

Mechanical Waves

The mechanical waves discussed require

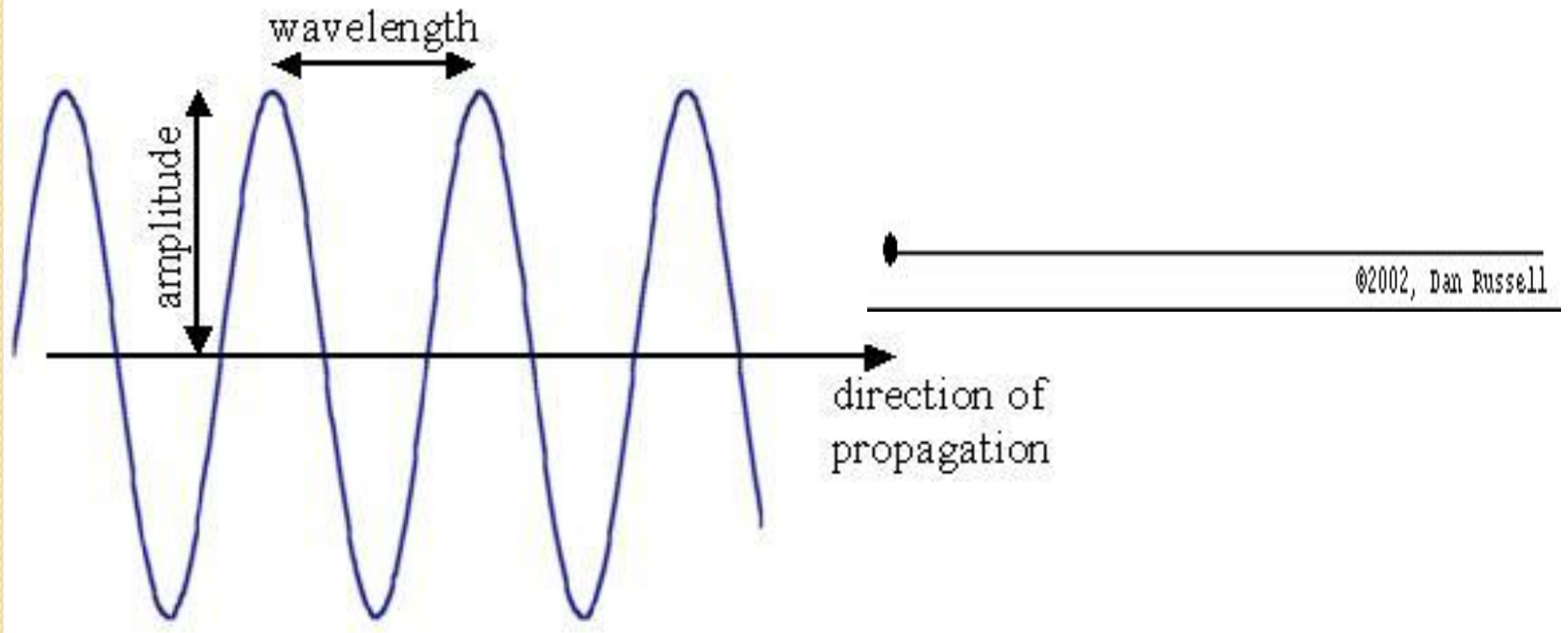
- some source of disturbance,
- a medium that can be disturbed, and
- some physical connection through which adjacent portions of the medium can influence each other.

Mechanical waves are disturbances in matter that carry energy from one place to another through medium.

- Medium can be a solid, liquid, or gas
- Some waves can travel through space...with no medium! e.g. Electromagnetic waves

Types of Mechanical Waves

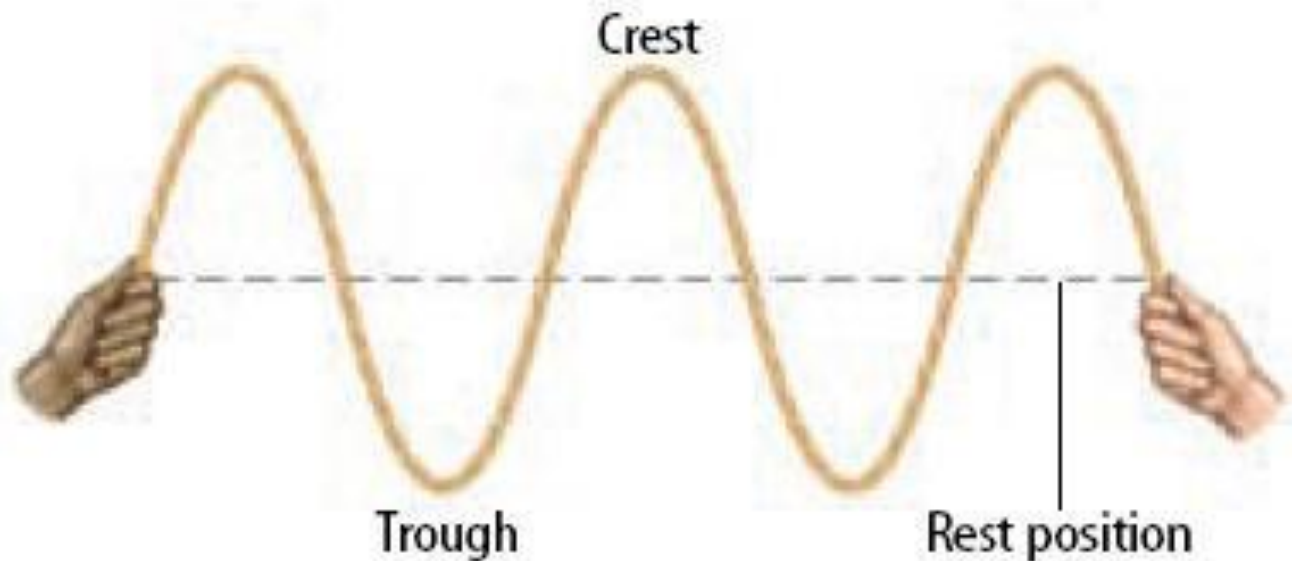
- Transverse:
 - A wave that causes the medium to vibrate at right angles to the direction of the wave



