

## Decision tree assignment.

1) chest pain decision tree for heart attack.

	id	chest pain	male	smokers	Exercise	Heart Attack.
1	yes	yes	no	no	yes	yes
2	yes	yes	yes	yes	no	yes
3	no	no	yes	yes	yes	no
4	no	yes	no	no	yes	yes
5	yes	no	yes	yes	yes	no
6	no	yes	yes	yes	yes	.

Step 1:- Find entropy of training set.

$$\text{Entropy} = E\{(-4/6)\log_2(4/6) + (-2/6)\log_2(2/6)\} \\ = 0.918295.$$

Step 2:- Information gain for chest pain.

$$G(S, \text{chest pain}) = 0.918295 - ((3/6)(-1\log_2(1)) + (3/6)((-1/3)\log_2(1/3) + (-2/3)\log_2(2/3))) \\ = 0.4591.$$

Step 3:- Information gain for male attribute.

$$G(S, \text{male}) = 0.918295 - ((4/6)(-2/4)\log_2(2/4) + (-2/4)\log_2(2/4) + (2/6)(-1\log_2(1))) \\ = 0.251.$$

Step 4:- Information gain for smokers attribute.

$$G(S, \text{smokers}) = 0.918295 - \left( \left(\frac{4}{6}\right) \left(\left(-\frac{3}{4}\right) \log_2 \left(\frac{3}{4}\right) + \left(-\frac{1}{4}\right) \log_2 \left(\frac{1}{4}\right)\right) + \left(\frac{2}{6}\right) \left(\left(-\frac{1}{2}\right) \log_2 \left(\frac{1}{2}\right) + \left(-\frac{1}{2}\right) \log_2 \left(\frac{1}{2}\right)\right) \right)$$
$$= 0.0441.$$

Step 5:- Information gain for exercise attribute.

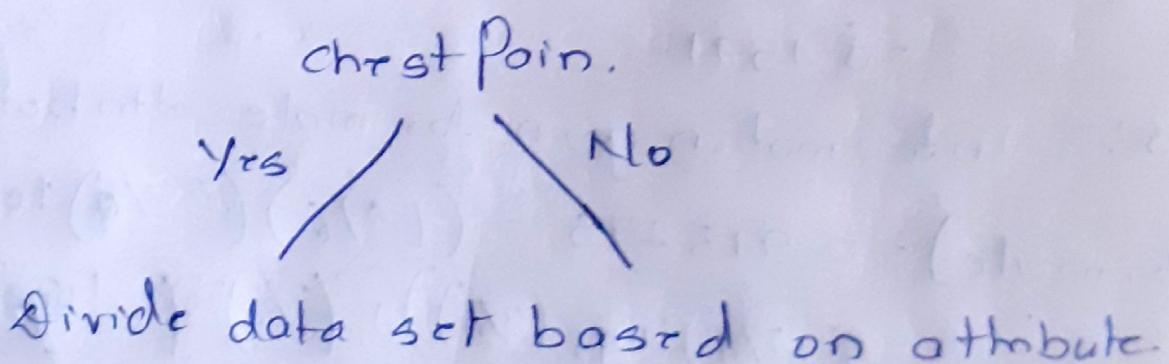
$$G(S, \text{exercise}) = 0.918295 - \left( \left(\frac{4}{6}\right) \left(\left(-\frac{3}{4}\right) \log_2 \left(\frac{2}{4}\right) + \left(-\frac{1}{4}\right) \log_2 \left(\frac{1}{4}\right)\right) + \left(\frac{2}{6}\right) \left(\left(-1\right) \log_2 \left(1\right)\right) \right)$$
$$= 0.2516.$$

Step 6:-

	Information gain
chest pain	0.4591
male	0.251629
smokes	0.0441
exercise	0.25169

chest pain will be root.

Step 7:- Construct tree



		Heart Attack		
		Yes	No	
Chest Pain	Yes	3	0	
	No	2	2	

Chest pain Male Smokes Exercise Heart Attack.

