11/11/22 UNIT 1 Introduction to Probability -

☐ Experiment

→ An experiment is an act or process that leads to a single outcome that can't be predicted with certainity. A result of an experiment is called outcome. Usually denoted by capital letters

eg Tossing a coin is an experiment.

→ It has two outcomes Heads or Tails

☐ Random Experiment.

→ Experiments which are performed essentially under same condⁿs and whose results can't be predicted are known as random experimen

☐ Sample Space

→ The set of all possible outcomes of random experiment is called sample space for that experiment. And is denoted by S.

eg On tossing a coin possible outcomes are head ~~or tails~~ and tail

□ Exhaustive Events (Total)

→ A set of events is said to be exhaustive if it includes all the possible events

eg In tossing a coin there are two exhaustive event head & ~~hee~~ tail & there is no therd

Mutually exclusive events

→ Events are said to be mutually exclusive if the occurance of one event rules out the happening of all other events

eg Tossing a coin, Head or tail can turn out but can't happen at the same time.

Probability

→ If an experiment can result in n exhaustive, mutually exclusive and equally likely cases and m of them are favorable to the event E denoted by P(E) and is defined as

$$\boxed{P(E) = \frac{m}{n}} = \frac{\text{favorable no. of outcomes}}{\text{~~Total~~ Exhaustive no. of cases}}$$

or

$$P = \frac{\text{No. of ways it can happen}}{\text{Total no. of ways.}}$$

Note :- ① Probability is a concept which tends to measure the degree of uncertainity.

② Probability does not tell us exactly what will happen, it is just a guide

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Conditional Probability -

→ The probability of happening of an event A, when another event B is known to have already happened is called conditional probability & it is denoted by $P(A/B)$ or $P\left(\frac{A}{B}\right)$.

→ If A & B are any two events in a sample space S then the conditional probability of A given B is defined as $P(A/B) = \frac{P(A \cap B)}{P(B)}$

where $P(B) > 0$

$||^{ly}$ $P(B/A)$ or $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$, $P(A) > 0$

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Addition theorem of Probability -

→ If A & B are any two events in sample space S then $\boxed{P(A \text{ or } B) = P(A\cup B) = P(A) + P(B) - P(A\cap B)}$

Multiplication Theorem on Probability -

→ If A & B are any 2 events in sample space S then $\boxed{P(A\cap B) = P(A)P(B/A)}$, $P(A)>0$

$\boxed{P(B\cap A) = P(B)P(A/B)}$, $P(B)>0$

Theorem of Total Probability -

Statement :- Let $B_1, B_2 \dots B_n$ be a set of exhaustive & mutually exclusive events of the sample space S with $P(B_i) \neq 0$, $i = 1,2,\dots n$ & let A be any event of S then

~~probability~~ $\boxed{P(A) = \sum_{i=1}^{n} P(B_i \cap A) = \sum_{i=1}^{n} P(B_i)P(A/B_i)}$

Proof

Since $B_1, B_2, B_3 \dots B_n$ are exhaustive and mutually exclusive events of S

ie $S = \bigcup_{i=1}^{n} B_i$ & $B_i \cap B_j = \phi$, $i \neq j$

let A be any event of S

$\therefore A\cap B_1, A\cap B_2 \dots\dots A\cap B_n$

are also mutually exclusive & ~~A=(A∩B)∪~~

$$A = (A\cap B_1)\cup(A\cap B_2)\cup \dots \cup (A\cap B_n)$$

Hence by additive rule ~~by~~ of mutually exclusive events we obtain

$$P(A) = P[(A\cap B_1)\cup(A\cap B_2)\cup(A\cap B_3)\cup\dots\cup(A\cap B_n)]$$

$$P(A) = P(A\cap B_1) + P(A\cap B_2) + \dots + P(A\cap B_n)$$

$$P(A) = P(B_1\cap A) + P(B_2\cap A) + \dots + P(B_n\cap A)$$

$$P(A) = \sum_{i=1}^{n} P(B_i\cap A) = \sum_{i=1}^{n} P(B_i)P(A/B_i)$$

Baye's Theorem

→ Let $B_1, B_2 \dots B_n$ be a set of exhaustive & mutually exclusive events of sample space S with $P(B_i)\neq 0$, $i = 1, 2 \dots n$ & A be any event of S with $P(A)\neq 0$ then ~~$P(B_i/A)$~~

$$\boxed{P(B_i/A) = \frac{P(B_i)P(A/B_i)}{\sum_{i=1}^{n} P(B_i)P(A/B_i)}}$$

Proof :- By the definition of cond^{nal} probability we have $P(B_i/A) = \dfrac{P(B\cap A)}{P(A)}$, $P(A)>0$ — 1

By Total probability theorem we have

$$P(A) = \sum_{i=1}^{n} P(B_i\cap A) = \sum_{i=1}^{n} P(B_i)P(A/B_i) \quad — 2$$

Also by multiplication laws we have

$$P(B_i\cap A) = P(B_i)P(A/B_i) \quad — 3$$

using ② & ③ in ①

$\therefore \cancel{P(B_i/A)}\ P(B_i/A) = \dfrac{P(B_i)P(A/B_i)}{\sum_{i=1}^{n} P(B_i)P(A/B_i)}$

Note :- Baye's theorem is known as forecasting theorem.

