

CSE-200 Final Presentation

Red Black Tree

Muhasina Ashraf - 2205002

Rifah Tasnia Islam - 2205008

Sufian Ashraf - 2205019

February 19, 2026

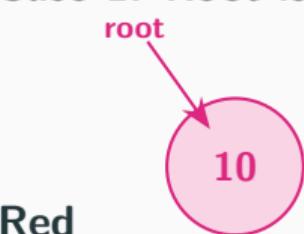
Department of CSE

Bangladesh University of Engineering and Technology

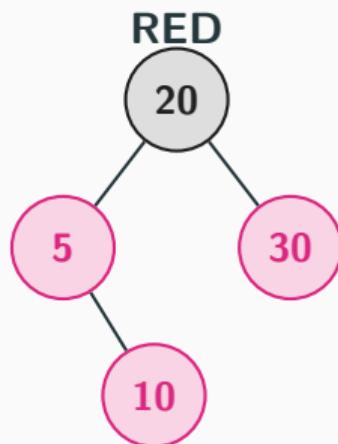
Insertion Violation Cases

Four main violation cases in Red-Black Tree insertion:

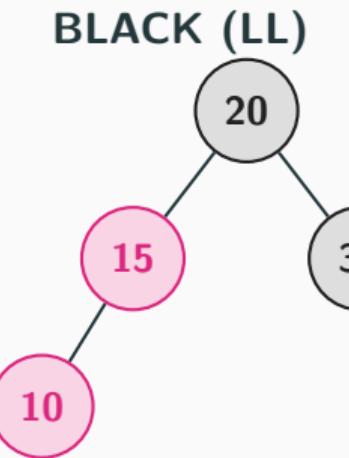
Case 1: Root is



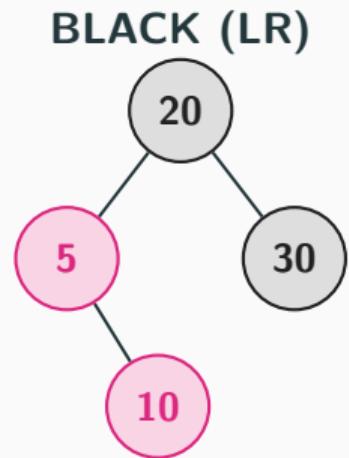
Case 2: Uncle is



Case 3: Uncle is



Case 4: Uncle is



We Need to Store and Search Data

- Everything is **tree-structured**

We Need to Store and Search Data

- Everything is **tree-structured**
- **Insert** data into the structure

We Need to Store and Search Data

- Everything is **tree-structured**
- **Insert** data into the structure
- **Delete** data efficiently

We Need to Store and Search Data

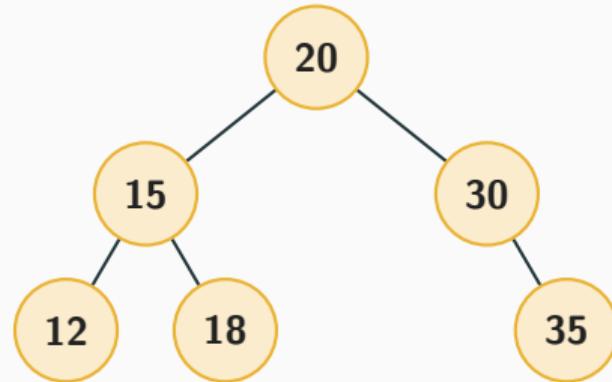
- Everything is **tree-structured**
- **Insert** data into the structure
- **Delete** data efficiently
- **Search** for data quickly

Good way to do all of this?

Use a BST!

The BST Rule

How does BST decide where to put a node?



The BST Rule

How does BST decide where to put a node?

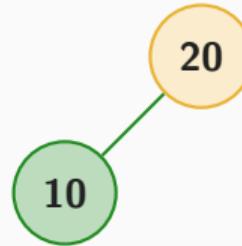


- Smaller than me? Go **Left**

The BST Rule

How does BST decide where to put a node?

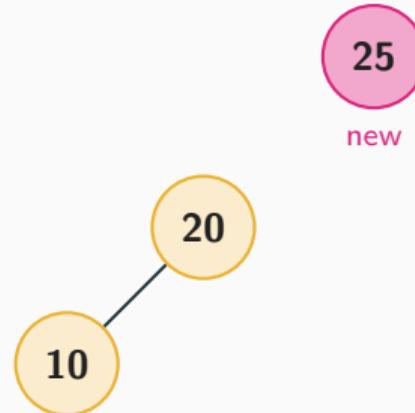
- Smaller than me? Go **Left**



The BST Rule

How does BST decide where to put a node?

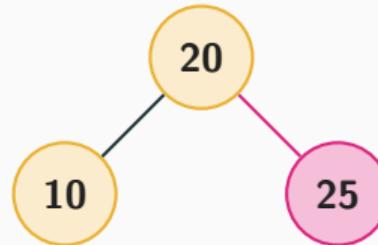
- Smaller than me? Go **Left**
- Larger than me? Go **Right**



The BST Rule

How does BST decide where to put a node?

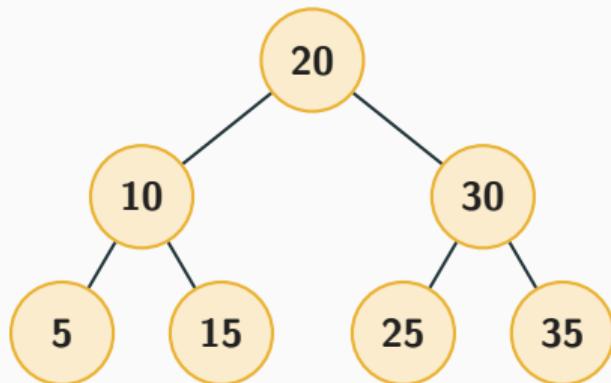
- Smaller than me? Go **Left**
- Larger than me? Go **Right**



The BST Rule

How does BST decide where to put a node?

- Smaller than me? Go **Left**
- Larger than me? Go **Right**



Good technique!

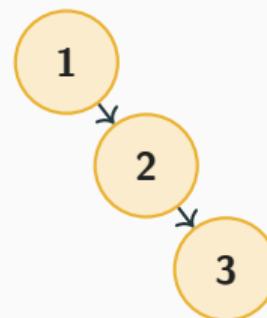
Insert the roll numbers in a class sequentially

1, 2, 3, 4 ...10

Insert the roll numbers in a class sequentially

1, 2, 3, 4 ...10

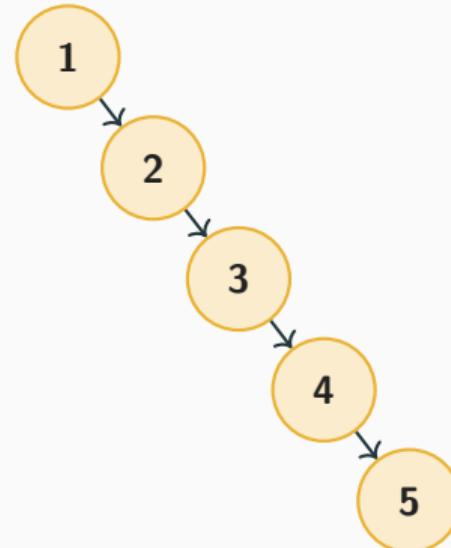
- Each goes to the **right** of the last



Insert the roll numbers in a class sequentially

1, 2, 3, 4 ...10

- Each goes to the **right** of the last
- The tree just keeps **growing** right...

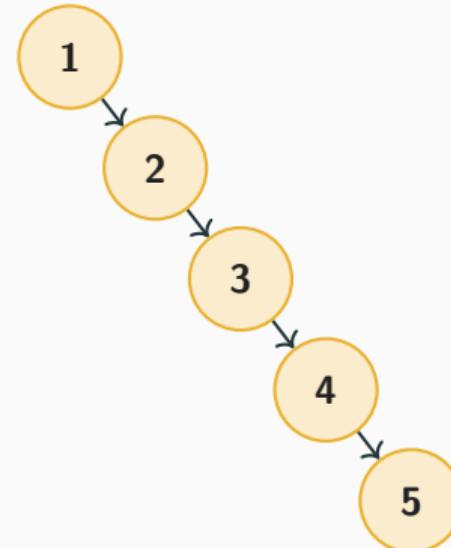


...and on

Insert the roll numbers in a class sequentially

1, 2, 3, 4 ...10

- Each goes to the **right** of the last
- The tree just keeps **growing** right...

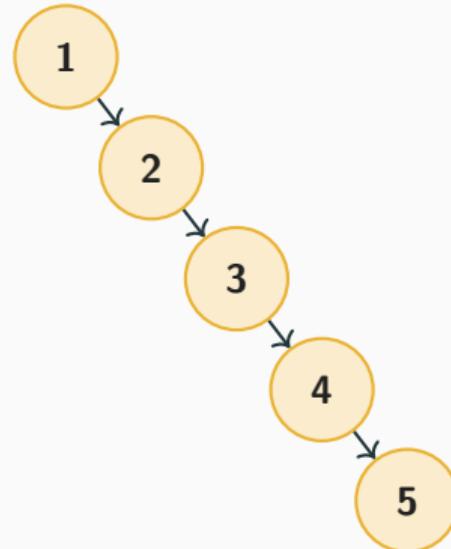


Still works!

...and on

But, What's the Problem?

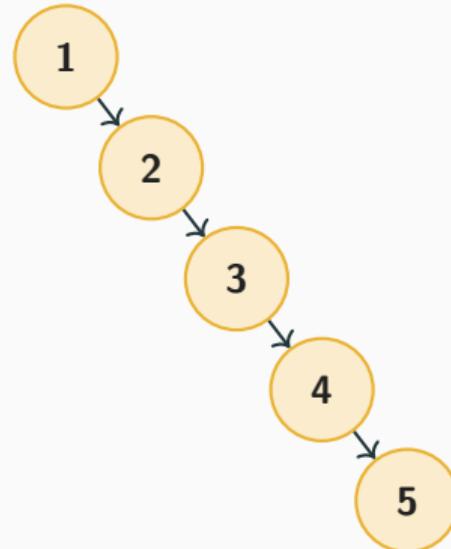
- Height becomes n



...and on

But, What's the Problem?

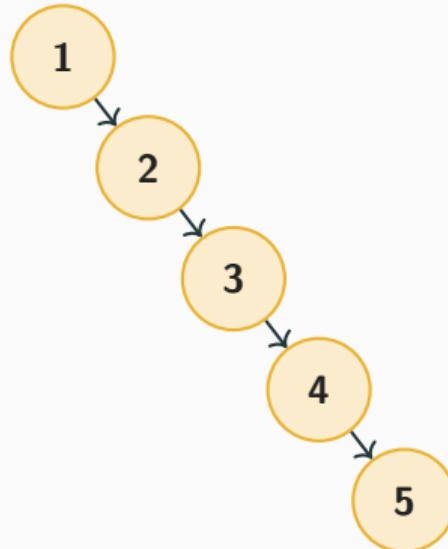
- Height becomes n
- Insertion takes $O(n)$



...and on

But, What's the Problem?

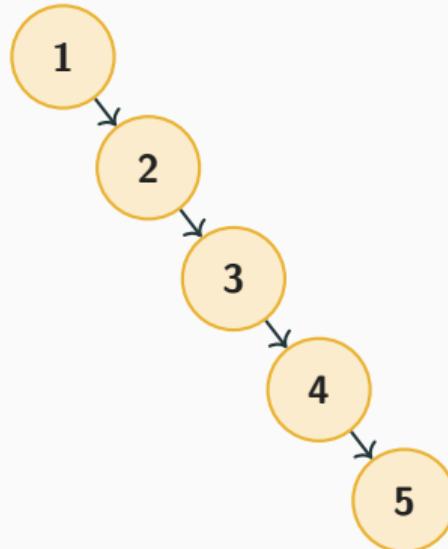
- Height becomes n
- Insertion takes $O(n)$
- Deletion takes $O(n)$



...and on

But, What's the Problem?

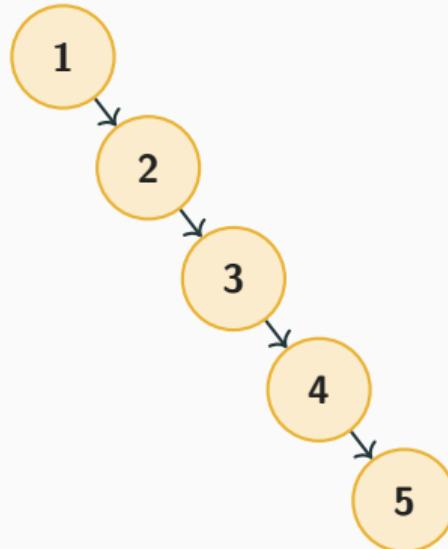
- Height becomes n
- Insertion takes $O(n)$
- Deletion takes $O(n)$
- Search takes $O(n)$



...and on

But, What's the Problem?

- Height becomes n
- Insertion takes $O(n)$
- Deletion takes $O(n)$
- Search takes $O(n)$
- A linked list in disguise



...and on

But, What's the Problem?

- Insertion takes $O(n)$
- Deletion takes $O(n)$
- Search takes $O(n)$
- A linked list in disguise

Time complexity becomes $O(n)$

The Solution?

Use a BST that **promises** to keep its height **logarithmic**
no matter how and what element you insert.

The Solution?

Examples of Self-Balancing Trees:

- AVL Tree
- **Red-Black Tree**
- Splay Tree
- B-Tree

Let's look at **Red-Black** Trees



What is Red-Black Tree

A Red-Black Tree rebalances itself by coloring nodes **red** and **black**, ensuring no two **red** nodes are **adjacent** and all **paths** have the same **black-height**, which keeps its height **logarithmic**.

What is Red-Black Tree

A Red-Black Tree rebalances itself by coloring nodes **red** and **black**, ensuring no two **red** nodes are **adjacent** and all **paths** have the same **black-height**, which keeps its height **logarithmic**.

Height becomes $\log(n)$ here!