①	Yes	Yes	No	Yes	Yes
	Yes	Yes	Yes	No	Yes
	Yes	No	Yes	Yes	Yes

Chest pain male Smokes Exercise Heart Attack

②	No	No	Yes	No	Yes
	No	Yes	No	Yes	No
	No	Yes	Yes	Yes	No

Entropy for ①.

$$E(s) = E((3)) = (-1) \log_2(1) = 0$$

Entropy for ②.

$$E(s) = E[(1, -2)] = \left( -\frac{1}{3} \log_2(\frac{1}{3}) + \left(-\frac{2}{3}\right) \log_2(\frac{2}{3}) \right)$$

$$\approx 0.918295$$

Step 8:- Information gain for all attributes w.r.t data set ②.

→ For male

$$G(S, \text{male}) = 0.9182 - \left( \left( \frac{2}{3} \right) \left[ \left( -\frac{2}{2} \right) \log_2(\frac{2}{2}) + \left( \frac{1}{2} \right) \log_2(\frac{1}{2}) \right] \right)$$

$$\approx 0.9182$$

For smokers

$$G(S, \text{smokes}) = 0.9182 - \left( \left(\frac{2}{3}\right) \left(-\frac{1}{2}\right) \log_2 \left(\frac{1}{2}\right) + \left(-\frac{1}{2}\right) \log_2 \left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)(0) \right)$$
$$= 0.2515.$$

For Exercise

$$G(S, \text{Exercise}) = 0.9182 - \left( \left(\frac{2}{3}\right)(0) + \left(\frac{1}{3}\right)(0) \right)$$
$$= 0.9182.$$

Male

Info gain.

0.9182

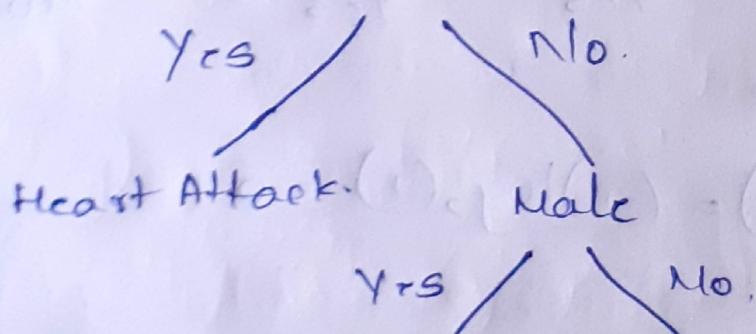
Smokers

0.2515

Exercise.

0.9182.

chest pain.

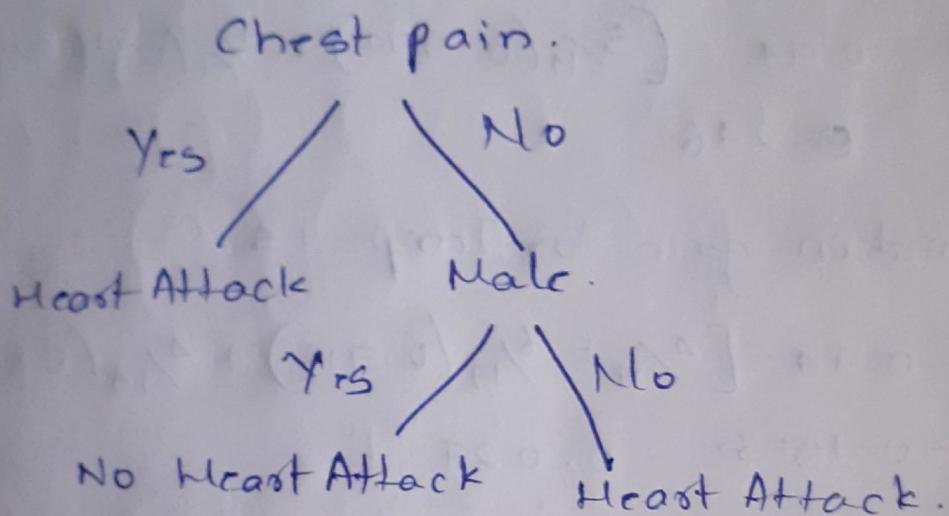


Step 9:- Divide data set.