Transverse Waves

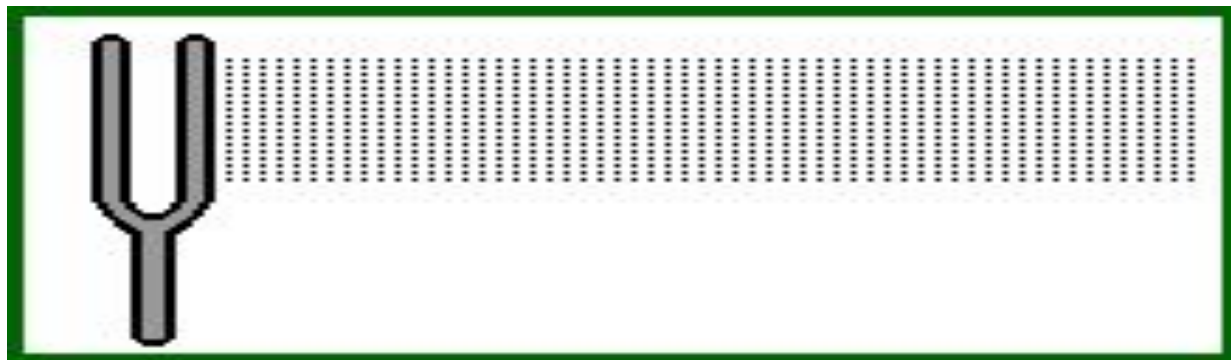
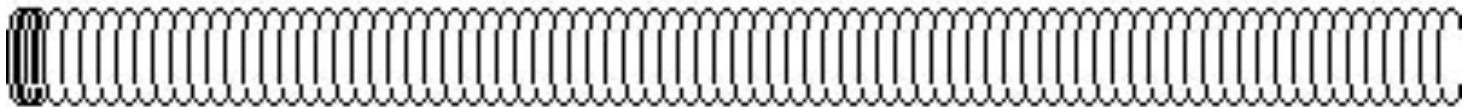


In a transverse wave the matter in the wave moves up and down at a right angle to the direction of the wave

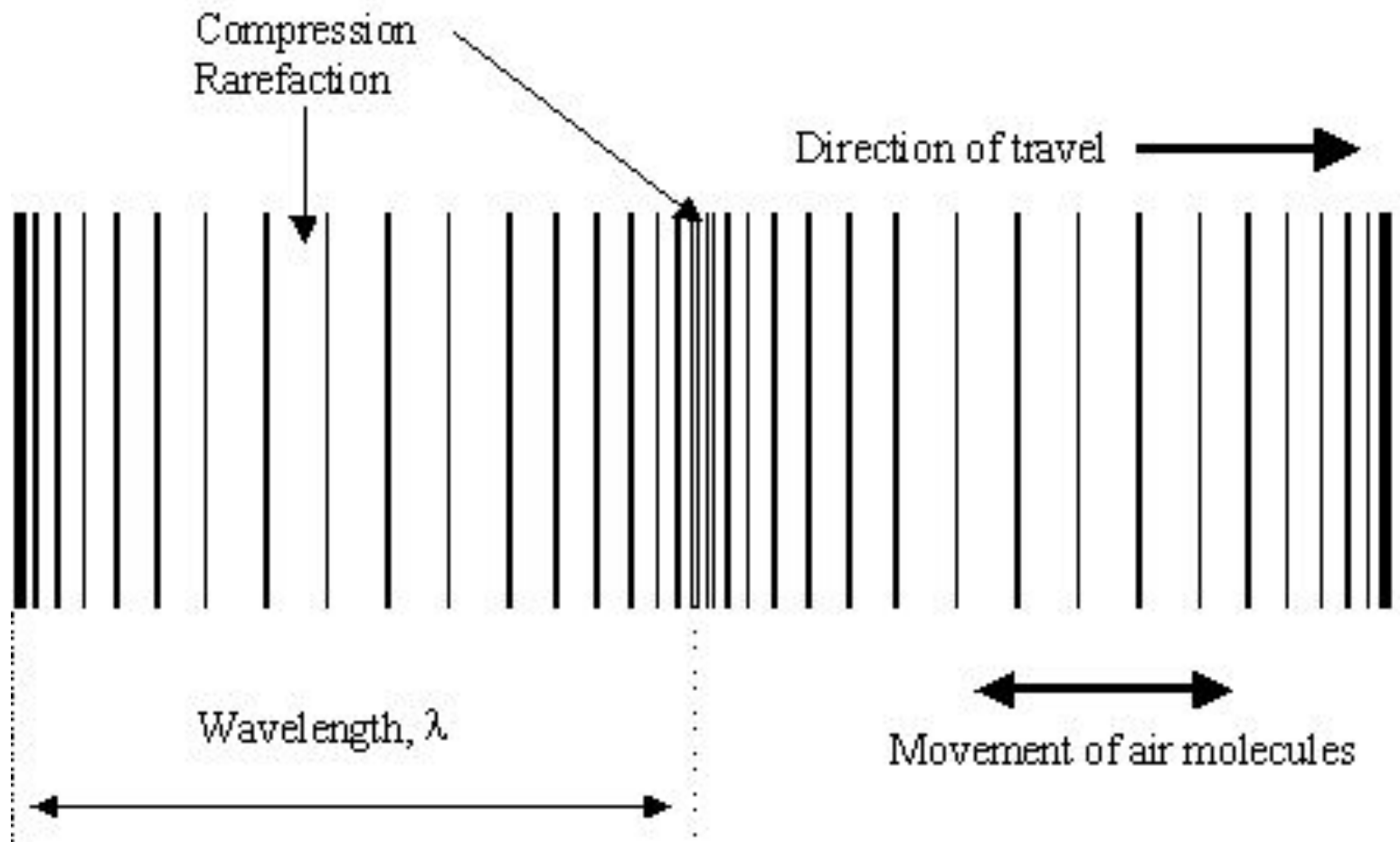


● Longitudinal:

- A wave in which the vibration of the medium is parallel to the direction the wave travels

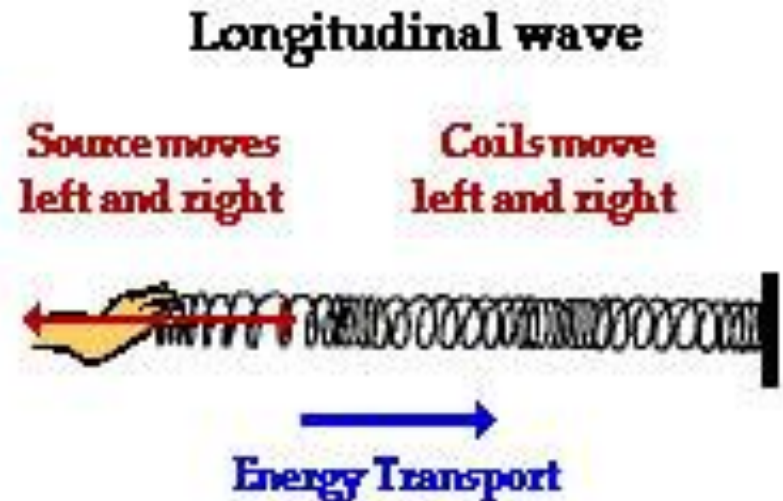
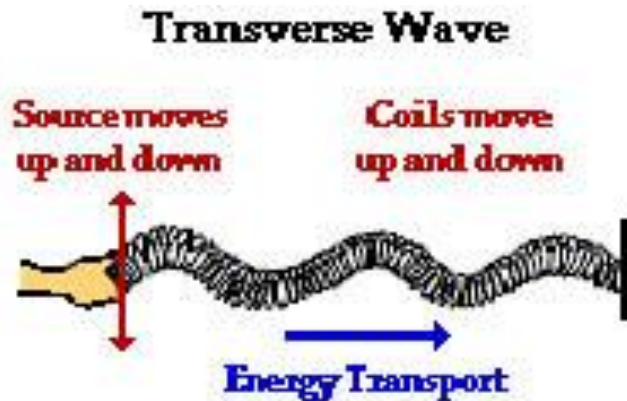