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Q A coin is tossed once find the probabili of getting head.

→ A coin is tossed once ∴ no. of equally likely cases $S = \{H, T\}$ ie $n = 2$

→ no. of favorable cases (getting head ie $m = 1$

→ By the defination we have probability equal $P = \dfrac{m}{n}$

$$\boxed{P = \frac{1}{2}}$$

Q Two coins are tossed once find the probability of getting (i) one head
(ii) atleast one head.

→ Two coins are tossed once ∴ no. of equally likely cases is 4 ie $S = \{HH, HT, TH, TT\}$
ie $n = 4$

(i) Let A be the event in which exactly one head turns up then $A = \{HT, TH\}$ $m = 2$

$\therefore P(A) = \frac{m}{n} = \frac{2}{4} = \frac{1}{2}$

(ii) Let B be the event in which atleast one head turns up then $B = \{HH, HT, TH\}$
$m = 3$

$\therefore P(B) = \frac{m}{n} = \frac{3}{4}$

Q A coin is tossed thrice find the probability of getting all tails.

→ A coin is tossed thrice ∴ no. of equally likely cases is 8 ie $S = \{HHH, HHT, HTH, TTH, THH, HTT, THT, TTT\}$ $n = 8$

→ Let A be the event of getting all tails $= m = 1$

$P(A) = \frac{m}{n} = \frac{1}{8}$

Q A die is rolled find the probability of getting an even no.

→ ~~A~~ A die has six ~~pho~~ faces

$S = \{1, 2, 3, 4, 5, 6\}$ ie $n = 6$

Let A be the event of getting an even number $A = \{2, 4, 6\}$ ie $m = 3$

$$P(A) = \frac{m}{n} = \frac{3}{6} = \frac{1}{3}$$

Q ~~A~~ A pair of dice is rolled and two numbers appearing on top are recorded find the probability of event in which the sum is 10 or more

→ ~~A~~ Two dice rolled simultaneously

$S = 6^2 = 36$ $\quad n = 36$

~~S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)~~
~~(2,1), (2,2) (2,3) (2,4) (2,5) (2,6)~~
~~3~~
~~4~~
~~5~~
~~6~~

$S = \{(4,6), (5,5), (6,4), (5,6), (6,6), (6,5)\}$

$m = 6$

$$P(A) = \frac{m}{n} = \frac{6}{36} = \frac{1}{6}$$

Q What is the chance that a non leap year selected at random will contain 53 sundays

→ A non leap year has 365 days

$52 \times 7 = 364d + 1 day$

Here $m = 1$ (sunday)

$n = 7$

$$P = \frac{m}{n} = \frac{1}{7}$$

Q → What is the chance that a leap year Selected at random will contain 53 Sundays

→ A leap year has 366 days

$\therefore 52 \times 7 = 364$ days $+ 2$ days

that 2 days can either be {(S,M), (M,T), (T,W), (W,Th), (Th,F), (F,Sa), (Sa,S)}

$n = 7$

$m = 2$ (Sunday)

$$P = \frac{m}{n} = \frac{2}{7}$$

Q What is the probability that a card is drawn at random from a pack of playing cards (i) King

(ii) Heart

→ Let S be the sample space associated with drawing of a card ie $n = 52$

(i) Let A be the event of drawing a king

ie $m = 4$

$$P(A) = \frac{m}{n} = \frac{4}{52} = \frac{1}{13}$$

(ii) Let B be the event of drawing a heart ie $m = 13$

$$P(B) = \frac{m}{n} = \frac{13}{52} = \frac{1}{4}$$

Q A bag contains 4 red, 3 green, & 5 black balls. If a ball is drawn at random from the bag. find the probability that the ball drawn is red.

→ No. of balls in a bag $= 4+3+5 = 12$

ie $n = 12$

then favorable (red ball) ie $m = 4$

$$P(R) = \frac{m}{n} = \frac{4}{12} = \frac{1}{3}$$

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Q If A & B are any two events, if $P(A) = \frac{1}{2}$ $P(B) = \frac{1}{4}$ $P(A \cap B) = \frac{1}{8}$ then find $P(A \cup B)$

A $P(A) = \frac{1}{2}$

$P(B) = \frac{1}{4}$

$P(A \cap B) = \frac{1}{8}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{4} - \frac{1}{8}$$

$$P(A \cup B) = \frac{5}{8}$$

$P(A) = 0.25$ $P(B) = 0.5$ $P(A \cup B) = 0.59$

$P(A \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.59 \; \cancel{P(A \cup B)} = 0.25 + 0.5 - P(A \cap B)$$

$$P(A \cap B) = 0.25 + 0.5 - 0.59$$

$$\boxed{P(A \cap B) = 0.16}$$

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Note :- All the outcomes that are not ever is compliment of event.

eg When the event is head its compliment is tail.