	Male	smokes	Exercise	Heart Attack
①	Yes	No	Yes	No
	Yes	Yes	Yes	No
②	Male	smokes	Exercise	Heart Attack
	No	Yes	No	Yes

Euthropy for ① is 0.  
 Euthropy for ② is 0.

Final tree:



### ⑥ Play Tennis Decision Tree.

	Day	Outdoor	Temp	Humidity	Wind	Play Tennis.
1	Sunny	Hot	High	Weak	No	No
2	Sunny	Hot	High	Strong	No	No
3	Overcast	Hot	High	Weak	Yes	Yes
4	Rain	Mild	High	Weak	Yes	Yes
5	Rain	Cool	Normal	Weak	Yes	Yes
6	Rain	Cool	Normal	Strong	No	No
7	Overcast	Cool	Normal	Strong	Yes	Yes
8	Sunny	Mild	High	Weak	No	No
9	Sunny	Cool	High	Weak	Yes	Yes
10	Rain	Mild	Normal	Weak	Yes	Yes
11	Sunny	Mild	Normal	Strong	Yes	Yes
12	Overcast	Mild	Normal	Strong	Yes	Yes
13	Overcast	Hot	High	Weak	No	No
14	Rain	Mild	Normal	Strong	No	No

i) Step 1:- Find Entropy of data set

$$E(S) = E((9, 5-1)) = (-\frac{4}{14} \log_2 \frac{4}{14}) + (\frac{5}{14} \log_2 \frac{5}{14}) \\ = 0.94$$

Step 2:- Information gain for outlook is

$$G(S, \text{outlook}) = 0.94 - [\frac{5}{14}(0.97) + \frac{9}{14}(0.5)(0.9)] \\ = 0.246$$

Step 3:- Information gain for Temp is

$$G(S, \text{Temp}) = 0.94 - [\frac{4}{14} + \frac{6}{14}(0.918) + \frac{7}{14}(0.8)] \\ = 0.1515 = 0.029$$

Step 4:- Information gain for Humidity is

$$G(S, \text{Humidity}) = 0.94 - [\frac{7}{14} \times 0.985 + \frac{7}{14}(0.892)] \\ = 0.1515$$

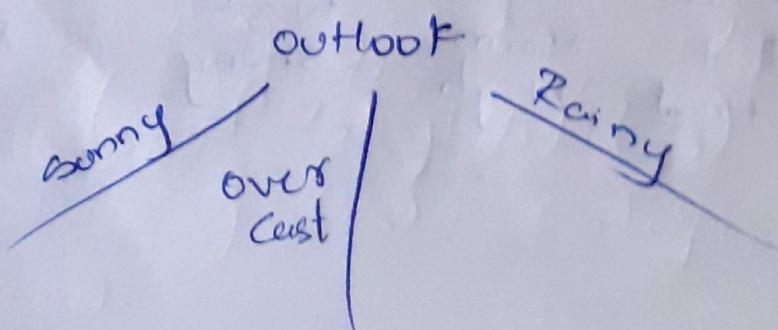
Step 5:- Information gain for windy is

$$G(S, \text{windy}) = 0.94 - [\frac{8}{14} \times 0.811 + \frac{6}{14} \times 1] \\ = 0.048$$

