Calculus

MT1003

Assignment 1 | CH 2

Teacher Name: Miss Javeria Iftikhar

Name: **Sufiyaan Usmani**

Roll no: **21K-3195**

Section: **J**

Sufiyaan Usmani
21K-3195
BCS 1-J

Exercise Set 2.3

1. $\frac{dy}{dx} = 28x^6$

2. $\frac{dy}{dx} = -36x^{11}$

3. $\frac{dy}{dx} = 24x^7 + 2$

4. $y = \frac{1}{2}x^4 + \frac{7}{2}$

$\frac{dy}{dx} = 2x^3$

5. $\frac{dy}{dx} = 0$

6. $\frac{dy}{dx} = \sqrt{2}$

7. $y = -\frac{1}{3}x^7 - \frac{2}{3}x + 3$

$\frac{dy}{dx} = -\frac{7}{3}x^6 - \frac{2}{3}$

$\frac{dy}{dx} = -\frac{1}{3}(7x^6 + 2)$

Sufiyaan Usmani
21K-3195
BCS 1-J

8. $y = \frac{1}{5}x^2 + \frac{1}{5}$

$\frac{dy}{dx} = \frac{2x}{5}$

9. $f(x) = x^{-3} + x^{-7}$

$f'(x) = -3x^{-4} - 7x^{-8}$

10. $f(x) = x^{\frac{1}{2}} + x^{-1}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - x^{-2}$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$

11. $f(x) = -3x^{-8} + 2x^{\frac{1}{2}}$

$f'(x) = 24x^{-9} + x^{-\frac{1}{2}}$

$f'(x) = \frac{24}{x^9} + \frac{1}{\sqrt{x}}$

12. $f(x) = 7x^{-6} - 5x^{\frac{1}{2}}$

$= -42x^{-7} - \frac{5}{2}x^{-\frac{1}{2}}$

$f'(x) = -\frac{42}{x^7} - \frac{5}{2\sqrt{x}}$

Sufiyaan Usmani
21K-3195
BCS 1-J

$\cancel{13.\ f(x) = x^e + x^{-5}}$

$\cancel{f'(x) = ex^{e-1} - 5x^{-6}}$

13. $f(x) = x^e + x^{-\sqrt{10}}$

$= x^e + x^{-\sqrt{10}}$

$f'(x) = ex^{e-1} - \sqrt{10}\, x^{-\sqrt{10}-1}$

14. $f(x) = \sqrt[3]{8x^{-1}}$

$= \sqrt[3]{8}\,\sqrt[3]{x^{-1}}$

$= 2x^{-\frac{1}{3}}$

$f'(x) = -\frac{2}{3}x^{-\frac{4}{3}}$

15. $f(x) = 9x^4 + 6x^2 + 1$

$f'(x) = 36x^3 + 12x$

16. $f'(x) = 3ax^2 + 2bx + c$

17. $y' = 10x - 3$

$y'(1) = 10(1) - 3$

$y'(1) = 7$

18. $y = x^{\frac{1}{2}} + 2x^{-1}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2}$

$y' = \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$

Sufiyaan Usmani
21K-3195
BCS 1-J

$$y'(1) = \frac{1}{2\sqrt{1}} - \frac{2}{1^2}$$

$$= \frac{1}{2} - 2$$

$$y'(1) = -\frac{3}{2}$$

19. $\frac{dx}{dt} = 2t - 1$

20. $x = \frac{1}{3}t + \frac{1}{3}t^{-1}$

$$\frac{dx}{dt} = \frac{1}{3} - \frac{1}{3}t^{-2}$$

$$\frac{dx}{dt} = \frac{1}{3} - \frac{1}{3t^2}$$

21. $\frac{dy}{dx} = 1 + 2x + 3x^2 + 4x^3 + 5x^4$

$$\left.\frac{dy}{dx}\right|_{x=1} = 1 + 2(1) + 3(1)^2 + 4(1)^3 + 5(1)^4$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 15$$

Sufiyaan Usmani
21K-3195
BCS 1-J

22. $y = \frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} + 1 + x + x^2 + x^3$

$y = x^{-3} + x^{-2} + x^{-1} + 1 + x + x^2 + x^3$

$\frac{dy}{dx} = -3x^{-4} - 2x^{-3} - x^{-2} + 1 + 2x + 3x^2$

$\frac{dy}{dx}\Big|_{x=1} = -3(1)^{-4} - 2(1)^{-3} - (1)^{-2} + 1 + 2(1) + 3(1)^2$

$= -3 - 2 - 1 + 1 + 2 + 3$

$\frac{dy}{dx}\Big|_{x=1} = 0$

23. $y = (1 - x^2)(1 + x^2)(1 + x^4)$

$= (1 + x^2 - x^2 - x^4)(1 + x^4)$

$= (1 - x^4)(1 + x^4)$

$y = 1 - x^8$

$\frac{dy}{dx} = -8x^7$

$\frac{dy}{dx}\Big|_{x=1} = -8(1)^7$

$\frac{dy}{dx}\Big|_{x=1} = -8$

Sufiyaan Usmani
21K-3195
BCS 1-J

24. $\frac{dy}{dx} = 24x^{23} + 24x^{11} + 24x^{7} + 24x^{5}$

$\left.\frac{dy}{dx}\right|_{x=1} = 24(1)^{23} + 24(1)^{11} + 24(1)^{7} + 24(1)^{5}$

$\left.\frac{dy}{dx}\right|_{x=1} = 96$

41a. $\frac{dy}{dx} = 21x^{2} - 10x + 1$

$\frac{d^2y}{dx^2} = 42x - 10$

b. $\frac{dy}{dx} = 24x - 2$

$\frac{d^2y}{dx^2} = 24$

c. $y = 1 + x^{-1}$

$\frac{dy}{dx} = -x^{-2}$

$\frac{d^2y}{dx^2} = 2x^{-3}$

$\frac{d^2y}{dx^2} = \frac{2}{x^3}$

Sufiyaan Usmani
21K-3195
BCS 1-J

d. $y = 35x^5 + 5x^3 - 21x^3 - 3x$

$y = 35x^5 - 16x^3 - 3x$

$\frac{dy}{dx} = 175x^4 - 48x^2 - 3$

~~$\frac{d^2y}{dx^2} = 700x^3 - 96x$~~

$\frac{d^2y}{dx^2} = 700x^3 - 96x$

42a. $\frac{dy}{dx} = 28x^6 - 15x^2 + 2$

$\frac{d^2y}{dx^2} = 168x^5 - 30x$

b. $\frac{dy}{dx} = 3$

$\frac{d^2y}{dx^2} = 0$

c. $y = \frac{3}{5} - \frac{2}{5}x^{-1}$

$\frac{dy}{dx} = \frac{2}{5}x^{-2}$

$\frac{d^2y}{dx^2} = -\frac{4}{5}x^{-3}$

$\frac{d^2y}{dx^2} = -\frac{4}{5x^3}$

Sufiyaan Usmani
21K-3195
BCS 1-J

d. $y = 2x^4 + 3x^3 - 10x - 15$

$\cancel{y=}$

$\frac{dy}{dx} = 8x^3 + 9x^2 - 10$

$\frac{d^2y}{dx^2} = 24x^2 + 18x$

43a. $y' = -5x^{-6} + 5x^4$

$y'' = 30x^{-7} + 20x^3$

$y''' = -210x^{-8} + 60x^2$

b. $y = x^{-1}$

$y' = -x^{-2}$

$y'' = 2x^{-3}$

$y''' = -6x^{-4}$

c. $y' = 3ax^2 + b$

$y'' = 6ax$

$y''' = 6a$

44a. $y' = 10x - 4$

$y'' = 10$

$y''' = 0$

Sufiyaan Usmani
21K-3195
BCS 1-J

b. $y' = -6x^{-3} - 4x^{-2} + 1$

$y'' = 18x^{-4} + 8x^{-3}$

$y''' = -72x^{-5} - 24x^{-4}$

c. $y' = 4ax^3 + 2bx$

$y'' = 12ax^2 + 2b$

$y''' = 24ax$

45a. $f(x) = 3x^2 - 2$

$f'(x) = 6x$

$f''(x) = 6$

$f'''(x) = 0$

$f'''(2) = 0$

b. $y = 6x^5 - 4x^2$

$\frac{dy}{dx} = 30x^4 - 8x$

$\frac{d^2y}{dx^2} = 120x^3 - 8$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 120(1)^3 - 8$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 112$

Sufiyaan Usmani
21K-3195
BCS 1-J

c. $\frac{d}{dx}(x^{-3}) = -3x^{-4}$

$\frac{d}{dx^2}(-3x^{-4}) = 12x^{-5}$

$\frac{d}{dx^3}(12x^{-5}) = -60x^{-6}$

$\frac{d}{dx^4}(-60x^{-6}) = 360x^{-7}$

$\frac{d^4}{dx^4}[x^{-3}]\Big|_{x=1} = 360(1)^{-7}$

$\frac{d^4}{dx^4}[x^{-3}] = 360$

46a. $y' = 16x^3 + 6x^2$

$y'' = 48x^2 + 12x$

$y''' = 96x + 12$

$y'''(0) = 96(0) + 12$

~~$y'''(0) = 12$~~

$y'''(0) = 12$

b. $y = 6x^{-4}$

$\frac{d}{dx}(6x^{-4}) = -24x^{-5}$

$\frac{d}{dx^2}(-24x^{-5}) = 120x^{-6}$

$\frac{d}{dx^3}(120x^{-6}) = -720x^{-7}$

Sufiyaan Usmani
21K-3195
BCS 1-J

$$\frac{d}{dx^4}(-720x^{-7}) = 5040x^{-8}$$

$$\left.\frac{d^4y}{dx^4}\right|_{x=1} = 5040(1)^{-8}$$

$$\left.\frac{d^4y}{dx^4}\right|_{x=1} = 5040$$

47. $y' = 3x^2 + 3$

$y'' = 6x$

$y''' = 6$

$$y''' + xy'' - 2y' = 0$$

$\cancel{6+6}$

$$6 + x(6x) - 2(3x^2+3) = 0$$

$$6 + 6x^2 - 6x^2 - 6 = 0$$

$$0 = 0 \quad \text{(Shown)}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