How does RBT do it:Properties

Five points to remember

How does RBT do it:Properties

- **Property 1:** Every node is either red or black

How does RBT do it:Properties

- **Property 1:** Every node is either red or black

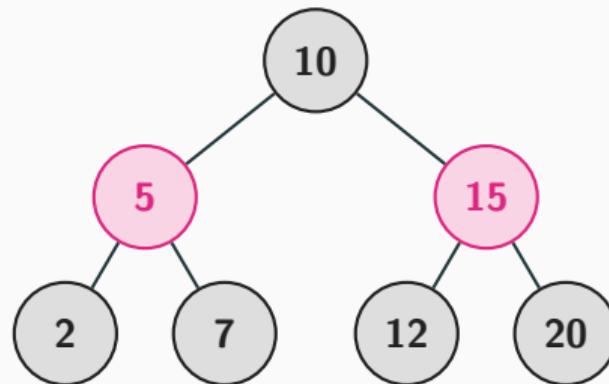
Hence, the name Red Black Tree

How does RBT do it:Properties

- **Property 2:** Root will always be a black node

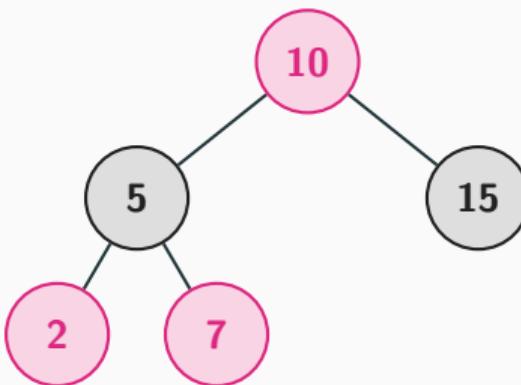
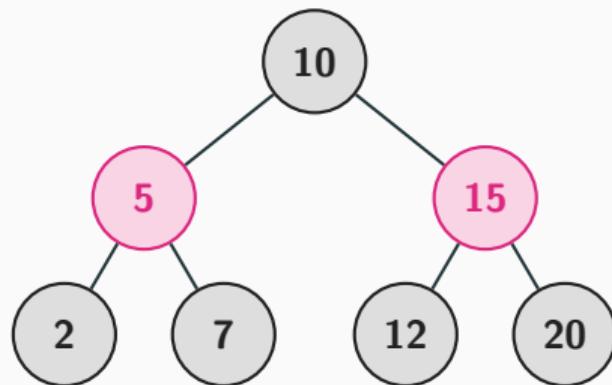
How does RBT do it:Properties

- **Property 2:** Root will always be a black node



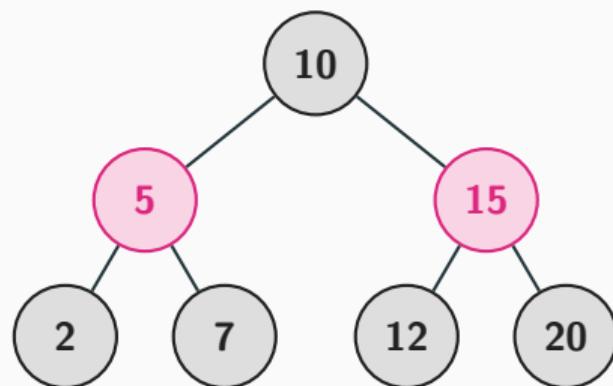
How does RBT do it:Properties

- **Property 2:** Root will always be a black node

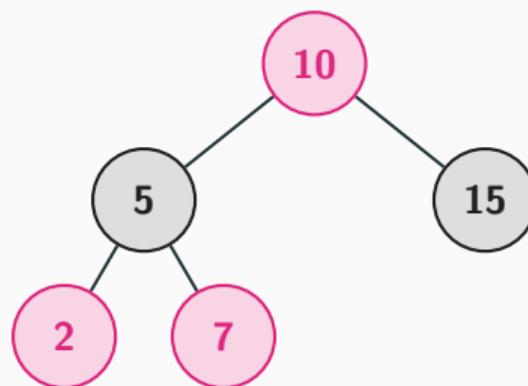


How does RBT do it:Properties

- **Property 2:** Root will always be a black node

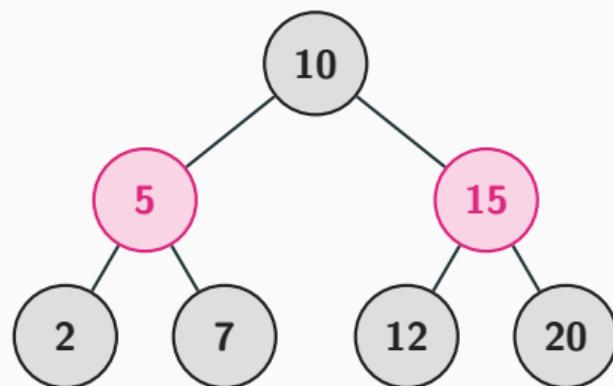


Correct

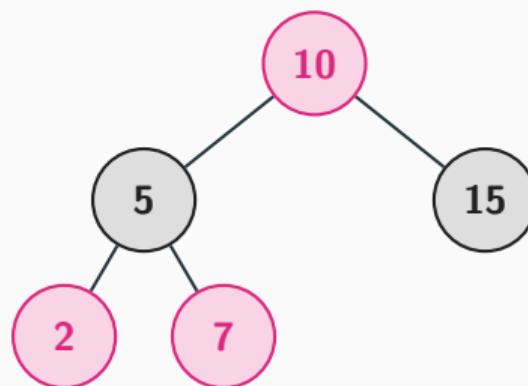


How does RBT do it:Properties

- **Property 2:** Root will always be a black node



Correct



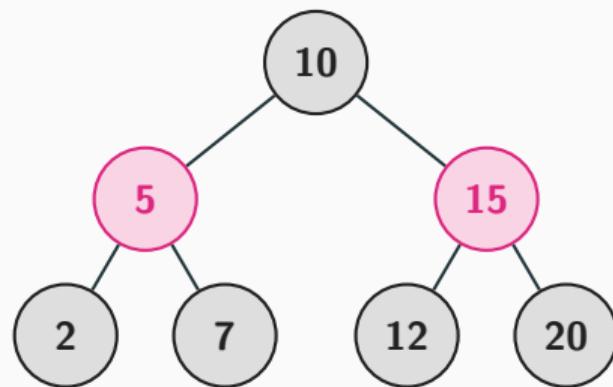
Incorrect

How does RBT do it:Properties

- **Property 3:** Leaves will either be black or NIL

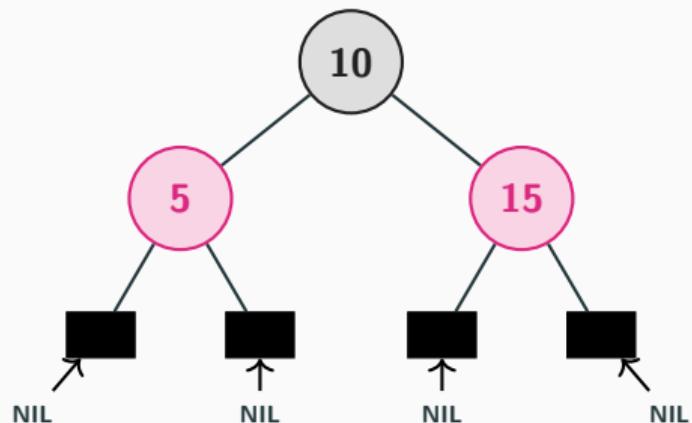
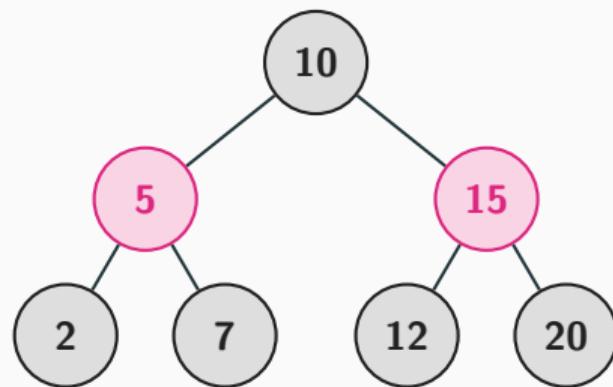
How does RBT do it:Properties

- **Property 3:** Leaves will either be black or NIL



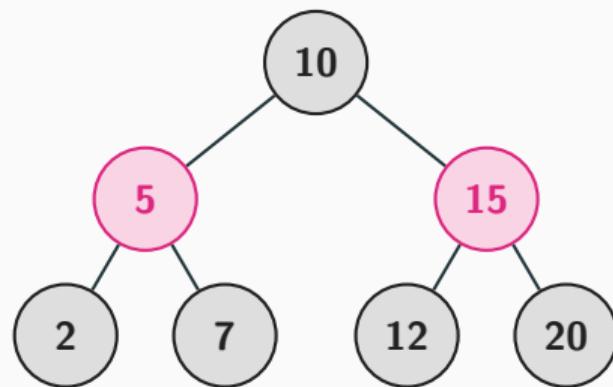
How does RBT do it:Properties

- **Property 3:** Leaves will either be black or NIL

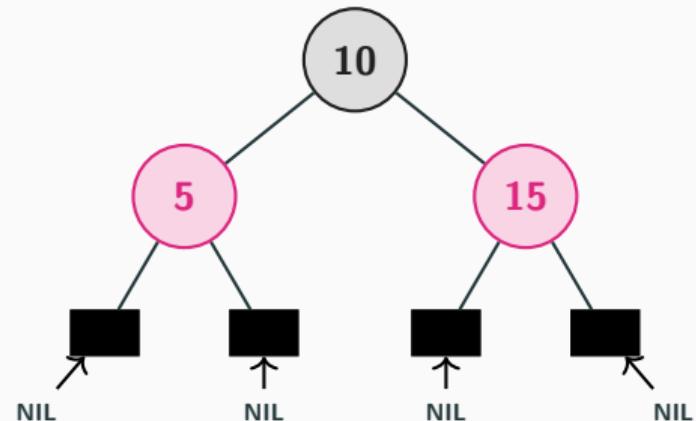


How does RBT do it:Properties

- **Property 3:** Leaves will either be black or NIL

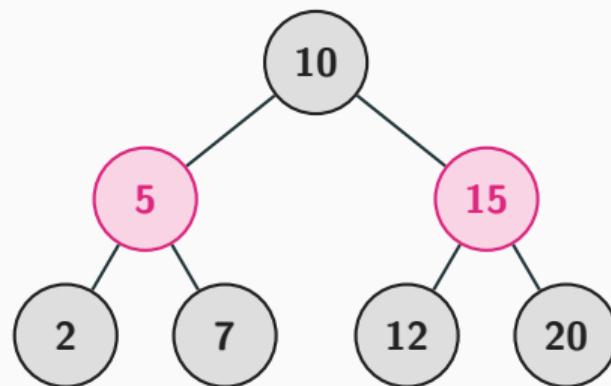


Black Leaves

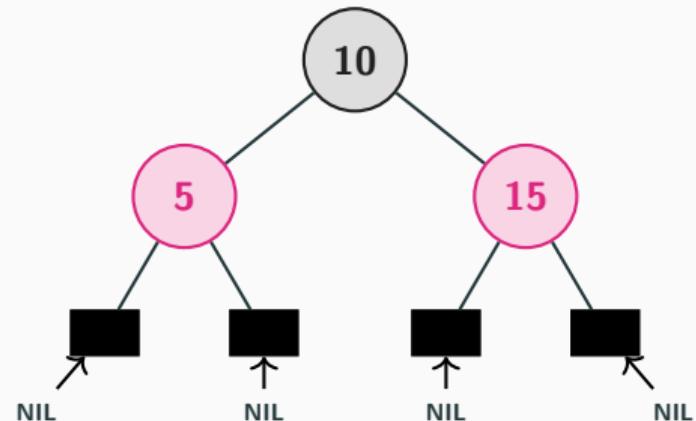


How does RBT do it:Properties

- **Property 3:** Leaves will either be black or NIL



Black Leaves



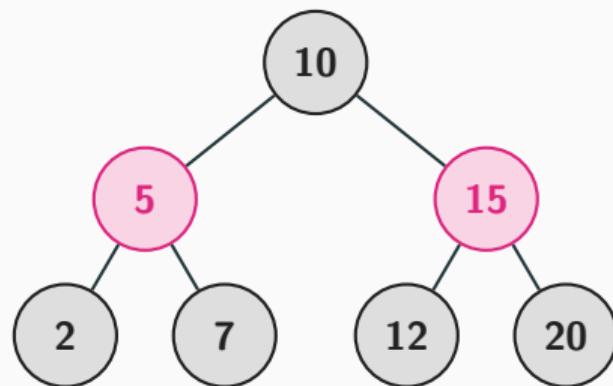
NIL nodes (counted as Black)

How does RBT do it:Properties

- **Property 4:** There will be no two consecutive red nodes

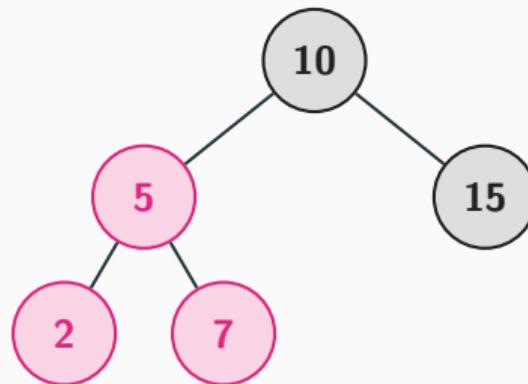
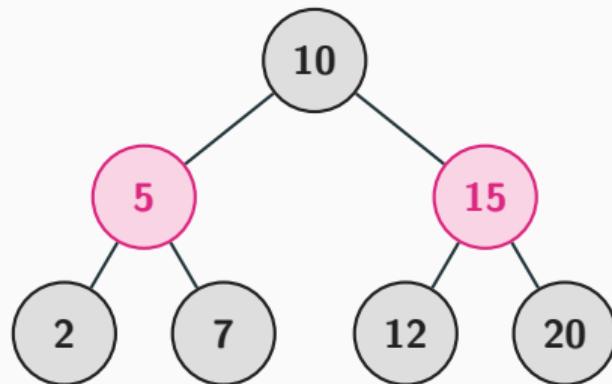
How does RBT do it:Properties

- **Property 4:** There will be no two consecutive red nodes



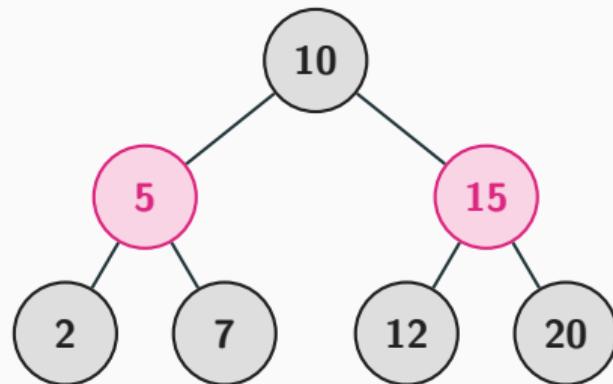
How does RBT do it:Properties

- **Property 4:** There will be no two consecutive red nodes

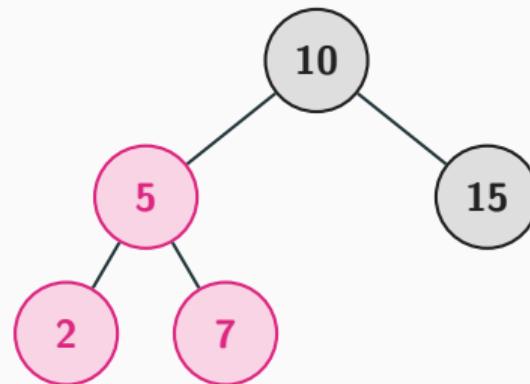


How does RBT do it:Properties

- **Property 4:** There will be no two consecutive red nodes

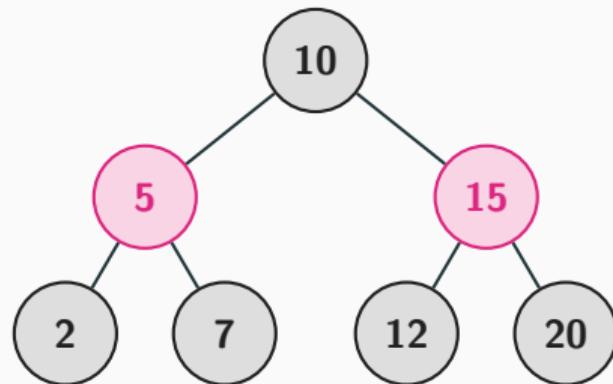


Correct

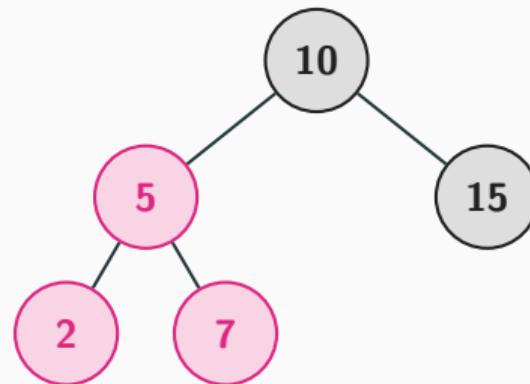


How does RBT do it:Properties

- **Property 4:** There will be no two consecutive red nodes



Correct



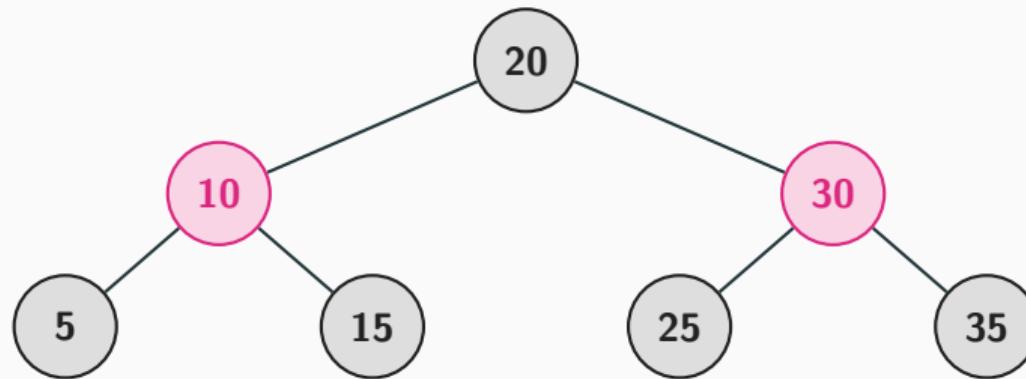
Incorrect

How does RBT do it:Properties

- **Property 5:** From a given node, the number of black nodes in any given path will always be same for that node