Information gain

Outlook	0.247
Temp	0.029
Humidity	0.152
Windy	0.048

outlook will be root



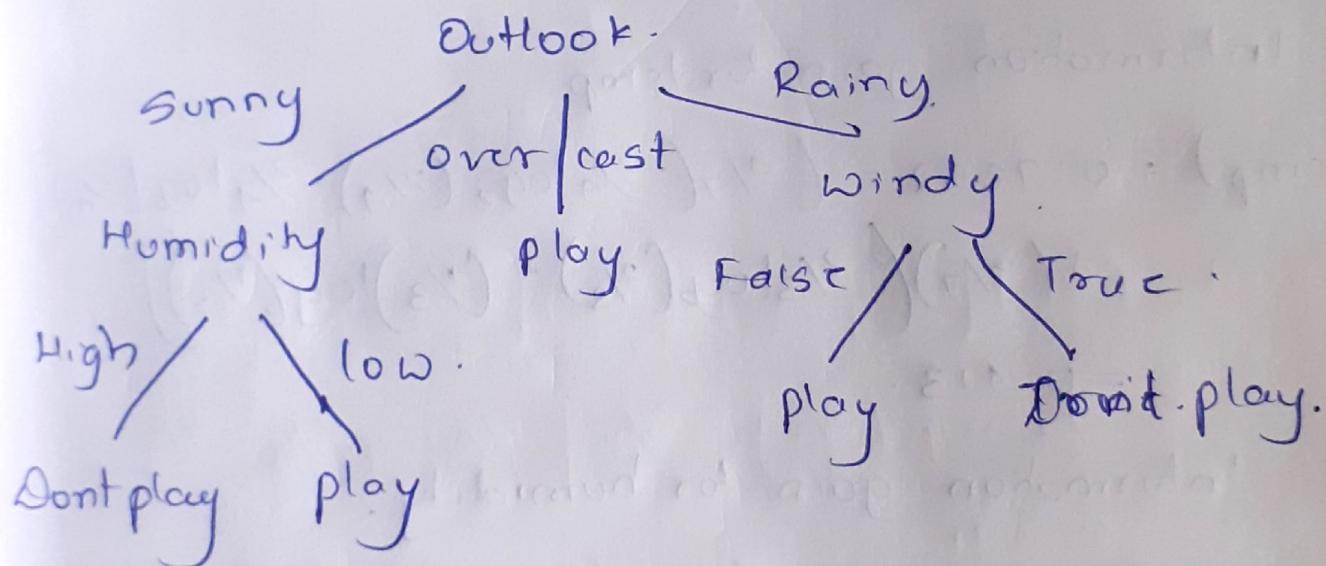
Entropy and Information gain after dividing the set.

Information gain.

Temperature	0.571
Humidity	0.971
Windy	0.020

Also entropy of overcast set is 0.

∴ Overcast will be leaf.



b) For day 1 to 7.

Day	Outlook	Temp	Humidity	Windy	Play
1	Sunny	Hot	High	False	No
2	Sunny	Hot	High	True	No
3	overcast	Hot	High	False	Yes
4	rainy	Mild	High	False	Yes
5	rainy	Cool	Normal	False	Yes
6	rainy	Cool	Normal	True	No
7	overcast	Cool	Normal	True	Yes

Step 1:- Entropy of Data set.

$$E(S) = E((A, S)) = (-\frac{4}{7} \log_2 \frac{4}{7} + (-\frac{3}{7}) \log_2 \frac{3}{7}) \\ = 0.9852.$$

Step 2:- Information gain for outlook

$$G(S, \text{outlook}) = 0.9852 - (\frac{2}{7}(0) + (\frac{2}{7})(0) + (\frac{3}{7}) \\ (-\frac{2}{3} \log_2 \frac{2}{3} + (-\frac{1}{3}) \log_2 \frac{1}{3})) \\ = 0.591$$

Step 3:- Information gain for temp.

$$G(S, \text{temp}) = 0.9852 - (\frac{4}{7}(-\frac{2}{4} \log_2 \frac{2}{4} - \frac{3}{4} \log_2 \frac{3}{4}) \\ + (\frac{3}{7})(-\frac{2}{3} \log_2 \frac{2}{3} - (\frac{1}{3}) \log_2 \frac{1}{3})) \\ = 0.413.$$

Step 4:- Information gain for humidity.

$$G(S, \text{humidity}) = 0.9852 - (\frac{4}{7}(-1/2 \log(1/2) - (1/2) \log(1/2)) \\ + \frac{3}{7}(-\frac{2}{3} \log(\frac{2}{3}) - \frac{1}{3} \log(\frac{1}{3}))) \\ = 0.9852 - (\frac{4}{7}(-\log(1/2) + \frac{3}{7}(-\frac{2}{3} \log(\frac{2}{3}) \\ - \frac{1}{3} \log(\frac{1}{3}))) \\ = 0.0202.$$

Step 5:- Information gain for windy

$$= 0.9852 - (\frac{3}{7}(-\frac{2}{3} \log(\frac{2}{3}) - (1/3) \log(1/3)) + \frac{4}{7} \\ (-0.75 \log(0.75) - 0.25 \log(0.25))) \\ = 0.089.$$

## Information gain

outlook	0.591
temp	0.413
windy.	0.089
Humidity	0.020

$\therefore$  outlook will be root

Step 7) - Find information gain based on outlook.