→ The probability of the compliment of the event is to one minus the probability of the event. since the sum of the probabilities of all possible ~~eq~~ event equals one

$$P(A^c) = 1 - P(A)$$

$$P(A^c \cap B) = P(B) - P(A \cap B)$$

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$P(A^c \cap B^c) = 1 - P(A \cup B)$$

Q If A & B are two events such that

$P(A) = 0.5$

$P(B) = 0.6$

$P(A \cup B) = 0.8$

find $P(A/B)$

Sol $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.5 + 0.6 - 0.8$

$P(A \cap B) = 0.3$

$$P(A/B) = \frac{0.3}{0.6}$$

$$\boxed{P(A/B) = 0.5}$$

Q → If A & B are any two events

$P(A) = 0.4$

$P(B) = 0.6$

$P(A/B) = 0.5$

find $P(B/A)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A/B) \times P(A)$$

$$= 0.5 \times 0.6$$

$$= 0.3$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.3}{0.4}$$

$$\boxed{P(B/A) = 0.75}$$

③ $P(A) = 1/5$

$P(B) = 2/3$

$P(A \cap B) = 1/15$

then find $P(A^c \cap B)$

$$P(A^c \cap B) = P(B) - P(A \cap B)$$

$$= \frac{2}{3} - \frac{1}{15}$$

$$= 0.6$$

④ $P(A) = 1/3$

$P(B) = 1/4$

$P(A \cup B) = 1/2$

find (i) $P(B/A)$

(ii) $P(A/B^c)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{3} + \frac{1}{4} - \frac{1}{2}$$

$$= 0.08$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.08}{0.33}$$

$$= \underline{0.24}$$

(ii) $P(A/B^c) = \frac{P(A \cap B^c)}{P(B^c)}$

$P(A \cap B^c) = P(A) - P(A \cap B)$

$= 0.3 - 0.08$

$= 0.2\cancel{5}$

$P(B^c) = 1 - P(B)$

$1 - 0.25$

$= 0.75$

$P(A/B^c) = \frac{0.2\cancel{5}}{0.75} = 0.33$

(5) $P(A) = 1/2$

$P(B) = 1/3$

$P(A \cap B) = 1/3$

find $P(A \cup B)$

(ii) $P(A^c \cap B)$

(iii) $P(A \cap B^c)$

(iv) $P(A^c \cap B^c)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{3}}$

$= \frac{1}{2}$

$P(A^c \cap B) = P(B) - P(A \cap B)$

$= \cancel{1/3} - \cancel{1/3}$

$= 0$

(iii) $P(A \cap B^c) = P(A) - P(A \cap B)$

$= \frac{1}{2} - \frac{1}{3}$

$= 1/6 = 0.166$

(iv) $P(A^c \cap B^c) = 1 - P(A \cup B)$

$= 1 - 1/2$

$= 0.5$

(6) find $P(A^c) + P(B^c)$. The probability that atleast one event A & B occur is 0.7 & that they occur simultaneousl is 0.2 ~~find~~

$P(A \cup B) = 0.7$

$P(A \cap B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 1 - P(A^c) + 1 - P(B^c) - P(A \cap B)$

$P(A^c) + P(B^c) = 2 - P(A \cap B) - P(A \cup B)$

$P(A^c) + P(B^c) = 2 - 0.7 - 0.2$

$P(A^c) + P(B^c) = 1.1$

Factorial

→ Factorial means to multiply a series of decending natural numbers

Combination

→ when the order does not matter

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

eg No.*in the lotteries

Permutation.

→ When the order does matter

$$^nP_r = \frac{n!}{(n-r)!}$$

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Q Bag A contains two white & 3 red balls & Bag B contains 4 white & 5 red balls One ball is drawn at random from one of bag & it is found to be red. Find the probability that the red ball drawn is from Bag B.

Sol Let A & B denote the events of selecting bag A & B respectively

$P(A) = \frac{1}{2}$

$P(B) = 1/2$

→ Let R denote the event of drawing a Red ball.

Having selected bag A the probability to draw a red ball from bag A is

$P(R/A) = 3/5$

$P(R/B) = 5/9$

One of the bag is selected at random & and ball selected it is found to be is red. The probability that selected Bag is B

By Bayes theorem $P\left(B_i/A\right) = \dfrac{P(B_i)\,P(A/B_i)}{\sum_{i=1}^{n} P(B_i)\,P(A/B_i)}$

$$P(B/R) = \frac{P(B)\,P(R/B)}{P(A)\,P(R/A) + P(B)\,P(R/B)}$$

$$= \frac{\frac{1}{2}\left(\frac{5}{9}\right)}{\left(\frac{1}{2} \times \frac{3}{5}\right) + \left(\frac{1}{2} \times \frac{5}{9}\right)}$$

$$= \frac{\frac{5}{18}}{\frac{3}{10} + \frac{5}{18}}$$

$$= \frac{25}{52}$$

Q → First box contains 2 black, 3 red and 1 white & 2nd box 1 black 1 red, 2 white balls 3rd box contains 5 black 3 Red & 4 white balls. Of these a box is selected at random. From it a red a ball is drawn. If the ball is red find the probability that it is from 2nd box.

