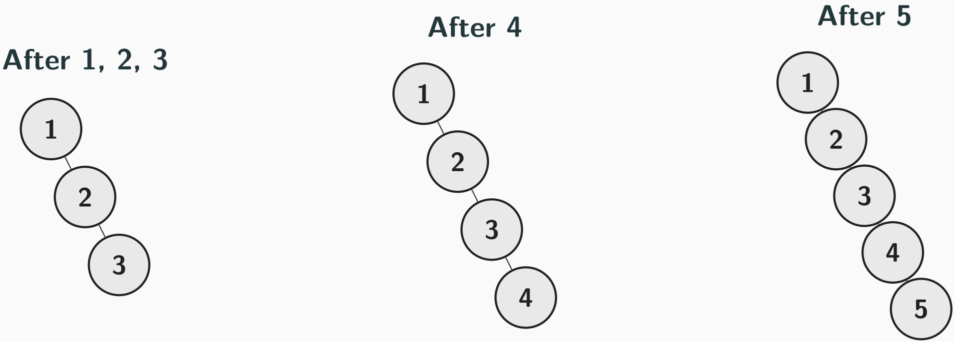

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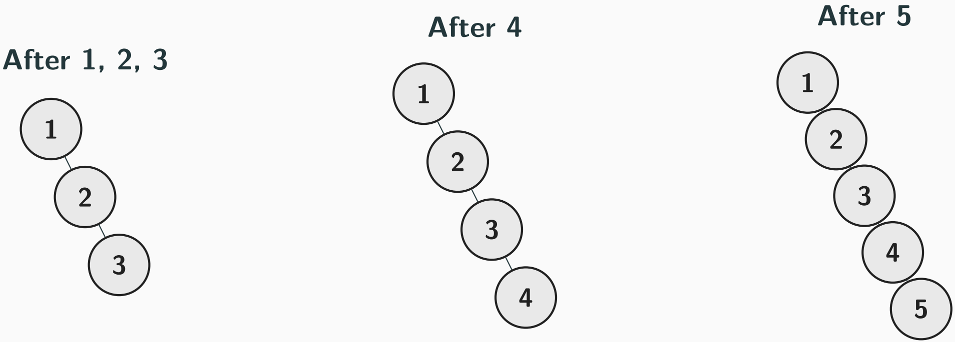


**Our tree became a... LINKED LIST!** Damn!

# From Sports Car to Bicycle

**Before (Balanced)**

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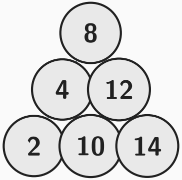


**After (Degenerate)**

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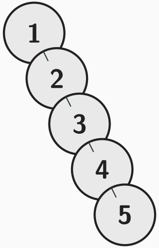

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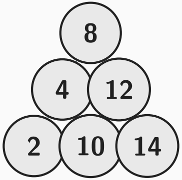


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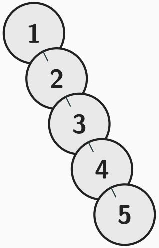


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**Red-Black Trees are used EVERYWHERE!**

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- Around since the 1970s

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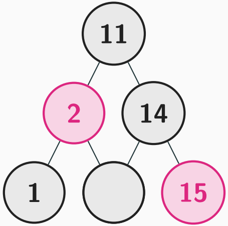

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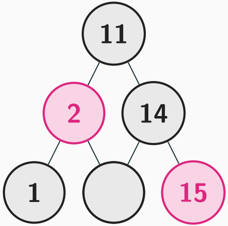


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**Let's get started, shall we?**

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**Red-Black Trees have 5 properties**

Let's see them one by one...