Parts of a longitudinal wave:

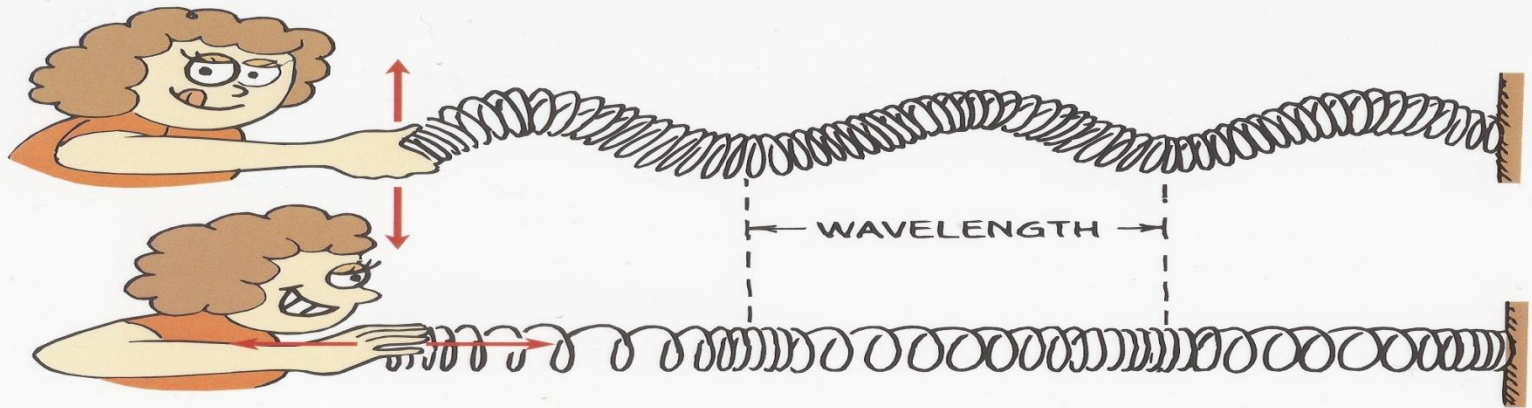
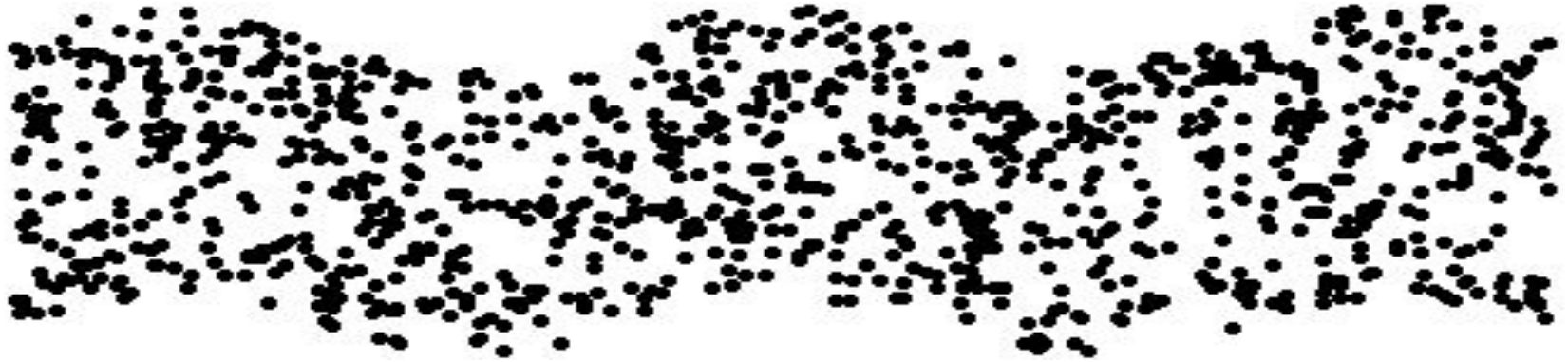


Remember!

- A wave doesn't move the medium...it's just energy traveling through the medium!