Sol Let A, B, C be the events of selecting a box 1, 2, 3 respectively

$P(A) = \frac{1}{3}$

$P(B) = \frac{1}{3}$

$P(C) = 1/3$

Let R be the event of drawing a red ball from a box

$P(R/A) = 3/6 = 1/2$

$P(R/B) = 1/4$

$P(R/C) = \frac{3}{12} = 1/4$

$$P(B/R) = \frac{P(B)\, P(R/B)}{P(A)\, P(R/A) + P(B)\, P(R/B) + P(C)\, P(R/C)}$$

$$= \underline{\underline{1/4}}$$

③ Of of 3 men chances that a politian, a businessman or an academician will be appointed will be appointed as vice chancellor (V.C) of a university are 0.5, 0.3, 0.2 respectively. probability that research is promoted by these person if they are appointed as V.C are 0.3, 0.7, 0.8 respectively.

(i) Determine the probability that research is Promoted

(ii) If research is promoted what is the probability that V.C is an academecian

Sol Let A, B, C be the events that a politican, bussnessman & academecian be appointed as V.C then of

$P(A) = 0.5$

$P(B) = 0.3$

$P(C) = 0.2$

Let R be the event of promoting research the probability that research is promoted if they are appointed as V.C are.

$P(R/A) = 0.3$

$P(R/B) = 0.7$

$P(R/C) = 0.8$

By Bayes theorem

(i) $P(R) = P(A)P(R/A) + P(B)P(R/B) + P(C)P(R/C)$

$= 0.5 \times 0.3 + 0.3 \times 0.7 + 0.2 \times 0.8$

$= 0.52$

(ii) $P(C/R) = \dfrac{P(C)\,P(R/C)}{P(A)\,P(R/A) + P(B)\,P(R/B) + P(C)\,P(R/C)}$

$= 0.30769$

Q Suppose 5 men out of 100 & 25 women out of 10000 are colourblind. A colourblind person is chosen at random what is the probability that persons being male (assume male & female be equal in no.)

Sol let A & B be male & female respectively

$P(A) = \cancel{\frac{5}{100}} \cancel{= \frac{1}{20}}\ \frac{1}{2}$

$P(B) = \cancel{\frac{25}{10000}} = \cancel{\frac{1}{400}}\ \frac{1}{2}$

~~Set C be~~ A colourblind person is chosen at random. the probability that chosen is, let C be event that person is colour blind

$P(C/A) = 5/100$

$P(C/B) = 25/1000$

The Probability that chosen person is male

$$P(A/C) = \frac{P(A)\,P(C/A)}{P(A)\,P(C/A) + P(B)\,P(C/B)}$$

$$= \frac{20}{21}$$

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Q → In a bolt factory machines A, B, C manufacture 20%, 30%, 50% of the total of their output & 6%, 3% & 2% are defective. A bolt is drawn at random & found to be defective find the probability it is manuctured from (i) Machine A

(ii) Machine B

(iii) Machine C.

<u>Sol</u> Let P(A), P(B), P(C) be the probabilitis of events that the bolts are manufactured by machine A, B, C respectively then

$P(A) = 20\% = \frac{20}{100} = 0.2$

$P(B) = 30\% = \frac{30}{100} = 0.3$

$P(C) = 50\% = \frac{50}{100} = 0.5$

Let D be event bolt is difectives

$P(D/A) = 6\% = 6/100 = 0.06$

$P(D/B) = 3\% = 3/100 = 0.03$

$P(D/C) = 2\% = 2/100 = 0.02$

By Baye's theorem we know that

$$P(B_i/A) = \frac{P(B_i)P(A/B_i)}{\sum_{i=1}^{n} P(B_i)P(A/B_i)}$$

(i) $$P(A/D) = \frac{P(A)P(D/A)}{P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)}$$

$$= \frac{(0.2)(0.06)}{(0.2)(0.06) + (0.3)(0.03) + (0.5)(0.02)}$$

$$= 0.38709$$

(ii) $$P(B/D) = \frac{P(B)P(D/B)}{P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)}$$

$$= \frac{0.3 \times 0.03}{0.2 \times 0.06 + 0.3 \times 0.03 + 0.5 \times 0.02}$$

$$= 0.2903$$

(iii) $$P(C/D) = \frac{P(C)P(D/C)}{P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)}$$

$$= \frac{0.5 \times 0.02}{0.2 \times 0.06 + 0.3 \times 0.03 + 0.5 \times 0.02}$$

$$= 0.3225$$

P_1 & P , significance ...

Q → 4 boxes A, B, C, D contains toys, the boxes contain 5000, 3000, 2000, 1000 toys respectively. The %s of toys in the boxes which are defective are 3%, 2%, 1%, 0.5% respectively. One toy is selected at random from one of the boxes & it is found to be defective. find the probability that it has come from box D.

Sol Let P(A), P(B), P(C), P(D) be the probability of events that toys are present in boxes A, B, C, D respectively

$P(A) = \frac{1}{4}$

$P(B) = \frac{1}{4}$

$P(C) = \frac{1}{4}$

$P(D) = \frac{1}{4}$

let E be the event of toy being defective

$P(E/A) = 3\% = 3/100 = 0.03$

$P(E/B) = 2\% = 2/100 = 0.02$

$P(E/C) = 1\% = 01/100 = 0.01$

$P(E/D) = 0.5\% = 0.5/100 = 0.005$

then from Baye's theorem

$$P(D/E) = \frac{P(D)\,P(E/D)}{P(A)\times P(E/A) + P(B)P(E/B) + P(C)\,P(E/C) + P(D)P(E/D)}$$

$$\frac{\overset{0.25}{\cancel{0.0909}} \times 0.005}{\underset{0.25}{\cancel{0.4545}} \times 0.03 + \underset{0.25}{\cancel{0.2727}} \times 0.02 + \underset{0.25}{\cancel{0.1818}} \times 0.01 + \underset{0.25}{\cancel{0.0909}} \times 0.0\ldots}$$

$= 0.0769$

☐ Random variables

→ A variable is a symbol that can assume any of a prescribed set of values. The term random variable is more meaningful than the term variable & it is one of the basic concept of probability theory. The values of random variables are real no. associated with the outcome of an experiment.