# The Five Properties

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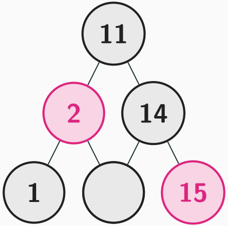

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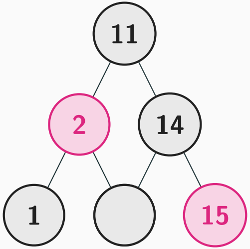

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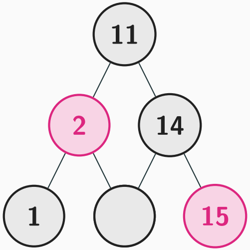


## Why This Matters

This property helps us understand the **balance** of the tree!

# The Amazing Height Bound

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*(I skipped the detailed proof - nobody wants to sit through that!)*

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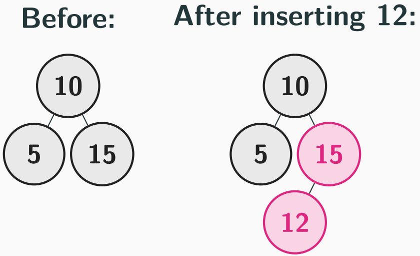

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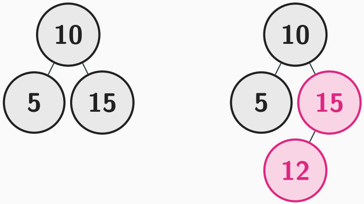
**Before:** **After inserting 12:**




**Problem!**

Two adjacent red nodes — Property 4 violated!

# Fixing Violations

## Uncle is RED

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This is where it gets tricky!

But it keeps the tree balanced!

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Let's try inserting **1, 2, 3, 4, 5** again...

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Let's see what happens...

Will it stay balanced?

🪄 Stay tuned!

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**Watch the magic happen!**

## Simulation — Step 1 of 5: Insert 1


- First node is always the root

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- First node is always the root
- Color it **BLACK**

# Simulation — Step 1 of 5: Insert 1

1 2 3 4 5 Inserting 1

- First node is always the root
- Color it **BLACK**
- Property 2: Root must be black ✔


**All properties satisfied!**

Height $= 1$ Black-height $= 1$

# Simulation — Step 2 of 5: Insert 2


Inserting 2

- Insert as right child of 1

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Inserting 2

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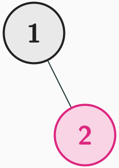
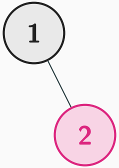


# Simulation — Step 2 of 5: Insert 2


Inserting 2

- Insert as right child of 1
- Color it **RED**
- Parent is **BLACK** $\Rightarrow$ no violation!




**Still balanced!**

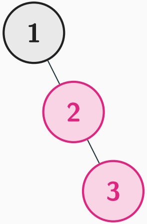
Height $= 2$ Black-height $= 1$

# Simulation — Step 3 of 5: Insert 3 (Fix needed!)

✓ ✓ 3 4 5

Inserting 3 — rotation required

**After Insert**




⚠ VIOLATION

# Simulation — Step 3 of 5: Insert 3 (Fix needed!)

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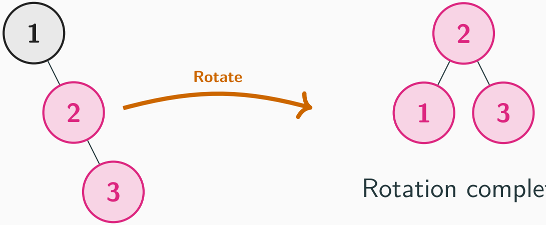
**After Insert** | **Left-Rotate at 1**




