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21K-3195
BCS 1-J

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Homework 1

1. For x-axis:

$\vec{A} = 5\hat{i} - 3\hat{j} + 7\hat{k}$

$\vec{X} = i + 0\hat{j} + 0\hat{k}$

$|\vec{A}| = \sqrt{5^2 + (-3)^2 + 7^2}$

$|\vec{A}| = \sqrt{83}$

$|\vec{X}| = \sqrt{1^2}$

$|\vec{X}| = 1$

$\vec{A}.\vec{X} = |\vec{A}||\vec{X}|\cos\theta$

$(5\hat{i} - 3\hat{j} + 7\hat{k}).(\hat{i} + 0\hat{j} + 0\hat{k}) = \sqrt{1}\sqrt{83}\cos\theta_x$

$5 = \sqrt{83}\cos\theta_x$

$\cos\theta_x = \frac{5}{\sqrt{83}}$

$\theta_x = \cos^{-1}\left(\frac{5}{\sqrt{83}}\right)$

$\theta_x = 56.7°$

Angle between vector $\vec{A}$ and x-axis $= 56.7°$

For y-axis:

$\vec{Y} = 0\hat{i} + \hat{j} + 0\hat{k}$

$|\vec{Y}| = 1$

$\vec{A}.\vec{Y} = |\vec{A}||\vec{Y}|\cos\theta_y$

$(5\hat{i} - 3\hat{j} + 7\hat{k}).(0\hat{i} + \hat{j} + 0\hat{k}) = \sqrt{83}\cos\theta_y$

$-3 = \sqrt{83}\cos\theta_y$

Sufiyaan Usmani
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$$\cos\theta_y = -\frac{3}{\sqrt{83}}$$

$$\theta_y = \cos^{-1}\left(-\frac{3}{\sqrt{83}}\right)$$

$$\theta_y = 109.2^\circ$$

Angle between vector $\vec{A}$ and y-axis $= 109.2^\circ$

For z-axis:

$$\vec{Z} = 0\hat{i} + 0\hat{j} + \cancel{1}\hat{k}$$

$$\cancel{A}|\vec{Z}| = 1$$

$$\vec{A}\cdot\vec{Z} = |\vec{A}||\vec{Z}|\cos\theta$$

$$(5\hat{i} - 3\hat{j} + 7\hat{k})\cdot(0\hat{i} + 0\hat{j} + \hat{k}) = \sqrt{83}\cos\theta_z$$

$$7 = \sqrt{83}\cos\theta_z$$

$$\cos\theta_z = \frac{7}{\sqrt{83}}$$

$$\theta_z = \cos^{-1}\left(\frac{7}{\sqrt{83}}\right)$$

$$\theta_z = \cancel{38}\, 39.8^\circ$$

Angle between vector $\vec{A}$ and z-axis $= 39.8^\circ$

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2. $\vec{F} = q(\vec{V} \times \vec{B})$

$\vec{V} = 2\hat{i} + 4\hat{j} + 6\hat{K}$, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{K}$

$$\vec{V} \times \vec{B} = \begin{vmatrix} i & j & K \\ 2 & 4 & 6 \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 4 & 6 \\ B_y & B_z \end{vmatrix} - j\begin{vmatrix} 2 & 6 \\ B_x & B_z \end{vmatrix} + K\begin{vmatrix} 2 & 4 \\ B_x & B_y \end{vmatrix}$$

$$= \hat{i}(4B_z - 6B_y) - \hat{j}(2B_z - 6B_x) + \hat{k}(2B_y - 4B_x)$$

As $B_x = B_y$,

$\vec{V} \times \vec{B} = (4B_z - 6B_x)\hat{i} - (2B_z - 6B_x)\hat{j} + (2B_x - 4B_x)\hat{K}$

$\vec{V} \times \vec{B} = (4B_z - 6B_x)\hat{i} - (2B_z - 6B_x)\hat{j} - 2B_x\hat{K}$

$\vec{F} = q(\vec{V} \times \vec{B})$, $q = 2$

$4\hat{i} - 20\hat{j} + 12\hat{K} = 2[(4B_z - 6B_x)\hat{i} - (2B_z - 6B_x)\hat{j} - 2B_x\hat{K}]$