→ A random variable is a numerical valued defined on sample space of an experiment.

→ Random variable is a rule that assigns one & only one numerical value to each single event of an experiment

→ Random variables are also known as Stochastic variables.

eg Consider an experiment in which 2 coins are tossed simultaneously. Let S be the sample space

$S = \{HH, HT, TH, TT\}$

If we define random variable X as the no. of heads then the values 2, 1, 1, 0 corresponding to the outcomes.

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Random variables are classified as (i) discrete random variables.

(ii) Continous random variables.

□ Discrete random Variable (DRV)

→ A DRV is one which can assume only isolated values in the interval of domain.

eg Tossing a coin, Throwing a die

→ The no. of heads in 4 tosses of a coin is DRV as it cannot assume values other than {0, 1, 2, 3, 4}.

□ Discrete probability Distribution (Probability mass func^n) (PMF)

→ Probability distribution of a random is a set of its possibles values together with their respective probabilities. Suppose X is a DRV with possible outcomes ie $x_1, x_2, x_3 \dots x_n$. The probability of each possible outcome x_i is $P_i = P(X = x_i) = P(x_i)$ where i takes values from 1, 2, ... n

→ If the no.s $P(x_i)$ satisfy the 2 cond^ns

(i) $P(x_i) > 0$ for all values of i ie $0 < P_i \leq 1$

(ii) $\sum P(x_i) = 1$

then the func^n $P(x)$ is called PMF of random variable X & the set $\{P(x_i)\}$

is called Discrete Probability Distribution of DRV X.

→ The probability distribution of random variable X is given by means of following table

x	x_1	x_2	x_3	....	x_i	.....	x_n
$P(X=x)$	P_1	P_2	P_3	.....	P_i	.....	P_n

Mean & Variance of DRV

If X is DRV then the mean & variance is defined as $\boxed{\text{Mean} = \mu = \sum_{i=1}^{n} x_i P_i}$

$$\boxed{\text{Variance} = \text{Var} = \sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 P_i}$$

$$= \sum_{i=1}^{n} (x_i^2 + \mu^2 - 2x_i\mu) P_i$$

$$= \sum_{i=1}^{n} x_i^2 P_i + \mu^2 \sum_{i=1}^{n} P_i - 2\mu \sum_{i=1}^{n} x_i P_i$$

$$= \sum_{i=1}^{n} x_i^2 P_i + \mu^2 - 2\mu^2$$

$$\boxed{\sigma^2 = \sum_{i=1}^{n} x_i^2 P_i - \mu^2}$$

Q A random variable X has the following Probability distribution

①

X	-2	-1	0	1	2	3
$P(X=x_i)$	0.1	K	0.2	$2K$	0.3	K

(i) find K

(ii) Mean

(iii) Variance

(i) $\therefore \sum P_i = 1$

$$0.1 + K + 0.2 + 2K + 0.3 + K = 1$$

$$4K + 0.6 = \cancel{0}1$$

$$4K = 0.4$$

$$\boxed{K = 0.1}$$

(ii) $\mu = \sum_{i=1}^{n} x_i P_i$

$$= (-2)(0.1) + (-1)(K) + 0(0.2) + 1(2K) + 2(0.3) + 3(K)$$

$$= -\cancel{0.2} - 0.1 + 0 + \cancel{0.2} + 0.6 + 0.3$$

$$\boxed{\mu = 0.8}$$

(iii) $\sigma^2 = \sum_{i=1}^{n} x_i^2 p_i - \mu^2$

$$-0.8$$

$$= 4(0.1) + 1(K) - \cancel{0.8} + 0(0.2) - \cancel{0.8} + 1(0.2) - \cancel{0.8}$$

$$+ 4(0.3) - \cancel{0.8} + 9(0.1) - \cancel{0.8}$$

~~$-0.4 + 0.7 - 0.6 + 0.4 + 0.1$~~

~~$= -1.2$~~

$0.4 + 0.1 + 0.2 + 1.2 + 0.9 - (0.8)^2$

$= 2.8 - 0.64$

$= \underline{\underline{2.16}}$

Q(2)

