Red-Black Trees

Why Even the Inventor Moved On...

Your Name

February 19, 2026

We All Love to Sort Things!

- Organizing bookshelves

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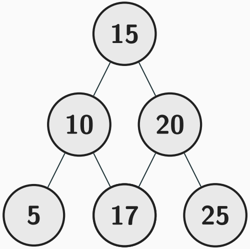
- **What's a really good way?**



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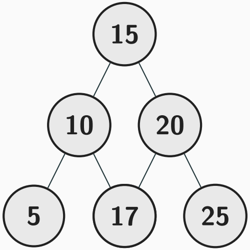
- **What's a really good way?** 🌲
- **TREES!** Specifically Binary Trees



Binary Trees are Awesome!

Why Binary Trees Are Good

Beautiful Structure

- Everything has its place
- Search: $O(\log n)$
- Insert: $O(\log n)$
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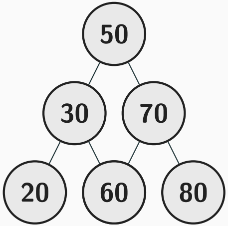
The Magic

Logarithmic time = **Sports car performance!**



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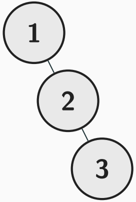

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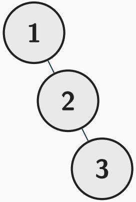

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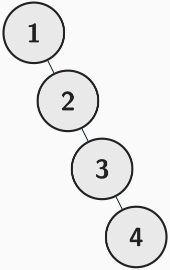

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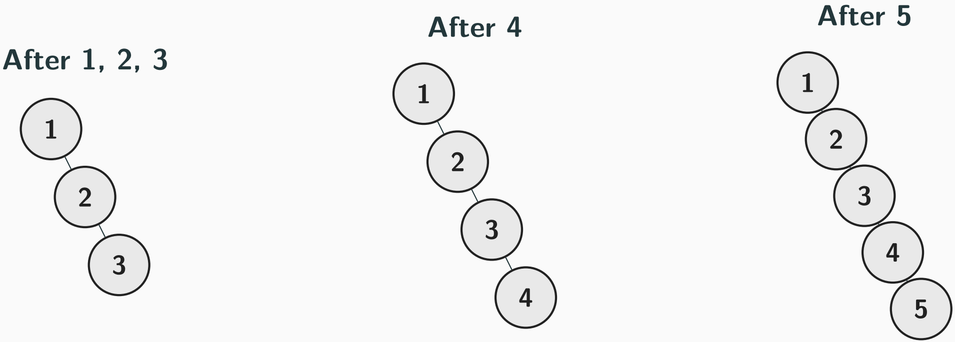

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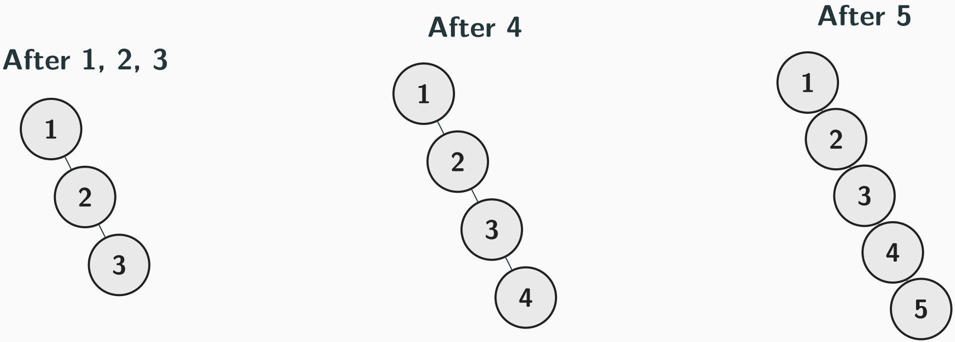

Our tree became a... LINKED LIST! Damn!