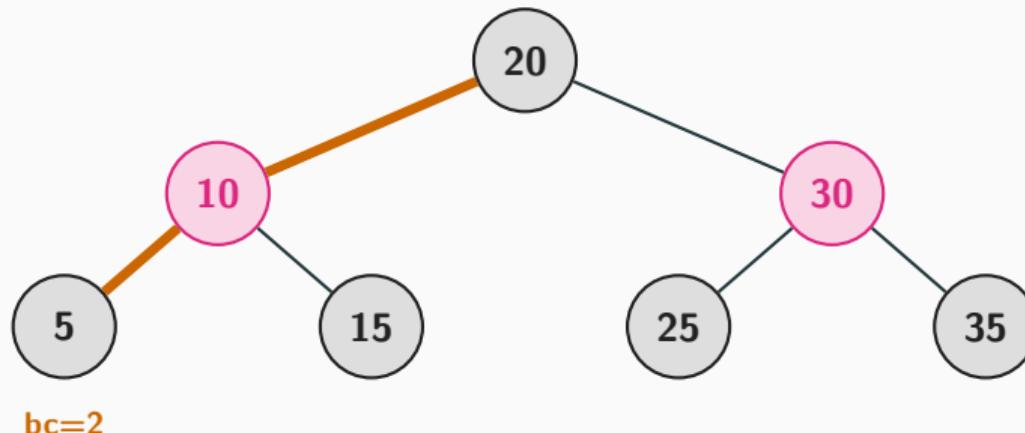
How does RBT do it:Properties

- **Property 5:** From a given node, the number of black nodes in any given path will always be same for that node



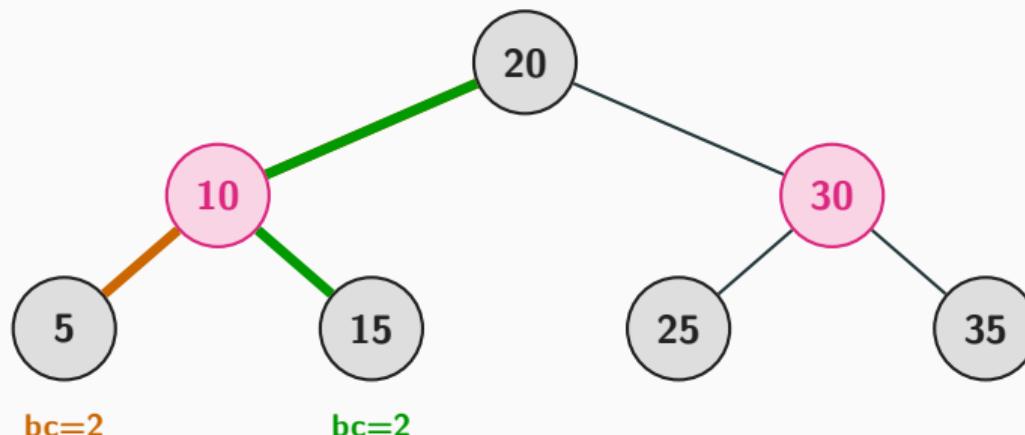
How does RBT do it:Properties

- **Property 5:** From a given node, the number of black nodes in any given path will always be same for that node



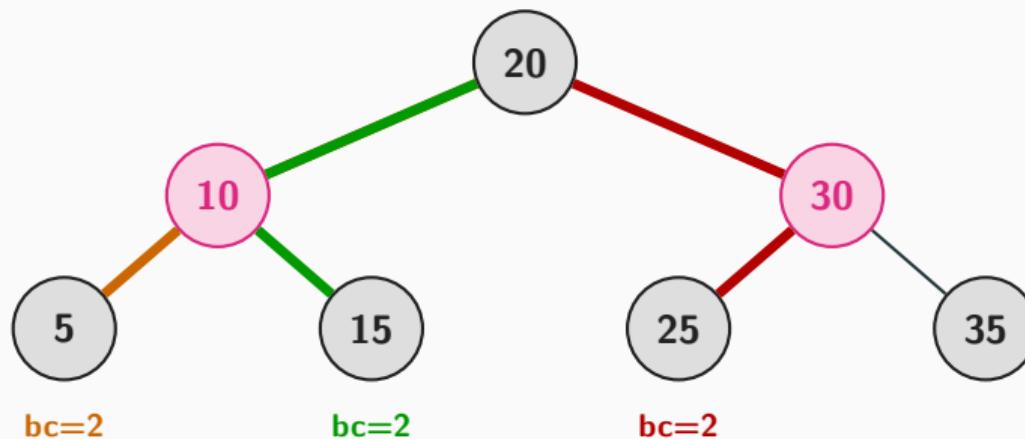
How does RBT do it:Properties

- **Property 5:** From a given node, the number of black nodes in any given path will always be same for that node



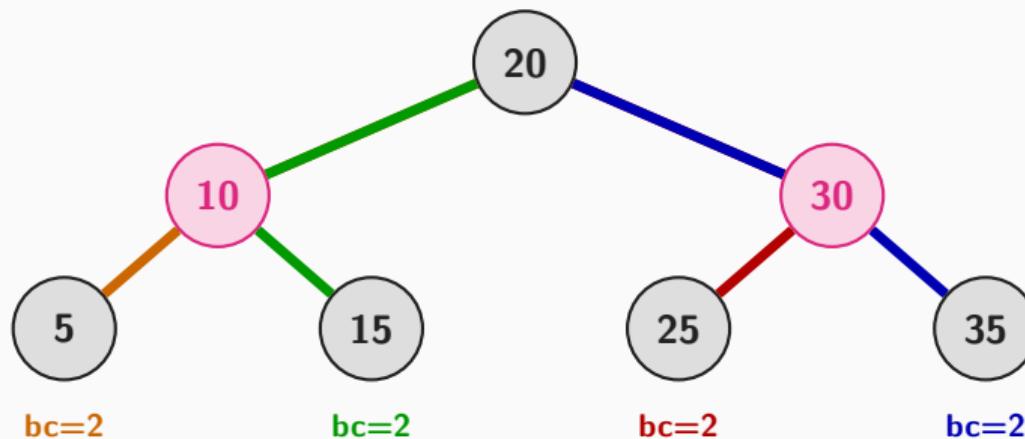
How does RBT do it:Properties

- **Property 5:** From a given node, the number of black nodes in any given path will always be same for that node



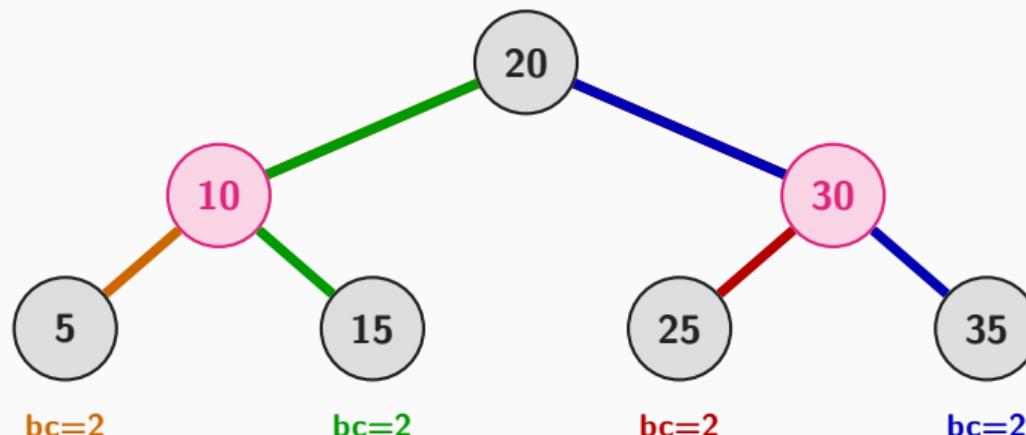
How does RBT do it:Properties

- **Property 5:** From a given node, the number of black nodes in any given path will always be same for that node



How does RBT do it:Properties

- **Property 5:** From a given node, the number of black nodes in any given path will always be same for that node



All paths from root have same black count = 2

RBT Operations

Now, How do these points ensure the "rebalancing" feature of Red Black Tree?

Now, How do these points ensure the "rebalancing" feature of Red Black Tree?

Let's see some operations....

Insertion

Insert node x in a Red Black Tree

Insertion

Insert node x in a Red Black Tree

Pseudocode

```
color[x] = RED  
  
y = root[T]  
  
while y ≠ NIL do  
    if key[x] > key[y]  
        y = right[y]  
    else  
        y = left[y]
```

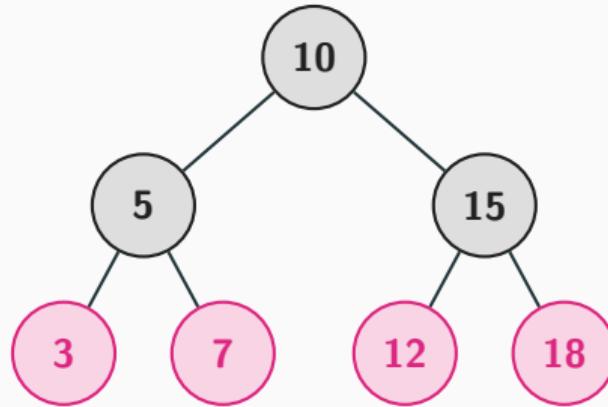
Insertion

Insert node x in a Red Black Tree

Pseudocode

```
color[x] = RED  
  
y = root[T]  
  
while y ≠ NIL do  
    if key[x] > key[y]  
        y = right[y]  
    else  
        y = left[y]
```

Existing RBT

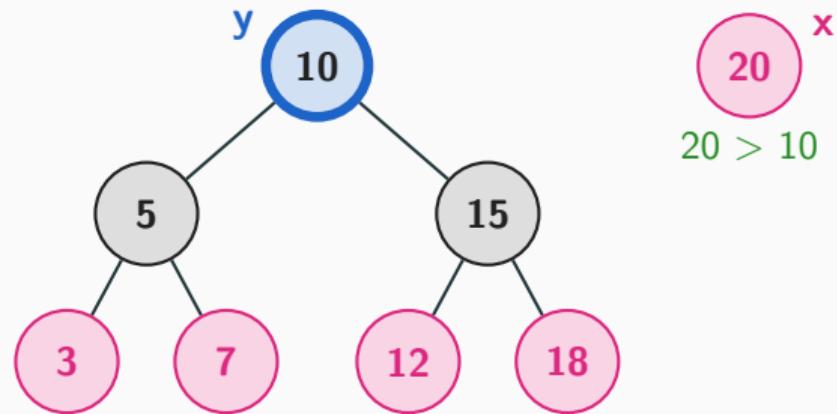


Insertion

Insert node x in a Red Black Tree

Pseudocode

```
color[x] = RED  
  
y = root[T]  
  
while y ≠ NIL do  
    if key[x] > key[y]  
        y = right[y]  
    else  
        y = left[y]
```

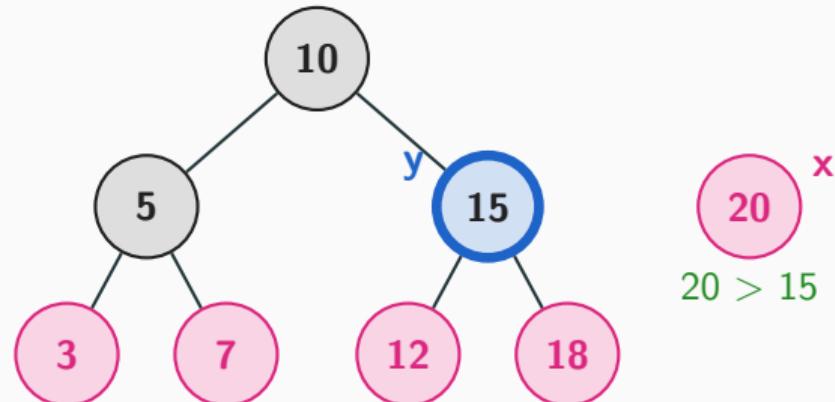


Insertion

Insert node x in a Red Black Tree

Pseudocode

```
color[x] = RED  
  
y = root[T]  
  
while y ≠ NIL do  
    if key[x] > key[y]  
        y = right[y]  
    else  
        y = left[y]
```

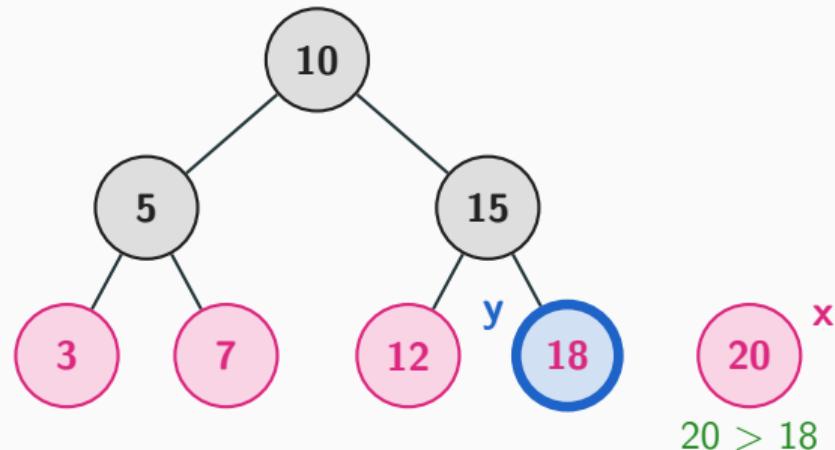


Insertion

Insert node x in a Red Black Tree

Pseudocode

```
color[x] = RED  
  
y = root[T]  
  
while y ≠ NIL do  
    if key[x] > key[y]  
        y = right[y]  
    else  
        y = left[y]
```

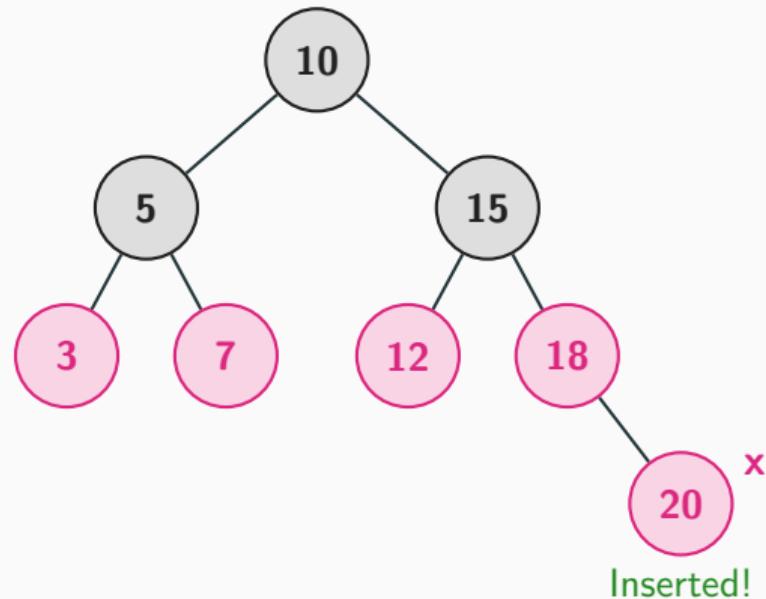


Insertion

Insert node x in a Red Black Tree

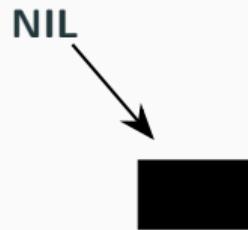
Pseudocode

```
color[x] = RED  
  
y = root[T]  
  
while y ≠ NIL do  
    if key[x] > key[y]  
        y = right[y]  
    else  
        y = left[y]
```