⚠ **VIOLATION**

Rotation complete

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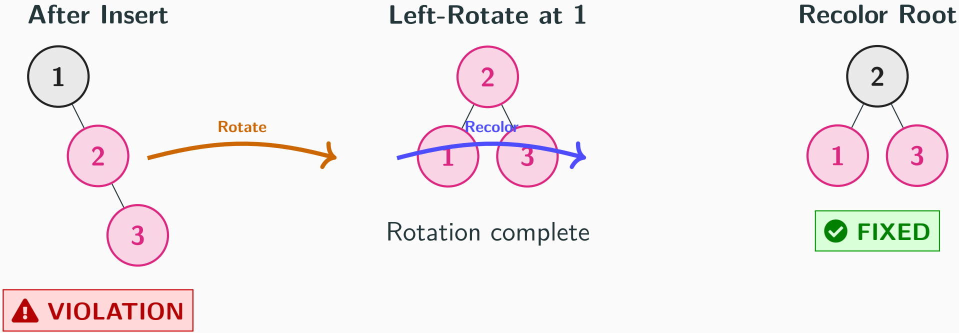


**Balanced!**

Height = 2 (BST would be 3)

# Simulation — Step 4 of 5: Insert 4

Inserting 4 — uncle recolor

- Insert as right child of 3

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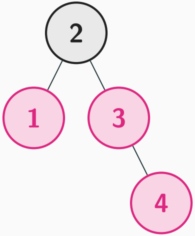
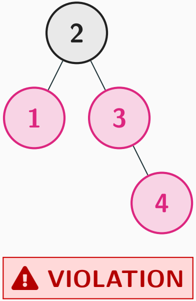


# Simulation — Step 4 of 5: Insert 4


Inserting 4 — uncle recolor

- Insert as right child of 3
- Color it **RED**
- Parent (3) is red — violation!

**After Insert**



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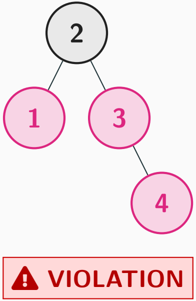

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✓ ✓ ✓ 4 5 Inserting 4 — uncle recolor

- Insert as right child of 3
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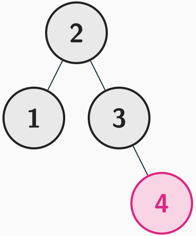
- Parent (3) is red — violation!
- Uncle (1) is also red
- Recolor: flip parent, uncle & grandparent

**After Recolor**




FIXED

**Still balanced!**

Height $= 3$ Black-height preserved

# Simulation — Step 5 of 5: Insert 5 (Final Fix!)


Inserting 5 — rotation + recolor

- Insert as right child of 4

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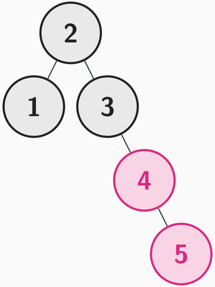

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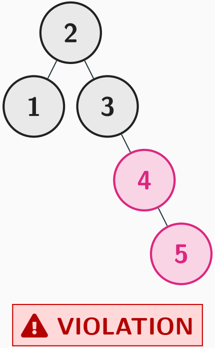
**Before Fix**



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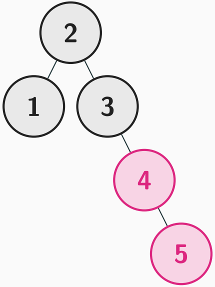


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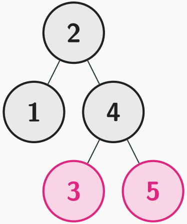
- Left-rotate at 3, then recolor

