

Empty Promises

I would like to dedicate this final discussion to vacuous statements - statements that are true *by default* - or by lack of counterexamples.

Let me show you.

Suppose you have an empty bucket. Consider the statement **all balls in this bucket are blue**. Is this statement true or false?

Well, can you produce a ball from the bucket that *isn't* blue? No, of course not. Thus the statement must be true.

Now you could argue that one cannot produce a blue ball either. But get this, my statement was **if a ball is in this bucket, it is blue**. You *cannot* disprove it, because doing so would require you to produce a counterexample from the bucket. And as we know, the bucket is empty. Hence there are no counterexamples.

A Little bit More

Vacuous statements have the form $\forall r, P(r) \implies Q(r)$. The difference is that r either comes from an empty collection, or for every r , $\neg P(r)$ holds. (Recall the equivalent of an implication).

Have some more examples.

1. Every even injective function from \mathbb{R} to itself is continuous
2. Every skew-symmetric nonzero 1×1 matrix is invertible.
3. Every even integer of the form $24k + 13$ is a perfect square

and so on. This is important because sometimes, it could be the case that a forall statement is vacuously true, but you use it the wrong way.

I really don't have any examples for this other than the footnote in PW-13.

Conclusion

That concludes my little course on proofwriting and logic. I hope it was useful to you, and that you learned something.

If you encounter questions, typos, or anything else, please feel free to reach out. Cya.