From Sports Car to Bicycle

Before (Balanced)

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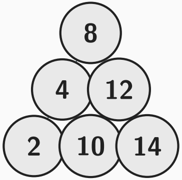

After (Degenerate)

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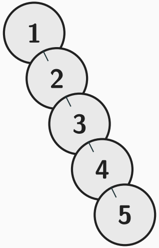

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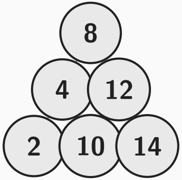

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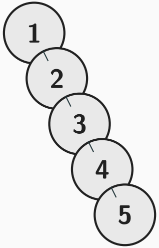

⚠ Not Great!

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We want to keep $O(\log n)$ operations **even in the worst case!**

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Red-Black Trees are used EVERYWHERE!

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- Around since the 1970s

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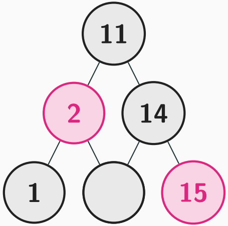

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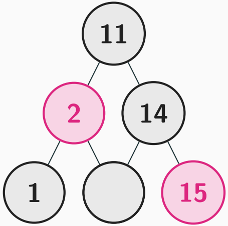

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Let's get started, shall we?

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Red-Black Trees have 5 properties

Let's see them one by one...

The Five Properties

1. Every node is either Red or Black

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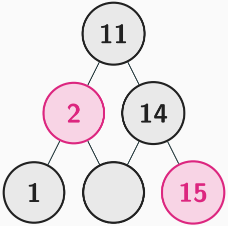

Black Height: The Core Property

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The **black-height** of a node is the number of black nodes on any path from that node to a leaf (not counting the node itself).

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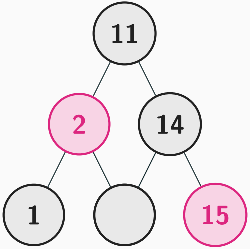

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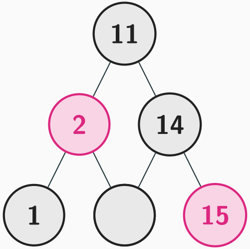

Why This Matters

This property helps us understand the **balance** of the tree!

The Amazing Height Bound

Because of black height, we can prove:

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(I skipped the detailed proof - nobody wants to sit through that!)

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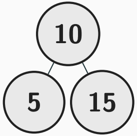
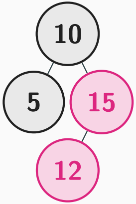


After inserting 12:



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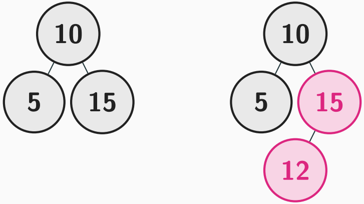

Problem!

Two adjacent red nodes — Property 4 violated!

Fixing Violations

Uncle is RED

- Easy case!
- Just recolor
- Flip parent, uncle & grandparent

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Uncle is BLACK

- Hard case!
- Need rotations
- Left rotate, right rotate
- Sometimes both!

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This is where it gets tricky!

But it keeps the tree balanced!

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Let's try inserting **1, 2, 3, 4, 5** again...

But this time in a Red-Black Tree!

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$O(n)$ operations ☹

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Let's see what happens...

Will it stay balanced?

🪄 Stay tuned!

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Watch the magic happen!

Simulation — Step 1 of 5: Insert 1

- First node is always the root

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Inserting 1

- First node is always the root
- Color it **BLACK**

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Inserting 1

- First node is always the root
- Color it **BLACK**
- Property 2: Root must be black ✔

All properties satisfied!

Height $= 1$ Black-height $= 1$

Simulation — Step 2 of 5: Insert 2

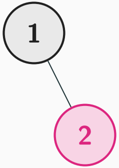
Inserting 2

- Insert as right child of 1

Simulation — Step 2 of 5: Insert 2

Inserting 2

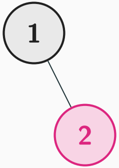
- Insert as right child of 1
- Color it **RED**



Simulation — Step 2 of 5: Insert 2

Inserting 2

- Insert as right child of 1
- Color it **RED**
- Parent is **BLACK** $\Rightarrow$ no violation!