Exercise Set 2.4

1 a. $f(x) = 2x^2 - x + 2x - 1$ 2.4

$f(x) = 2x^2 + x - 1$

$f'(x) = 4x + 1$

b. $f'(x) = vu' + uv'$

$= (2x-1)(1) + (x+1)(2)$

$= 2x - 1 + 2x + 2$

$f'(x) = 4x + 1$

2a. $f(x) = 3x^4 + 6x^2 - x^2 - 2$

$f(x) = 3x^4 + 5x^2 - 2$

$f'(x) = 12x^3 + 10x$

b. $f'(x) = (x^2+2)(6x) + (3x^2-1)(2x)$

$= 6x^3 + 12x + 6x^3 - 2x$

$f'(x) = 12x^3 + 10x$

3a. $f(x) = x^4 - 1$

$f'(x) = 4x^3$

b. $f'(x) = (x^2-1)(2x) + (x^2+1)(2x)$

$= 2x^3 - 2x + 2x^3 + 2x$

$f'(x) = 4x^3$

Sufiyaan Usmani
21K-3195
BCS 1-J

4a. $f(x) = x^3 - x^2 + x + x^2 - x + 1$

$f(x) = x^3 + 1$

$f'(x) = 3x^2$

b. $f'(x) = (x^2 - x + 1)(1) + (x+1)(2x-1)$

$= x^2 - x + 1 + 2x^2 - x + 2x - 1$

$f'(x) = 3x^2$

5. $u = 3x^2 + 6$ $\quad$ $v = 2x - \frac{1}{4}$

$u' = 6x$

$v' = 2$

$f'(x) = \left(2x - \frac{1}{4}\right)(6x) + (3x^2+6)(2)$

$= 12x^2 - \frac{3}{2}x + 6x^2 + 12$

$= 18x^2 - \frac{3}{2}x + 12$

6. $\cancel{u = 3x^2}$ $u = 2 - x - 3x^3$ $\quad$ $v = 7 + x^5$

$u' = -1 - 9x^2$ $\quad$ $v' = 5x^4$

$f'(x) = (7+x^5)(-1-9x^2) + (2-x-3x^3)(5x^4)$

$= -7 - 63x^2 - x^5 - 9x^7 + 10x^4 - 5x^5 - 15x^7$

$f'(x) = -24x^7 - 6x^5 + 10x^4 - 63x^2 - 7$

Sufiyaan Usmani
21K-3195
BCS 1-J

7. $u = x^3 + 7x^2 - 8$ $\quad$ $V = 2x^{-3} + x^{-4}$

$u' = 3x^2 + 14x$ $\quad$ $V' = -6x^{-4} - 4x^{-5}$

$$f'(x) = (2x^{-3} + x^{-4})(3x^2 + 14x) + (x^3 + 7x^2 - 8)(-6x^{-4} - 4x^{-5})$$
$$= 6x^{-1} + 28x^{-2} + 3x^{-2} + 14x^{-3} - 6x^{-1} - 4x^{-2} - 42x^{-2} - 28x^{-3}$$
$$+ 48x^{-4} + 32x^{-5}$$
$$f'(x) = -15x^{-2} - 14x^{-3} + 48x^{-4} + 32x^{-5}$$

8. $u = x^{-1} + x^{-2}$ $\quad$ $V = 3x^3 + 27$

$u' = -x^{-2} - 2x^{-3}$ $\quad$ $V' = 9x^2$

$$f'(x) = (3x^3 + 27)(-x^{-2} - 2x^{-3}) + (x^{-1} + x^{-2})(9x^2)$$
$$= -3x - 6 - 27x^{-2} - 54x^{-3} + 9x + 9$$
$$= 3 + 6x - 27x^{-2} - 54x^{-3}$$

9. $u = x - 2$ $\quad$ $V = x^2 + 2x + 4$

$u' = 1$ $\quad$ $V' = 2x + 2$

$$f'(x) = (x^2 + 2x + 4)(1) + (x - 2)(2x + 2)$$
$$= x^2 + 2x + 4 + 2x^2 + 2x - 4x - 4$$
$$f'(x) = 3x^2$$

10

Sufiyaan Usmani
21K-3195
BCS 1-J

10. $u = x^2 + x$ $\quad$ $v = x^2 - x$

$u' = 2x + 1$ $\quad$ $v' = 2x - 1$

$$f'(x) = (x^2 - x)(2x+1) + (x^2+x)(2x-1)$$
$$= 2x^3 + x^2 - 2x^2 - x + 2x^3 - x^2 + 2x^2 - x$$
$$f'(x) = 4x^3 - 2x$$

11. $u = 3x + 4$ $\quad$ $v = x^2 + 1$

$u' = 3$ $\quad$ $v' = 2x$

$$f'(x) = \frac{vu' - uv'}{v^2}$$
$$= \frac{(x^2+1)(3) - (3x+4)(2x)}{(x^2+1)^2}$$
$$= \frac{3x^2 + 3 - 6x^2 - 8x}{(x^2+1)^2}$$
$$f'(x) = \frac{-3x^2 - 8x + 3}{(x^2+1)^2}$$

12. $u = x - 2$ $\quad$ $v = x^4 + x + 1$

$u' = 1$ $\quad$ $v' = 4x^3 + 1$

$$f'(x) = \frac{(x^4+x+1)(1) - (x-2)(4x^3+1)}{(x^4+x+1)^2}$$
$$= \frac{x^4 + x + 1 - 4x^4 - x + 8x^3 + 2}{(x^4+x+1)^2}$$
$$f'(x) = \frac{-3x^4 + 8x^3 + 3}{(x^4 + x + 1)^2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

13. $u = x^2$ $\qquad V = 3x-4$

$u' = 2x$ $\qquad V' = 3$

$$f'(x) = \frac{(3x-4)(2x) - (x^2)(3)}{(3x-4)^2}$$

$$= \frac{6x^2 - 8x - 3x^2}{(3x-4)^2}$$

$$f'(x) = \frac{3x^2 - 8x}{(3x-4)^2}$$

14. $u = 2x^2+5$ $\qquad V = 3x-4$

$u' = 4x$ $\qquad V' = 3$

$$f'(x) = \frac{(3x-4)(4x) - (2x^2+5)(3)}{(3x-4)^2}$$

$$= \frac{12x^2 - 16x - 6x^2 - 15}{(3x-4)^2}$$

$$f'(x) = \frac{6x^2 - 16x - 15}{(3x-4)^2}$$

15. $\cancel{u} = f(x) = \frac{2x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x - 1}{x+3}$

$u = 2x^{\frac{3}{2}} + x - 2x^{\frac{1}{2}} - 1$ $\qquad V = x+3$

$u' = 3x^{\frac{1}{2}} + 1 - \cancel{2}x^{-\frac{1}{2}}$ $\qquad V' = 1$

$$f'(x) = \frac{(x+3)(3x^{\frac{1}{2}} + 1 - x^{-\frac{1}{2}}) - (2x^{\frac{3}{2}} + x - 2x^{+\frac{1}{2}} - 1)(1)}{(x+3)^2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

$$= \frac{3x^{\frac{3}{2}} + x - x^{+\frac{1}{2}} + 9x^{\frac{1}{2}} + 3 - 3x^{-\frac{1}{2}} - 2x^{\frac{3}{2}} - x + 2x^{+\frac{1}{2}} + 1}{(x+3)^2}$$

$$= \frac{x^{\frac{3}{2}} + 10x^{\frac{1}{2}} + 4 - 3x^{-\frac{1}{2}}}{(x+3)^2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

16. $f(x) = \dfrac{(2\sqrt{x}+1)(2-x)}{x^2+3x}$

$f(x) = \dfrac{4x^{\frac{1}{2}} - 2x^{\frac{3}{2}} + 2 - x}{x^2+3x}$

$u = 4x^{\frac{1}{2}} - 2x^{\frac{3}{2}} + 2 - x$ $\qquad$ $v = x^2+3x$

$u' = 2x^{-\frac{1}{2}} - 3x^{\frac{1}{2}} - 1$ $\qquad$ $v' = 2x+3$

$$f'(x) = \frac{(x^2+3x)(2x^{-\frac{1}{2}} - 3x^{\frac{1}{2}} - 1) - (4x^{\frac{1}{2}} - 2x^{\frac{3}{2}} + 2 - x)(2x+3)}{(x^2+3x)^2}$$

$$= \frac{2x^{\frac{3}{2}} - 3x^{\frac{5}{2}} - x^2 + 6x^{\frac{1}{2}} - 9x^{\frac{3}{2}} - 3x - (8x^{\frac{3}{2}} + 12x^{\frac{1}{2}} - 4x^{\frac{5}{2}} - 6x^{\frac{3}{2}} + 4x + 6 - 2x^2 - 3x)}{(x^2+3x)^2}$$

$$= \frac{-7x^{\frac{3}{2}} - 3x^{\frac{5}{2}} - x^2 + 6x^{\frac{1}{2}} - 3x - (2x^{\frac{3}{2}} + 12x^{\frac{1}{2}} - 4x^{\frac{5}{2}} + x + 6 - 2x^2)}{(x^2+3x)^2}$$

$$= \frac{-9x^{\frac{3}{2}} + x^{\frac{5}{2}} + x^2 - 6x^{\frac{1}{2}} - 4x - 6}{(x^2+3x)^2}$$