X	0	1	2	3	4
$P(X=x_i)$	3K	3K	K	2K	6K

(i) K

(ii) Mean

(iii) Variance

(iv) $P(X>2)$

(v) $P(X<3$

(vi) $P(0<X<4)$

(vii) $P(0\le X<4)$

(viii) $P(X\le 3)$

(ix) $P(X\geq 2)$

Sol (i) $\sum P_i = 1$

$3K + 3K + K + 2K + 6K = 1$

$15K = 1$

$K = 1/15 \Rightarrow \boxed{K = 0.06}$

(ii) Mean $\mu = \sum_{i=1}^{n} x_i p_i$

$\mu = 0(3K) + 1(3K) + 2(K) + 3(2K) + 4(6K)$

$\mu = 0.18 + 0.12 + 0.36 + 1.44$

$\boxed{\mu = 2.1}$

(iii) $\sigma^2 = \sum_{i=1}^{n} x_i^2 \cdot P_i - \mu^2$

$= 1(3K) + 4(K) + 9(2K) + 16(6K) - (2.1)^2$

$= 0.18 + 0.24 + 1.08 + 5.76 - 4.41$

$\boxed{\sigma^2 = 2.85}$

(iv) $P(X > 2)$

$P(X > 2) = P(X = 3) + P(X = 4)$

$= 2K + 6K$

$= 8K$

$\boxed{P(X > 2) = 0.48}$

(v) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 3K + 3K + K$

$= 7K$

$= 0.42$

(vi) $P(0 < X < 4) = P(X = 1) + P(X = 2) + P(X = 3)$

$= 3K + K + 2K$

$= 6K$

$= 0.36$

(vii) $P(0 \le X \le 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X$

$= 3K + 3K + K + 2K + 6K$

$= 15K$

$= 0.9$

(viii) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 3K + 3K + K + 2K$

$= 9K$

$= 0.54$

(ix) $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$= K + 2K + 6K$

$= 9K$

$= 0.54$

24/11/2022

Q - A random variable X has a following Probability distribution

X	0	1	2	3	4	5	6	7
P(X)	0	a	2a	2a	3a	a^2	$2a^2$	$2a^2+a$

find (i) a

(ii) Mean

(iii) Variance

(iv) $P(X>2)$

(v) $P(X<6)$

(vi) $P(0<X<6)$

(vii) $P(0\le X\le 6)$

(viii) $P(0<X\le 5)$

(ix) $P(X\geq 5)$

(x) $P(X\le 3)$

Sol $\sum P_i = 1$

$0 + a + 2a + 2a + 3a + a^2 + 2a^2 + 2a^2 + a$

$5a^2 + 9a = 1$

$5a^2 + 9a - 1 = 0$

$a = 0.1049$

$= -1.904$ ✗ $\therefore$ probability can't be -ve

$\boxed{a = 0.1049}$

... Samples of Secee n d ...

(ii) Mean $(\mu) = \sum_{i=1}^{n} x_i p_i$

$= 1(a) + 2(2a) + 3(2a) + 4(3a) + 5(a^2) + 6(2a^2)$
$+ 7(2a^2 + a)$

$0.1049 + 0.4196 + 0.6294 + 1.2588 + 0.7343$
$+ 0.077 + 0.05 + 0.132$

$\boxed{\mu = 3.488}$

(iii) $\sigma^2 = \sum_{i=1}^{n} x_i^2 p_i - \mu^2$

$= 1(a) + 4(2a) + 9(2a) + 16(3a) + 25(a^2) + 36(2a^2)$
$+ 49(2a^2 + a) - (3.488)^2$ '98

$\Rightarrow$ ~~... 14.658~~ $\Rightarrow 1.899$

(iv) $P(X > 2) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$
$+ P(X=7)$

$= 2a + 3a + a^2 + 2a^2 + 2a^2 + a$

$= 5a^2 + 6a$

$P(X>2) = 0.6844$

(v) $P(X < 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$+ P(X=4) + P(X=5)$

$0 + a + 2a + 2a + 3a + a^2$

$a^2 + 8a = 0.8502$

(vi) $P(0 < X < 6) = P(X=1) + P(X=2) + P(X=3)$

$+ P(X=4) + P(X=5)$

$= a + 2a + 2a + 3a + a^2$

$= a^2 + 8a$

$= 0.8502$

(vii) $P(0 \leq X \leq 6) = P(X = 0, 1, 2, 3, 4, 5, 6)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3) +$

$P(X=4) + P(X=5) + P(X=6)$

$= a + 2a + 2a + 3a + a^2 + 2a^2$

$= 3a^2 + 8a$

(1)

X	0	1	2	3	4
P(X)	K	K	2K	K^2	$3K^2$

find (i) K

(ii) Mean

(iii) Variance

Sol $\Sigma P_i = 1$

$K + K + 2K + K^2 + 3K^2 = 1$

$4K^2 + 4K = 1$

$\boxed{K = 0.2071}$

$= -1.2071$ neglected

(ii) Mean $(\mu) = \sum_{i=1}^{n} P_i x_i$

$0(K) + 1(K) + 2(\cancel{2K}) + 3(K^2) + 4(3K^2)$

$\boxed{\mu = 1.6788}$

116
3
48

(iii) $\sigma^2 = \sum_{i=1}^{n} x_i^2 P_i - \mu^2$

$= 1^2(K) + 2^2(2K) + 3^2(K^2) + 4^2(3K^2) - (1.6788)^2$

$\boxed{\sigma^2 = 1.49}$

(3)

X	0	1	2	3	4
P(X)	0	2c	2c	c^2	$5c^2$

find (i) c

(ii) $P(X < 3)$

(iii) $P(0 < X < 4)$

(iv) $P(0 \leq X \leq 3)$

Sol $\Sigma P_i = 1$

$2c + 2c + c^2 + 5c^2$

$6c^2 + 4c = 1$

$\boxed{c = 0.1937}$

(ii) $P(X<3) = P(X=1) + P(X=2)$

$2c + 2c = 4c$

$= 0.7748$

(iii) $P(0 < X < 4) = P(X=1) + P(X=2) + P(X=3)$

$2c + 2c + c^2$

$= c^2 + 4c$

$= 0.8123$

(iv) $P(0 \leq X \leq 3)$

$P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$2c + 2c + c^2 \Rightarrow c^2 + 4c$

$= 0.8123$

25/11/2022

Continuous Random Variable (CRV)

→ Let X be a random variable, if X takes non countable ∞ no. of values then X is called CRV.