Information gain.

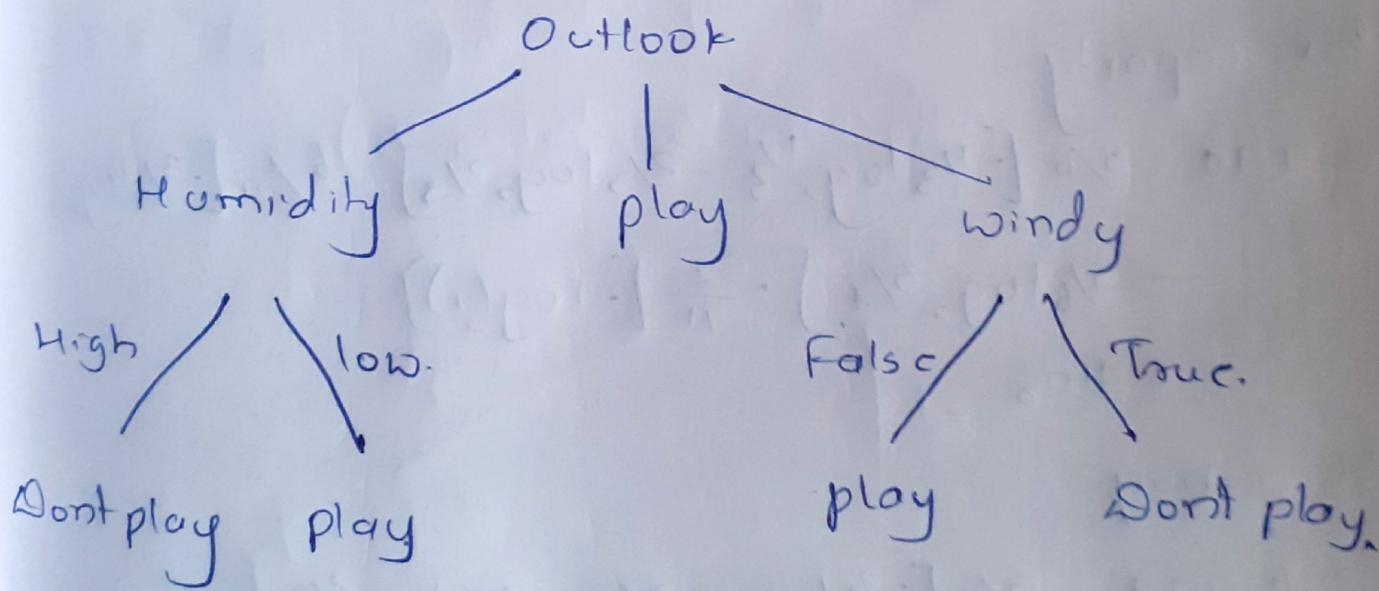
Temp 0.98

Humidity. 0.71

Windy 0.017.

Entropy of overcast is 0.

$\therefore$  it will be leaf



$$3) \text{ Entropy} = E([6, -5]) = -(6/11) \log_2(6/11) + (-5/11) \log_2(-5/11) = 0.994.$$

$$\begin{aligned} \text{IG for price} &= [0.994 - (4/11) [-\frac{2}{4} \log_2(\frac{2}{4}) - \frac{2}{4} \log_2(\frac{2}{4})] - \\ &- (4/11) [-\frac{2}{4} \log_2(\frac{2}{4}) \cdot \frac{2}{4} \log_2(\frac{2}{4})] - \\ &-\frac{3}{11} [-\frac{2}{3} \log_2(\frac{2}{3}) - \frac{1}{3} \log_2(\frac{1}{3})] \\ &= 0.0167. \end{aligned}$$