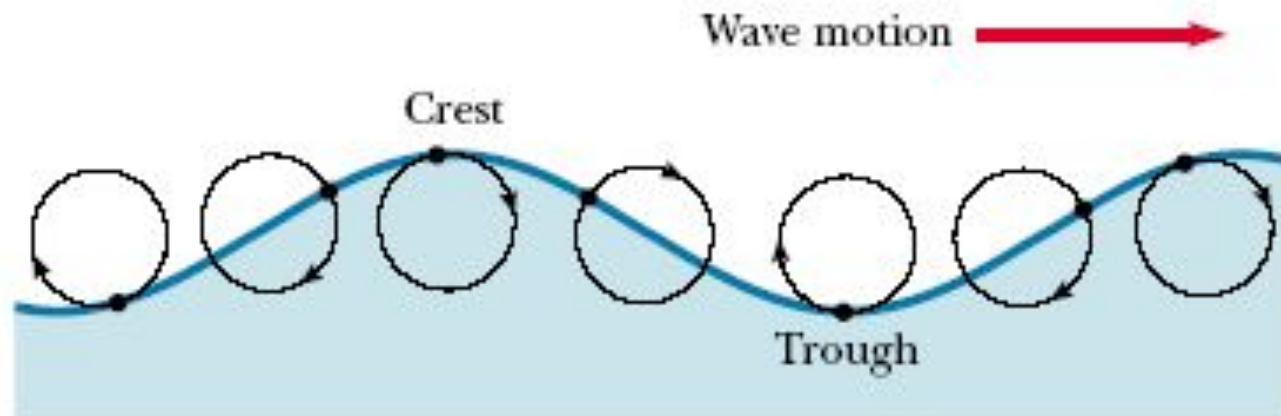
Transverse and Longitudinal Wave



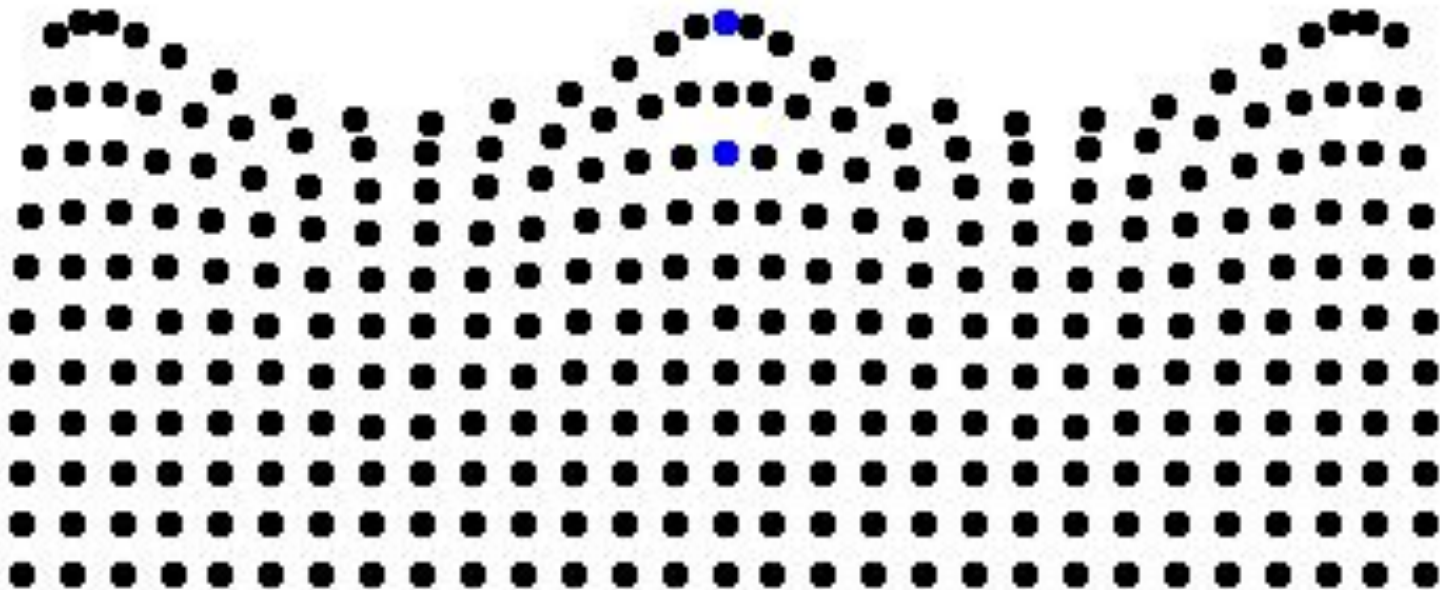
Combination of transverse and longitudinal



- The motion of water molecules on the surface of deep water in which a wave is propagating is a combination of transverse and longitudinal displacements, with the result that molecules at the surface move in nearly circular paths. Each molecule is displaced both horizontally and vertically from its equilibrium position.

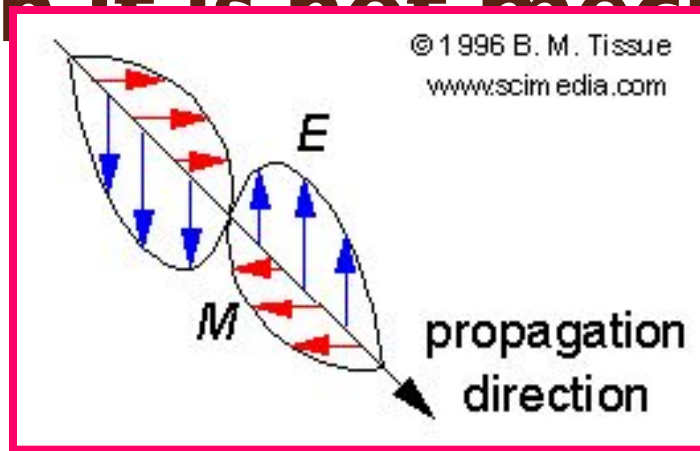


Combination of transverse and longitudinal surface of deep water



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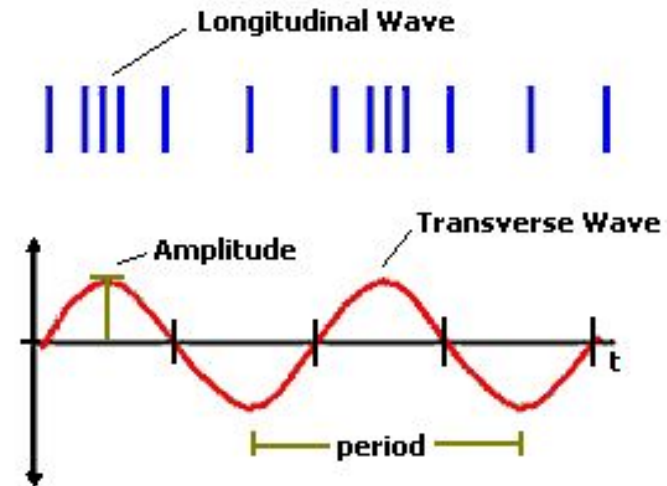
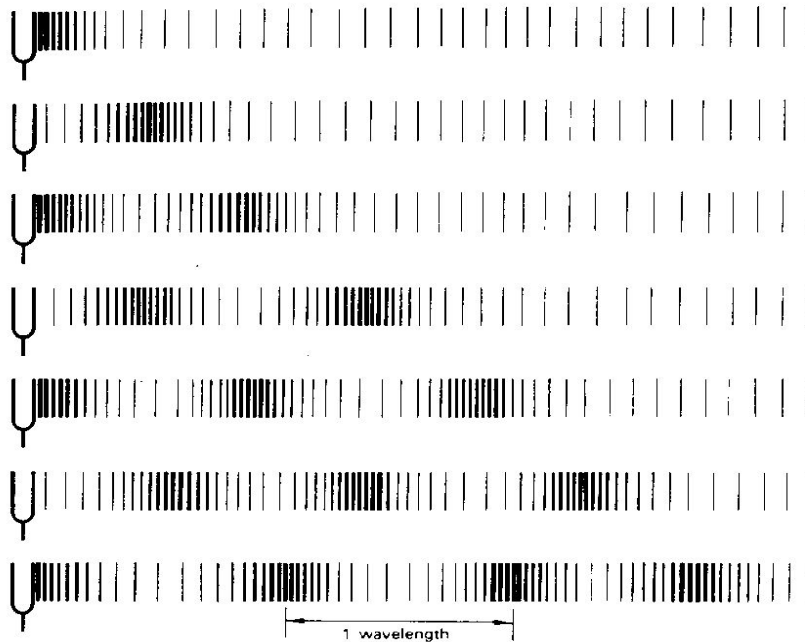
Light is a Transverse Waves although it is not mechanical wave



- Energy is perpendicular to direction of motion
- Moving photon creates electric & magnetic field
- Light has BOTH Electric & Magnetic fields at right angles and perpendicular to the direction of propagation of wave.

Sound Waves

Sound waves are Longitudinal wave.



Sound travels through different media.

We hear sound which usually travels through air. Sound travels through other media as well, such as water and various solids.

Sound travels different speeds in different media. Sound typically travels faster in a solid than a liquid and faster in a liquid than a gas.

The denser the medium, the faster sound will travel.

The higher the temperature, the faster the particles of the medium will move and the faster the particles will carry the sound.