→ If X is a CRV then the range of X in an ~~real~~ interval on real line.

eg :- life of electric bulb
detection range of radar
temp., height, weight etc are eg of CRV

→ **Probability density func^n (Pdf)**

→ Let X be a CRV. If for every X in the range of X we assign a real no. $f(x)$ satisfying the cond^n,

(i) $f(x) \geq 0$, $-\infty < x < \infty$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

then the func^n $f(x)$ is called Pdf of X

Sometimes it is also referred as density func^n.

Note :- 1) If $f(x)$ is a pdf of X then

$P(a \leq x \leq b) = \int_a^b f(x)\,dx$ for any real const.

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2) Unlike in the case of DRV we cannot represent CRV by table.

Mean & Variance of CRV

Mean $= \mu = \int_{-\infty}^{\infty} x f(x)\,dx$

Variance $= \sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

S.D $= \sqrt{\sigma^2}$

Q → If the pdf of CRV is given by $f(x) = e^{-x}$ where $0 \leq x \leq \infty$ find mean & variance

Sol Mean $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$\mu = \int_{0}^{\infty} x e^{-x}\,dx$

$\mu = \left[-x e^{-x} - e^{-x}\right]_0^{\infty}$

$$\begin{array}{cc} x & e^{-x} \\ 1 & -e^{-x} \\ 0 & e^{-x} \end{array}$$

$\mu = [0 - (-1)]$

$\boxed{\mu = 1}$

$$\begin{array}{cc} x^2 & e^{-x} \\ 2x & -e^{-x} \\ 2 & e^{-x} \\ 0 & -e^{-x} \end{array}$$

$\sigma^2 = \int_{0}^{\infty} x^2 f(x)\,dx - 1^2$

$\sigma^2 = \left[-x^2 e^{-x} - 2x e^{-x} - 2e^{-x}\right]_0^{\infty}$

$\boxed{\sigma^2 = 1}$

② $f(x) = Kxe^{-x}$, $0 \le x \le 1$ find K & mean

Sol

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 Kxe^{-x}\,dx = 1$$

$$K\int_0^1 xe^{-x}\,dx = 1 \Rightarrow K\left[-xe^{-x} - e^{-x}\right]_0^1$$

$$\Rightarrow K\left[(-e^{-1} - e^{-1}) - (-1)\right] = 1$$

$$\Rightarrow K\left[(-2e^{-1} + 1)\right] = 1$$

$$K = \frac{1}{1 - 2e^{-1}} \Rightarrow \frac{e}{e-2}$$

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\mu = \int_{-\infty}^{\infty} x\left[Kxe^{-x}\right]dx$$

$$\int_{-\infty}^{\infty} Kx^2 e^{-x}\,dx$$

$$\mu = K\int_{-\infty}^{\infty} x^2 e^{-x}\,dx$$

$$\mu = \left[\frac{e}{e-2}\right]\left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^1$$

$$\mu = \left[\frac{e}{e-2}\right]\left[-e^{-1} - 2e^{-1} - 2e^{-1} - (-2)\right]$$

$$\left[\frac{e}{e-2}\right]\left[-5e^{-1} + 2\right] \Rightarrow \mu = \frac{2e-5}{e-2}$$