IG for maintenance.

$$\begin{aligned} &= 0.994 - (2/11) [-1 \log_2(1)] - (4/11) [-\frac{2}{4} \log_2(\frac{2}{4}) - \\ &-\frac{2}{4} \log_2(\frac{2}{4})] - \frac{5}{11} [-\frac{2}{5} \log_2(\frac{2}{5}) - \frac{3}{5} \log_2(\frac{3}{5})] \\ &= 0.591 \end{aligned}$$

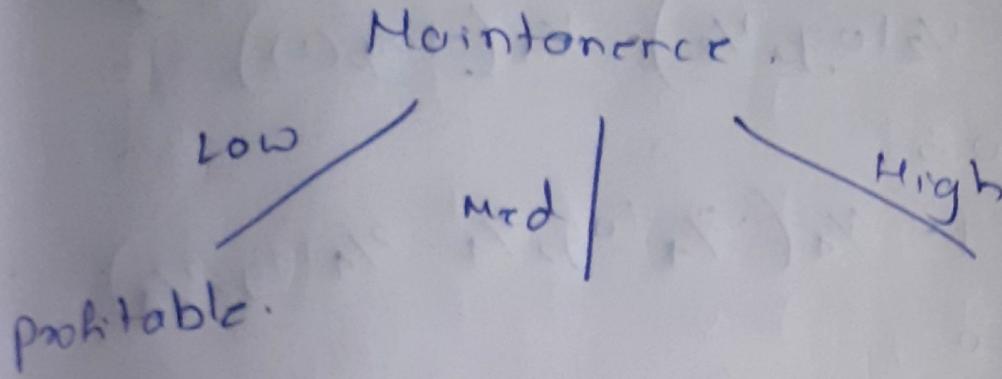
IG for capacity

$$\begin{aligned} &= 0.994 - \frac{3}{11} [-\frac{1}{3} \log_2(\frac{1}{3}) - \frac{2}{3} \log_2(\frac{2}{3})] - \frac{6}{11} [-\frac{3}{6} \log_2(\frac{3}{6}) - \\ &-\frac{3}{6} \log_2(\frac{3}{6})] - \frac{2}{11} [-1 \log_2(1)] \\ &= 0.4108 \end{aligned}$$

IG for orbbag

$$\begin{aligned} &= 0.994 - \frac{5}{11} [-\frac{3}{5} \log_2(\frac{3}{5}) - \frac{2}{5} \log_2(\frac{2}{5})] - \\ &-\frac{6}{11} [-\frac{3}{6} \log_2(\frac{3}{6}) - \frac{3}{6} \log_2(\frac{3}{6})] \\ &= 0.0075. \end{aligned}$$

Maintainence is root



Maintaince Price Capacity Airbag Profitable.

(i)	Low	low	2	No	Yes
	Low	low	4	No	Yes

(ii)	Med	low	4	Yes	No
	Med	med	4	No	No
	Med	med	4	Yes	Yes
	Med	high	4	Yes	Yes

(iii)	High	low	4	No	No
	High	med	2	yes	No
	High	med	5	No	Yes
	High	high	2	yes	No
	High	high	5	Yes	Yes

(i) Entropy =  $E [3J] = -1 \log_2 1 = 0$  leaf.

(ii) Entropy =  $E [2, -2] = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log \frac{1}{3} = 1.$



$$IG \text{ for price} = 1 - \frac{1}{4}(-\log_2) - \frac{2}{4}(-\frac{1}{2}\log_2(\frac{1}{2}) - \frac{1}{2}\log_2(\frac{1}{2})) - \frac{1}{4}(-\log_2(1)) \\ = 0.5.$$

