Red-Black Trees

Why Even the Inventor Moved On...

Your Name

February 8, 2026

We All Love to Sort Things!

- Organizing bookshelves

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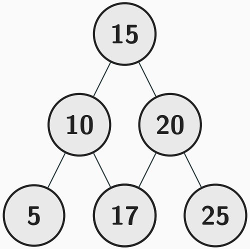

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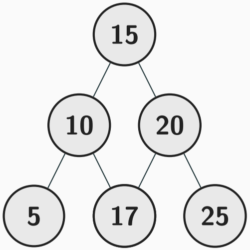

Binary Trees are Awesome!

Why Binary Trees Are Good

Beautiful Structure

- Everything has its place
- Search: $O(\log n)$
- Insert: $O(\log n)$
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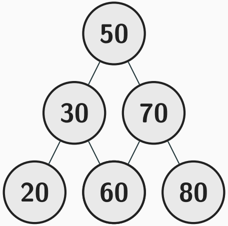
The Magic

Logarithmic time = **Sports car performance!**



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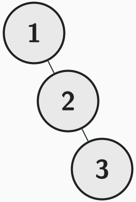

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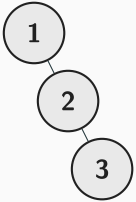

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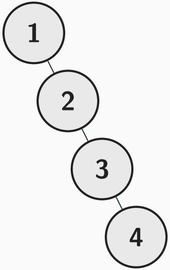

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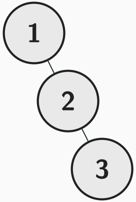

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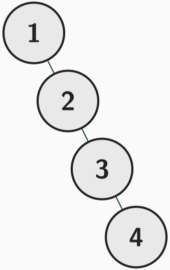

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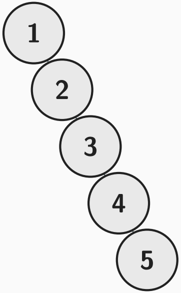

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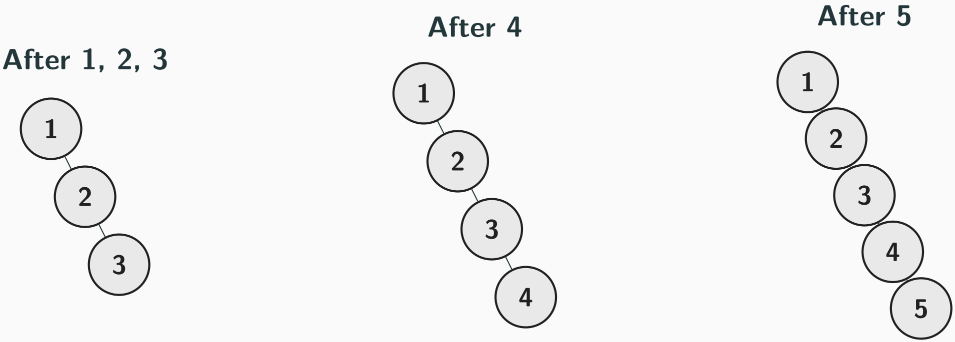

Our tree became a... LINKED LIST!

From Sports Car to Bicycle

Before (Balanced)

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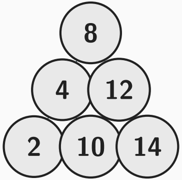

After (Degenerate)

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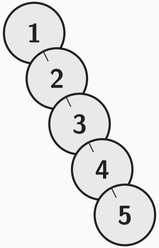

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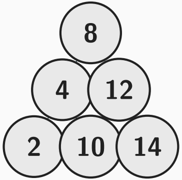

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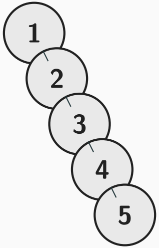

⚠ Not Great!

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Red-Black Trees are used EVERYWHERE!