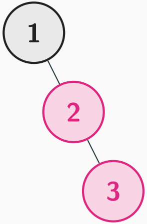
Still balanced!

Height $= 2$ Black-height $= 1$

Simulation — Step 3 of 5: Insert 3 (Fix needed!)

Inserting 3 — rotation required

After Insert



VIOLATION

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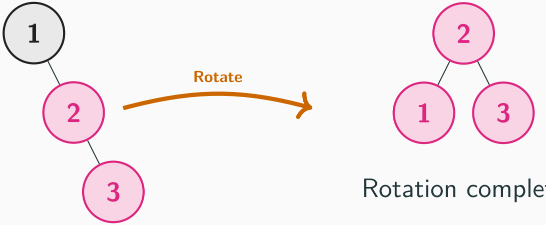
Left-Rotate at 1



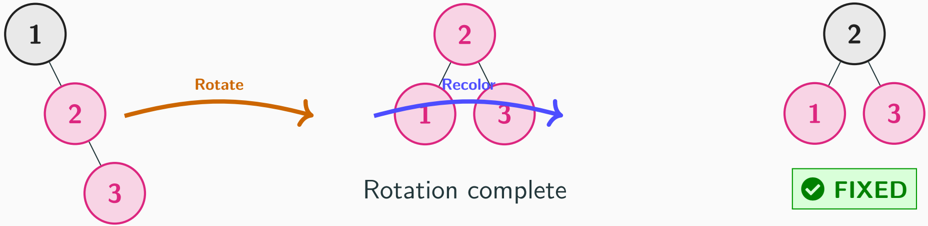
Rotation complete

⚠ **VIOLATION**

Simulation — Step 3 of 5: Insert 3 (Fix needed!)

✓ ✓ 3 4 5 Inserting 3 — rotation required

After Insert | **Left-Rotate at 1** | **Recolor Root**



VIOLATION

Rotation complete

FIXED

Balanced!

Height = 2 (BST would be 3)

Simulation — Step 4 of 5: Insert 4

Inserting 4 — uncle recolor

- Insert as right child of 3

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Inserting 4 — uncle recolor

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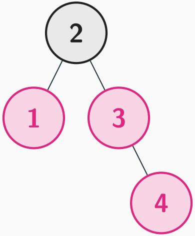
After Insert

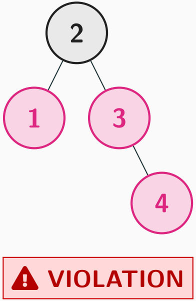


Simulation — Step 4 of 5: Insert 4

Inserting 4 — uncle recolor

- Insert as right child of 3
- Color it **RED**
- Parent (3) is red — violation!

After Insert



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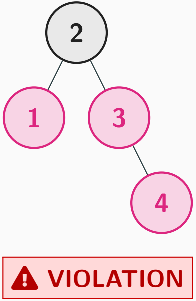
After Insert



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Inserting 4 — uncle recolor

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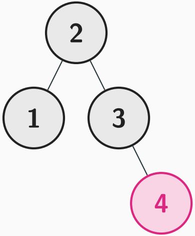
- Uncle (1) is also red
- Recolor: flip parent, uncle & grandparent

After Recolor



FIXED

Still balanced!

Height $= 3$ Black-height preserved

Simulation — Step 5 of 5: Insert 5 (Final Fix!)

Inserting 5 — rotation + recolor

- Insert as right child of 4

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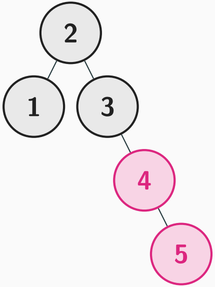

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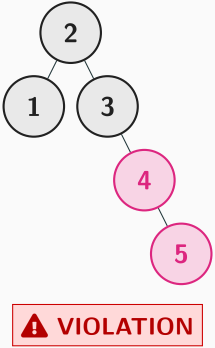
- Parent (4) is red — violation!