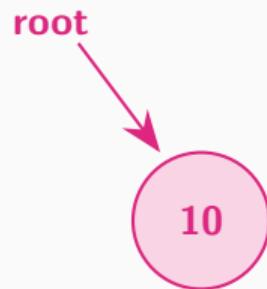
What can go wrong

Insert 10



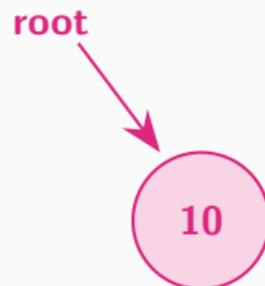
What can go wrong

Insert 10



What can go wrong

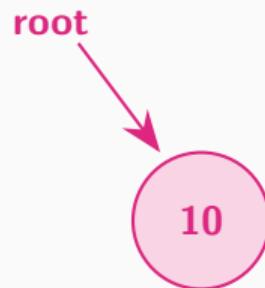
Insert 10



Root can't be RED

What can go wrong

Insert 10

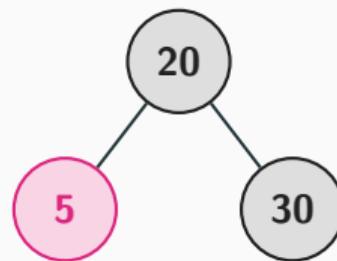
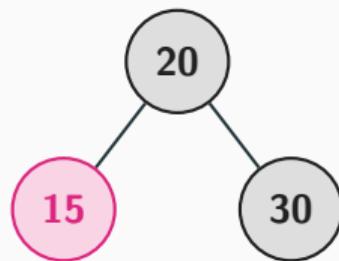
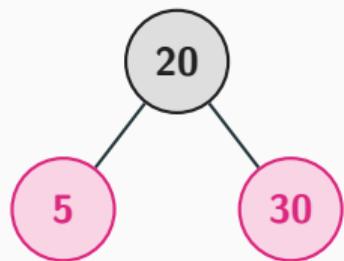


Root can't be RED

Case 1

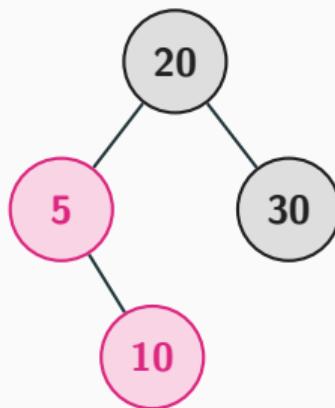
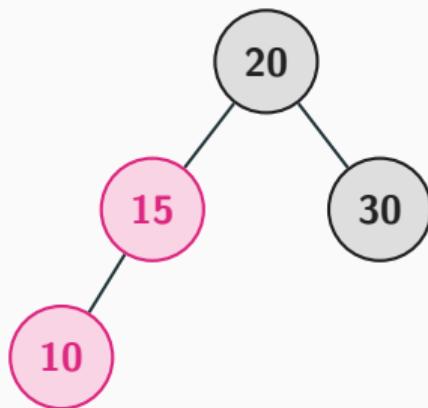
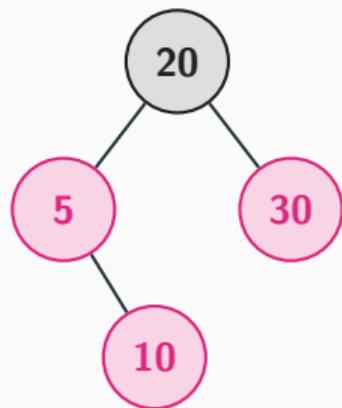
What can go wrong

Insert 10



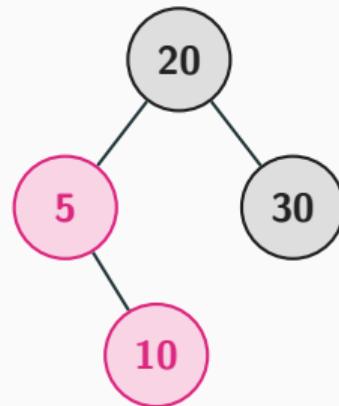
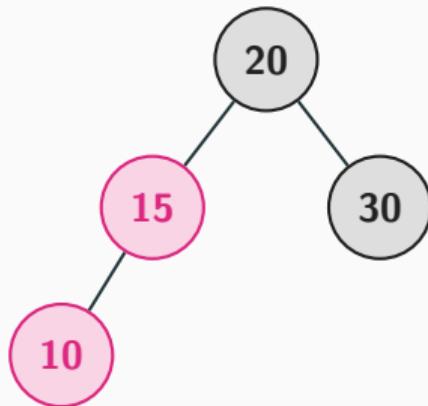
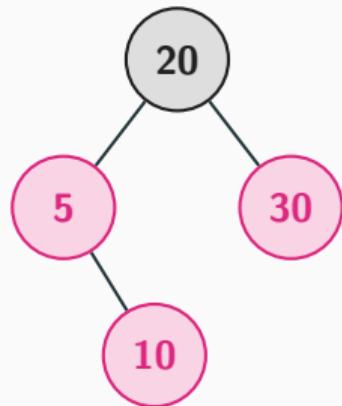
What can go wrong

Insert 10



What can go wrong

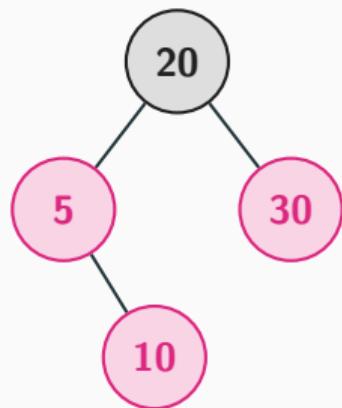
Insert 10



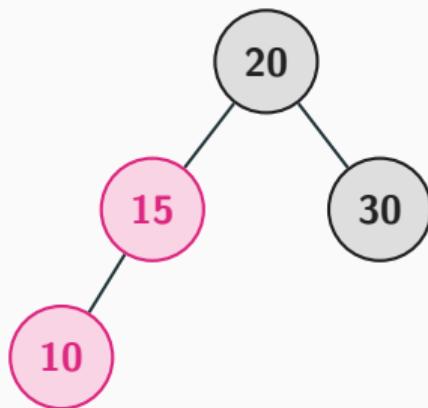
Case 2

What can go wrong

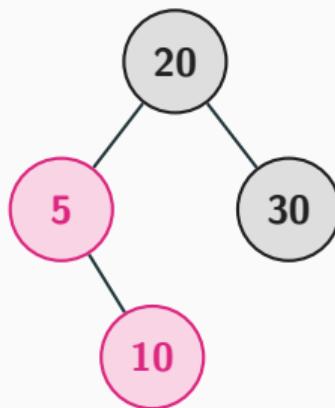
Insert 10



Case 2

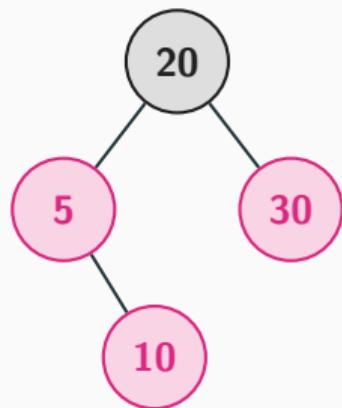


Case 3

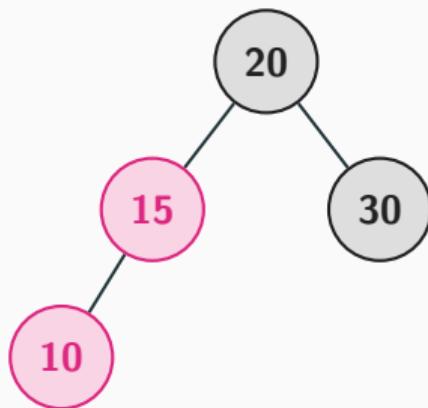


What can go wrong

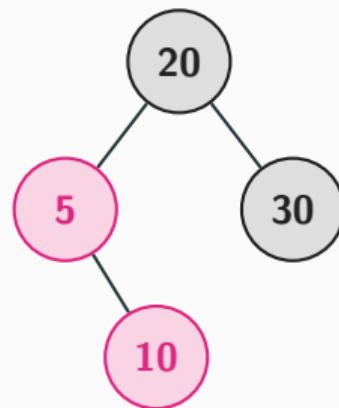
Insert 10



Case 2



Case 3

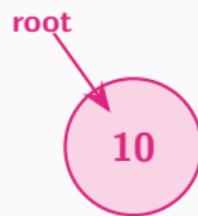


Case 4

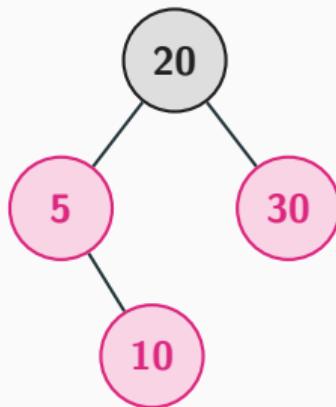
Insertion Violation Cases

Four main violation cases in Red-Black Tree insertion:

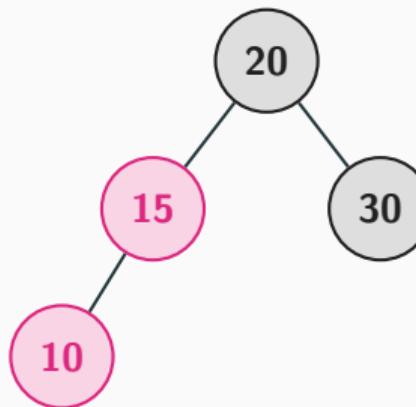
Case 1



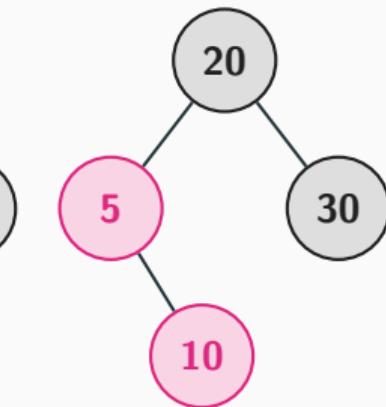
Case 2



Case 3



Case 4



How to solve this?

Solutions to Violations

Case 1



Violation

Root is **RED** — Property 2 broken



Fix

Recolor root to **BLACK**

Solutions to Violations

Case 1

Violation

Root is **RED** — Property 2 broken

Fix

Recolor root to **BLACK**

Case 2

Violation

Uncle is **RED** — two reds adjacent

Fix

Recolor parent, uncle **BLACK**;
grandparent **RED**

Solutions to Violations

Case 1

⚠️ Violation

Root is **RED** — Property 2 broken

🔧 Fix

Recolor root to **BLACK**

Case 2

⚠️ Violation

Uncle is **RED** — two reds adjacent

🔧 Fix

Recolor parent, uncle **BLACK**;
grandparent **RED**

Case 3

⚠️ Violation

Uncle is **BLACK** — Right-Right

🔧 Fix

Left rotate at grandparent, then
recolor

Solutions to Violations

Case 1

⚠️ Violation

Root is **RED** — Property 2 broken

🔧 Fix

Recolor root to **BLACK**

Case 2

⚠️ Violation

Uncle is **RED** — two reds adjacent

🔧 Fix

Recolor parent, uncle **BLACK**;
grandparent **RED**

Case 3

⚠️ Violation

Uncle is **BLACK** — Right-Right

🔧 Fix

Left rotate at grandparent, then
recolor

Case 4

⚠️ Violation

Uncle is **BLACK** — Left-Right

🔧 Fix

Left rotate at parent, then apply Case
3

Remember This Problem?

Let's try inserting **1, 2, 3, 4, 5** again...

But this time in a **Red-Black Tree!**

Remember This Problem?

Let's try inserting **1, 2, 3, 4, 5** again...

But this time in a **Red-Black Tree!**

Watch the magic happen!

Insert 1

- First node is always **root**
- Insert as **RED** (default color)

Insert 1

- First node is always **root**
- Insert as **RED** (default color)

After Insert



Insert 1

- First node is always **root**
- Insert as **RED** (default color)
- But Root cannot be **RED!**

Property 2 violated - Case : 1

Violation — Root is RED!



Insert 1

- First node is always **root**
 - Insert as **RED** (default color)
 - But Root cannot be **RED!**
- Property 2 violated - Case : 1**
- Recolor root to **BLACK**
 - **Fixed!**

After Recolor



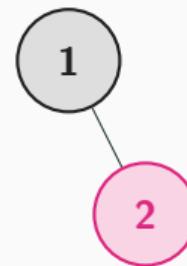
Insert 2

- Right child of 1
- Insert as **RED** (default color)

Insert 2

- Right child of 1
- Insert as **RED** (default color)
- Parent is BLACK — **no violation ✓**

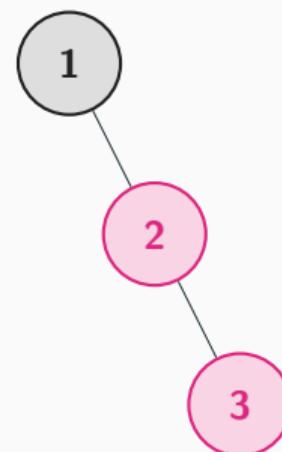
After Insert



Insert 3

- Right child of 2
- Insert as **RED** (default)

After Insert

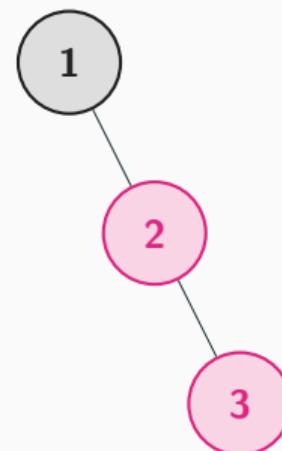


Insert 3

- Right child of 2
- Insert as **RED** (default)
- Uncle is **NIL/BLACK**
- **Case: 3**

Left rotate at node 1

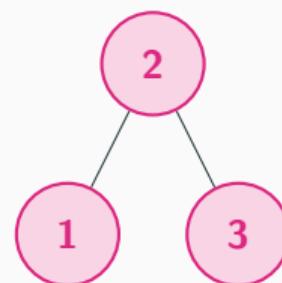
Violation Found — Two RED in a row!



Insert 3

- Right child of 2
- Insert as **RED** (default)
- Uncle is **NIL/BLACK**
- **Case: 3**
Left rotate at node 1
- Recolor: 2 → **BLACK**, children
→ **RED**

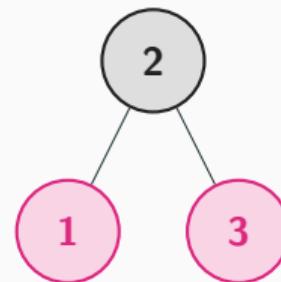
After Left Rotation



Insert 3

- Right child of 2
- Insert as **RED** (default)
- Uncle is **NIL/BLACK**
- **Case: 3**
 - Left rotate at node 1
- Recolor: 2 → **BLACK**, children → **RED**
- **Fixed!**

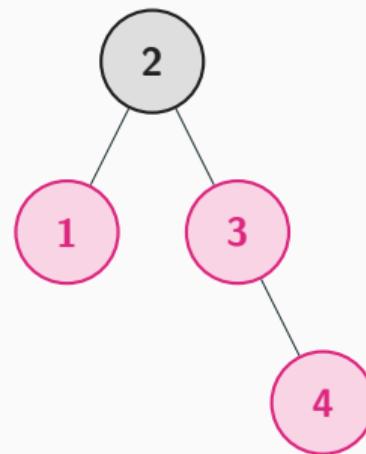
After Recolor



Insert 4

- Right child of 3
- Insert as **RED** (default)

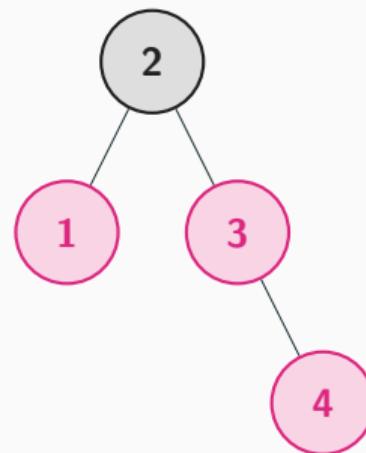
After Insert



Insert 4

- Right child of 3
- Insert as **RED** (default)
- Uncle (node 1) is **RED**
- **CASE 2**
Uncle is RED — just recolor!