17. $f(x) = (2x+1)(1+x^{-1})(x^{-3}+7)$

$= (2x+2+1+x^{-1})(x^{-3}+7)$

$f(x) = (2x+3+x^{-1})(x^{-3}+7)$

$\cancel{= 2x^{-2} + 14x + 3x^{-3} + 21}$

$u = 2x+3+x^{-1}$ $\qquad$ $v = x^{-3}+7$

$u' = 2 - x^{-2}$ $\qquad$ $v' = -3x^{-4}$

$f'(x) = (x^{-3}+7)(2-x^{-2}) + (2x+3+x^{-1})(-3x^{-4})$

$= 2x^{-3} - x^{-5} + 14 - 7x^{-2} - 6x^{-3} - 9x^{-4} - 3x^{-5}$

$f'(x) = 14 - 7x^{-2} - 4x^{-3} - 9x^{-4} - 4x^{-5}$

Sufiyaan Usmani
21K-3195
BCS 1-J

18. $f(x) = (x^{-3} + 2x^{-4})(8x^9 + 4 - 6x^{10} - 3x)$

$u = x^{-3} + 2x^{-4}$ $\quad$ $v = 8x^9 + 4 - 6x^{10} - 3x$

$u' = -3x^{-4} - 8x^{-5}$ $\quad$ $v' = 72x^8 - 60x^9 - 3$

$f'(x) = (8x^9 + 4 - 6x^{10} - 3x)(-3x^{-4} - 8x^{-5}) + (x^{-3} + 2x^{-4})(72x^8 - 60x^9 - 3)$

$= -24x^5 - 64x^4 - 12x^{-4} - 32x^{-5} + 18x^6 + 48x^5 + 9x^{-3} + 24x^{-4} + 72x^5$

$- 60x^6 - 3x^{-3} + 144x^4 - 120x^5 - 6x^{-4}$

$f'(x) = -42x^6 - 24x^5 + 80x^4 + 6x^{-3} + 6x^{-4} - 32x^{-5}$

19. $f'(x) = 3(x^7 + 2x - 3)^2 \cdot (7x^6 + 2)$

20. $f'(x) = 4(x^2+1)^3(2x)$

$f'(x) = 8x(x^2+1)^3$

21. $u = 2x - 1$ $\quad$ $v = x + 3$

$u' = 2$ $\quad$ $v' = 1$

$$\frac{dy}{dx} = \frac{(x+3)(2) - (2x-1)}{(x+3)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(1+3)(2) - (2(1)-1)}{(1+3)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{7}{16}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

22. $u = 4x+1$ $\quad$ $v = x^2-5$

$u' = 4$ $\quad$ $v' = 2x$

$$\frac{dy}{dx} = \frac{(x^2-5)(4) - (4x+1)(2x)}{(x^2-5)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(1^2-5)(4) - (4(1)+1)(2(1))}{(1^2-5)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -\frac{13}{8}$$

23. $y = (3+2x^{-1})(x^{-5}+1)$

$u = 3+2x^{-1}$ $\quad$ $v = x^{-5}+1$

$u' = -2x^{-2}$ $\quad$ $v' = -5x^{-6}$

$$\frac{dy}{dx} = (x^{-5}+1)(-2x^{-2}) + (3+2x^{-1})(-5x^{-6})$$

$$\left.\frac{dy}{dx}\right|_{x=1} = (1^{-5}+1)(-2(1)^{-2}) + (3+2(1)^{-1})(-5(1^{-6}))$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -29$$

Sufiyaan Usmani
21K-3195
BCS 1-J

24. $y = \dfrac{(2x^7 - x^2)(x-1)}{(x+1)}$

$$y = \frac{2x^8 - 2x^7 - x^3 + x^2}{x+1}$$

$u = 2x^8 - 2x^7 - x^3 + x^2$ $\qquad$ $V = x + 1$

$u' = \cancel{8}16x^7 - 14x^6 - 3x^2 + 2x$ $\qquad$ $V' = 1$

$$\frac{dy}{dx} = \frac{(x+1)(16x^7 - 14x^6 - 3x^2 + 2x) - (2x^8 - 2x^7 - x^3 + x^2)(1)}{(x+1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{(1+1)(16 - 14 - 3 + 2) - (2 - 2 - 1 + 1)}{(1+1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

Exercise Set 2.5

1. $f'(x) = -4\sin x + 2\cos x$ 2.5

2. $f(x) = 5x^{-2} + \sin x$

$f'(x) = -10x^{-3} \cancel{+\cos x \sin x} + \cos x$

$f'(x) = -\dfrac{10}{x^3} + \cos x$

3. $u = -4x^2$ $v = \cos x$

$u' = -8x$ $v' = -\sin x$

$f'(x) = (\cos x)(-8x) + (-4x^2)(-\sin x)$

$f'(x) = -8x\cos x + 4x^2\sin x$

4. $f(x) = 2(\sin x)^2$

$f'(x) = 4\sin x\cos x$

5. $u = 5 - \cos x$ $v = 5 + \sin x$

$\cancel{u' = 5 + \sin x}$ $v' = \cos x$

$u' = \sin x$

$$f'(x) = \frac{(5+\sin x)(\sin x) - (5-\cos x)(\cos x)}{(5+\sin x)^2}$$

$$= \frac{5\sin x + \sin^2 x - 5\cos x + \cos^2 x}{(5+\sin x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x + 5\sin x - 5\cos x}{(5+\sin x)^2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

$$f'(x) = \frac{1 + 5(\sin x - \cos x)}{(5 + \sin x)^2}$$

6. $u = \sin x$ $\qquad v = x^2 + \sin x$

$u' = \cos x$ $\qquad v' = 2x + \cos x$

$$f'(x) = \frac{(x^2+\sin x)(\cos x) - (\sin x)(2x + \cos x)}{(x^2 + \sin x)^2}$$

$$= \frac{x^2\cos x + \sin x\cos x - 2x\sin x - \sin x\cos x}{(x^2+\sin x)^2}$$

$$f'(x) = \frac{x^2\cos x - 2x\sin x}{(x^2 + \sin x)^2}$$

7. $f'(x) = \sec x\tan x - \sqrt{2}\,\cancel{\tan x}\,\sec^2 x$

8. $u = x^2 + 1$ $\qquad v = \sec x$

$u' = 2x$ $\qquad v' = \sec x\tan x$

$f'(x) = (\cancel{\sin}(\sec x)(2x) + (x^2+1)(\sec x\tan x)$

$f'(x) = 2x\sec x + (x^2+1)(\sec x\tan x)$

9. $f'(x) = -4\operatorname{cosec} x\cot x + \operatorname{cosec}^2 x$

10. $f'(x) = -\sin x - \begin{bmatrix} u = x & v = \operatorname{cosec} x \\ u' = 1 & v' = -\operatorname{cosec} x \cdot \cot x \end{bmatrix}$

$= -\sin x - [\operatorname{cosec} x - x\operatorname{cosec} x\cot x]$

$f'(x) = -\sin x - \operatorname{cosec} x + x\operatorname{cosec} x\cot x$

Sufiyaan Usmani
21K-3195
BCS 1-J

11. $u = \sec x$ $\quad$ $v = \tan x$

$u' = \sec x \tan x$ $\quad$ $v' = \sec^2 x$

$$f'(x) = (\tan x)(\sec x \tan x) + (\sec x)(\sec^2 x)$$
$$= \sec x \tan^2 x + \sec^3 x$$

12. $u = \operatorname{cosec} x$ $\quad$ $v = \cot x$

$u' = -\operatorname{cosec} x \cot x$ $\quad$ $v' = -\operatorname{cosec}^2 x$

$$f'(x) = (\cot x)(-\operatorname{cosec} x \cot x) + (\operatorname{cosec} x)(-\operatorname{cosec}^2 x)$$
$$= -\cot^2 x \operatorname{cosec} x - \operatorname{cosec}^3 x$$

13. $u = \cot x$ $\quad$ $v = 1 + \operatorname{cosec} x$

$u' = -\operatorname{cosec}^2 x$ $\quad$ $v' = -\operatorname{cosec} x \cot x$

$$f'(x) = \frac{(1+\operatorname{cosec} x)(-\operatorname{cosec}^2 x) - (\cot x)(-\operatorname{cosec} x \cot x)}{(1+\operatorname{cosec} x)^2}$$

$$f'(x) = \frac{-\operatorname{cosec}^2 x - \operatorname{cosec}^3 x + \operatorname{cosec} x \cot^2 x}{(1+\operatorname{cosec} x)^2}$$

$$= \frac{\operatorname{cosec} x\,(-\operatorname{cosec} x - \operatorname{cosec}^2 x + \cot^2 x)}{(1+\operatorname{cosec} x)^2}$$

As $\cot^2 x - \operatorname{cosec}^2 x = -1$,

$$= \frac{\operatorname{cosec} x(-\operatorname{cosec} x - 1)}{(1+\operatorname{cosec} x)^2} = \frac{-\operatorname{cosec} x\cancel{(1+\operatorname{cosec} x)}}{(1+\operatorname{cosec} x)^{\cancel{2}}}$$

$$f'(x) = \frac{-\operatorname{cosec} x}{1+\operatorname{cosec} x}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