**Final Tree**

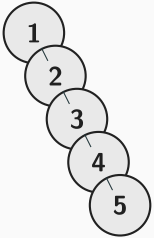



**PERFECT**

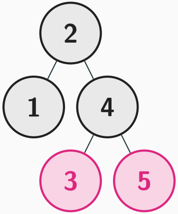
# BST vs. Red-Black Tree

Inserting $\{1, 2, 3, 4, 5\}$ in order

**Regular BST**




**Red-Black Tree**




**Bad**

Height $= 5$ ▪ Degenerate ▪ $O(n)$ ops

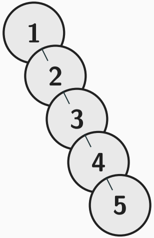
**Excellent!**

Height $= 3$ ▪ Balanced ▪ $O(\log n)$ ops

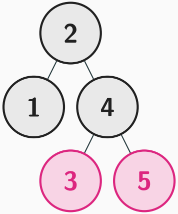
# BST vs. Red-Black Tree

Inserting $\{1, 2, 3, 4, 5\}$ in order

### Regular BST




### Red-Black Tree




**Bad**

Height $= 5$ ▪ Degenerate ▪ $O(n)$ ops

**Excellent!**

Height $= 3$ ▪ Balanced ▪ $O(\log n)$ ops

# Deletion: Even More Fun!

**Deletion is even more... interesting! ☻**

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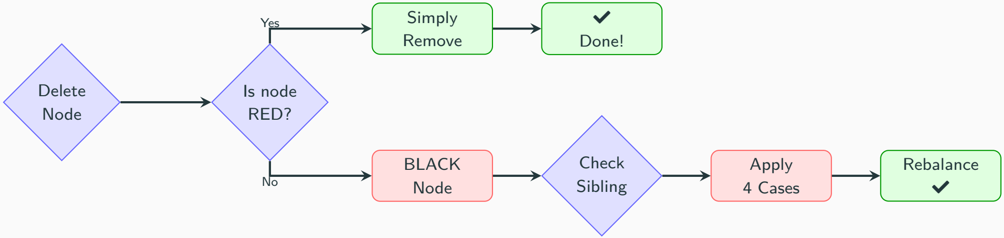
## Let's see both cases...

# Deletion Decision Flowchart

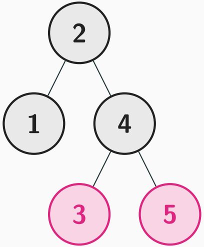



**Top path (RED)** = easy **Bottom path (BLACK)** = complex

# Case 1: Deleting a RED Node (Easy!)

Delete **5** from our tree

**Before**



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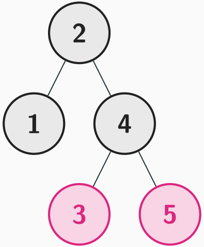
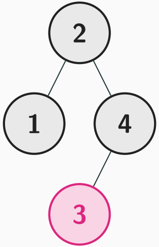



**After**



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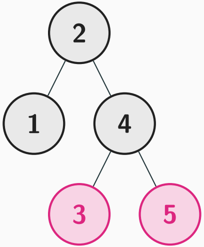


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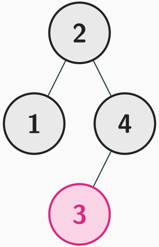


**Why It's Easy**

- Node 5 is RED and a leaf

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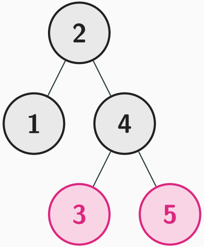


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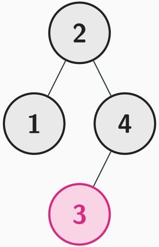


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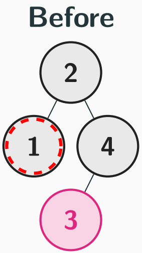
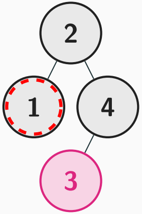
# Case 2: Deleting a BLACK Node (Uh oh...)

Delete **1** from our tree

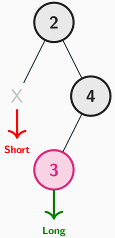


Delete **1** from our tree

**Before**




**After Delete**




✕ IMBALANCED

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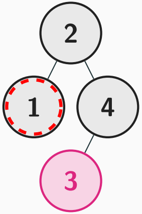
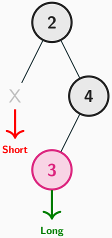
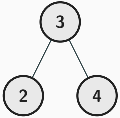