Before Fix



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Inserting 5 — rotation + recolor

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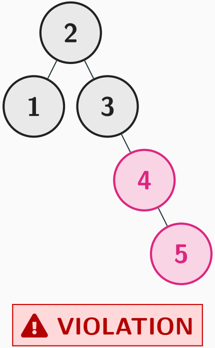
- Parent (4) is red — violation!
- Uncle is **BLACK** (NIL)

Before Fix



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- Insert as right child of 4
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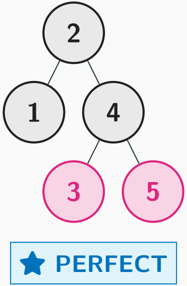
- Left-rotate at 3, then recolor

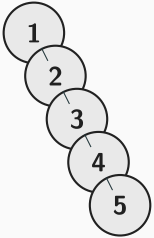
Final Tree



BST vs. Red-Black Tree

Inserting $\{1, 2, 3, 4, 5\}$ in order

Regular BST



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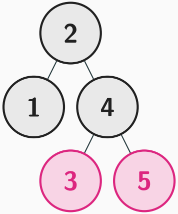


Bad

Height $= 5$ ▪ Degenerate ▪ $O(n)$ ops

Excellent!

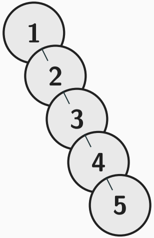
Height $= 3$ ▪ Balanced ▪ $O(\log n)$ ops

The Difference is HUGE!

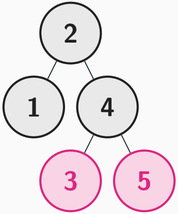
BST vs. Red-Black Tree

Inserting $\{1, 2, 3, 4, 5\}$ in order

Regular BST



Red-Black Tree



Bad

Height $= 5$ ▪ Degenerate ▪ $O(n)$ ops

Excellent!

Height $= 3$ ▪ Balanced ▪ $O(\log n)$ ops

Deletion: Even More Fun!

Deletion is even more... interesting! ☻

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- No problem!
- Just remove it
- Properties still hold

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- Complex cases

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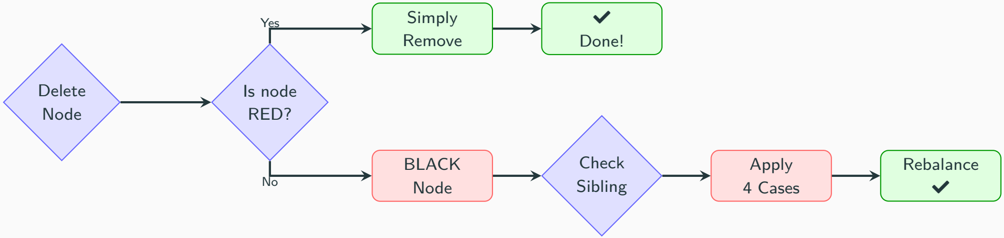
Let's see both cases...

Deletion Decision Flowchart

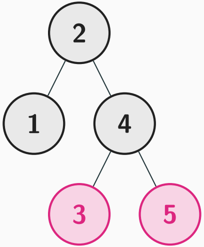


Top path (RED) = easy **Bottom path (BLACK)** = complex

Case 1: Deleting a RED Node (Easy!)

Delete **5** from our tree

Before



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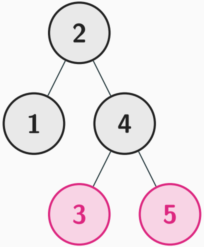
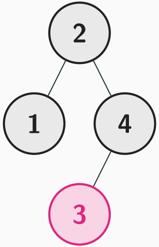


After



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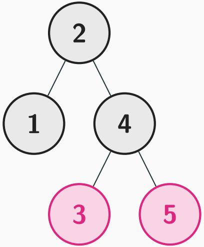

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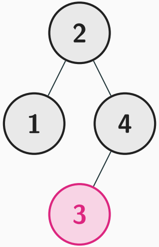

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Case 2: Deleting a BLACK Node (Uh oh...)