14. $u = \sec x$ $\qquad v = 1 + \tan x$

$u' = \sec x \tan x$ $\qquad v' = \sec^2 x$

$$f'(x) = \frac{(1+\tan x)(\sec x \tan x) - (\sec x)(\sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{\sec x \tan x + \sec x \tan^2 x - \sec^3 x}{(1+\tan x)^2}$$

$$= \frac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2}$$

As $\tan^2 x - \sec^2 x = -1$,

$$f'(x) = \frac{\sec x(\tan x - 1)}{(1+\tan x)^2}$$

15. $f(x) = \sin^2 x + \cos^2 x$

As $\sin^2 x + \cos^2 x = 1$,

$f(x) = 1$

$f'(x) = 0$

16. $f(x) = \sec^2 x - \tan^2 x$

$= 1 + \tan^2 x - \tan^2 x$

$f(x) = 1$

$f'(x) = 0$

Sufiyaan Usmani
21K-3195
BCS 1-J

17. $f(x) = \dfrac{\sin x \sec x}{1 + x\tan x}$

$$= \frac{\frac{\sin x}{\cos x}}{1 + x\tan x}$$

$$f(x) = \frac{\tan x}{1 + x\tan x}$$

$u = \tan x$

$u' = \sec^2 x$

$v = 1 + x\tan x$

$v' = \begin{bmatrix} u = x & v = \tan x \\ u' = 1 & v' = \sec^2 x \end{bmatrix}$

$v' = \tan x + x\sec^2 x$

$$f'(x) = \cancel{t+1}\frac{(1 + x\tan x)(\sec^2 x) - \tan x(\tan x + x\sec^2 x)}{(1 + x\tan x)^2}$$

$$= \frac{\sec^2 x + x\tan x\sec^2 x - \tan^2 x - x\tan x\sec^2 x}{(1 + x\tan x)^2}$$

$$= \frac{\sec^2 x - \tan^2 x}{(1 + x\tan x)^2}$$

As $\sec^2 x - \tan^2 x = 1$,

$$f'(x) = \frac{1}{(1 + x\tan x)^2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

18. $u = (x^2+1)\cot x$ $\qquad v = \cancel{v} = 3 - \cos x \operatorname{cosec} x$

$u' = \begin{bmatrix} u = x^2+1 & v = \cot x \\ u' = 2x & v' = -\operatorname{cosec}^2 x \end{bmatrix}$ $\qquad v = 3 - \cot x$

$\qquad v' = \cancel{3} + \operatorname{cosec}^2 x$

$u' = 2x\cot x - (x^2+1)(\operatorname{cosec}^2 x)$

$\cancel{f'(x) =}$

$$f'(x) = \frac{(3-\cot x)(2x\cot x - (x^2+1)(\operatorname{cosec}^2 x)) - (x^2+1)(\cot x)(\cancel{x}\operatorname{cosec}^2 x)}{(3-\cot x)^2}$$

$$= \cancel{6x\cot x}$$

$$= \frac{(3-\cot x)(2x\cot x - x^2\operatorname{cosec}^2 x - \operatorname{cosec}^2 x) - (x^2+1)(\cancel{3\cot x} + \cot x\operatorname{cosec}^2 x)}{(3-\cot x)^2}$$

$$= \frac{(6x\cot x - 3x^2\operatorname{cosec}^2 x - 3\operatorname{cosec}^2 x - 2x\cot^2 x \cancel{+ x^2\operatorname{cosec}^2 x\cot x + \cot x\operatorname{cosec}^2 x} - \cancel{3x^2\cot x} - \cancel{x^2\cot x\operatorname{cosec}^2 x} - \cancel{3\cot x} - \cancel{\cot x\operatorname{cosec}^2 x})}{(3-\cot x)^2}$$

$$= \frac{6x\cot x - 3x^2\operatorname{cosec}^2 x - 3\operatorname{cosec}^2 x - 2x\cot^2 x}{(3-\cot x)^2} \cancel{- x^2\cot x - 3\cot x}$$

$$f'(x) = \frac{6x\cot x - 2x\cot^2 x - 3\operatorname{cosec}^2 x(x^2+1)}{(3-\cot x)^2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

~~18~~

19. $u = x \qquad\qquad v = \cos x$

$u' = 1 \qquad\qquad v' = -\sin x$

$\frac{dy}{dx} = \cos x - x\sin x$

$\frac{d^2y}{dx^2} = -\sin x - \left[\begin{matrix} u = x & v = \sin x \\ u' = 1 & v' = \cos x \end{matrix}\right]$

$= -\sin x - [\sin x + x\cos x]$

$= -\sin x - \sin x \mp x\cos x$

$\frac{d^2y}{dx^2} = -x\cos x - 2\sin x$

20. $\frac{dy}{dx} = -\text{cosec}\, x \cot x$

$u = -\text{cosec}\, x \qquad\qquad v = \cot x$

$u' = \text{cosec}\, x \cot x \qquad\qquad v' = -\text{cosec}^2 x$

$\frac{d^2y}{dx^2} = \cot x(\text{cosec}\, x \cot x) + (-\text{cosec}\, x)(-\text{cosec}^2 x)$

$\frac{d^2y}{dx^2} = \text{cosec}\, x \cot^2 x + \text{cosec}^3 x$

Sufiyaan Usmani
21K-3195
BCS 1-J

21. $\frac{dy}{dx} = \begin{bmatrix} u=x & v=\sin x \\ u'=1 & v'=\cos x \end{bmatrix} + 3\sin x$

$= \sin x + x\cos x + 3\sin x$

$\frac{dy}{dx} = 4\sin x + x\cos x$

$\frac{d^2y}{dx^2} = 4\cos x + \begin{bmatrix} u=x & v=\cos x \\ u'=1 & v'=-\sin x \end{bmatrix}$

$= 4\cos x + \cos x - x\sin x$

$\frac{d^2y}{dx^2} = 5\cos x - x\sin x$

22. $\frac{dy}{dx} = \begin{bmatrix} u=x^2 & v=\cos x \\ u'=2x & v'=-\sin x \end{bmatrix} + 4\cos x$

$\frac{dy}{dx} = 2x\cos x - x^2\sin x + 4\cos x$

$\frac{d^2y}{dx^2} = \begin{bmatrix} u=2x & v=\cos x \\ u'=2 & v'=-\sin x \end{bmatrix} - \begin{bmatrix} u=x^2 & v=\sin x \\ u'=2x & v'=\cos x \end{bmatrix} - 4\sin x$

$= 2\cos x - 2x\sin x - (2x\sin x + x^2\cos x) - 4\sin x$

$= 2\cos x - 2x\sin x - 2x\sin x - x^2\cos x - 4\sin x$

$\frac{d^2y}{dx^2} = 2\cos x - 4x\sin x - x^2\cos x - 4\sin x$

$= 2\cos x - x^2\cos x \cancel{4\sin x} - 4x\sin x - 4\sin x$

$\frac{d^2y}{dx^2} = (2-x^2)\cos x - 4(x+1)\sin x$

Sufiyaan Usmani
21K-3195
BCS 1-J

23. $u = \sin x$ $\qquad$ $v = \cos x$

$u' = \cos x$ $\qquad$ $v' = -\sin x$

$$\frac{dy}{dx} = \cos^2 x - \sin^2 x$$

$$\frac{dy}{dx} = (\cos x)(\cos x) - (\sin x)(\sin x)$$

$$\frac{d^2y}{dx^2} = \begin{bmatrix} u = \cos x & v = \cos x \\ u' = -\sin x & v' = -\sin x \end{bmatrix} - \begin{bmatrix} u = \sin x & v = \sin x \\ u' = \cos x & v' = \cos x \end{bmatrix}$$

$$= (\cos x)(-\sin x) + (\cos x)(-\sin x) - [(\sin x)(\cos x) + (\sin x)(\cos x)]$$

$$= -\sin x \cos x - \sin x \cos x - \sin x \cos x - \sin x \cos x$$

$$\frac{d^2y}{dx^2} = -4 \sin x \cos x$$

24. $\frac{dy}{dx} = \sec^2 x$

$$\frac{dy}{dx} = (\sec x)(\sec x)$$

$$\frac{d^2y}{dx^2} = \begin{bmatrix} u = \sec x & v = \sec x \\ u' = \sec x \tan x & v' = \sec x \tan x \end{bmatrix}$$

$$= \sec x(\sec x \tan x) + \sec x(\sec x \tan x)$$

$$= \sec^2 x \tan x + \sec^2 x \tan x$$

$$\frac{d^2y}{dx^2} = 2\sec^2 x \tan x$$

Sufiyaan Usmani
21K-3195
BCS 1-J

7. $f'(x) = 37(x^3+2x)^{36} \cdot \frac{d}{dx}(x^3+2x)$

$f'(x) = 37(x^3+2x)^{36}(3x^2+2)$

8. $f'(x) = 6(3x^2+2x-1)^5 \cdot \frac{d}{dx}(3x^2+2x-1)$

$= 6(3x^2+2x-1)^5(6x+2)$

$f'(x) = 12(3x^2+2x-1)^5(3x+1)$

9. $f'(x) = -2\left(x^3-\frac{7}{x}\right)^{-3} \cdot \frac{d}{dx}(x^3-7x^{-1})$

$= -2\left(x^3-\frac{7}{x}\right)^{-3}(3x^2+7x^{-2})$

$f'(x) = -2\left(x^3-\frac{7}{x}\right)^{-3}\left(3x^2+\frac{7}{x^2}\right)$

10. $f(x) = (x^5-x+1)^{-9}$

$f'(x) = -9(x^5-x+1)^{-10} \cdot \frac{d}{dx}(x^5-x+1)$

$f'(x) = -9(x^5-x+1)^{-10}(5x^4-1)$

$f'(x) = \frac{-9(5x^4-1)}{(x^5-x+1)^{10}}$

Sufiyaan Usmani
21K-3195
BCS 1-J

11. $f(x) = 4(3x^2-2x+1)^{-3}$

$$f'(x) = -12(3x^2-2x+1)^{-4}(6x-2)$$

$$= -12(3x^2-2x+1)^{-4} \cdot -2(1-3x)$$

$$= \frac{24(1-3x)}{(3x^2-2x+1)^4}$$

12. $f(x) = (x^3-2x+5)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(x^3-2x+5)^{-\frac{1}{2}} \cdot (3x^2-2)$$

$$f'(x) = \frac{3x^2-2}{2\sqrt{x^3-2x+5}}$$

~~13. $f(x) = \sqrt{\ }$~~

13. $f(x) = (4+\sqrt{3}x^{\frac{1}{2}})^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(4+\sqrt{3}x^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(\frac{\sqrt{3}}{2}x^{-\frac{1}{2}}\right)$$

$$f'(x) = \frac{1}{2\sqrt{4+\sqrt{3x}}} \cdot \frac{\sqrt{3}}{2\sqrt{x}}$$

$$f'(x) = \frac{\sqrt{3}}{4\sqrt{x}\sqrt{4+\sqrt{3x}}}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

