第八次作业

(1)

Consider a potential

$$V = 0 \quad \text{for } r > R, \qquad V = V_0 = \text{constant} \quad \text{for } r < R,$$

where V_0 may be positive or negative. Using the method of partial waves, show that for $|V_0| \ll E = \hbar^2 k^2/2m$ and $kR \ll 1$ the differential cross section is isotropic and that the total cross section is given by

$$\sigma_{\text{tot}} = \left(\frac{16\pi}{9}\right)\frac{m^2 V_0^2 R^6}{\hbar^4}.$$

Suppose the energy is raised slightly. Show that the angular distribution can then be written as

$$\frac{d\sigma}{d\Omega} = A + B\cos\theta.$$

Obtain an approximate expression for B/A.

(2)

Consider the scattering of a particle by an impenetrable sphere

$$V(r) = \begin{cases} 0 & \text{for } r > a \\ \infty & \text{for } r < a. \end{cases}$$

a. Derive an expression for the s-wave ($l = 0$) phase shift. (You need not know the detailed properties of the spherical Bessel functions to be able to do this simple problem!)

b. What is the total cross section σ $[\sigma = \int (d\sigma/d\Omega)\, d\Omega]$ in the extreme low-energy limit $k \to 0$? Compare your answer with the geometric cross section πa^2. You may assume without proof:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2,$$

$$f(\theta) = \left(\frac{1}{k}\right)\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\cos\theta).$$

(3)

The Lippmann-Schwinger formalism can also be applied to a *one*-dimensional transmission-reflection problem with a finite-range potential, $V(x) \neq 0$ for $0 < |x| < a$ only.

ı. Suppose we have an incident wave coming from the left: $\langle x|\phi\rangle = e^{ikx}/\sqrt{2\pi}$. How must we handle the singular $1/(E - H_0)$ operator if we are to have a transmitted wave only for $x > a$ and a reflected wave and the original wave for $x < -a$? Is the $E \to E + i\varepsilon$ prescription still correct? Obtain an expression for the appropriate Green's function and write an integral equation for $\langle x|\psi^{(+)}\rangle$.

(4)

Consider scattering by a repulsive δ-shell potential:

$$\left(\frac{2m}{\hbar^2}\right)V(r) = \gamma\delta(r - R), \quad (\gamma > 0).$$

a. Set up an equation that determines the s-wave phase shift δ_0 as a function of k $(E = \hbar^2 k^2/2m)$.

b. Assume now that γ is very large,

$$\gamma \gg \frac{1}{R}, k.$$

Show that if $\tan kR$ is *not* close to zero, the s-wave phase shift resembles the hard-sphere result discussed in the text. Show also that for $\tan kR$ close to (but not exactly equal to) zero, resonance behavior is possible; that is, $\cot \delta_0$ goes through zero from the positive side as k increases.

第九次作业

（1）

Consider scattering by a repulsive δ-shell potential:

$$\left(\frac{2m}{\hbar^2}\right)V(r) = \gamma\delta(r-R), \quad (\gamma > 0).$$

Assume now that γ is very large,

$$\gamma \gg \frac{1}{R}, k.$$

Determine approximately the positions of the resonances keeping terms of order $1/\gamma$; compare them with the bound-state energies for a particle confined *inside* a spherical wall of the same radius,

$$V = 0, \quad r < R; \qquad V = \infty, \quad r > R.$$

（2）

The Lippmann-Schwinger formalism can also be applied to a *one*-dimensional transmission-reflection problem.

Consider the special case of an attractive δ-function potential

$$V = -\left(\frac{\gamma\hbar^2}{2m}\right)\delta(x) \quad (\gamma > 0).$$

Solve the integral equation to obtain the transmission and reflection amplitudes.

The one-dimensional δ-function potential with $\gamma > 0$ admits one (and only one) bound state for any value of γ. Show that the transmission and reflection amplitudes you computed have bound-state poles at the expected positions when k is regarded as a complex variable.

（3）证明波算符满足关系式 $\hat{H}\hat{U}(0,\pm\infty) = \hat{U}(0,\pm\infty)\hat{H}_0$

（4）证明散射矩阵满足关系式 $\hat{S}|E,l,m\rangle = e^{2i\delta_l}|E,l,m\rangle$