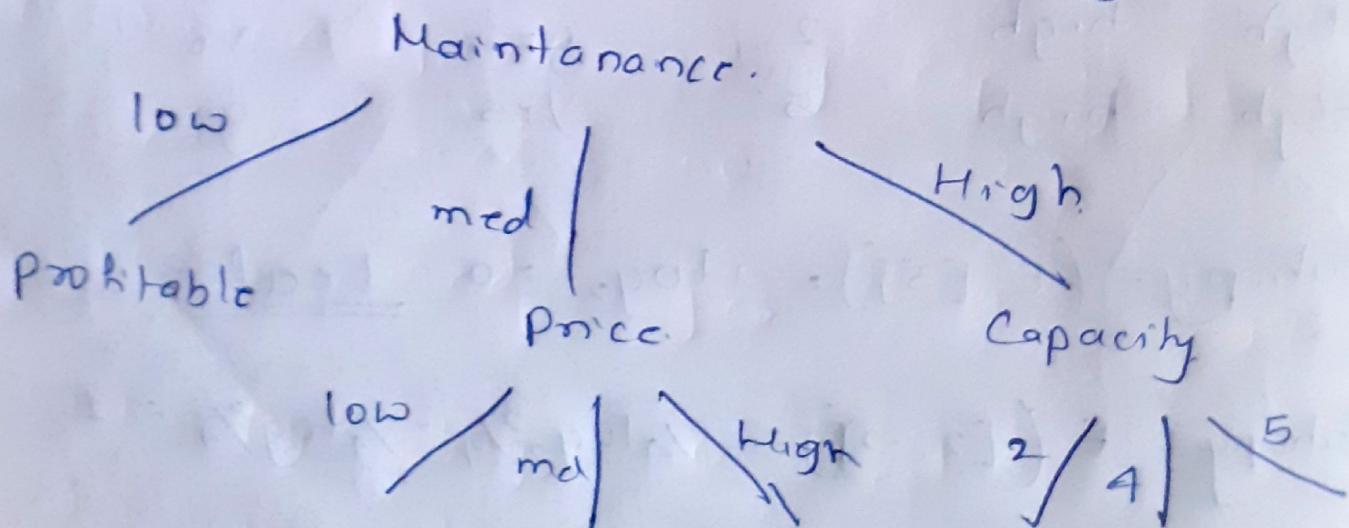
$$IG \text{ for capacity} = 1 - \frac{1}{4}(-\frac{3}{4}\log_2 \frac{3}{4} - \frac{1}{4}\log_2 \frac{1}{4}) \\ = 0.$$

$$IG \text{ for airbag} = 1 - \frac{3}{4} \left[ -\frac{2}{3}\log_2 \frac{2}{3} - \frac{1}{3}\log_2 \frac{1}{3} \right] - \frac{1}{4}(-\log_2(1)) \\ = 0.3113.$$

Prize vs Price next Node.

$$\text{Entropy} = E([2, 3]) = -\frac{2}{5}\log_2 \frac{2}{5} - \frac{3}{5}\log_2 \frac{3}{5} \\ = 0.971$$

$$IG \text{ for price} = 0.971 - \frac{1}{5}(-\log_2(1)) - \frac{2}{5}(-\frac{1}{2}\log_2 \frac{1}{2} - \frac{1}{2}\log_2 \frac{1}{2}) \\ - \frac{2}{5}(-\frac{1}{2}\log_2 \frac{1}{2} - \frac{1}{2}\log_2 \frac{1}{2})$$



	Maintenance	Price	Capacity	Airbag	Profitab.
i)	med.	low	4	yes	no
ii)	med	med	4	no	no
iii)	med	med.	4	yes	yes.
iv)	med	High	4	yes	yes

b) Maintenance Capacity Price airbag profitab.

High	2	med	yes	no
High	2	High	yes	no
High	4	low	no	no
High	5	med	yes	yes
High	5	High	yes	yes.

a) i) Entropy =  $E[-1] = -1 \log_2 1 = 0$  Lcat.

ii) Entropy =  $E[1, -1] = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} = 1$ .

IG for capacity =  $1 - 1 \left( -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} \right)$   
 $= 0$ .

IG for airbag =  $1 - \frac{1}{2} (-\log_2 \frac{1}{2}) - \frac{1}{2} (-\log_2 \frac{1}{2})$   
 $= 1$ .