14. $f(x) = \left(12+x^{\frac{1}{2}}\right)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}\left(12+x^{\frac{1}{2}}\right)^{-\frac{2}{3}} \cdot \left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$= \frac{1}{3\left(12+\sqrt{x}\right)^{\frac{2}{3}}} \cdot \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{1}{6\sqrt{x}\left(12+\sqrt{x}\right)^{\frac{2}{3}}}$

15. $f(x) = \sin\left(x^{-2}\right)$

$f'(x) = \cos\left(x^{-2}\right) \cdot -2x^{-3}$

$f'(x) = -\frac{2}{x^3}\cos\left(\frac{1}{x^2}\right)$

16. $f(x) = \tan\left(x^{\frac{1}{2}}\right)$

$f'(x) = \left(\sec^2\sqrt{x}\right) \cdot \frac{dy}{dx}\left(x^{\frac{1}{2}}\right)$

$= \sec^2\sqrt{x} \cdot \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = \frac{\sec^2\sqrt{x}}{2\sqrt{x}}$

17. $f(x) = 4(\cos x)^5$

$f'(x) = 20(\cos x)^4 \cdot -\sin x$

$f'(x) = -20\cos^4 x \sin x$

Sufiyaan Usmani
21K-3195
BCS 1-J

18. $f(x) = 4x + 5(\sin x)^4$

$f'(x) = 4 + 20(\sin x)^3 \cdot \cos x$

$f'(x) = 4 + 20\sin^3 x \cos x$

19. $f(x) = [\cos(3x^{\frac{1}{2}})]^2$

$f'(x) = 2\cos(3\sqrt{x}) \cdot \frac{d}{dx}(\cos(3x^{\frac{1}{2}}))$

$= 2\cos(3\sqrt{x}) \cdot -\sin(3\sqrt{x}) \cdot \frac{d}{dx}(3x^{\frac{1}{2}})$

$= -\cancel{2}\cos(3\sqrt{x})\sin(3\sqrt{x}) \cdot \frac{3}{\cancel{2}}x^{-\frac{1}{2}}$

~~= -cos~~

$f'(x) = \frac{-3\cos(3\sqrt{x})\sin(3\sqrt{x})}{\sqrt{x}}$

20. $f(x) = [\tan(x^3)]^4$

$f'(x) = 4[\tan(x^3)]^3 \cdot \frac{d}{dx}(\tan(x^3))$

$= 4(\tan(x^3))^3 \cdot \sec^2(x^3) \cdot \frac{d}{dx}(x^3)$

$= 4\tan^3(x^3)\sec^2(x^3) \cdot 3x^2$

$f'(x) = 12x^2\tan^3(x^3)\sec^2(x^3)$

Sufiyaan Usmani
21K-3195
BCS 1-J

21. $f(x) = 2[\sec(x^7)]^2$

$f'(x) = 4(\sec(x^7)) \cdot \frac{d}{dx}(\sec(x^7))$

$= 4\sec(x^7) \cdot \sec(x^7)\tan(x^7) \cdot \frac{d}{dx}(x^7)$

$= 4\sec^2(x^7)\tan(x^7) \cdot 7x^6$

$f'(x) = 28x^6\sec^2(x^7)\tan(x^7)$

22. $f(x) = \left[\cos\left(\frac{x}{x+1}\right)\right]^3$

$f'(x) = 3\left[\cos\left(\frac{x}{x+1}\right)\right]^2 \cdot \frac{d}{dx}\left[\cos\left(\frac{x}{x+1}\right)\right]$

$= 3\left[\cos\left(\frac{x}{x+1}\right)\right]^2 \cdot -\sin\left(\frac{x}{x+1}\right) \cdot \frac{d}{dx}\left(\frac{x}{x+1}\right)$

$= -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right) \cdot \begin{bmatrix} u = x & v = x+1 \\ u' = 1 & v = 1 \end{bmatrix}$

$= -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right) \cdot \left(\frac{x+1-x}{(x+1)^2}\right)$

$= -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right) \cdot \left(\frac{1}{(x+1)^2}\right)$

$f'(x) = \frac{-3}{(x+1)^2}\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)$

Sufiyaan Usmani
21K-3195
BCS 1-J

23. $f(x) = [\cos(5x)]^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}[\cos(5x)]^{-\frac{1}{2}} \cdot \frac{d}{dx}[\cos(5x)]$$

$$= \frac{1}{2\sqrt{\cos(5x)}} \cdot -5\sin(5x)$$

$$f'(x) = \frac{-5\sin(5x)}{2\sqrt{\cos(5x)}}$$

24. $f(x) = [3x - (\sin(4x))^2]^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(3x - (\sin(4x))^2]^{-\frac{1}{2}} \cdot \frac{d}{dx}(3x - (\sin(4x))^2)$$

$$= \frac{1}{2\sqrt{3x - \sin^2(4x)}} \cdot \left(3 - \frac{d}{dx}[\sin(4x)]^2\right)$$

$$= \frac{1}{2\sqrt{3x - \sin^2(4x)}} \cdot \left(3 - 2\sin 4x \cdot 4\cos 4x\right)$$

$$f'(x) = \frac{3 - 8\sin(4x)\cos(4x)}{2\sqrt{3x - \sin^2(4x)}}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

25. $f(x) = [x + \text{cosec}(x^3+3)]^{-3}$

$$f'(x) = -3[x + \text{cosec}(x^3+3)]^{-4} \cdot \frac{d}{dx}[x + \text{cosec}(x^3+3)]$$

$$= \frac{-3}{[x + \text{cosed}(x^3+3)]^4} \cdot [1 + (-\text{cosed}(x^2+3)\cot(x^2+3) \cdot 3x^2)]$$

$$f'(x) = \frac{-3[1 - 3x^2 \text{cosec}(x^2+3)\cot(x^2+3)]}{[x + \text{cosec}(x^3+3)]^4}$$

26. $f(x) = [x^4 - \sec(4x^2-2)]^{-4}$

$$f'(x) = -4[x^4 - \sec(4x^2-2)]^{-5} \cdot \frac{d}{dx}[x^4 - \sec(4x^2-2)]$$

$$= \frac{-4}{[x^4 - \sec(4x^2-2)]^5} \cdot \left[4x^3 - \frac{d}{dx}[\sec(4x^2-2)]\right]$$

$$= \frac{-4}{[x^4 - \sec(4x^2-2)]^5} \cdot \left[4x^3 - [\sec(4x^2-2)\tan(4x^2-2) \cdot 8x]\right]$$

$$= \frac{-4}{[x^4 - \sec(4x^2-2)]^5} \cdot 4[x^3 - 2x\sec(4x^2-2)\tan(4x^2-2)]$$

$$f'(x) = \frac{-16x[x^2 - 2\sec(4x^2-2)\tan(4x^2-2)]}{[x^4 - \sec(4x^2-2)]^5}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

27. $u = x^3$
$u' = 3x^2$

$v = [\sin(5x)]^2$
$v' = 2\sin(5x) \cdot 5\cos 5x$
$v' = 10\sin 5x \cos 5x$

$\frac{dy}{dx} = vu' + uv'$

$= \sin^2(5x)(3x^2) + (x^3)(10\sin 5x \cos 5x)$

$\frac{dy}{dx} = 3x^2\sin^2(5x) + 10x^3\sin(5x)\cos(5x)$

28. $u = x^{\frac{1}{2}}$
$u' = \frac{1}{2}x^{-\frac{1}{2}}$

$u' = \frac{1}{2\sqrt{x}}$

$v = [\tan(x^{\frac{1}{2}})]^3$
$v' = 3[\tan(\sqrt{x})]^2 \cdot \frac{d(\tan(x^{\frac{1}{2}}))}{dx}$

$= 3[\tan(\sqrt{x})]^2 \cdot \sec^2(\sqrt{x}) \cdot \frac{d}{dx}(x^{\frac{1}{2}})$

$= 3\tan^2(\sqrt{x})\sec^2(\sqrt{x}) \cdot \frac{1}{2}x^{-\frac{1}{2}}$

$v' = \frac{3\tan^2(\sqrt{x})\sec^2(\sqrt{x})}{2\sqrt{x}}$

$\frac{dy}{dx} = \tan^3(\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right) + \sqrt{x}\left(\frac{3\tan^2(\sqrt{x})\sec^2(\sqrt{x})}{2\sqrt{x}}\right)$