$2\hat{i} - 10\hat{j} + 6\hat{K} = (4B_z - 6B_x)\hat{i} - (2B_z - 6B_x)\hat{j} - 2B_x\hat{K}$

$-2B_x = 6$

$B_x = -3$

$B_y = B_x$

$B_y = -3$

$4B_z - 6(-3) = 2$

$4B_z + 18 = 2$

$4B_z = -16$

$B_z = -4$

$\vec{B} = -3\hat{i} - 3\hat{j} - 4\hat{K}$

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3. $|\vec{A}| = |\vec{B}| = |\vec{A}+\vec{B}|$

Let $|\vec{A}| = x$

$|\vec{A}| = |\vec{B}| = |\vec{A}+\vec{B}| = x$

$$x^2 = x^2 + x^2 + 2x^2\cos\theta$$

$$x^2 - 2x^2 = 2x^2\cos\theta$$

$$-x^2 = 2x^2\cos\theta$$

$$2x^2\cos\theta = -x^2$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$$

~~$\theta = 120^\circ$~~

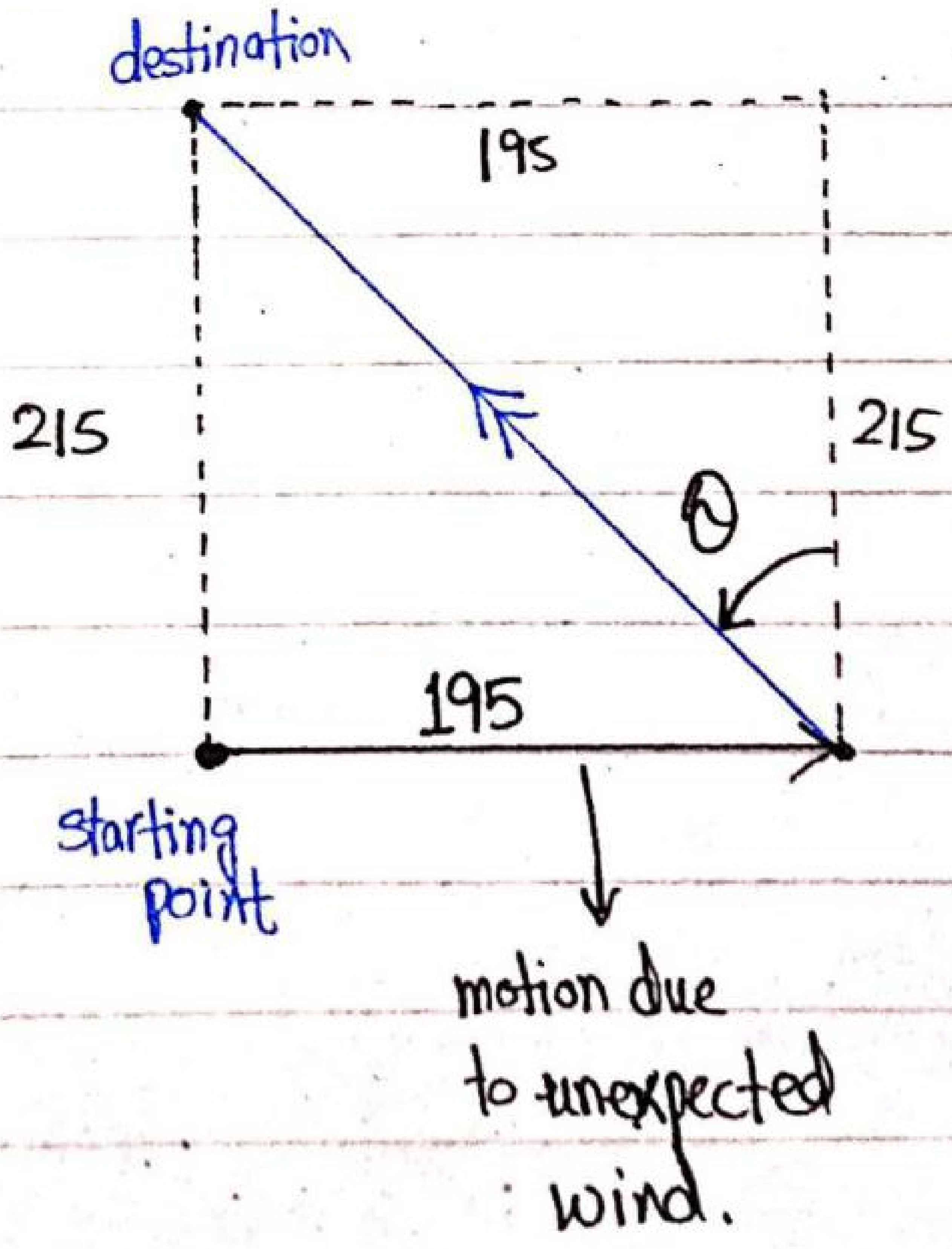
$$\theta = 120^\circ$$

4i. $d = \sqrt{215^2 + 195^2}$

$d = 290$ Km

ii. $\theta = \tan^{-1}\left(\frac{195}{215}\right)$

$\theta = 42.2^\circ$ west of due north



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5 i. $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta$

