Solid State Physics

Homework Ch.5-No.1, Due on May 20th, Friday

1. Problem No.4 in Chapter 31 of "Solid State Physics" by Ashcroft/Mermin, page 668.
2. Problem No.9 in Chapter 31 of "Solid State Physics" by Ashcroft/Mermin, page 669.
3. From the magnetization M of the formula (5.10) in the lecture notes Ch5-No.2, compute the susceptibility at any temperature and show that the susceptibility is analytic generically. This implies that there is no phase transition in this system.
4. Answer the question presented in page 10 in the PPT of Lecture Notes, Ch5. No.2 (see also the Remarks in page 6 in the PPT of Lecture Notes, Ch5. No.1). Provide a quantitative proof.

Solid State Physics

Homework Ch5-No.2, Due on May 27th, Friday

1. Problem No.5 in Chapter 32 of "Solid State Physics" by Ashcroft/Mermin, page 667: "Hubbard model of the hydrogen molecule".

2. From the Hartree-Fock theory, the energy of a gas of electron gas with Coulomb interactions is given by $E = E_\uparrow + E_\downarrow$, where

$$E_\uparrow = N_\uparrow\big[\frac{3}{5}(k_{F,\uparrow}a_0)^2 - \frac{3}{2\pi}(k_{F,\uparrow}a_0)\big]\mathrm{Ry}, \quad E_\downarrow = N_\downarrow\big[\frac{3}{5}(k_{F,\downarrow}a_0)^2 - \frac{3}{2\pi}(k_{F,\downarrow}a_0)\big]\mathrm{Ry}. \tag{1}$$

   For fixed $N = N_\uparrow + N_\downarrow$, denote by $\alpha = |N_\uparrow - N_\downarrow|/(N_\uparrow + N_\downarrow)$ the polarization ratio of the gas. a) Find the total energy as a function of N and α. b) Show that the total energy satisfies $E(N,\alpha) \geq \min\big[E(N,\alpha=0), E(N,\alpha=1)\big]$. Namely, the minimal energy corresponds to the case with either $\alpha = 0$ or $\alpha = 1$.

3. For a system with three spins located at the three vertices of an equilateral triangle. The Heisenberg spin model reads

$$H = -\sum_{i<j}^{3} J_{ij}\vec{S}_i \cdot \vec{S}_j. \tag{2}$$

   Let the spin $S_i = 1/2$ and $J_{ij} \equiv J$. Solve the ground state and the corresponding energy of the total system for a) $J > 0$, and b) $J < 0$. Compute the spin polarization at each site for the ground states in the two cases. You may diagonalize the Hamiltonian numerically if necessary.

Solid State Physics

Homework Ch5-No.3, Due on June 03rd, Friday

1. Problem No.5 in Chapter 33 of "Solid State Physics" by Ashcroft/Mermin, page 667: "Anisotropic Heisenberg Model".

2. Problem No.6 in Chapter 33 of "Solid State Physics" by Ashcroft/Mermin, page 667: "Mean field near critical point".

3. Consider the Hamiltonian $H = -g\mu_B \vec{H} \cdot \vec{J} = -g\mu_B H_z J_z$, with $J_z = -Ns, -Ns+1, ..., Ns$. Calculate the spontaneous magnetization in the two different ways:

$$\langle M \rangle \ = \ g\mu_B \lim_{N\to\infty} \lim_{H_z \to 0} \frac{1}{V} \frac{\text{Tr } e^{-\beta g\mu_B H_z J_z} J_z}{\text{Tr } e^{-\beta g\mu_B H_z J_z}}, \tag{1}$$

$$\langle M \rangle \ = \ g\mu_B \lim_{H_z \to 0} \lim_{N\to\infty} \frac{1}{V} \frac{\text{Tr } e^{-\beta g\mu_B H_z J_z} J_z}{\text{Tr } e^{-\beta g\mu_B H_z J_z}}, \tag{2}$$

   with $V = Nv_0$. Find the results through the two different formulas, and compare the two results. This result gives a clear illustration about what the *spontaneous symmetry-breaking* is. (Think about why this result is not dependent on temperature β.)

4. From the mean-field theory, Find the magnetization when temperature $T \to 0$, and compare your solution with the result in Eq. (33.34) of "Solid State Physics" by Ashcroft/Mermin.