Table 1 Speed of Sound in Different Mediums

Medium	Speed of Sound (in m/s)
Air	347
Cork	500
Water	1,498
Brick	3,650
Aluminum	4,877

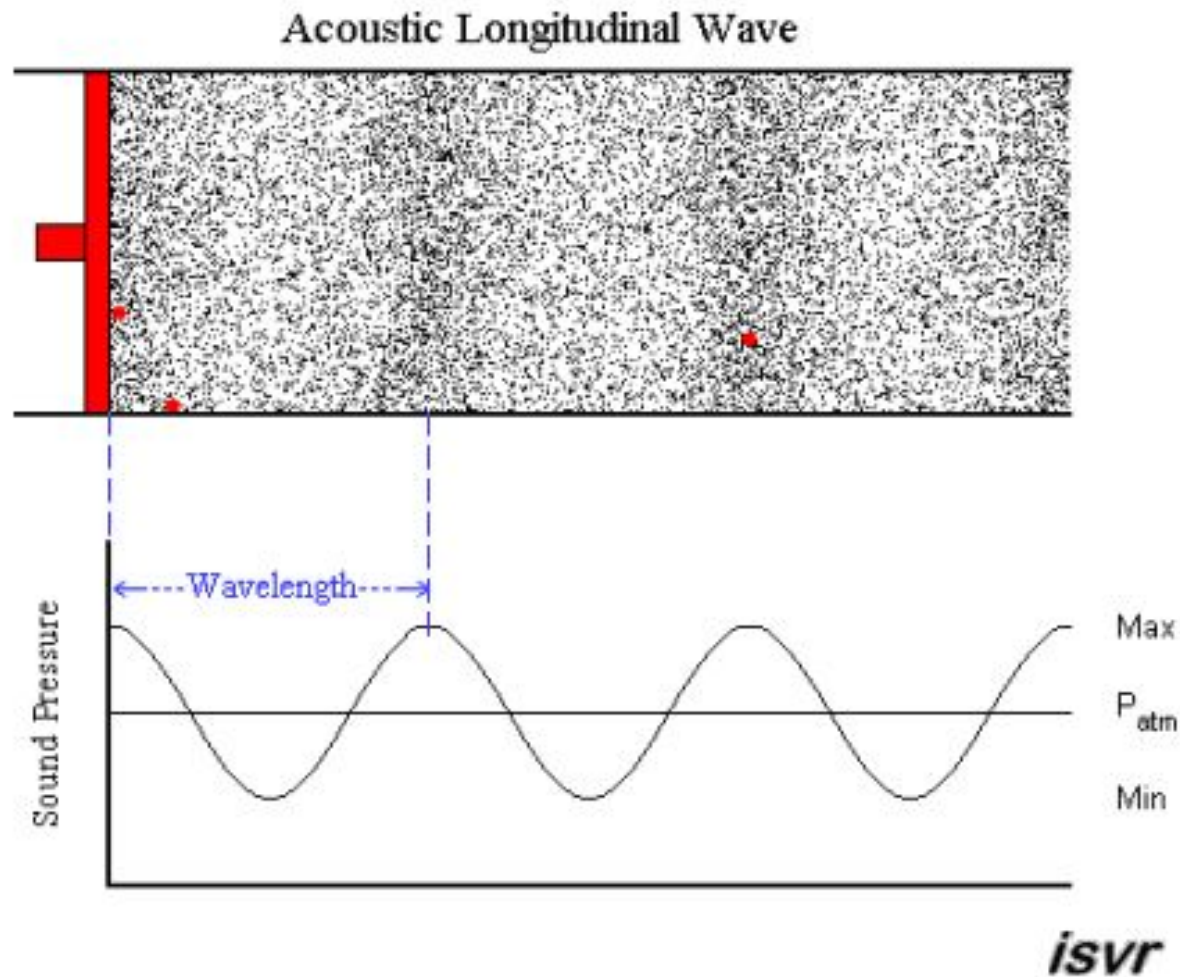
Infrasonic and Ultrasonic

A healthy human ear can hear frequencies in the range of **20 Hz to 20,000 Hz**. Humans cannot hear below 20 Hz. Sounds below this frequency are termed ***infrasonic***.

Sounds above 20,000 Hz are termed ***ultrasonic***. Some animals, such as dogs, can hear frequencies in this range in which humans cannot hear.

Waves transfer energy

ter.



Frequency= waves/time

Speed of Sound

Medium	velocity m/sec
air (20 C)	343
air (0 C)	331
water (25 C)	1493
sea water	1533
diamond	12000
iron	5130
copper	3560
glass	5640

Wave on a string

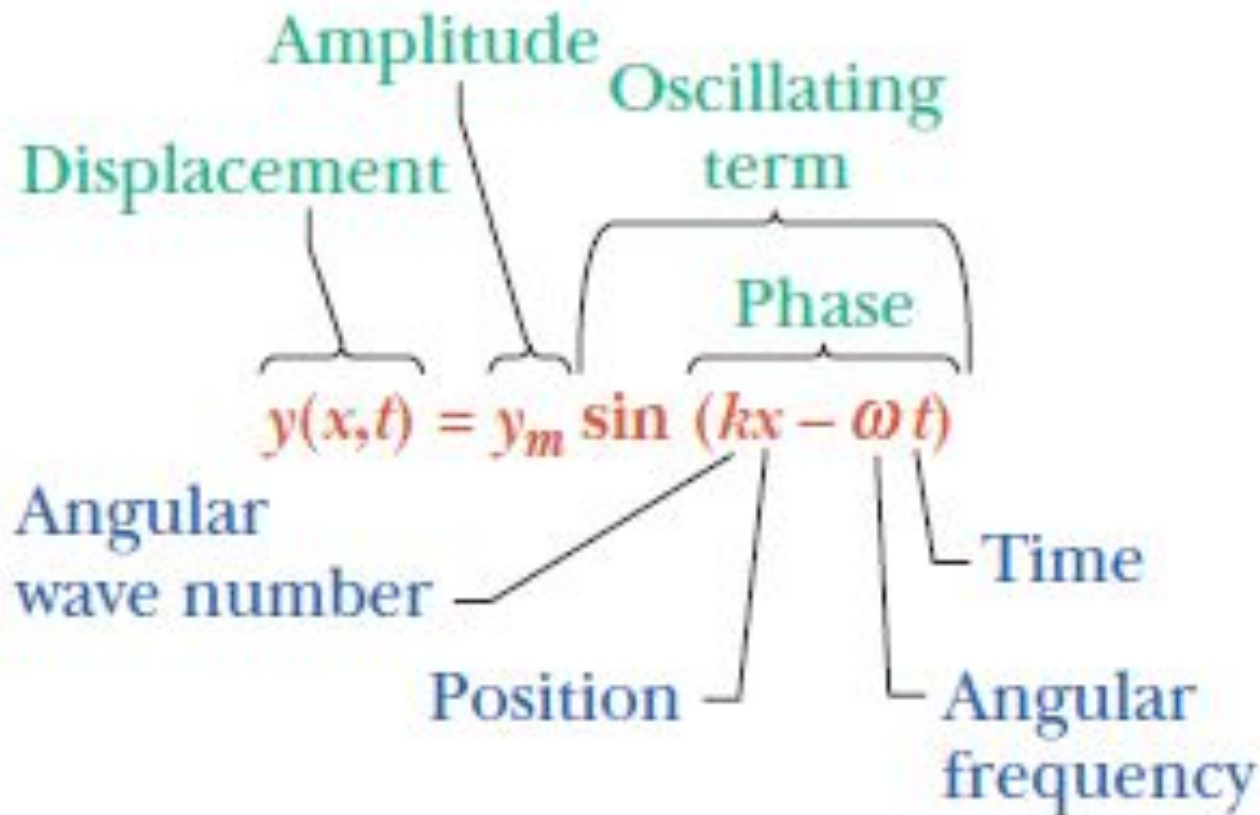
To completely describe a wave on a string, we need a function that gives the shape of the wave.