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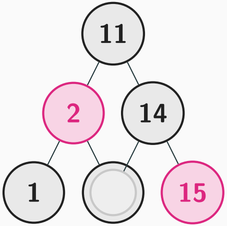

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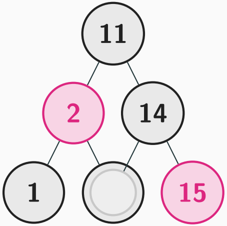

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Let's get started, shall we?

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Red-Black Trees have 5 properties

Let's see them one by one...

The Five Properties

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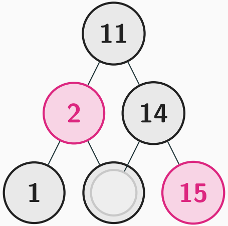

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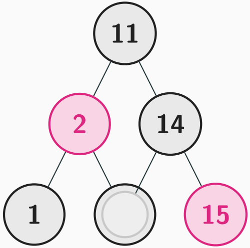

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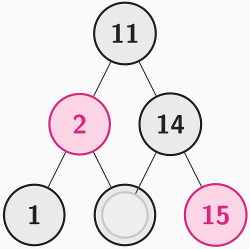

Why This Matters

This property helps us understand the **balance** of the tree!

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(I skipped the detailed proof - nobody wants to sit through that!)

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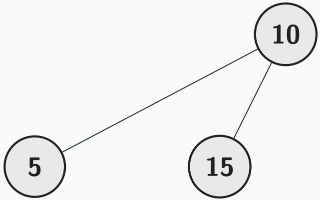
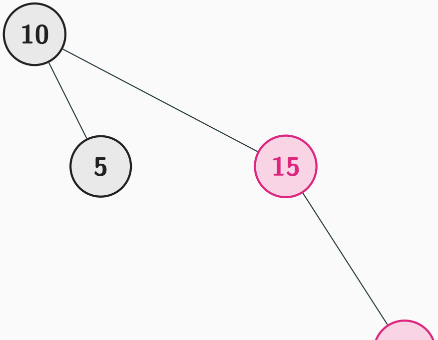


After inserting 12:



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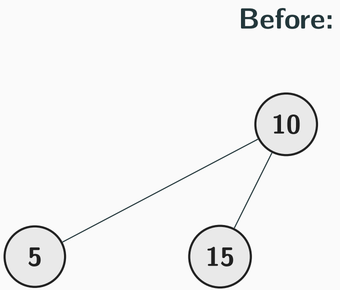

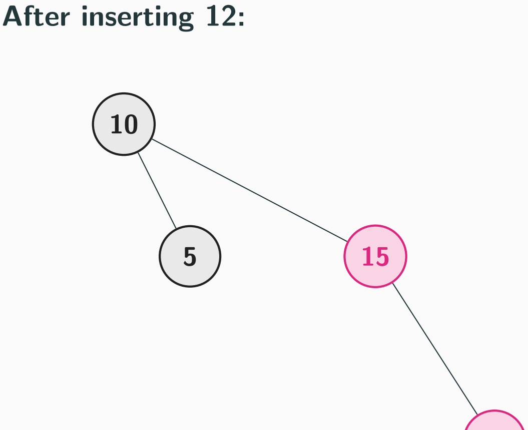

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This is where it gets tricky!

But it keeps the tree balanced!

Deletion: Even More Fun!

Deletion is even more... interesting! ☻

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The key: Maintain black height at all costs!

(We'll skip the gory details - you get the idea!)

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Every time you use these...

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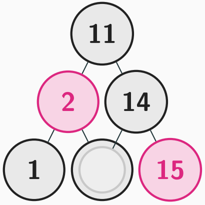

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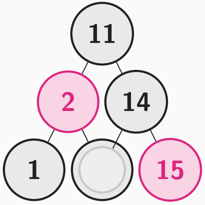

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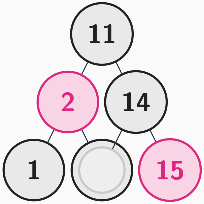

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Any Questions?