$= (5)(6) \sin 90$

$|\vec{a} \times \vec{b}| = 30$ out of the page $(+\hat{z})$

ii. $|\vec{c}| = \sqrt{5^2 + 6^2}$

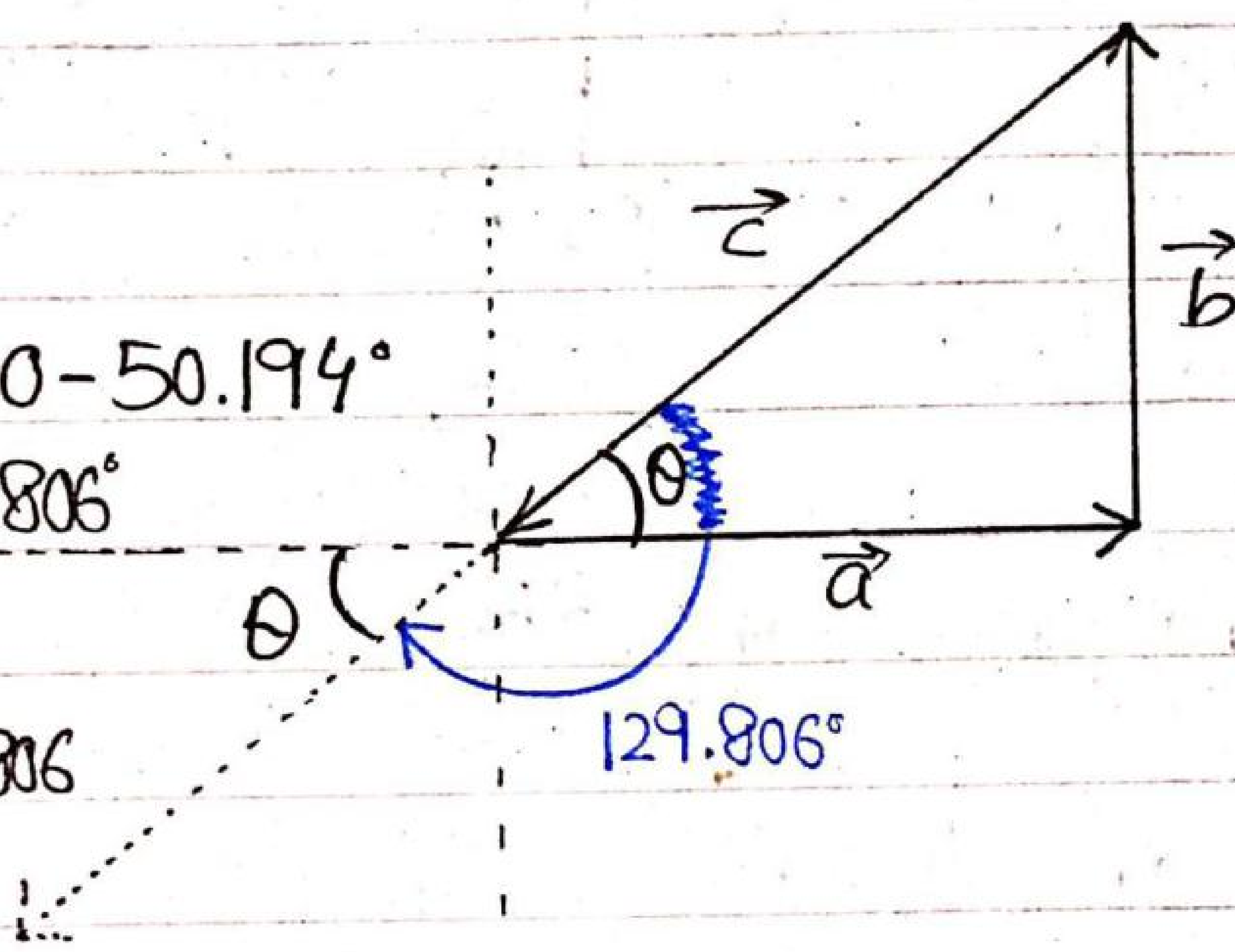
$|\vec{c}| = \sqrt{61}$

$\theta = \tan^{-1}\left(\frac{6}{5}\right)$

$\theta = 50.194°$

angle between $\vec{a}$ and $\vec{c} = 180 - 50.194°$

$= 129.806°$



$|\vec{a} \times \vec{c}| = (5)(\sqrt{61}) \sin 129.806$

$\vec{a} \times \vec{c} = 30$

$|\vec{a} \times \vec{c}| = 30$ into the page $(-z)$

6. $\vec{A} \cdot \vec{B} = |A||B| \cos 90$

$\vec{A} \cdot \vec{B} = 0$

$(2i + aj + K) \cdot (4i - 2j - 2K)$

$8 - 2a - 2 = 0$

$2a = 6$

$a = 3$

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7. Solution is same as Q3

8 i. Area of parallelogram $= |\vec{A} \times \vec{B}|$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix}$$

$$= (6+9)\hat{i} - (-2+12)\hat{j} + (6+24)\hat{k}$$

$$= 15\hat{i} - 10\hat{j} + 30K$$

$$= \sqrt{15^2 + (-10)^2 + (30)^2}$$

Area of parallelogram = 35 units²

ii. Area of triangle $= \frac{1}{2}|\vec{A} \times \vec{B}|$

$$= \frac{1}{2}(35)$$

Area of triangle = 17.5 units²

9. $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$

$$\sqrt{(-5)^2 + (2)^2} = (3)(3)\sin\theta$$

$$9\sin\theta = \sqrt{29}$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{29}}{9}\right)$$

$$\theta = 36.75^\circ$$

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10. $|\vec{w}| = \sqrt{3^2+4^2}$