$$y = h(x, t)$$

Where y is the transverse displacement of any string element as a function h of the time t and the position x of the element along the string.

h can be a sine or cosine function

Wave Variables



Amplitude

Oscillating term

Displacement

Phase

$$y(x,t) = y_m \sin(kx - \omega t)$$

Angular wave number

Position

Time

Angular frequency

Fig. 16-3 The names of the quantities in Eq. 16-2, for a transverse sinusoidal wave.

Wave Variables

$$y(x, t) = y_m \sin(kx - \omega t).$$

The **amplitude** y_m of a wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them.

The **phase of the wave** is the argument ($kx - \omega t$) of the sine function. As the wave sweeps through a string element at a particular position x , the phase changes linearly with time t .

The **wavelength** λ of a wave is the distance parallel to the direction of the wave's travel between repetitions of the shape of the wave (or wave shape). It is related to the angular wave number, k , by

$$k = \frac{2\pi}{\lambda} \quad (\text{angular wave number}).$$

The **period of oscillation** T of a wave is the time for an element to move through one full oscillation. It is related to the angular frequency, ω , by

$$\omega = \frac{2\pi}{T}$$

The **frequency** f of a wave is defined as $1/T$ and is related to the angular frequency ω by

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

A phase constant ϕ in the wave function:
 $y = y_m \sin(kx - \omega t + \phi)$. The value of ϕ can be chosen so that the function gives some other displacement and slope at $x = 0$ when $t = 0$.

The Speed of a Traveling Wave

$$y(x, t) = y_m \sin(kx - \omega t).$$

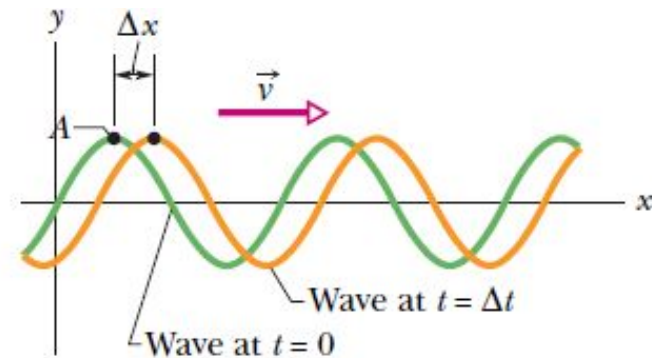


Fig. 16-7 Two snapshots of the wave of Fig. 16-4, at time $t = 0$ and then at time $t = \Delta t$. As the wave moves to the right at velocity \vec{v} , the entire curve shifts a distance Δx during Δt . Point A “rides” with the wave form, but the string elements move only up and down.

$$kx - \omega t = \text{a constant.}$$

$$k \frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = v = \frac{\omega}{k}.$$



$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad (\text{wave speed}).$$

Example, Transverse Wave

A wave traveling along a string is described by

$$y(x, t) = 0.00327 \sin(72.1x - 2.72t), \quad (16-18)$$

in which the numerical constants are in SI units (0.00327 m, 72.1 rad/m, and 2.72 rad/s).

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(a) What is the amplitude of this wave?

$$y_m = 0.00327 \text{ m} = 3.27 \text{ mm.} \quad (\text{Answer})$$

(b) What are the wavelength, period, and frequency of this wave?

$$k = 72.1 \text{ rad/m} \quad \text{and} \quad \omega = 2.72 \text{ rad/s.}$$

We then relate wavelength λ to k via Eq. 16-5:

$$\begin{aligned} \lambda &= \frac{2\pi}{k} = \frac{2\pi \text{ rad}}{72.1 \text{ rad/m}} \\ &= 0.0871 \text{ m} = 8.71 \text{ cm.} \end{aligned} \quad (\text{Answer})$$

Next, we relate T to ω with Eq. 16-8:

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{2.72 \text{ rad/s}} = 2.31 \text{ s,} \quad (\text{Answer})$$

and from Eq. 16-9 we have

$$f = \frac{1}{T} = \frac{1}{2.31 \text{ s}} = 0.433 \text{ Hz.} \quad (\text{Answer})$$

(c) What is the velocity of this wave?

$$\begin{aligned} v &= \frac{\omega}{k} = \frac{2.72 \text{ rad/s}}{72.1 \text{ rad/m}} = 0.0377 \text{ m/s} \\ &= 3.77 \text{ cm/s.} \end{aligned} \quad (\text{Answer})$$

(d) What is the displacement y of the string at $x = 22.5 \text{ cm}$ and $t = 18.9 \text{ s}$?

$$\begin{aligned} y &= 0.00327 \sin(72.1 \times 0.225 - 2.72 \times 18.9) \\ &= (0.00327 \text{ m}) \sin(-35.1855 \text{ rad}) \\ &= (0.00327 \text{ m})(0.588) \\ &= 0.00192 \text{ m} = 1.92 \text{ mm.} \end{aligned} \quad (\text{Answer})$$

Thus, the displacement is positive. (Be sure to change your calculator mode to radians before evaluating the sine. Also, note that we do *not* round off the sine's argument before evaluating the sine. Also note that both terms in the argument are properly in radians, a dimensionless quantity.)

Example, Transverse Wave, Transverse Velocity, and Acceleration

In the preceding sample problem, we showed that at $t = 18.9$ s the transverse displacement y of the element of the string at $x = 22.5$ cm due to the wave of Eq. 16-18 is 1.92 mm.

(a) What is u , the transverse velocity of the same element of the string, at that time? (This velocity, which is associated with the transverse oscillation of an element of the string, is in the y direction. Do not confuse it with v , the constant velocity at which the *wave form* travels along the x axis.)

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KEY IDEAS

The transverse velocity u is the rate at which the displacement y of the element is changing. In general, that displacement is given by

$$y(x, t) = y_m \sin(kx - \omega t). \quad (16-20)$$

For an element at a certain location x , we find the rate of change of y by taking the derivative of Eq. 16-20 with respect to t while treating x as a constant. A derivative taken while one (or more) of the variables is treated as a constant is called a *partial derivative* and is represented by the symbol $\partial/\partial x$ rather than d/dx .

