

## Q1.1

This problem can be solved using binomial distribution. Probability of winning a match is 0.6.  
Matches left.

$$P = P(\text{win}) + P(\text{lose})$$

$$\text{Formula} = p^k(1-p)^{(n-k)}$$

$$P = \binom{8}{1}(.6)^1(.4)^7 + \binom{8}{1}(.6)^7(.4)^1 = 0.097$$

## Q1.2

In this question we were required to find the winning probability of a player in a game of tennis. It was given that the game is tied at 40:40. This problem can also be solved using binomial distribution. We have two scenarios:

$$p = p(w), 1-p = p(l)$$

1)WLWLWWLWL....

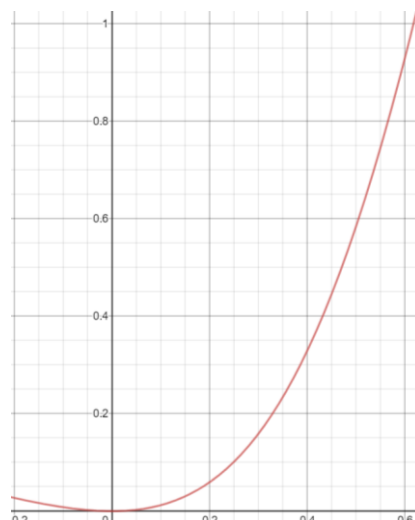
2)WW,WIWWW.....

So adding both probabilities:

$$P(W) = p^2 * (1 + (1-p)p + (1-p)^2 * p^2 + \dots) + ((1-p)*p^3) * (1 + (1-p)p + (1-p)^2 * p^2 + \dots)$$

$$\text{Using the summation formula of the infinite series: } P(W) = (p^2 + p^3 + p^4) / (1 - p + p^2)$$

Graph:



**Q4.1**

For the uniqueness of the solution we are given  $\|w_1\| = 1$ , and also  $\Sigma = \text{COV}(x)$ .

So considering this and that  $w_1^T x = z_1$

$$w_1^T E[(x - \mu)(x - \mu)^T] w_1 = w_1^T \Sigma w_1 = \text{Var}(z_1).$$

If we take derivative with respect to  $\Sigma w_1$  and equating it to zero, we have  $\Sigma w_1 = \lambda w_1$

Hence  $w_1$  can be considered as eigen vector of  $\Sigma$  and  $\lambda$  is the corresponding Eigen value.

Hence the largest  $\lambda = \lambda_1$

While maximizing the  $w$ , it is necessary to use Lagrange method.

**Q4.2**

In this part we are to show, that the second component is the eigenvector of the covariance matrix with the second largest eigenvalue. So to prove this the second principal component  $w_2$  should also maximize variance, be unit length, and orthogonal to  $w_1$ .

So in this part we also maximize  $w^T \Sigma w$  called as  $w_2$ . Taking the derivative and equating it to zero, and then multiply with the transpose, we get  $\Sigma w_2 = \lambda_2 w_2$

Hence  $\lambda_2$  is the second largest eigen value.