Violation Found — Uncle is RED



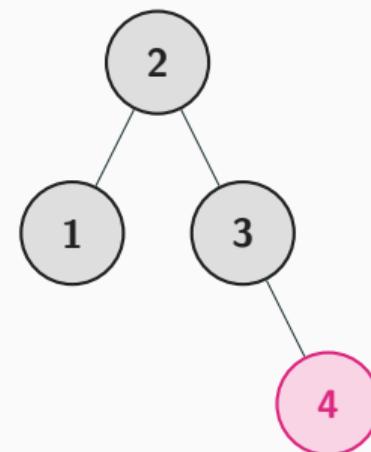
Insert 4

- Right child of 3
- Insert as **RED** (default)
- Uncle (node 1) is **RED**
- **CASE 2**
 - Uncle is RED — just recolor!
- Recolor: parent & uncle →
BLACK
- Grandparent stays **BLACK**

Insert 4

- Right child of 3
- Insert as **RED** (default)
- Uncle (node 1) is **RED**
- **CASE 2**
 - Uncle is RED — just recolor!
- Recolor: parent & uncle → **BLACK**
- Grandparent stays **BLACK**
- **Fixed!**

After Recolor



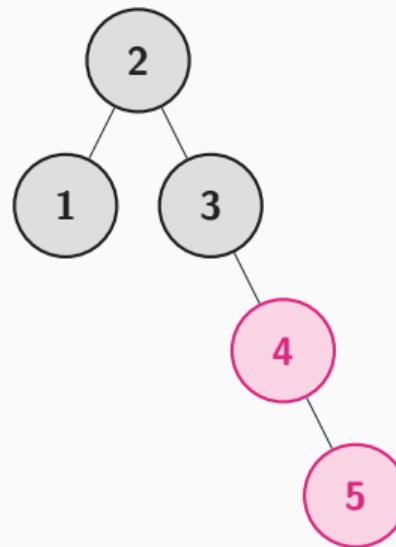
Insert 5

- Right child of 4
- Insert as **RED** (default)

Insert 5

- Right child of 4
- Insert as **RED** (default)

After Insert

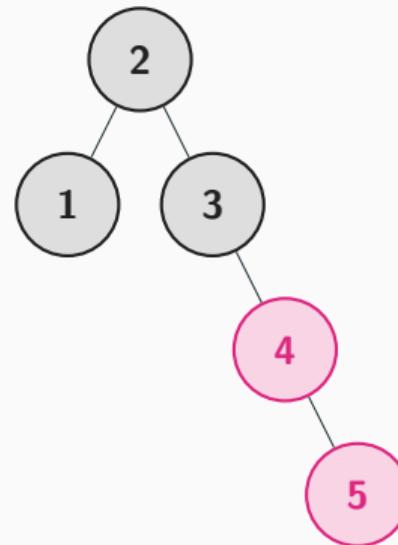


Insert 5

- Right child of 4
- Insert as **RED** (default)
- Uncle (node 1) is **BLACK**
- **CASE 3**

Uncle is **BLACK** — rotate & recolor!

Violation — Two RED in a row!



Insert 5

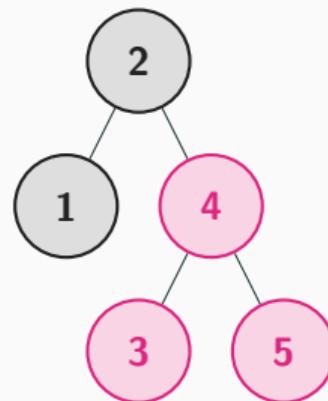
- Right child of 4
- Insert as **RED** (default)
- Uncle (node 1) is **BLACK**

CASE 3

Uncle is **BLACK** — rotate & recolor!

- Left rotate at node 3 → 4 moves up

After Left Rotation



Insert 5

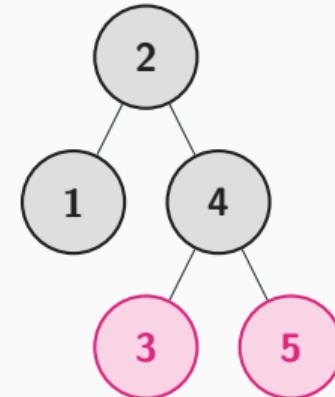
- Right child of 4
- Insert as **RED** (default)
- Uncle (node 1) is **BLACK**

CASE 3

Uncle is **BLACK** — rotate & recolor!

- Left rotate at node 3 → 4 moves up
- Recolor: 4 → **BLACK**, children → **RED**
- **We're done!**

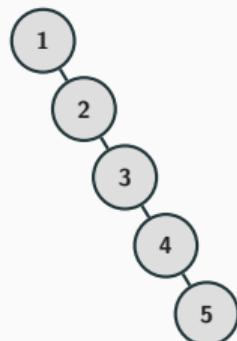
After Recolor



BST vs. Red-Black Tree

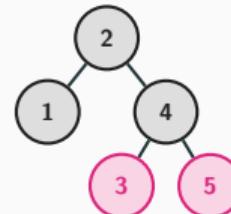
Inserting $\{1, 2, 3, 4, 5\}$ in order

Regular BST



Height = 5 • $O(n)$

Red-Black Tree



Height = 3 • $O(\log n)$

Insertion is the easy half

Now, What happens when we delete a node?

Deletion

Deletion is even more... interesting!

Deletion is even more... interesting!

Deleting a **RED** node

Deleting a **BLACK** node

Deletion is even more... interesting!

Deleting a **RED** node

Deleting a **BLACK** node

- No problem!

Deletion is even more... interesting!

Deleting a **RED** node

- No problem!
- Just remove it

Deleting a **BLACK** node

Deletion is even more... interesting!

Deleting a **RED** node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a **BLACK** node

Deletion is even more... interesting!

Deleting a **RED** node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a **BLACK** node

- Oh boy...

Deletion is even more... interesting!

Deleting a **RED** node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a **BLACK** node

- Oh boy...
- Black height changes!

Deletion is even more... interesting!

Deleting a **RED** node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a **BLACK** node

- Oh boy...
- Black height changes!
- **Need “double black” fix**

Deletion is even more... interesting!

Deleting a **RED** node

- No problem!
- Just remove it
- **Properties still hold**

Deleting a **BLACK** node

- Oh boy...
- Black height changes!
- **Need “double black” fix**
- Complex cases

Deletion is even more... interesting!

Deleting a **RED** node

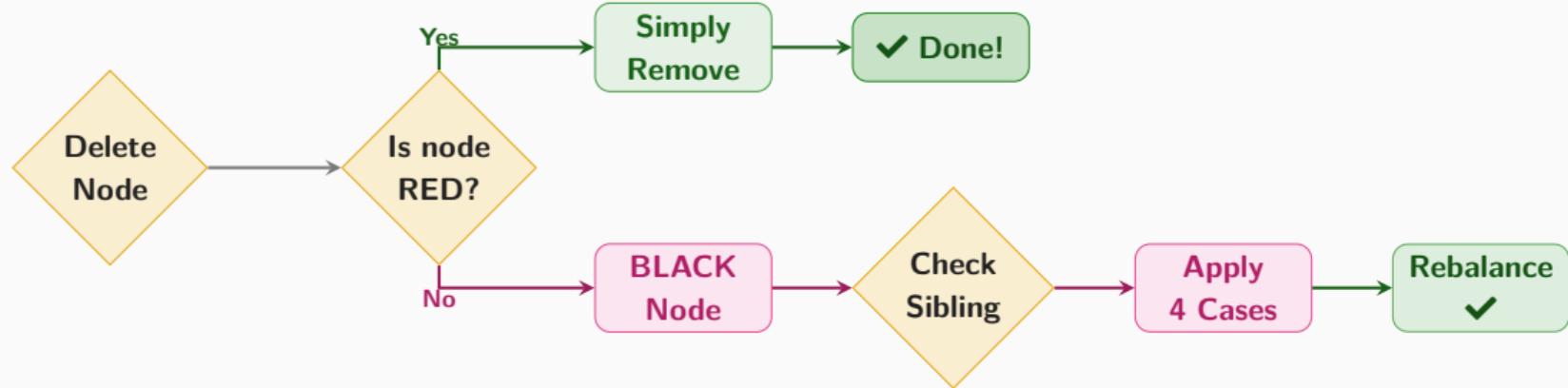
- No problem!
- Just remove it
- **Properties still hold**

Deleting a **BLACK** node

- Oh boy...
- Black height changes!
- **Need “double black” fix**
- Complex cases

Let's see both cases...

Deletion Decision Flowchart

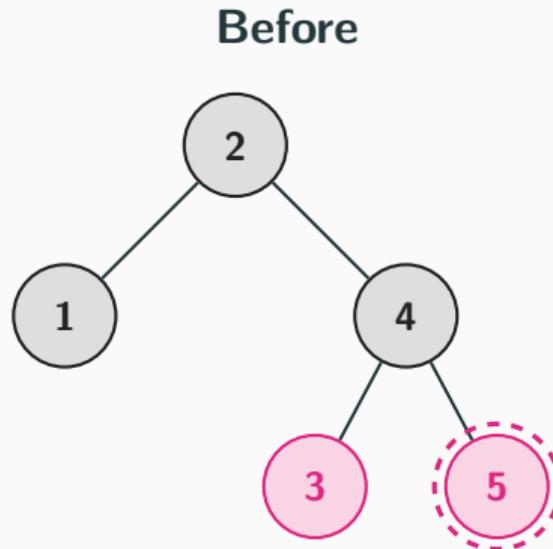


Top path (RED node) = straightforward

Bottom path (BLACK node) = complex

Case 1: Deleting a RED Node

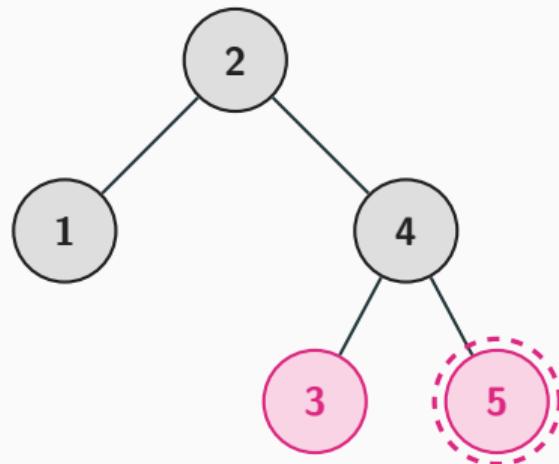
Delete node **5** from the tree



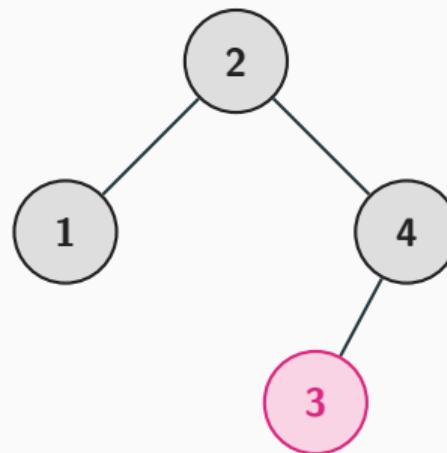
Case 1: Deleting a RED Node

Delete node **5** from the tree

Before



After



Case 1: Deleting a **RED** Node

Is it done?

Case 1: Deleting a **RED** Node

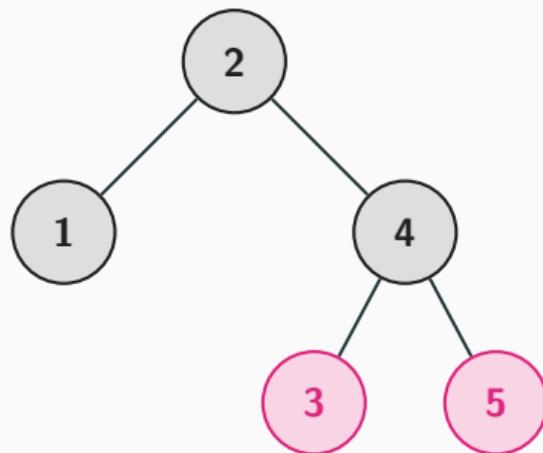
Is it done?

Let's check the black height.

Case 1: Black-Height Stays the Same

Every path still has $\text{bc} = 2$ black nodes after removing 5

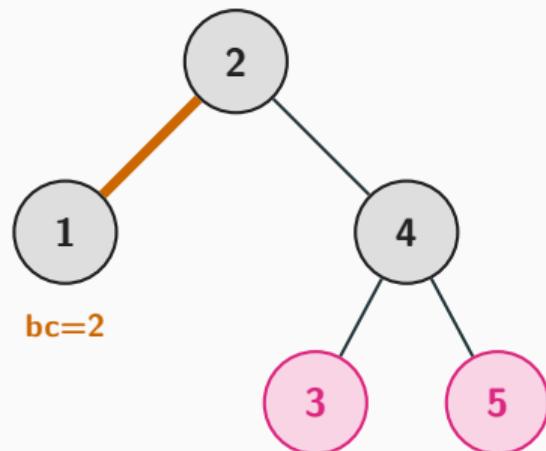
Before (with node 5)



Case 1: Black-Height Stays the Same

Every path still has $\text{bc} = 2$ black nodes after removing 5

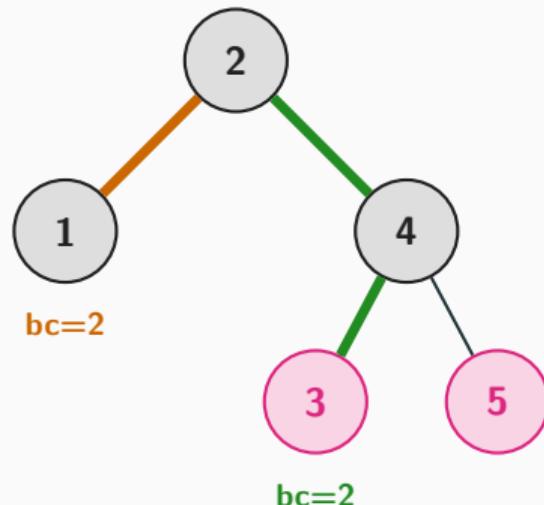
Before (with node 5)



Case 1: Black-Height Stays the Same

Every path still has $\text{bc} = 2$ black nodes after removing 5

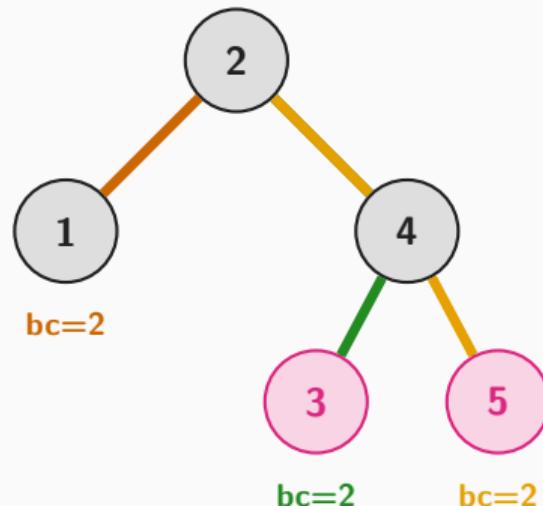
Before (with node 5)



Case 1: Black-Height Stays the Same

Every path still has $\text{bc} = 2$ black nodes after removing 5

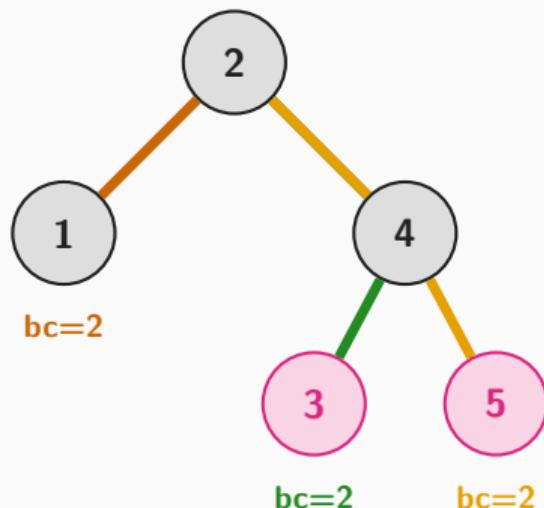
Before (with node 5)



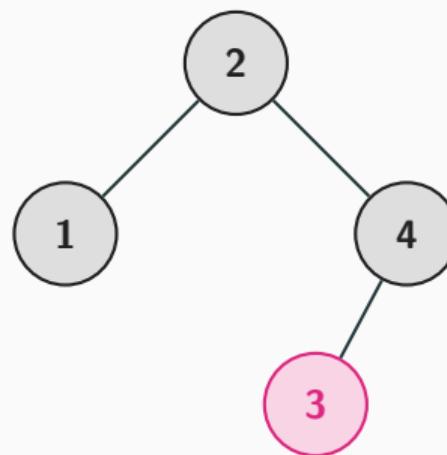
Case 1: Black-Height Stays the Same

Every path still has $\text{bc} = 2$ black nodes after removing 5

Before (with node 5)