Calculations: Here we have

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t). \quad (16-21)$$

Next, substituting numerical values from the preceding sample problem, we obtain

$$\begin{aligned} u &= (-2.72 \text{ rad/s})(3.27 \text{ mm}) \cos(-35.1855 \text{ rad}) \\ &= 7.20 \text{ mm/s}. \end{aligned} \quad (\text{Answer})$$

Thus, at $t = 18.9$ s, the element of the string at $x = 22.5$ cm is moving in the positive direction of y with a speed of 7.20 mm/s.

(b) What is the transverse acceleration a_y of the same element at that time?

KEY IDEA

The transverse acceleration a_y is the rate at which the transverse velocity of the element is changing.

Calculations: From Eq. 16-21, again treating x as a constant but allowing t to vary, we find

$$a_y = \frac{\partial u}{\partial t} = -\omega^2 y_m \sin(kx - \omega t).$$

Comparison with Eq. 16-20 shows that we can write this as

$$a_y = -\omega^2 y.$$

We see that the transverse acceleration of an oscillating string element is proportional to its transverse displacement but opposite in sign. This is completely consistent with the action of the element itself—namely, that it is moving transversely in simple harmonic motion. Substituting numerical values yields

$$\begin{aligned} a_y &= -(2.72 \text{ rad/s})^2(1.92 \text{ mm}) \\ &= -14.2 \text{ mm/s}^2. \end{aligned} \quad (\text{Answer})$$

Thus, at $t = 18.9$ s, the element of string at $x = 22.5$ cm is displaced from its equilibrium position by 1.92 mm in the positive y direction and has an acceleration of magnitude 14.2 mm/s² in the negative y direction.

The Wave Equation

A travelling wave is always in the following form:

$$y(x, t) = f(x \pm vt)$$

Such functions are solutions of the wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad (\text{wave equation}).$$

It is a **linear** partial differential equation; when y_1 and y_2 are solutions, any **linear** combination of y_1 and y_2 (like $ay_1 + by_2$) is also a solution.

The Superposition of Waves

$$y'(x, t) = y_1(x, t) + y_2(x, t).$$

Overlapping waves algebraically add to produce a **resultant wave (or net wave)**.

Overlapping waves do not in any way alter the travel of each other.

They interfere but do not interact.

The principle of linear superposition is valid only when the amplitude is small.

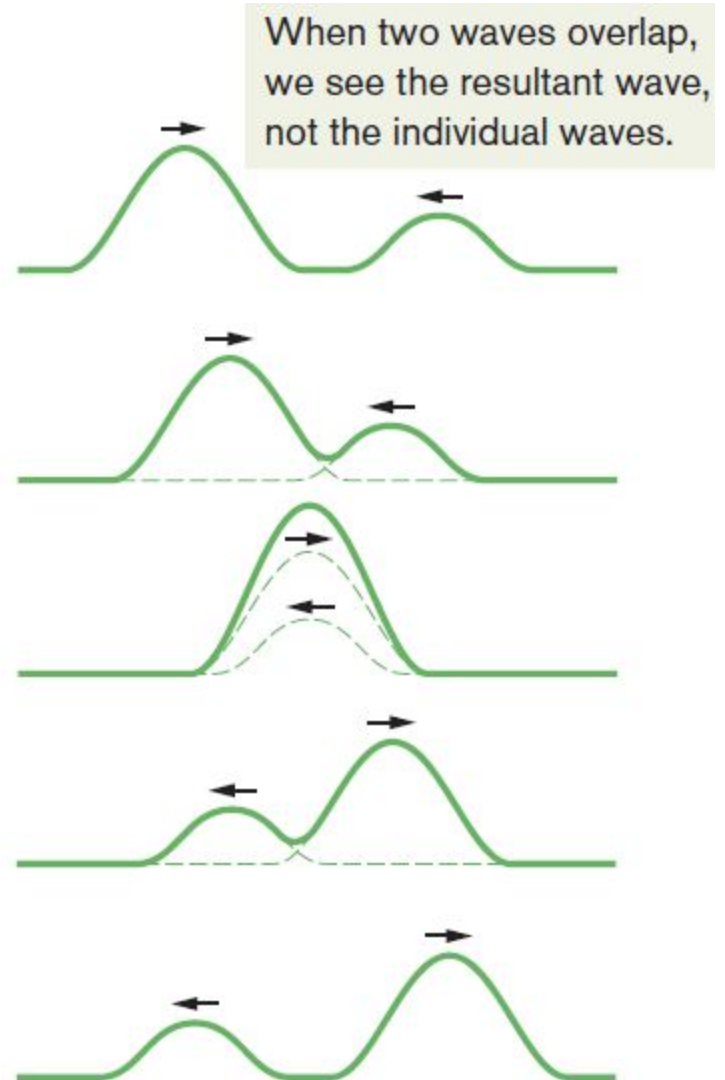


Fig. 16-11 A series of snapshots that show two pulses traveling in opposite directions along a stretched string. The superposition principle applies as the pulses move through each other.

Interference of Waves

If two sinusoidal waves of the same amplitude and wavelength travel in the *same* direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.

$$y_1(x, t) = y_m \sin(kx - \omega t)$$



$$y_2(x, t) = y_m \sin(kx - \omega t + \phi).$$

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$

$$= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi).$$



Displacement

$$y'(x, t) = [2y_m \cos \frac{1}{2}\phi] \sin(kx - \omega t + \frac{1}{2}\phi)$$

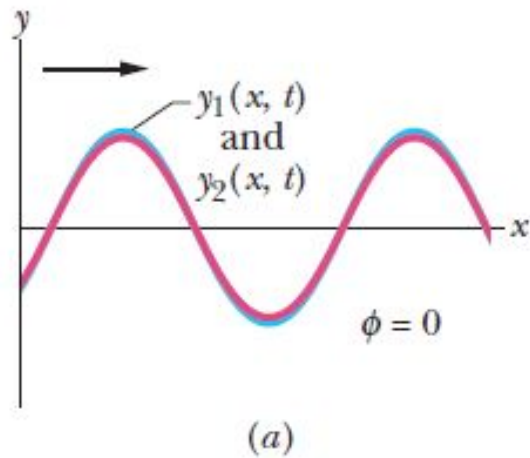
Magnitude
gives
amplitude

Oscillating
term

Fig. 16-12 The resultant wave of Eq. 16-51, due to the interference of two sinusoidal transverse waves, is also a sinusoidal transverse wave, with an amplitude and an oscillating term.