$\frac{dy}{dx} = \frac{\tan^3(\sqrt{x})}{2\sqrt{x}} + \frac{3}{2}\tan^2(\sqrt{x})\sec^2(\sqrt{x})$

Sufiyaan Usmani
21K-3195
BCS 1-J

29. $u = x^5$ $\qquad$ $v = \sec(x^{-1})$

$u' = 5x^4$ $\qquad$ $v' = \sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right) \cdot -x^{-2}$

$v' = \dfrac{-\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)}{x^2}$

$$\frac{dy}{dx} = \sec\left(\frac{1}{x}\right)(5x^4) + (x^{\cancel{5}\,3})\left(\frac{-\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)}{\cancel{x^2}}\right)$$

$$\frac{dy}{dx} = 5x^4\sec\left(\frac{1}{x}\right) - x^3\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)$$

30. $u = \sin x$ $\qquad$ $v = \sec(3x+1)$

$u' = \cos x$ $\qquad$ $v' = \sec(3x+1)\tan(3x+1) \cdot 3$

$v' = 3\sec(3x+1)\tan(3x+1)$

$$\frac{dy}{dx} = \frac{\sec(3x+1)\cos x - 3\sin x\sec(3x+1)\tan(3x+1)}{\sec^2(3x+1)}$$

$$= \frac{\cancel{\sec(3x+1)}\cos(x)}{\sec^{\cancel{2}}(3x+1)} - \frac{3\sin x\cancel{\sec(3x+1)}\tan(3x+1)}{\sec^{\cancel{2}}(3x+1)}$$

$$= \cos x\cos(3x+1) - 3\sin x \cdot \cancel{\cos(3x+1)} \cdot \frac{\sin(3x+1)}{\cancel{\cos(3x+1)}}$$

$$\frac{dy}{dx} = \cos x\cos(3x+1) - 3\sin x\sin(3x+1)$$

Sufiyaan Usmani
21K-3195
BCS 1-J

31. $y = \cos(\cos x)$

$$\frac{dy}{dx} = -\sin(\cos x) \cdot \frac{d}{dx}(\cos x)$$

$$= -\sin(\cos x) \cdot -\sin x$$

$$\frac{dy}{dx} = \sin(\cos x)\sin x$$

32. $y = \sin(\tan 3x)$

$$\frac{dy}{dx} = \cos(\tan 3x) \cdot \frac{d}{dx}(\tan 3x)$$

$$= \cos(\tan 3x) \cdot 3\sec^2 3x$$

$$\frac{dy}{dx} = 3\sec^2 3x \cos(\tan 3x)$$

33. $y = [\cos(\sin 2x)]^3$

$$\frac{dy}{dx} = 3[\cos(\sin 2x)]^2 \cdot \frac{d}{dx}[\cos(\sin 2x)]$$

$$= 3[\cos(\sin 2x)]^2 \cdot -\sin(\sin 2x) \cdot \frac{d}{dx}(\sin 2x)$$

$$= -3\cos^2(\sin 2x)\sin(\sin 2x) \cdot 2\cos 2x$$

$$\frac{dy}{dx} = -6\cos^2(\sin 2x)\sin(\sin 2x)\cos 2x$$

Sufiyaan Usmani
21K-3195
BCS 1-J

34. $u = 1 + \operatorname{cosec}(x^2)$ $\quad V = 1 - \cot(x^2)$

$u' = 1 - \operatorname{cosec}(x^2)\cot(x^2) \cdot 2x$ $\quad V' = 1 + \operatorname{cosec}(x^2) \cdot 2x$

$u' = 1 - 2x\operatorname{cosec}(x^2)\cot(x^2)$ $\quad V' = 1 + 2x\operatorname{cosec}(x^2)$

$$\frac{dy}{dx} = \frac{(1-\cot(x^2))(1-2x\operatorname{cosec}(x^2)\cot(x^2)) - (1+\operatorname{cosec}(x^2))(1+2x\operatorname{cosec}(x^2))}{[1-\cot(x^2)]^2}$$

$$= \frac{\cancel{(1-\cot(x^2))}(1-2x\operatorname{cosec}(x^2)\cot(x^2))}{(1-\cot(x^2))^{\cancel{2}}} - \frac{(1+\operatorname{cosec}(x^2))(1+2x\operatorname{cosec}(x^2))}{(1-\cot(x^2))^2}$$

$$\frac{dy}{dx} = \frac{1-2x\operatorname{cosec}(x^2)\cot(x^2)}{1-\cot(x^2)} - \frac{(1+\operatorname{cosec}(x^2))(1+2x\operatorname{cosec}(x^2))}{(1-\cot(x^2))^2}$$

~~35. 5x+~~

35. $u = (5x+8)^7$ $\quad V = (1-x^{\frac{1}{2}})^6$

$u' = 7(5x+8)^6 \cdot 5$ $\quad V' = \overset{3}{\cancel{6}}(1-\sqrt{x})^5 \cdot -\frac{1}{\cancel{2}}x^{-\frac{1}{2}}$

$u' = 35(5x+8)^6$

$$V' = \frac{-3(1-\sqrt{x})^5}{\sqrt{x}}$$

$$\frac{dy}{dx} = (1-\sqrt{x})^6[35(5x+8)^6] + (5x+8)^7\left[\frac{-3(1-\sqrt{x})}{\sqrt{x}}\right]$$

$$\frac{dy}{dx} = 35(1-\sqrt{x})^6(5x+8)^6 - \frac{3(5x+8)^7(1-\sqrt{x})^5}{\sqrt{x}}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

36. $u = (x^2+x)^5$

$u' = 5(x^2+x)^4 \cdot (2x+1)$

$u' = 5(2x+1)(x^2+x)^4$

$v = (\sin x)^8$

$v' = 8(\sin x)^7 \cdot \cos x$

$v' = 8\cos x \sin^7 x$

$$\frac{dy}{dx} = (\sin^8 x)[5(2x+1)(x^2+x)^4] + (x^2+x)^5(8\cos x \sin^7 x)$$

$$\frac{dy}{dx} = 5(\sin^8 x)(x^2+x)^4(2x+1) + 8(x^2+x)^5 \sin^7 x \cos x$$

37. $$\frac{dy}{dx} = 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \frac{d}{dx}\left(\frac{x-5}{2x+1}\right)$$

$$= 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \begin{bmatrix} u = x-5 & v = 2x+1 \\ u' = 1 & v' = 2 \end{bmatrix}$$

$$= 3\left(\frac{x-5}{2x+1}\right)^2 \cdot \left(\frac{(2x+1) - 2(x-5)}{(2x+1)^2}\right)$$

$$= \frac{3(x-5)^2}{(2x+1)^2} \cdot \frac{2x+1-2x+10}{(2x+1)^2}$$

$$= \frac{3(x-5)^2 \cdot 11}{(2x+1)^4}$$

$$\frac{dy}{dx} = \frac{33(x-5)^2}{(2x+1)^4}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

$$\cancel{38.\ \frac{dy}{dx} = 17\left(\frac{1+x^2}{1-x^2}\right.}$$

38. $$\frac{dy}{dx} = 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \cdot \frac{d}{dx}\left(\frac{1+x^2}{1-x^2}\right)$$

$$= 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \cdot \frac{(1-x^2)(+2x) + (1+x^2)(-2x)}{(1-x^2)^2}$$

$$= \frac{17(1+x^2)^{16}}{(1-x^2)^{16}} \cdot \frac{(1-x^2)(2x) + (1+x^2)(2x)}{(1-x^2)^2}$$

$$= \frac{17(1+x^2)^{16} \cdot 2x(1-x^2+1+x^2)}{(1-x^2)^{18}}$$

$$\frac{dy}{dx} = \frac{68x(1+x^2)^{16}}{(1-x^2)^{18}}$$

39. $u = (2x+3)^3$ $\qquad$ $v = (4x^2-1)^8$

$u' = 3(2x+3)^2 \cdot 2$ $\qquad$ $v' = 8(4x^2-1)^7 \cdot 8x$

$u' = 6(2x+3)^2$ $\qquad$ $v' = 64x(4x^2-1)^7$

$$\frac{dy}{dx} = \frac{(4x^2-1)^8(6(2x+3)^2) - (2x+3)^3(64x(4x^2-1)^7)}{[(4x^2-1)^8]^2}$$

$$\cancel{= 6(4x^2-1)^7}$$

$$= \frac{2\cancel{4}\cancel{(4x^2-1)^7}(2x+3)^2[(4x^2-1)(3) - (2x+3)(32x)]}{(4x^2-1)^{\cancel{16}9}}$$

$$= \frac{2(2x+3)^2[12x^2-3-64x^2-96x]}{(4x^2-1)^9}$$

$$= \frac{2(2x+3)^2(-52x^2-96x-3)}{(4x^2-1)^9}$$

$$\frac{dy}{dx} = \frac{-2(2x+3)^2(52x^2+96x+3)}{(4x^2-1)^9}$$

40. $\frac{dy}{dx} = 12[1+(\sin(x^5))^3]^{11} \cdot \frac{d}{dx}[1+(\sin(x^5))^3]$

$= 12[1+\sin^3(x^5)]^{11} \cdot [3(\sin(x^5))^2 \cdot \frac{d}{dx}(\sin(x^5))]$

$= 36[1+\sin^3(x^5)]^{11} \cdot \sin^2(x^5) \cdot \cos(x^5) \cdot 5x^4$

$\frac{dy}{dx} = 180x^4[1+\sin^3(x^5)]^{11}\sin^2(x^5)\cos(x^5)$

Sufiyaan Usmani
21K-3195
BCS 1-J

3.1

3. $2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{x}{y}$

4. $3x^2 + 3y^2\frac{dy}{dx} = \left[\begin{array}{ll} u = 3x & v = y^2 \\ u' = 3 & v' = 2y\frac{dy}{dx} \end{array}\right]$