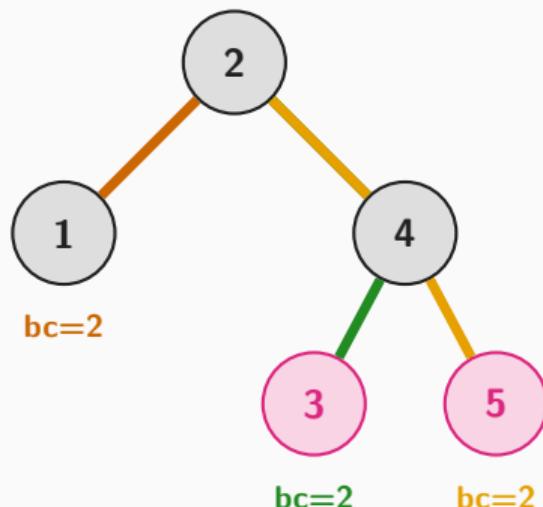
After (node 5 removed)



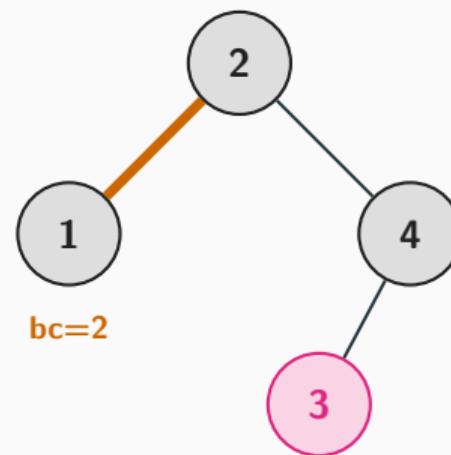
Case 1: Black-Height Stays the Same

Every path still has $\text{bc} = 2$ black nodes after removing 5

Before (with node 5)



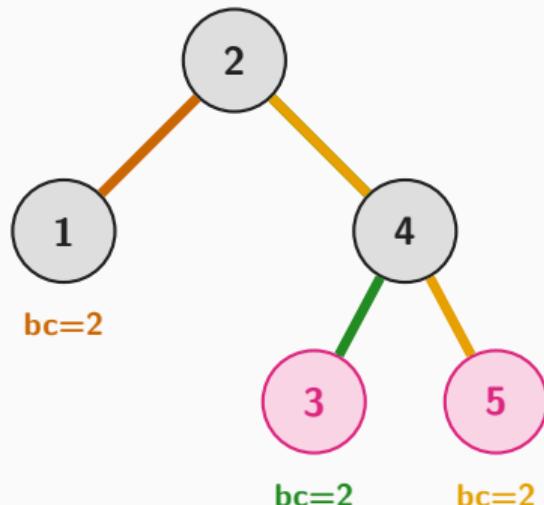
After (node 5 removed)



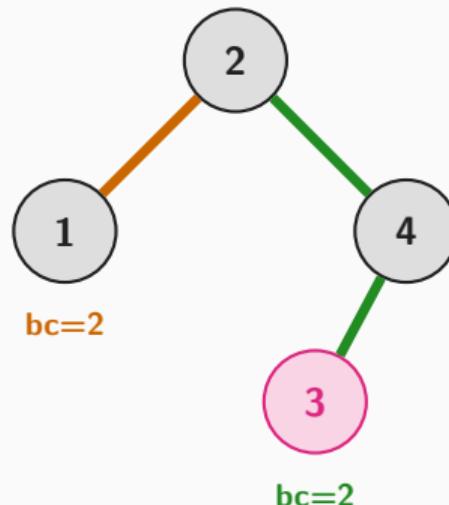
Case 1: Black-Height Stays the Same

Every path still has $\text{bc} = 2$ black nodes after removing 5

Before (with node 5)



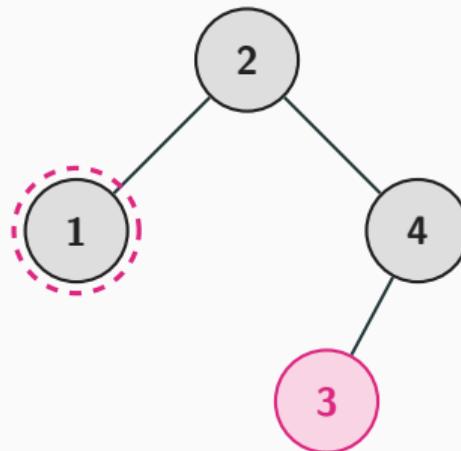
After (node 5 removed)



Case 2: Deleting a BLACK Node

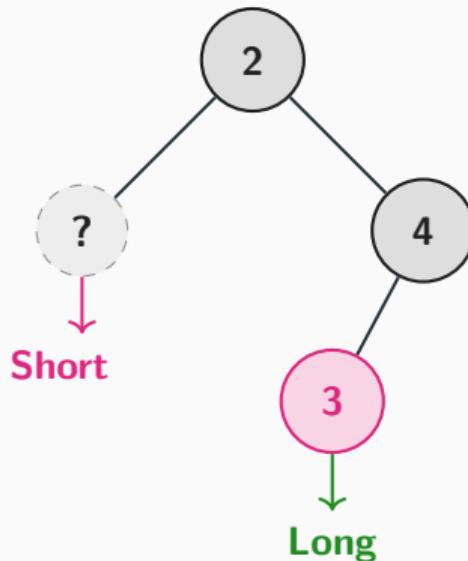
Delete node **1** from the tree

Before deletion:



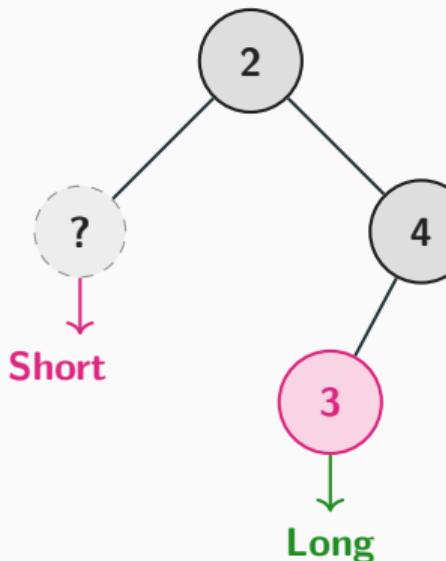
Case 2: After Deleting the BLACK Node

The node is gone - but now we have a **problem**



Case 2: After Deleting the BLACK Node

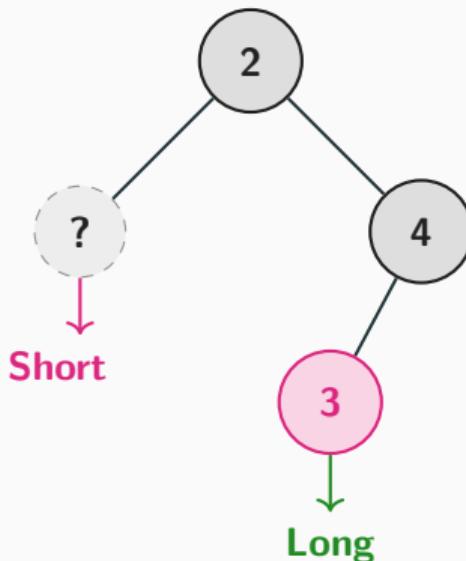
The node is gone - but now we have a **problem**



- Left path is now **shorter**

Case 2: After Deleting the BLACK Node

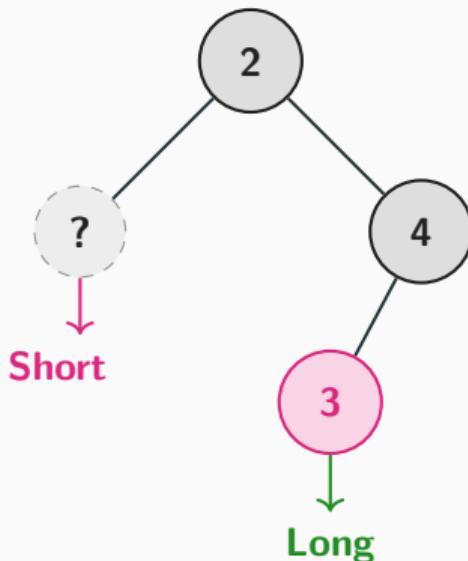
The node is gone - but now we have a **problem**



- Left path is now **shorter**
- Black-height **violated!**

Case 2: After Deleting the BLACK Node

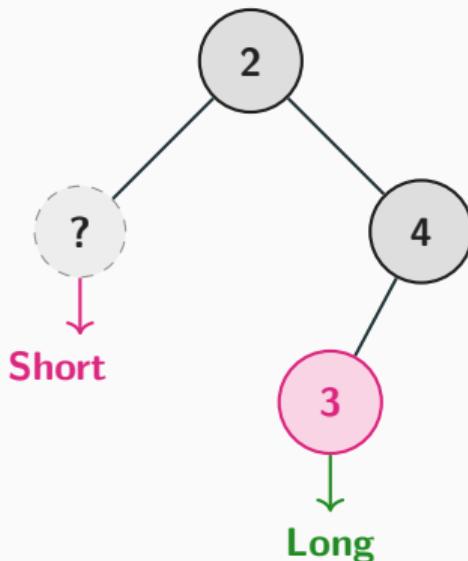
The node is gone - but now we have a **problem**



- Left path is now **shorter**
- Black-height **violated!**
- We call this a
“Double-Black” node

Case 2: After Deleting the BLACK Node

The node is gone - but now we have a **problem**



- Left path is now **shorter**
- Black-height **violated!**
- We call this a
“Double-Black” node

IMBALANCED - must fix!

Case 2: The Fix-up

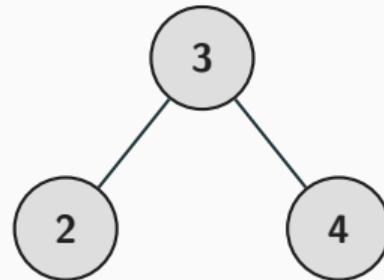
- **Rotate:** Right at 4,
then left at 2

Case 2: The Fix-up

- **Rotate:** Right at 4,
then left at 2
- **Recolor:** Node 3 → Black

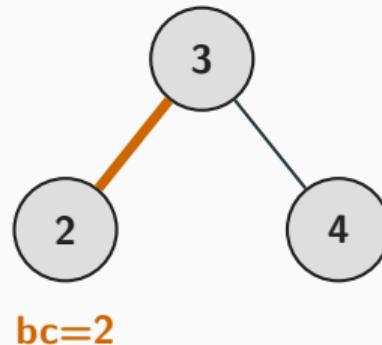
Case 2: The Fix-up

- **Rotate:** Right at 4,
then left at 2
- **Recolor:** Node 3 → Black



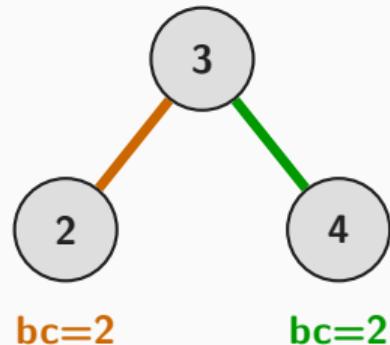
Case 2: The Fix-up

- **Rotate:** Right at 4,
then left at 2
- **Recolor:** Node 3 → Black



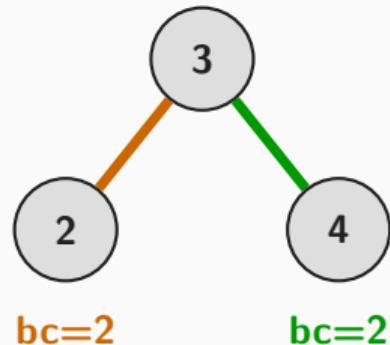
Case 2: The Fix-up

- **Rotate:** Right at 4,
then left at 2
- **Recolor:** Node 3 → Black



Case 2: The Fix-up

- **Rotate:** Right at 4,
then left at 2
- **Recolor:** Node 3 → Black
- **Tree is balanced!**



All paths: $bc = 2$ - Black-height restored!

Fixing Double-Black: 4 Cases

When we have a **Double-Black** node,
the fix depends on the **sibling's color and children**.

P = Parent **S** = Sibling **L / R** = S's children

 = Double-Black node

Fixing Double-Black: 4 Cases

When we have a **Double-Black** node,
the fix depends on the **sibling's color and children**.

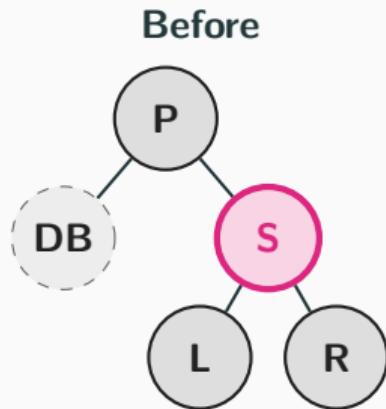
P = Parent **S** = Sibling **L / R** = S's children

 = Double-Black node

4 cases - let's go through them one by one!

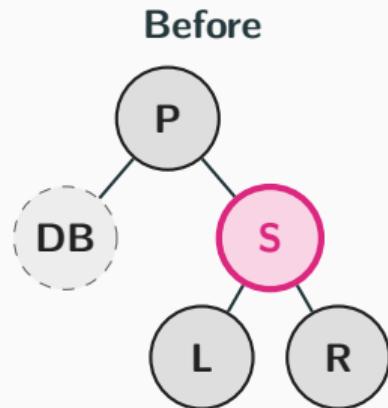
Fix Case 1 of 4: Sibling is RED

The Sibling S is RED



Fix Case 1 of 4: Sibling is RED

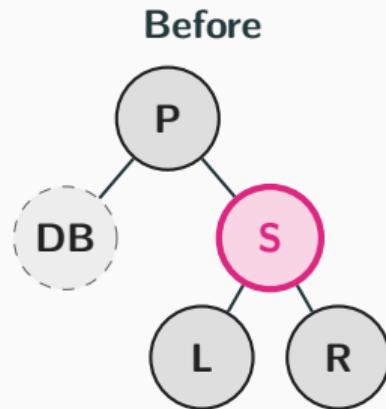
The Sibling S is RED



- **Rotate P to the left**

Fix Case 1 of 4: Sibling is RED

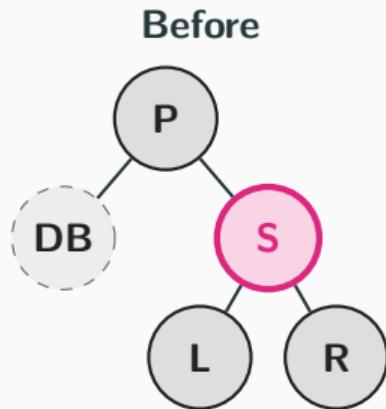
The Sibling S is RED



- **Rotate P to the left**
- **Recolor:** S → Black, P → Red

Fix Case 1 of 4: Sibling is RED

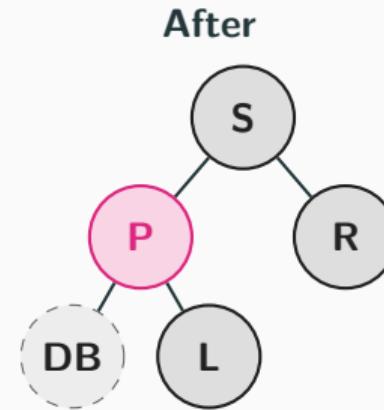
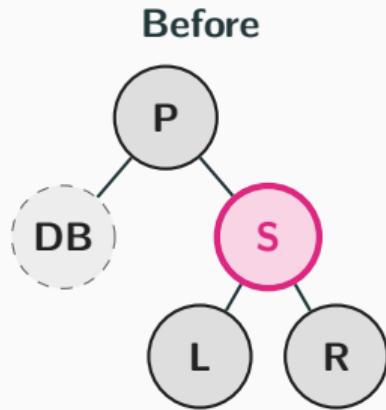
The Sibling S is RED