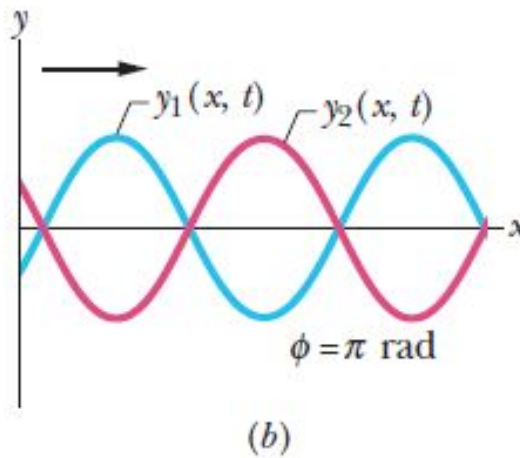
Interference of Waves

Being exactly in phase, the waves produce a large resultant wave.



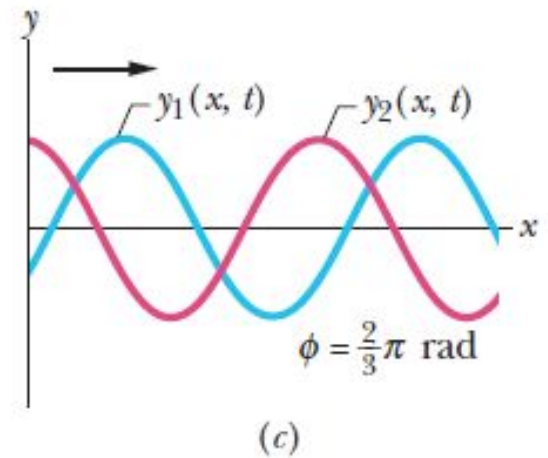
(a)

Being exactly out of phase, they produce a flat string.

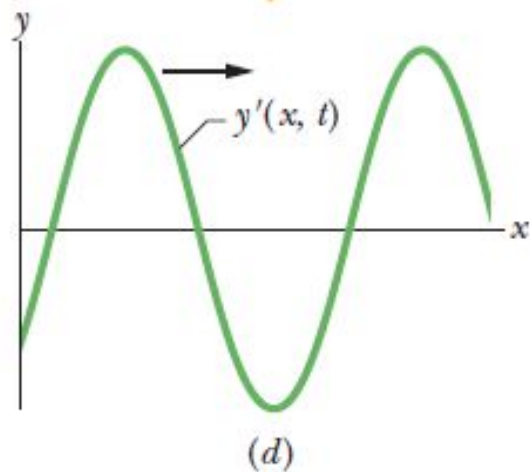


(b)

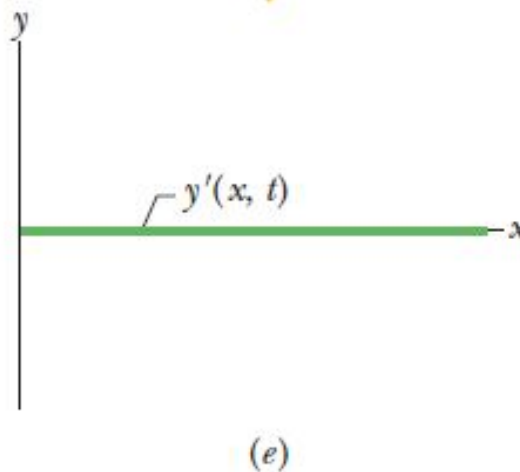
This is an intermediate situation, with an intermediate result.



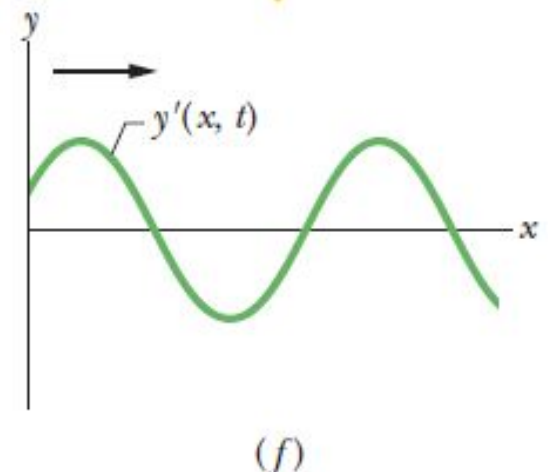
(c)



(d)



(e)



(f)

Interference of Waves

Table 16-1

Phase Difference and Resulting Interference Types^a

Phase Difference, in			Amplitude of Resultant Wave	Type of Interference
Degrees	Radians	Wavelengths		
0	0	0	$2y_m$	Fully constructive
120	$\frac{2}{3}\pi$	0.33	y_m	Intermediate
180	π	0.50	0	Fully destructive
240	$\frac{4}{3}\pi$	0.67	y_m	Intermediate
360	2π	1.00	$2y_m$	Fully constructive
865	15.1	2.40	$0.60y_m$	Intermediate

^aThe phase difference is between two otherwise identical waves, with amplitude y_m , moving in the same direction.

Example, Transverse Wave:

Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100° .

(a) What is the amplitude y'_m of the resultant wave due to the interference, and what is the type of this interference?

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(a) What is the amplitude y'_m of the resultant wave due to the interference, and what is the type of this interference?

KEY IDEA

These are identical sinusoidal waves traveling in the *same direction* along a string, so they interfere to produce a sinusoidal traveling wave.

Calculations: Because they are identical, the waves have the *same amplitude*. Thus, the amplitude y'_m of the resultant wave is given by Eq. 16-52:

$$\begin{aligned}y'_m &= |2y_m \cos \tfrac{1}{2}\phi| = |(2)(9.8 \text{ mm}) \cos(100^\circ/2)| \\ &= 13 \text{ mm.} \quad (\text{Answer})\end{aligned}$$

We can tell that the interference is *intermediate* in two ways. The phase difference is between 0 and 180° , and, correspondingly, the amplitude y'_m is between 0 and $2y_m$ ($= 19.6 \text{ mm}$).

(b) What phase difference, in radians and wavelengths, will give the resultant wave an amplitude of 4.9 mm?

Calculations: Now we are given y'_m and seek ϕ . From Eq. 16-52,

$$y'_m = |2y_m \cos \tfrac{1}{2}\phi|,$$

we now have

$$4.9 \text{ mm} = (2)(9.8 \text{ mm}) \cos \tfrac{1}{2}\phi,$$

which gives us (with a calculator in the radian mode)

$$\begin{aligned}\phi &= 2 \cos^{-1} \frac{4.9 \text{ mm}}{(2)(9.8 \text{ mm})} \\ &= \pm 2.636 \text{ rad} \approx \pm 2.6 \text{ rad.} \quad (\text{Answer})\end{aligned}$$

There are two solutions because we can obtain the same resultant wave by letting the first wave *lead* (travel ahead of) or *lag* (travel behind) the second wave by 2.6 rad. In wavelengths, the phase difference is

$$\begin{aligned}\frac{\phi}{2\pi \text{ rad/wavelength}} &= \frac{\pm 2.636 \text{ rad}}{2\pi \text{ rad/wavelength}} \\ &= \pm 0.42 \text{ wavelength.} \quad (\text{Answer})\end{aligned}$$