Airbag next child.

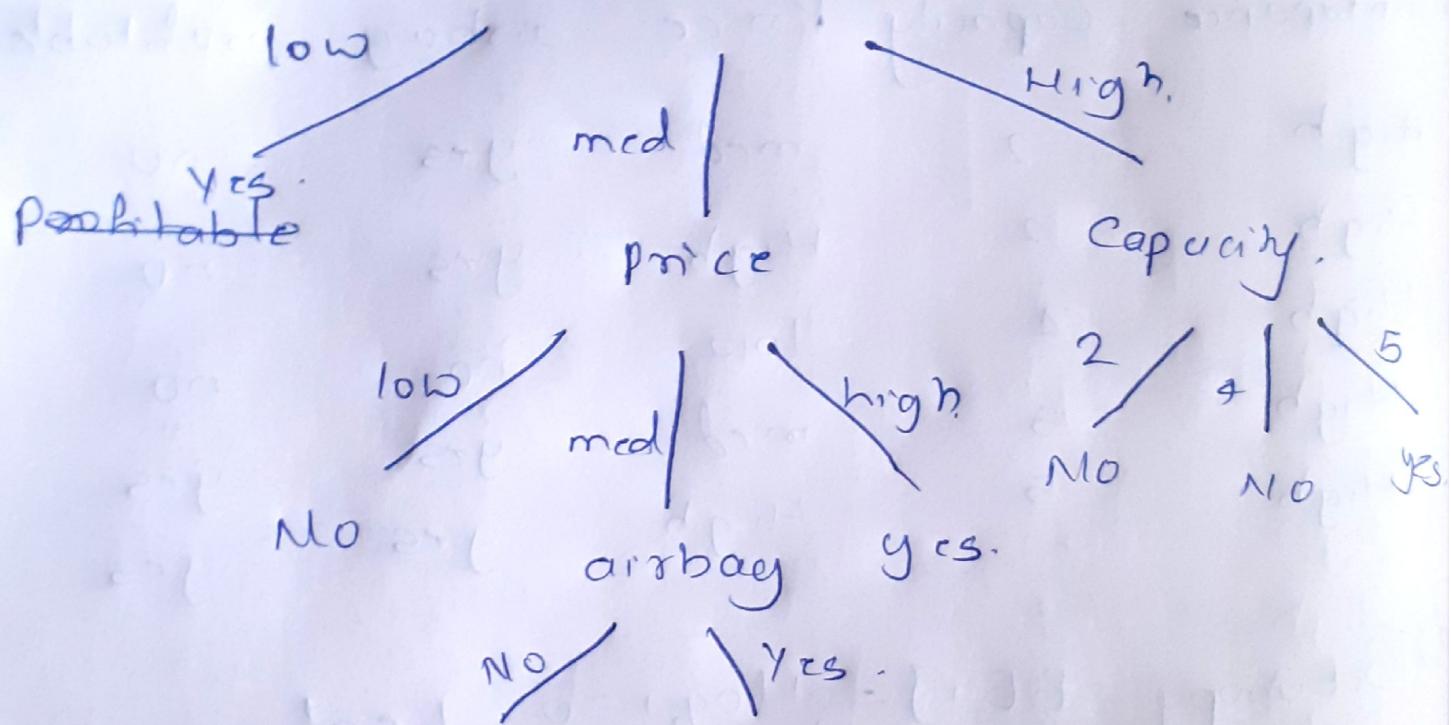
(iii) Entropy =  $E[I] = -\log_2(1) = 0$ . Leaf.

b.) (i) Entropy =  $-\log_2(1) = 0$ . Leaf

(ii) Entropy =  $-\log_2(1) = 0$ . Leaf

(iii) Entropy =  $-\log_2(1) = 0$ . Leaf.

Maintainance.



Maintainance price airbag capacity prohibitabl

(i) med med no 4 no

(ii) med med yes 4 yes

(i) Entropy =  $-\log_2 1 = 0$ . Leaf

(ii) Entropy =  $-\log_2 1 = 0$ . Leaf

## Maintenance.