$|\vec{w}| = 5$

$|\vec{v}| = |\vec{w}|$ and $\vec{v} = 0\hat{i} + a\hat{j}$

$, a = 5$

$\vec{v} = 5\hat{j}$

$\vec{u} + \vec{w} = \vec{v}$

$\vec{u} = \vec{v} - \vec{w}$

$= 5\hat{j} - (3\hat{i} + 4\hat{j})$

~~$\vec{u} = -3\hat{i}$~~

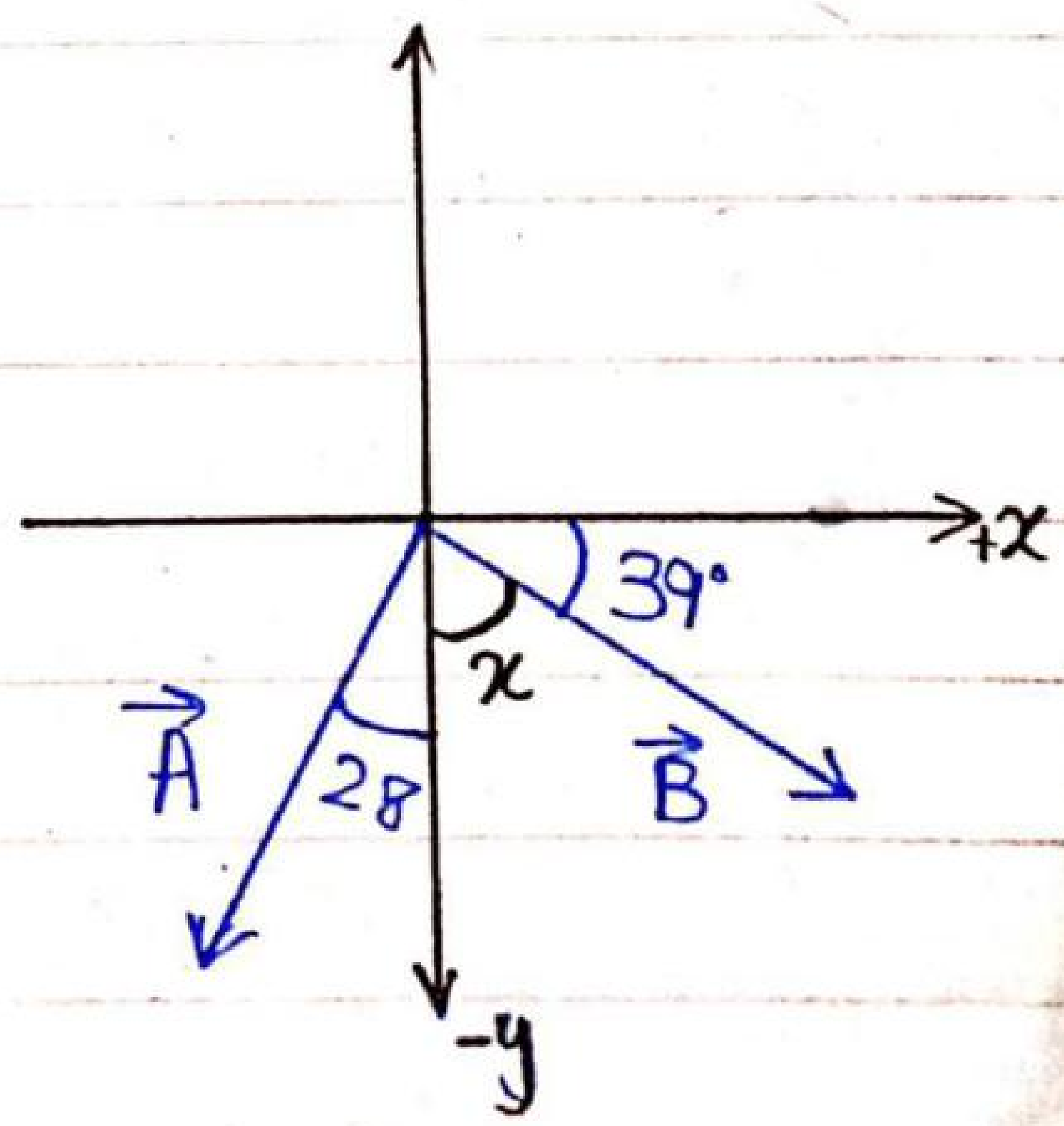
$\vec{u} = -3\hat{i} + \hat{j}$

$|\vec{u}| = \sqrt{(-3)^2 + (1)^2}$

$= \sqrt{9+1}$

$\boxed{|\vec{u}| = \sqrt{10}}$

11. $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$



$x = 90 - 39$

↳ see diagram

$x = 51°$

$\theta = 51 + 28$

$\theta = 79°$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$48 = 9|\vec{B}|\cos 79$

$|\vec{B}| = \dfrac{48}{9\cos 79}$

$|\vec{B}| = 27.95 \text{ m}$

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21K-3195
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12 i. $\hat{i} \cdot \hat{k} = |\hat{i}||\hat{k}|\cos\theta$

$= (1)(1)\cos 90$

$\hat{i} \cdot \hat{k} = 0$

$\hat{j} \cdot (-\hat{j}) = |1||1|\cos 180$

$\hat{j} \cdot (-\hat{j}) = -1$

ii. direction of $\hat{k} \times \hat{j}$ = in the direction of $-\hat{i}$ (west)

direction of $(-\hat{i}) \times (-\hat{j})$ = in the direction of $+\hat{k}$ (out of the page)