- **Rotate** P to the left
- **Recolor:** S → Black, P → Red

Fix Case 1 of 4: Sibling is RED

The Sibling S is RED

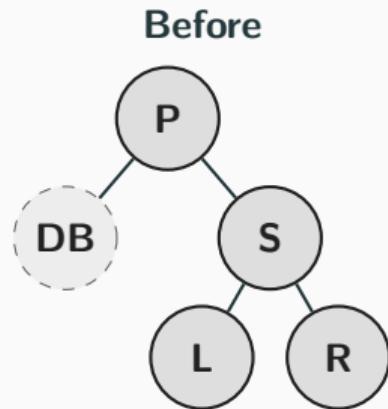


Now apply Case 2, 3, or 4 to DB

- **Rotate P to the left**
- **Recolor:** S → Black, P → Red

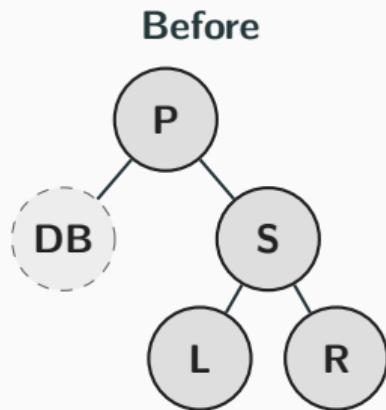
Fix Case 2 of 4: Sibling & Children All BLACK

S and both children are BLACK



Fix Case 2 of 4: Sibling & Children All BLACK

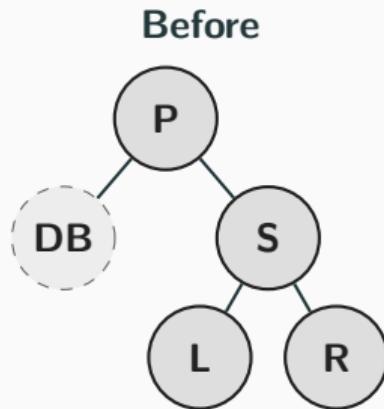
S and both children are BLACK



- Recolor S → Red

Fix Case 2 of 4: Sibling & Children All BLACK

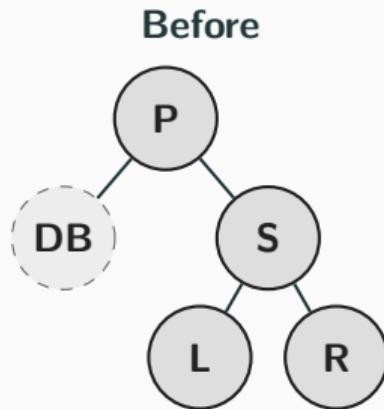
S and both children are BLACK



- Recolor S → Red
- Push the Double-Black up to P

Fix Case 2 of 4: Sibling & Children All BLACK

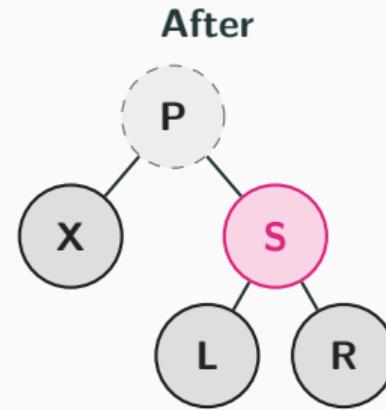
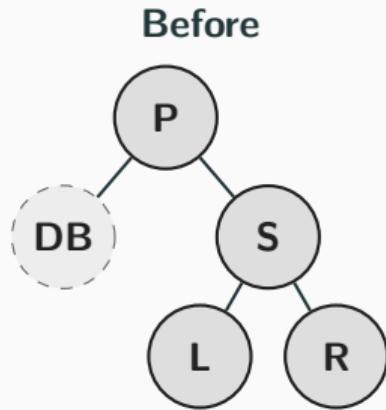
S and both children are BLACK



- Recolor S → Red
- Push the Double-Black up to P

Fix Case 2 of 4: Sibling & Children All BLACK

S and both children are BLACK

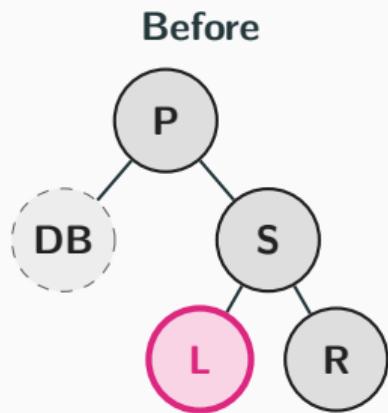


DB pushed to P — continue fixing

- Recolor S → Red
- Push the Double-Black up to P

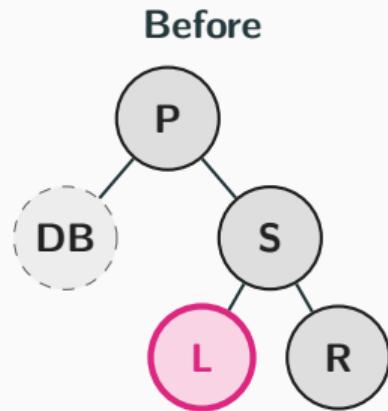
Fix Case 3 of 4: Sibling's Left Child is RED

S is Black, S's Left child is RED



Fix Case 3 of 4: Sibling's Left Child is RED

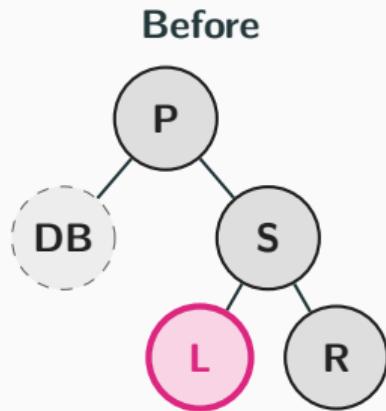
S is Black, S's Left child is RED



- Right-rotate at S, & Swap colors of S and L

Fix Case 3 of 4: Sibling's Left Child is RED

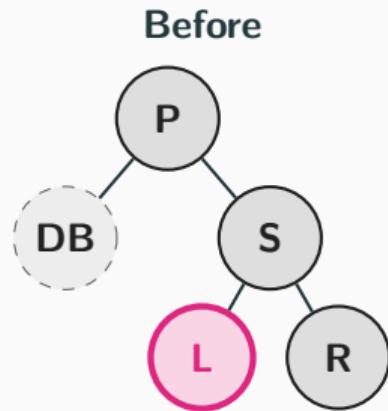
S is Black, S's Left child is RED



- Right-rotate at S, & Swap colors of S and L

Fix Case 3 of 4: Sibling's Left Child is RED

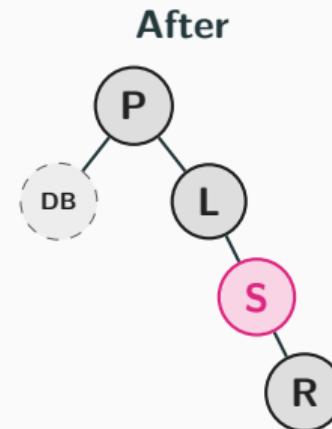
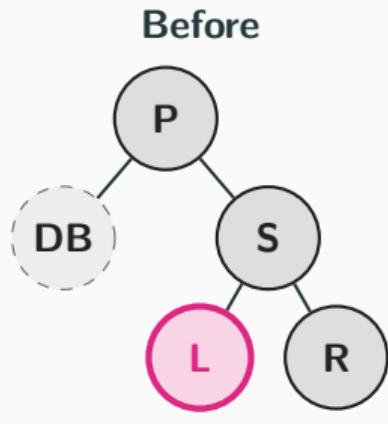
S is Black, S's Left child is RED



- Right-rotate at S, & Swap colors of S and L

Fix Case 3 of 4: Sibling's Left Child is RED

S is Black, S's Left child is RED

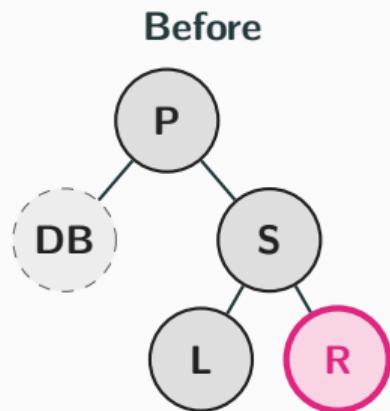


Now proceed with Case 4

- Right-rotate at S, & Swap colors of S and L

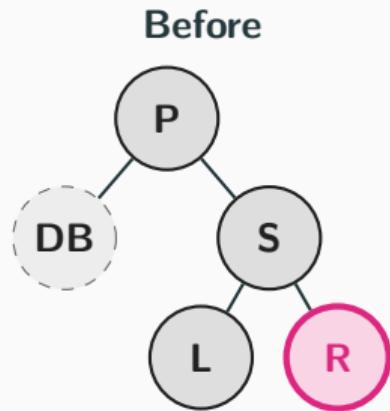
Fix Case 4 of 4: Sibling's Right Child is RED

S is Black, S's Right child is RED



Fix Case 4 of 4: Sibling's Right Child is RED

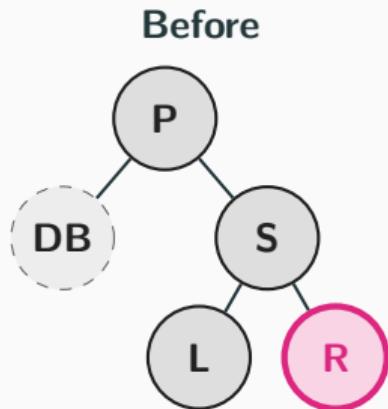
S is Black, S's Right child is RED



- Left-rotate at P

Fix Case 4 of 4: Sibling's Right Child is RED

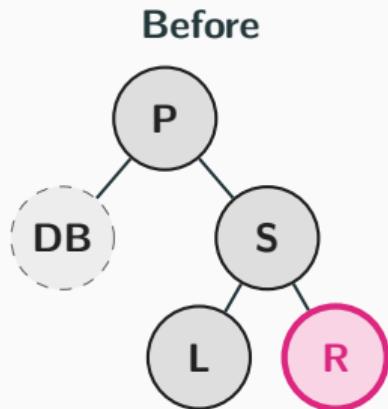
S is Black, S's Right child is RED



- **Left-rotate at P**
- **Recolor R → Black**

Fix Case 4 of 4: Sibling's Right Child is RED

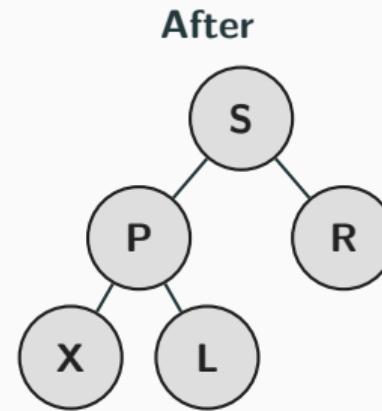
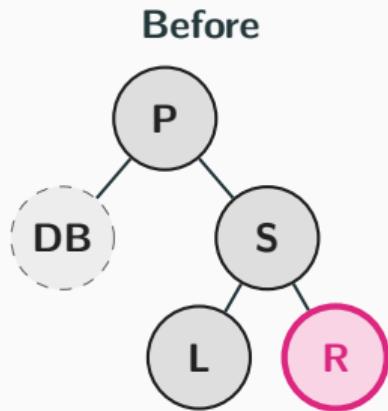
S is Black, S's Right child is RED



- **Left-rotate at P**
- **Recolor R → Black**

Fix Case 4 of 4: Sibling's Right Child is RED

S is Black, S's Right child is RED

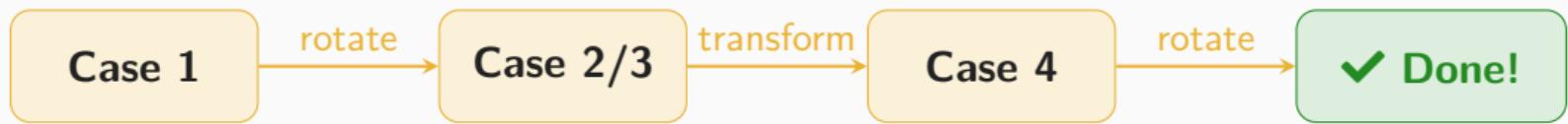


Double-Black fully resolved!

- Left-rotate at P
- Recolor R → Black

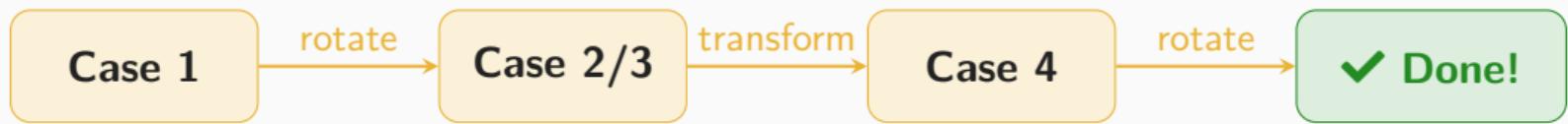
Summary

The cases form a **chain**:



Summary

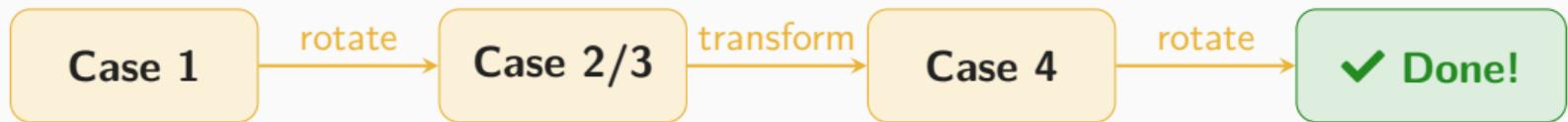
The cases form a **chain**:



Goal: Eventually reach Case 4 to fully eliminate Double-Black

Summary

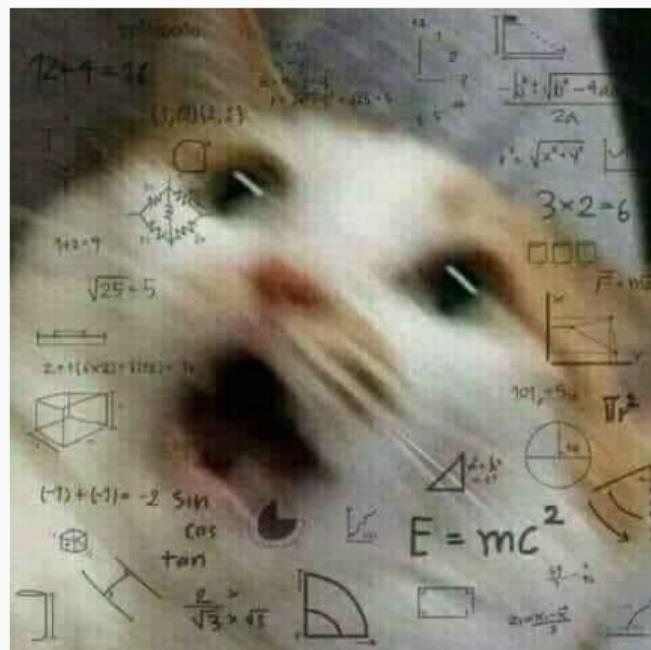
The cases form a **chain**:



Goal: Eventually reach Case 4 to fully eliminate Double-Black

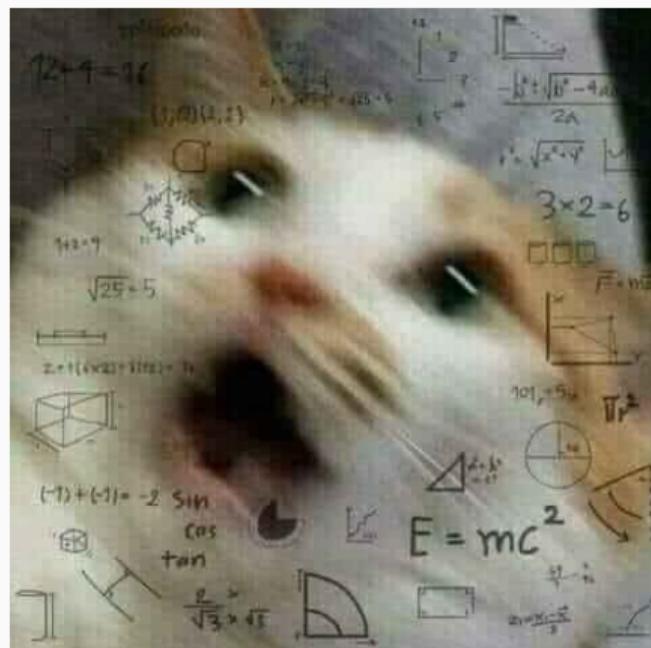
Case 2 may propagate upward; Cases 1 & 3 always lead to Case 4

Too Many Cases?