$3x^2 + 3y^2\frac{dy}{dx} = 3y^2 + 6xy\frac{dy}{dx}$

$3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} = 3y^2 - 3x^2$

$\frac{dy}{dx}(3y^2 - 6xy) = 3y^2 - 3x^2$

$\frac{dy}{dx} = \frac{\cancel{3}(y^2 - x^2)}{\cancel{3}(y^2 - 2xy)}$

$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 - 2xy}$

Sufiyaan Usmani
21K-3195
BCS 1-J

5. $\begin{bmatrix} u = x^2 & v = y \\ u' = 2x & v' = \frac{dy}{dx} \end{bmatrix} + \begin{bmatrix} u = 3x & v = y^3 \\ u' = 3 & v' = 3y^2\frac{dy}{dx} \end{bmatrix} - 1 = 0$

$$2xy + x^2\frac{dy}{dx} + 3y^3 + 9xy^2\frac{dy}{dx} = 1$$

$$x^2\frac{dy}{dx} + 9xy^2\frac{dy}{dx} = 1 - 3y^3 - 2xy$$

$$\frac{dy}{dx}(x^2 + 9xy^2) = 1 - 3y^3 - 2xy$$

$$\frac{dy}{dx} = \frac{1 - 3y^3 - 2xy}{x^2 + 9xy^2}$$

6. $\begin{bmatrix} u = x^3 & v = y^2 \\ u' = 3x^2 & v' = 2y\frac{dy}{dx} \end{bmatrix} - \begin{bmatrix} u = 5x^2 & v = y \\ u' = 10x & v' = \frac{dy}{dx} \end{bmatrix} + 1 = 0$

$$3x^2y^2 + 2x^3y\frac{dy}{dx} - \left[10xy + 5x^2\frac{dy}{dx}\right] = -1$$

$$3x^2y^2 + 2x^3y\frac{dy}{dx} - 10xy - 5x^2\frac{dy}{dx} = -1$$

$$2x^3y\frac{dy}{dx} - 5x^2\frac{dy}{dx} = 10xy - 3x^2y^2 - 1$$

$$\frac{dy}{dx}(2x^3y - 5x^2) = 10xy - 3x^2y^2 - 1$$

$$\frac{dy}{dx} = \frac{10xy - 3x^2y^2 - 1}{2x^3y - 5x^2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

7. $x^{-\frac{1}{2}} + y^{-\frac{1}{2}} = 1$

$$-\frac{1}{2}x^{-\frac{3}{2}} - \frac{1}{2}y^{-\frac{3}{2}}\frac{dy}{dx} = 0$$

$$\frac{1}{\cancel{2}}y^{-\frac{3}{2}}\frac{dy}{dx} = -\frac{1}{\cancel{2}}x^{-\frac{3}{2}}$$

$$\frac{1}{y^{\frac{3}{2}}}\frac{dy}{dx} = -\frac{1}{x^{\frac{3}{2}}}$$

$$\frac{dy}{dx} = \frac{y^{\frac{3}{2}}}{x^{\frac{3}{2}}}$$

8. $2x = \begin{bmatrix} u = x+y & v = x-y \\ u' = 1+\frac{dy}{dx} & v' = 1-\frac{dy}{dx} \end{bmatrix}$

$$2x = \frac{(x-y)\left(1+\frac{dy}{dx}\right) - (x+y)\left(1-\frac{dy}{dx}\right)}{(x-y)^2}$$

$$2x(x-y)^2 = \left[x + x\frac{dy}{dx} - y - y\frac{dy}{dx}\right] - \left[x - x\frac{dy}{dx} + y - y\frac{dy}{dx}\right]$$

$$2x(x-y)^2 = x + x\frac{dy}{dx} - y - y\frac{dy}{dx} - x + x\frac{dy}{dx} - y + y\frac{dy}{dx}$$

$$2x(x-y)^2 = 2x\frac{dy}{dx} - 2y$$

$$2x\frac{dy}{dx} = 2x(x-y)^2 + 2y$$

Sufiyaan Usmani
21K-3195
BCS 1-J

$$\frac{dy}{dx} = \frac{\cancel{2}[x(x-y)^2+y]}{\cancel{2}x}$$

$$\frac{dy}{dx} = \frac{x(x-y)^2+y}{x}$$

9. $\cos(x^2y^2) \cdot \frac{d}{dx}(x^2y^2) = 1$

$$\cos(x^2y^2) \cdot \begin{bmatrix} u = x^2 & v = y^2 \\ u' = 2x & v' = 2y\frac{dy}{dx} \end{bmatrix} = 1$$

$$\cos(x^2y^2) \cdot \left[2xy^2 + 2x^2y\frac{dy}{dx}\right] = 1$$

$$2xy^2 + 2x^2y\frac{dy}{dx} = \frac{1}{\cos(x^2y^2)}$$

$$2x^2y\frac{dy}{dx} = \frac{1}{\cos(x^2y^2)} - 2xy^2$$

$$\cancel{2x^2y\frac{dy}{dx}} =$$

$$\frac{dy}{dx} = \left[\frac{1}{\cos(x^2y^2)} - 2xy^2\right] \times \frac{1}{2x^2y}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

10. $-\sin(xy^2)\cdot\frac{d}{dx}(xy^2) = \frac{dy}{dx}$

$$-\sin(xy^2)\cdot\begin{bmatrix} u=x & v=y^2 \\ u'=1 & v'=2y\frac{dy}{dx}\end{bmatrix} = \frac{dy}{dx}$$

$$-\sin(xy^2)\cdot\left(y^2+2xy\frac{dy}{dx}\right) = \frac{dy}{dx}$$

$\cancel{y^2+2xy}$

$$-y^2\sin(xy^2) - 2xy\sin(xy^2)\frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx} + 2xy\sin(xy^2)\frac{dy}{dx} = -y^2\sin(xy^2)$$

$$\frac{dy}{dx}\left[2xy\sin(xy^2)+1\right] = -y^2\sin(xy^2)$$

$$\frac{dy}{dx} = -\frac{y^2\sin(xy^2)}{2xy\sin(xy^2)+1}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

11. $[\tan(xy^2+y)]^3 = x$

$$3[\tan(xy^2+y)]^2 \cdot \frac{d}{dx}[\tan(xy^2+y)] = 1$$

$$3\tan^2(xy^2+y) \cdot \sec^2(xy^2+y) \cdot \frac{d}{dx}(xy^2+y) = 1$$

$$3\tan^2(xy^2+y)\sec^2(xy^2+y) \cdot \left[y^2 + 2xy\frac{dy}{dx} + \frac{dy}{dx}\right] = 1$$

$$y^2 + \frac{dy}{dx}(2xy+1) = \frac{1}{3\tan^2(xy^2+y)\sec^2(xy^2+y)}$$

$$\frac{dy}{dx}(2xy+1) = \frac{1}{3\tan^2(xy^2+y)\sec^2(xy^2+y)} - y^2$$

$$\frac{dy}{dx} = \left[\frac{1}{3\tan^2(xy^2+y)\sec^2(xy^2+y)} - y^2\right] \times \left[\frac{1}{2xy+1}\right]$$

Sufiyaan Usmani
21K-3195
BCS 1-J

12. $$\left[\begin{array}{ll} u = xy^3 & v = 1+\sec y \\ u' = y^3+3xy^2\frac{dy}{dx} & v' = \sec y\tan y\,\frac{dy}{dx} \end{array}\right] = 4y^3\frac{dy}{dx}$$

$$\frac{(1+\sec y)\left(y^3+3xy^2\frac{dy}{dx}\right)-(xy^3)\left(\sec y\tan y\frac{dy}{dx}\right)}{(1+\sec y)^2} = 4y^3\frac{dy}{dx}$$

$$(1+\sec y)\left(y^3+3xy^2\frac{dy}{dx}\right) - xy^3\left(\sec y\tan y\frac{dy}{dx}\right) = (1+\sec y)^2 4y^3\frac{dy}{dx}$$

$$y^3+3xy^2\frac{dy}{dx}+y^3\sec y+3xy^2\sec y\frac{dy}{dx} - xy^3\sec y\tan y\frac{dy}{dx} = (1+\sec y)^2 4y^3\frac{dy}{dx}$$

$$3xy^2\frac{dy}{dx}+3xy^2\sec y\frac{dy}{dx}-xy^3\sec y\tan y\frac{dy}{dx}-(1+\sec y)^2 4y^3\frac{dy}{dx} = -y^3-y^3\sec y$$

$$\frac{dy}{dx}\left[3xy^2+3xy^2\sec y-xy^3\sec y\tan y-4y^3(1+\sec y)^2\right] = -y^3-y^3\sec y$$

$$\cancel{-}y^2\frac{dy}{dx}\left[4y(1+\sec y)^2+xy\sec y\tan y-3x\sec y-3x\right] = \cancel{-}y^{\cancel{3}}(1+\sec y)$$

$\cancel{dy}$

$$\frac{dy}{dx} = \frac{y(1+\sec y)}{4y(1+\sec y)^2+xy\sec y\tan y-3x\sec y-3x}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