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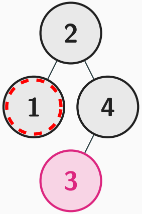

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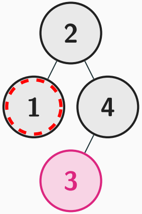
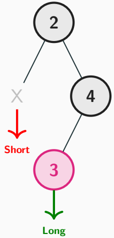


After Delete



✗ IMBALANCED

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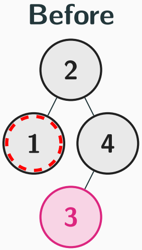

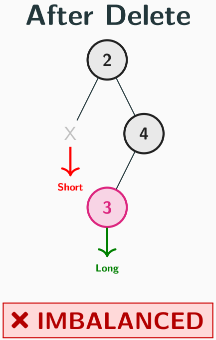

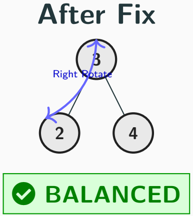

- **Rotate:** Right at 4, then 2
- **Recolor:** 3 → Black

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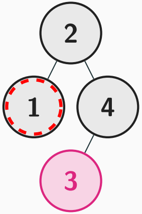
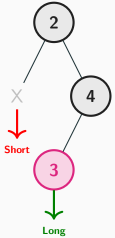
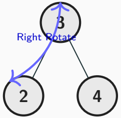


After Delete



✖ IMBALANCED

After Fix



✔ BALANCED

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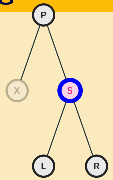
Black Node Deletion: The 4 Cases

Case 1: Sibling RED



Rotate & Recolor

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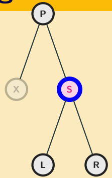
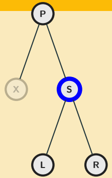


Rotate & Recolor

Case 2: All BLACK



Recolor S to RED

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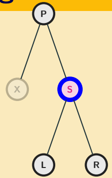

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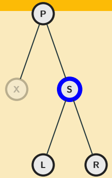
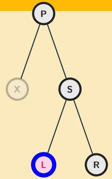


Recolor S to RED

Case 3: Left RED



Right-rotate at S

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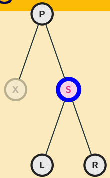
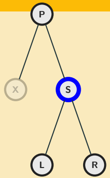


Rotate & Recolor

Case 2: All BLACK



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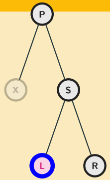
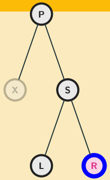


Right-rotate at S

Case 4: Right RED



Left-rotate at P

Black Node Deletion: The 4 Cases

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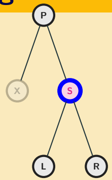
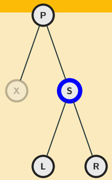


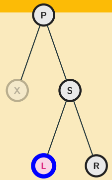
Rotate & Recolor

Case 2: All BLACK



Recolor S to RED

Case 3: Left RED



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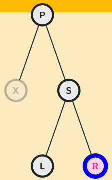

Left-rotate at P

Goal: Move RED to short path or recolor

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Everywhere!

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Every time you use these...

You're benefiting from **Red-Black Trees!**

So There You Have It!

Red-Black Trees in a Nutshell

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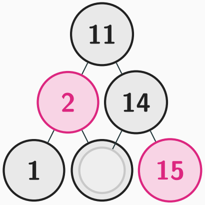

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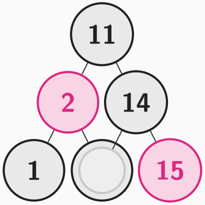

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The next time you're struggling with RBT implementation...

Even the **inventor** moved on to Left-Leaning Red-Black Trees! ☻

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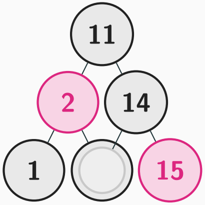

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Any Questions?