Confused?

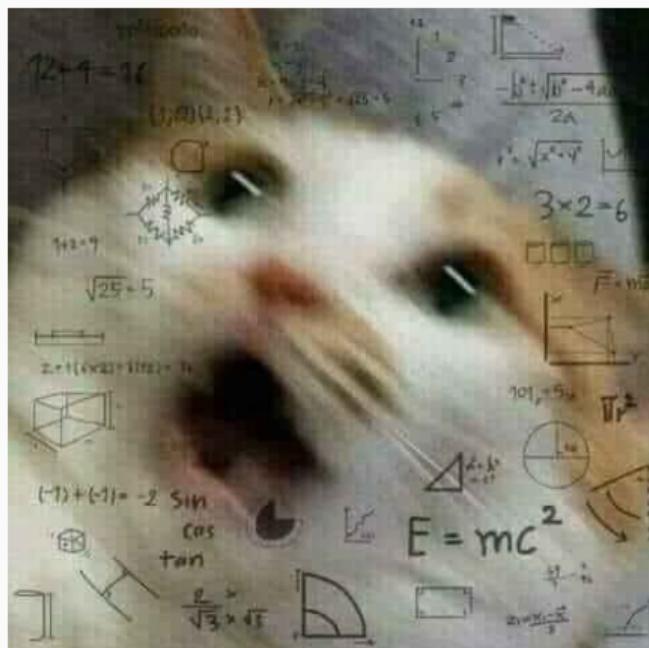
Too Many Cases?



Confused?

If this felt like **a lot** at once -
that's because **it is !**

Too Many Cases?



Confused?

If this felt like **a lot** at once -
that's because **it is !**

We know deletion is
complex - and that's **okay!**

Was All That Worth It?

All that work.

Was All That Worth It?

All that work.

What did it actually buy us?

Was All That Worth It?

All that work.

What did it actually buy us?

Let's finally see the payoff.

Why Rotations Work: The Magic Behind Balance

Rotation Complexity: $O(1)$ Operations

Rotation Complexity: $O(1)$ Operations

Left Rotation:

- Restructures tree locally

Right Rotation:

- Mirror of left rotation

Rotation Complexity: $O(1)$ Operations

Left Rotation:

- Restructures tree locally
- Preserves binary search property

Right Rotation:

- Mirror of left rotation
- Same time complexity

Rotation Complexity: $O(1)$ Operations

Left Rotation:

- Restructures tree locally
- Preserves binary search property
- Height changes by at most 1

Right Rotation:

- Mirror of left rotation
- Same time complexity
- Maintains red-black properties

Rotation Complexity: $O(1)$ Operations

Left Rotation:

- Restructures tree locally
- Preserves binary search property
- Height changes by at most 1

Right Rotation:

- Mirror of left rotation
- Same time complexity
- Maintains red-black properties

Key Insight

Rotations are $O(1)$ because they only change a constant number of pointers! No tree traversal needed - just pointer gymnastics!

Why Rotations Work: The Magic Behind Balance

Rotation Complexity: $O(1)$ Operations

Left Rotation:

- Restructures tree locally
- Preserves binary search property
- Height changes by at most 1

Right Rotation:

- Mirror of left rotation
- Same time complexity
- Maintains red-black properties

Key Insight

Rotations are $O(1)$ because they only change a constant number of pointers! No tree traversal needed - just pointer gymnastics!



Height Proof: Why $O(\log n)$?

Height Proof: Why $O(\log n)$?

Key Insight

A red-black tree with n internal nodes has height $\leq 2 \log_2(n + 1)$

Height Proof: Why $O(\log n)$?

Key Insight

A red-black tree with n internal nodes has height $\leq 2 \log_2(n + 1)$

Proof Sketch:

1. Every path from root to leaf has same number of black nodes
2. Red nodes can't have red children
3. At least half nodes on any path are black

Why Red-Black Trees Work: The Math Behind It

Height Proof: Why $O(\log n)$?

Key Insight

A red-black tree with n internal nodes has height $\leq 2 \log_2(n + 1)$

Proof Sketch:

1. Every path from root to leaf has same number of black nodes
2. Red nodes can't have red children
3. At least half nodes on any path are black
4. Height $\leq 2 \times$ black-height

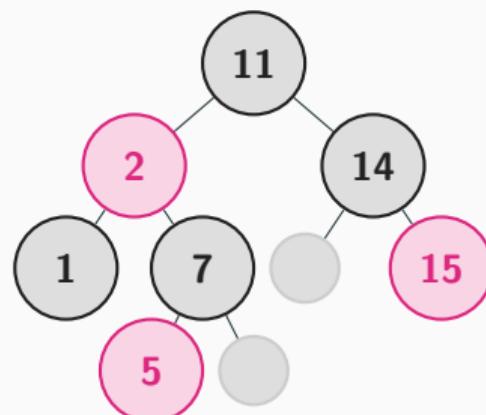
Height Proof: Why $O(\log n)$?

Key Insight

A red-black tree with n internal nodes has height $\leq 2 \log_2(n + 1)$

Proof Sketch:

1. Every path from root to leaf has same number of black nodes
2. Red nodes can't have red children
3. At least half nodes on any path are black
4. Height $\leq 2 \times$ black-height



Black height = 2, Total height = 4

Why Red-Black Trees Are Space Efficient

Space Complexity: $O(n)$ Memory Usage

Space Complexity: $O(n)$ Memory Usage

Memory Requirements:

- Each node stores: key, color and 2 pointers

Comparison:

- AVL trees: height field per node

Space Complexity: $O(n)$ Memory Usage

Memory Requirements:

- Each node stores: key, color and 2 pointers
- Total: $O(n)$ space

Comparison:

- AVL trees: height field per node
- B-trees: multiple keys per node

Space Complexity: $O(n)$ Memory Usage

Memory Requirements:

- Each node stores: key, color and 2 pointers
- Total: $O(n)$ space
- No extra storage for balance info

Comparison:

- AVL trees: height field per node
- B-trees: multiple keys per node
- RBTs: minimal overhead
[squirrel-level efficient]

Why Red-Black Trees Are Space Efficient

Space Complexity: $O(n)$ Memory Usage

Memory Requirements:

- Each node stores: key, color and 2 pointers
- Total: $O(n)$ space
- No extra storage for balance info

Comparison:

- AVL trees: height field per node
- B-trees: multiple keys per node
- RBTs: minimal overhead
[squirrel-level efficient]

Key Insight

Red-black trees achieve $O(n)$ space with just 1 extra bit per node (the color)!

That's the definition of space-efficient data structures!

50+ Years of Tree Balancing Innovation

50+ Years of Tree Balancing Innovation

- 1972: Red-Black Trees (Guibas & Sedgewick) [guibas1972]

50+ Years of Tree Balancing Innovation

- **1972:** Red-Black Trees (Guibas & Sedgewick) [guibas1972]
- **2008:** Left-Leaning Red-Black Trees (Sedgewick) [sedgewick2008]

50+ Years of Tree Balancing Innovation

- **1972:** Red-Black Trees (Guibas & Sedgewick) [guibas1972]
- **2008:** Left-Leaning Red-Black Trees (Sedgewick) [sedgewick2008]
- **2013:** WAVL Trees (Bronson et al.) [bronson2013]

50+ Years of Tree Balancing Innovation

- **1972:** Red-Black Trees (Guibas & Sedgewick) [guibas1972]
- **2008:** Left-Leaning Red-Black Trees (Sedgewick) [sedgewick2008]
- **2013:** WAVL Trees (Bronson et al.) [bronson2013]

What to do with these trees?

"Yikes! Trees evolving faster than my code." - Some sad developer

50+ Years of Tree Balancing Innovation

- **1972:** Red-Black Trees (Guibas & Sedgewick) [guibas1972]
- **2008:** Left-Leaning Red-Black Trees (Sedgewick) [sedgewick2008]
- **2013:** WAVL Trees (Bronson et al.) [bronson2013]

What to do with these trees?

"Yikes! Trees evolving faster than my code." - Some sad developer

Fun fact: Robert Sedgewick (co-inventor of RBT) later said: "*I prefer left-leaning red-black trees now - they're simpler!*"

Red-Black Trees in Modern Tech

Red-Black Trees in Modern Tech

Traditional Uses:

- CPU scheduling algorithms

Red-Black Trees in Modern Tech

Traditional Uses:

- CPU scheduling algorithms
- Memory management

Red-Black Trees in Modern Tech

Traditional Uses:

- CPU scheduling algorithms
- Memory management
- Network routing tables

Red-Black Trees in Modern Tech

Traditional Uses:

- CPU scheduling algorithms
- Memory management
- Network routing tables
- Game engines (spatial indexing)

Red-Black Trees in Modern Tech

Traditional Uses:

- CPU scheduling algorithms
- Memory management
- Network routing tables
- Game engines (spatial indexing)

Modern Applications:

Red-Black Trees in Modern Tech

Traditional Uses:

- CPU scheduling algorithms
- Memory management
- Network routing tables
- Game engines (spatial indexing)

Modern Applications:

- ML indexing, cloud storage, blockchain, AI pathfinding!



Red-Black Trees in Modern Tech

Traditional Uses:

- CPU scheduling algorithms
- Memory management
- Network routing tables
- Game engines (spatial indexing)



Modern Applications:

- ML indexing, cloud storage, blockchain, AI pathfinding!
- RBTs: Keanu Reeves of data structures - always reliable!



Why Choose Red-Black Trees?

The Ultimate Showdown: RBT vs Other Trees

Why Choose Red-Black Trees?

Tree Type	Search	Insert	Delete	Space
Red-Black	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$
AVL	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$
B-Tree	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$

The Ultimate Showdown: RBT vs Other Trees

Why Choose Red-Black Trees?

Tree Type	Search	Insert	Delete	Space
Red-Black	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$
AVL	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$
B-Tree	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(n)$

Red-Black Advantages: The Meme Edition

Why Red-Black Trees Are the Perfect Choice

Red-Black Advantages: The Meme Edition

Why Red-Black Trees Are the Perfect Choice



The Goldilocks Solution

Not too fast, not too slow,
just right!

Red-Black Advantages: The Meme Edition

Why Red-Black Trees Are the Perfect Choice



The Goldilocks Solution

Not too fast, not too slow,
just right!



Rock-Solid Guarantees

Faster insertions than AVL,
simpler than B-trees!

Red-Black Advantages: The Meme Edition

Why Red-Black Trees Are the Perfect Choice



The Goldilocks Solution

Not too fast, not too slow,
just right!



Rock-Solid Guarantees

Faster insertions than AVL,
simpler than B-trees!



Perfect Balance

Like coffee - balanced and
reliable!

The Real Story Behind the Scenes

The Real Story Behind the Scenes

Tech Giants & RBTs:

- **Google:** Uses RBTs in MapReduce

The Real Story Behind the Scenes

Tech Giants & RBTs:

- **Google:** Uses RBTs in MapReduce
- **Facebook:** News feed ranking

Secret Sauce

Many companies use hybrid approaches - RBTs for small datasets, B-trees for large ones. It's complicated... but mostly red-black trees!

The Real Story Behind the Scenes

Tech Giants & RBTs:

- **Google:** Uses RBTs in MapReduce
- **Facebook:** News feed ranking
- **Amazon:** Product recommendations

Secret Sauce

Many companies use hybrid approaches - RBTs for small datasets, B-trees for large ones. It's complicated... but mostly red-black trees!

The Real Story Behind the Scenes

Tech Giants & RBTs:

- **Google:** Uses RBTs in MapReduce
- **Facebook:** News feed ranking
- **Amazon:** Product recommendations
- **Netflix:** Content delivery networks

Secret Sauce

Many companies use hybrid approaches - RBTs for small datasets, B-trees for large ones. It's complicated... but mostly red-black trees!

The Real Story Behind the Scenes

Tech Giants & RBTs:

- **Google:** Uses RBTs in MapReduce
- **Facebook:** News feed ranking
- **Amazon:** Product recommendations
- **Netflix:** Content delivery networks
- **Microsoft:** Windows kernel

Secret Sauce

Many companies use hybrid approaches - RBTs for small datasets, B-trees for large ones. It's complicated... but mostly red-black trees!

The Real Story Behind the Scenes

Tech Giants & RBTs:

- **Google:** Uses RBTs in MapReduce
- **Facebook:** News feed ranking
- **Amazon:** Product recommendations
- **Netflix:** Content delivery networks
- **Microsoft:** Windows kernel

Secret Sauce

Many companies use hybrid approaches - RBTs for small datasets, B-trees for large ones. It's complicated... but mostly red-black trees!

The Real Story Behind the Scenes

Tech Giants & RBTs:

- **Google:** Uses RBTs in MapReduce
- **Facebook:** News feed ranking
- **Amazon:** Product recommendations
- **Netflix:** Content delivery networks
- **Microsoft:** Windows kernel

Secret Sauce

Many companies use hybrid approaches - RBTs for small datasets, B-trees for large ones. It's complicated... but mostly red-black trees!



Next Generation Data Structures

Next Generation Data Structures

Emerging Trends:

- **Persistent Trees** (functional programming)

Next Generation Data Structures

Emerging Trends:

- **Persistent Trees** (functional programming)
- **Concurrent RBTs** (multi-threading)

Next Generation Data Structures

Emerging Trends:

- **Persistent Trees** (functional programming)
- **Concurrent RBTs** (multi-threading)
- **GPU-accelerated trees**

Next Generation Data Structures

Emerging Trends:

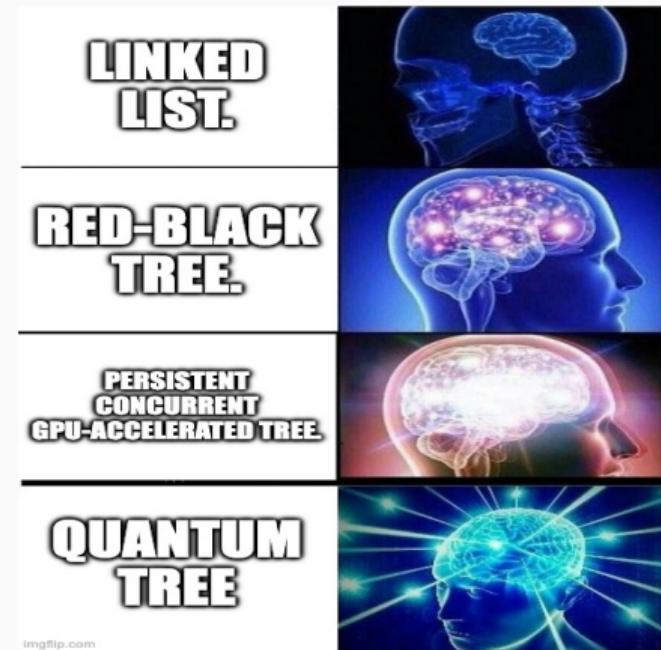
- **Persistent Trees** (functional programming)
- **Concurrent RBTs** (multi-threading)
- **GPU-accelerated trees**
- **Quantum-inspired data structures** - because we're not sure what they do, but they sound cool!

The Future: Where Are Trees Heading?

Next Generation Data Structures

Emerging Trends:

- **Persistent Trees** (functional programming)
- **Concurrent RBTs** (multi-threading)
- **GPU-accelerated trees**
- **Quantum-inspired data structures** - because we're not sure what they do, but they sound cool!



So There You Have It!

Red-Black Trees: The Unsung Heroes

Red-Black Trees: The Unsung Heroes

- Mathematical elegance

Red-Black Trees: The Unsung Heroes

- Mathematical elegance
- Practical performance

Red-Black Trees: The Unsung Heroes

- Mathematical elegance
- Practical performance
- Everywhere in computing

Red-Black Trees: The Unsung Heroes

- Mathematical elegance
- Practical performance
- Everywhere in computing

Hero Status

Red-Black Trees: Not all heroes wear capes!

Red-Black Trees: The Unsung Heroes

- Mathematical elegance
- Practical performance
- Everywhere in computing

Hero Status

Red-Black Trees: Not all heroes wear capes!

Remember

The next time your code runs in $O(\log n)$ time...

Thank a red-black tree! ❤

Thanks for listening!

Thanks for listening!

One Last thing before we go...

Thanks for listening!

One Last thing before we go...