13. $4x - 6y\frac{dy}{dx} = 0$

$$6y\frac{dy}{dx} = 4x$$

$$\frac{dy}{dx} = \frac{2x}{3y}$$

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{2x}{3y}\right]$$

$$\frac{d^2y}{dx^2} = \frac{3y\frac{d}{dx}(2x) - 2x\frac{d}{dx}(3y)}{(3y)^2}$$

$$= \frac{6y - 6x\frac{dy}{dx}}{9y^2}$$

$$= \frac{6y - \cancel{6}^{2}x\left[\frac{2x}{\cancel{3}y}\right]}{9y^2}$$

$$= \frac{6y - \frac{4x^2}{y}}{9y^2}$$

$$\frac{d^2y}{dx^2} = \frac{6y^2 - 4x^2}{9y^3}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

14. $3x^2 + 3y^2\frac{dy}{dx} = 0$

$$3y^2\frac{dy}{dx} = -3x^2$$

$$y^2\frac{dy}{dx} = -x^2$$

$$\frac{dy}{dx} = -\frac{x^2}{y^2}$$

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[-\frac{x^2}{y^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{y^2\frac{d}{dx}(-x^2) - \left[-x^2\frac{d}{dx}(y^2)\right]}{(y^2)^2}$$

$$= \frac{-2xy^2 + 2x^2y\frac{dy}{dx}}{y^4}$$

$$= \frac{-2xy^2 + 2x^2y\left[-\frac{x^2}{y^2}\right]}{y^4}$$

$$= \frac{-2xy^2 - \frac{2x^4}{y}}{y^4}$$

$$\frac{d^2y}{dx^2} = \frac{-2xy^3 - 2x^4}{y^5}$$

15. $y^3\frac{d}{dx}[x^3] + x^3\frac{d}{dx}[y^3] - 0 = 0$

$$y^3(3x^2) + x^3\left(3y^2\frac{dy}{dx}\right) = 0$$

$$3x^2y^3 + 3x^3y^2\frac{dy}{dx} = 0$$

$$\cancel{3}x^{\cancel{3}}\cancel{y^2}\frac{dy}{dx} = -\cancel{3}x^{\cancel{2}}y^{\cancel{3}}$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{-y}{x}\right]$$

$$\frac{d^2y}{dx^2} = \frac{x\frac{d}{dx}[-y] - \left[-y\frac{d}{dx}(x)\right]}{x^2}$$

$$= \frac{-x\frac{dy}{dx} + y}{x^2}$$

$$= \frac{-\cancel{x}\left[\frac{-y}{\cancel{x}}\right] + y}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{2y}{x^2}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

16. $y\frac{d}{dx}[x] + x\frac{d}{dx}[y] + 2y\frac{dy}{dx} = 0$

$$y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x+2y) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x+2y}$$

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{-y}{x+2y}\right]$$

$$\frac{d^2y}{dx^2} = \frac{(x+2y)\frac{d}{dx}[-y] - \left[-y\frac{d}{dx}[x+2y]\right]}{(x+2y)^2}$$

$$= \frac{-(x+2y)\frac{dy}{dx} - \left[\cancel{+2}-y\left(1+2\frac{dy}{dx}\right)\right]}{(x+2y)^2}$$

$$= \frac{\cancel{-(x+2y)}\left[\frac{-y}{\cancel{(x+2y)}}\right] + y\left[1+2\left[\frac{-y}{(x+2y)}\right]\right]}{(x+2y)^2}$$

$$= \frac{y + y - \frac{2y^2}{x+2y}}{(x+2y)^2}$$

$$= \frac{2y(x+2y) - 2y^2}{(x+2y)^3} = \frac{2y(x+2y-y)}{(x+2y)^3}$$

$$\frac{d^2y}{dx^2} = \frac{2y(x+y)}{(x+2y)^3}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

17. $\frac{dy}{dx} + \cos y \frac{dy}{dx} = 1$

$$\frac{dy}{dx}[\cos y + 1] = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y + 1}$$

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}[\cos y + 1]^{-1}$$

$$\frac{d^2y}{dx^2} = -(\cos y+1)^{-2} \cdot -\sin y \frac{dy}{dx}$$

$$= \frac{\sin y}{(\cos y+1)^2} \cdot \frac{dy}{dx}$$

$$= \frac{\sin y}{(\cos y+1)^2} \cdot \left[\frac{1}{(\cos y+1)}\right]$$

$$\frac{d^2y}{dx^2} = \frac{\sin y}{(\cos y+1)^3}$$

18. $\cos y \frac{d}{dx}[x] + x\frac{d}{dx}[\cos y] = \frac{d}{dx}[y]$

$$\cancel{-\sin y}\cos y - x\sin y \frac{dy}{dx} = \frac{dy}{dx}$$

$$\frac{dy}{dx}(x\sin y + 1) = \cos y$$

$$\frac{dy}{dx} = \frac{\cos y}{x\sin y + 1}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

$$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{\cos y}{x\sin y+1}\right]$$

$$\frac{d^2y}{dx^2} = \frac{(x\sin y+1)\frac{d}{dx}[\cos y] - (\cos y)\frac{d}{dx}[x\sin y+1]}{(x\sin y+1)^2}$$

$$= \frac{(x\sin y+1)\left[-\sin y\frac{dy}{dx}\right] - \cos y\left[\sin y\frac{d}{dx}[x] + x\frac{d}{dx}[\sin y]\right]}{(x\sin y+1)^2}$$

$$= \frac{-\sin y(x\sin y+1)\frac{dy}{dx} - \cos y\left[\sin y + x\cos y\frac{dy}{dx}\right]}{(x\sin y+1)^2}$$

$$= \cancel{-\sin y(x\sin y+1)\left[\frac{\cos y}{(x\sin y+1)}\right] - \cos y\left[\sin y + x\cos y\left(\frac{\cos y}{x\sin y+1}\right)\right]}$$

$$= \frac{-\frac{\sin y\cos y(x\sin y+1)}{(x\sin y+1)} - \sin y\cos y - \frac{x\cos^3 y}{(x\sin y+1)}}{(x\sin y+1)^2}$$

$$= \frac{-\sin y\cos y(x\sin y+1) - \sin y\cos y(x\sin y+1) - x\cos^3 y}{(x\sin y+1)^3}$$

$$\frac{d^2y}{dx^2} = \frac{-2\sin y\cos y(x\sin y+1) - x\cos^3 y}{(x\sin y+1)^3}$$

Sufiyaan Usmani
21K-3195
BCS 1-J

25. $4x^3 + 4y^3 \frac{dy}{dx} = 0$

$$4y^3 \frac{dy}{dx} = -4x^3$$

$$\frac{dy}{dx} = -\frac{x^3}{y^3}$$

$$\left.\frac{dy}{dx}\right|_{x=1,\, y=\sqrt[4]{15}} = \frac{-(1)^3}{[\sqrt[4]{15}]^3}$$

$$\left.\frac{dy}{dx}\right|_{x=1,\, y=\sqrt[4]{15}} = -0.131$$

$$\cancel{y - \sqrt[4]{15} = -0.131(x-1)}$$

$$\cancel{y - 1.96799 = -0.131x + 0.131}$$

$$\cancel{y =}$$

$$m_{tan} = -0.131$$

Sufiyaan Usmani
21K-3195
BCS 1-J

26. $3y^2\frac{dy}{dx} + x^2\frac{d}{dx}[y] + y\frac{d}{dx}[x^2] + 2x - \cancel{8}6y\frac{dy}{dx} = 0$

$3y^2\frac{dy}{dx} + x^2y\frac{dy}{dx} + \cancel{y} 2xy\frac{dy}{dx} + 2\cancel{y} - 6y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 + x^2y + 2xy - 6y) = -2\cancel{y}x$

$\frac{dy}{dx} = \frac{-2\cancel{y}x}{3y^2 + x^2y + 2xy - 6y}$

$\frac{dy}{dx}\Big|_{x=0,y=3} = \frac{-2(0)}{3(3)^2 + (0)^2(3) + 2(0)^3(3) - 6(3)}$

$= -\frac{\cancel{6}}{9}$

$\frac{dy}{dx}\Big|_{x=0,y=3} = 0$

$m_{tan} = 0$

27. $4(x^2+y^2)\cdot\frac{d}{dx}[x^2+y^2] = 25\left[2x - 2y\frac{dy}{dx}\right]$

$4(x^2+y^2)\cdot\left[2x + 2y\frac{dy}{dx}\right] = 25\left[2x - 2y\frac{dy}{dx}\right]$

$4(3^2+1^2)\cdot\left[2(3) + 2(1)\frac{dy}{dx}\right] = 25\left[2(3) - 2(1)\frac{dy}{dx}\right]$

$40\left(6 + 2\frac{dy}{dx}\right) = 25\left(6 - 2\frac{dy}{dx}\right)$

$240 + 80\frac{dy}{dx} = 150 - \cancel{2}50\frac{dy}{dx}$

Sufiyaan Usmani
21K-3195
BCS 1-J

$$\cancel{290\frac{dy}{dx} = 70}-90$$

$$130\frac{dy}{dx} = -90$$

$$\frac{dy}{dx} = \frac{-9}{13}$$

28. $\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$

$$\frac{2}{3}(-1)^{-\frac{1}{3}} + \frac{2}{3}(3\sqrt{3})^{-\frac{1}{3}}\frac{dy}{dx} = 0$$

$$\frac{-2}{3} + \frac{2}{3[3\sqrt{3}]^{\frac{1}{3}}}\frac{dy}{dx} = 0$$

$$\frac{\cancel{2}}{\cancel{3}[3\sqrt{3}]^{\frac{1}{3}}}\frac{dy}{dx} = \frac{\cancel{2}}{\cancel{3}}$$

$$\frac{dy}{dx} = [3\sqrt{3}]^{\frac{1}{3}}$$

$$= (3^{\frac{3}{2}})^{\frac{1}{3}}$$

$$= 3^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \sqrt{3}$$