### After Delete




**✗ IMBALANCED**

### After Fix




**✓ BALANCED**

- **Rotate:** Right at 4, then 2
- **Recolor:** 3 $\longrightarrow$ Black

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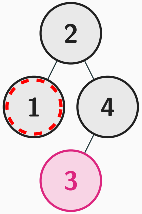


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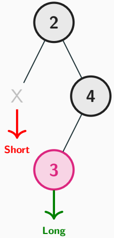
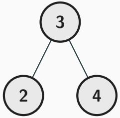



✖ **IMBALANCED**

**After Fix**




✔ **BALANCED**

- **Rotate:** Right at 4, then 2
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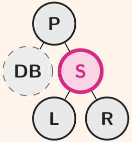
# Black Node Deletion Cases 1 & 2

**P**=Parent **S**=Sibling **L/R**=S's children DB = Double-Black node

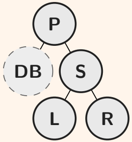
## ⇄ Case 1: Sibling is RED




**Fix:** Rotate P left, recolor S → Black, P → Red

*Converts to Case 2, 3, or 4*

## Case 2: Sibling & Children all BLACK




↑ **Fix:** Recolor S → Red,
push Double-Black up to P

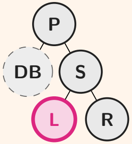
*Repeat fix from P if P was Black*

*Case 1 always leads to Case 2, 3, or 4 after rotation*

# Black Node Deletion Cases 3 & 4

**P**=Parent **S**=Sibling **L/R**=S's children DB = Double-Black node

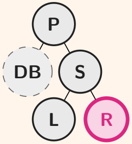
## Case 3: Sibling's Left child is RED




**Fix:** Right-rotate at S, swap colors of S & L

*Transforms into Case 4*

## Case 4: Sibling's Right child is RED

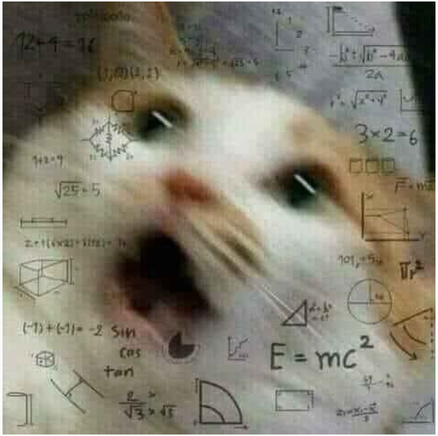



**Fix:** Left-rotate at P, recolor R $\rightarrow$ Black

**Double-Black resolved!**

**Goal:** Always eventually reach Case 4 to fully eliminate Double-Black

# Umm... What?




## What just happened?

*"Four cases?*

*Rotations?*

*Recoloring?*

*Help!"*

Don't worry — even textbooks span 20+ pages on this.

# Don’t Worry!

**We know deletion is complex — and that’s *okay*!**

## The Reality

- The deletion algorithm is **huge** with many edge cases
- Each of the 4 cases has intricate implementation details
- Full implementation can span **hundreds** of lines

**We’ll skip the gory details for now!**

## If You’re Interested...

- CLRS Chapter 13 — full pseudocode & proofs
- Online visualizer: `visualgo.net`
- GitHub implementations in your favourite language

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  TreeMap, TreeSet
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**Every time you use these...**

You're benefiting from **Red-Black Trees!**

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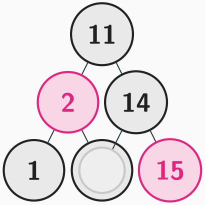

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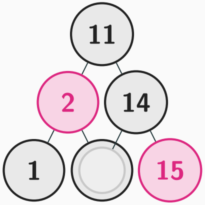


**Remember**

The next time you're struggling with RBT implementation...

Even the **inventor** moved on to Left-Leaning Red-Black Trees! ☻

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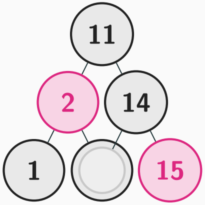


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# Any Questions?