③ $f(x) = 6x(1-x) \quad 0 \le x \le 1$ find mean variance S.D

<u>Sol</u> $\int_0^1 x f(x)\, dx$

$\int_0^1 x\, 6x(1-x)$

$\int_0^1 6x^2 - 6x^3$

$\int_0^1 6x^2 - \int_0^1 6x^3$

$6\left[\frac{x^3}{3}\right]_0^1 - 6\left[\frac{x^4}{4}\right]_0^1$

$\frac{2\cancel{6}}{\cancel{3}} - \cancel{6}_3 \frac{1}{\cancel{4}_2}$

$2 - \frac{3}{2}$

$\boxed{\mu = \frac{1}{2}}$

$\sigma^2 = \int_0^1 x^2 [6x(1-x)]$

$= \int_0^1 6x^3(1-x)$

$\int_0^1 6x^3 - 6x^4$

$6\int_0^1 x^3 - 6\int_0^1 x^4$

$$6\left[\frac{x^4}{4}\right] - 6\left[\frac{x^5}{5}\right] - \frac{1}{4}$$

$$6\left(\frac{1}{4}\right) - 6\left(\frac{1}{5}\right) - \frac{1}{4}$$

$$\frac{\cancel{6}^{3}}{\cancel{4}_{2}} - \frac{6}{5} - \frac{1}{4}$$

$$\frac{15-12}{10} - \frac{1}{4}$$

$$= \frac{3}{10} - \frac{1}{4}$$

$$= \frac{12-10}{40}$$

$$= \frac{\cancel{2}^{1}}{\cancel{40}_{20}} = \frac{1}{20}$$

$$SD = \sqrt{\sigma} \Rightarrow \sqrt{\frac{1}{20}}$$

$$= \frac{1}{\sqrt{20}}$$

④ $f(x) = Kx^2 e^{-x}$ when $x \geq 0$

M/A 2022

(i) K

(ii) Mean

(iii) Variance

PDF $\int_0^\infty Kx^2 e^{-x}\,dx = 1$

$K \int_0^\infty Kx^2 e^{-x}\,dx = 1$

$K\left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^\infty = 1$

$x^2 \quad e^{-x}$

$2x \quad -e^{-x}$

$2 \quad e^{-x}$

$0 \quad -e^{-x}$

$K[0-(-2)) = 1$

$2K = 1$

$\boxed{K = 1/2}$

Mean $= \int_0^\infty x f(x)\,dx$

$\int_0^\infty x\, Kx^2 e^{-x}$

$K\int_0^\infty x^3 e^{-x}$

$\frac{1}{2}\left[-x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x}\right]_0^\infty$

$x^3 \quad e^{-x}$

$3x^2 \quad -e^{-x}$

$6x \quad e^{-x}$

$6 \quad -e^{-x}$

$0 \quad e^{-x}$

$\frac{1}{2}(0-(-6))$

$\boxed{\mu = 3}$

$$\sigma^2 = \int_0^{\infty} x^2 \, k x^2 e^{-x} \, dx - \mu^2$$

$$\frac{1}{2} \int_0^{\infty} x^4 e^{-x} \, dx - 9$$

x^4	e^{-x}
$4x^3$	$-e^{-x}$
$12x^2$	e^{-x}
$24x$	$-e^{-x}$
24	e^{-x}
0	$-e^{-x}$

$$\frac{1}{2} \left[-x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24x e^{-x} - 24 e^{-x} \right]_0^{\infty}$$

$$\frac{1}{2} [0 - (-24)] - 9$$

$$12 - 9$$

$$\boxed{\sigma^2 = 3}$$

(9) $f(x) = \begin{cases} x/6 + k & , 0 < x < 3 \\ 0 & , \text{otherwise} \end{cases}$ find K

$$\int_0^3 f(x) dx = 1 \qquad 0 < x < 3$$

$$\int_0^3 \frac{x}{6} + k$$

$$\int_0^3 \frac{x}{6} + \int_0^3 k = 1$$

$$\left[\frac{x^2}{12} \right]_0^3 + \left\{ \frac{k}{\cancel{6}} \right\} \left[x \right]_0^3 = 1$$

$$\frac{9}{12} + 3k = 1$$

$$3k = 1 - \frac{9}{12} \Rightarrow k \left(\frac{\cancel{3}}{12} \right) \times \frac{1}{\cancel{3}} \quad \boxed{k = \frac{1}{12}}$$

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⑥ If $f(x) = \begin{cases} Kxe^{-\lambda x} & x>0,\ \lambda>0 \\ 0 & \text{elsewhere} \end{cases}$

find $K \rightarrow \lambda^2$

mean $\rightarrow 2/\lambda$

variance $\rightarrow 2/\lambda^2$

$$\int_0^\infty Kxe^{-\lambda x} = 1$$

$$K\int_0^\infty xe^{-\lambda x} = 1$$

x	$e^{-\lambda x}$
1	$-\dfrac{e^{-\lambda x}}{\lambda}$
0	$\dfrac{e^{-\lambda x}}{\lambda^2}$

$$K\left[-x\frac{e^{-\lambda x}}{\lambda} - \frac{e^{-\lambda x}}{\lambda^2}\right]_0^\infty = 1$$

$$\cancel{\lambda\left[0+\lambda^2\right] = 1}$$

$K =$

$$K\left(\frac{1}{\lambda^2}\right) = 1$$

$$\boxed{K = \lambda^2}$$

Mean $\displaystyle\int_0^\infty Kx^2 e^{-\lambda x}$

x^2	$e^{-\lambda x}$
$2x$	$-\dfrac{e^{-\lambda x}}{\lambda}$
2	$\dfrac{e^{-\lambda x}}{\lambda^2}$
0	$-\dfrac{e^{-\lambda x}}{\lambda^3}$

$$K\left[\frac{-x^2 e^{-\lambda x}}{\lambda} - \frac{2x e^{-\lambda x}}{\lambda^2} - \frac{2e^{-\lambda x}}{\lambda^3}\right]_0^\infty$$

$$\mu = (\lambda^2)\left[0 - \left(-\frac{2}{\lambda^3}\right)\right)$$

$$\mu = \lambda^2\left(\frac{2}{\lambda^3}\right) \Rightarrow \boxed{\mu = \frac{2}{\lambda}}$$

(iii) $\sigma^2 = \int_0^\infty k x^3 e^{-\lambda x} - \frac{4}{\lambda^2}$

x^3	$e^{-\lambda x}$
$3x^2$	$-\frac{e^{-\lambda x}}{\lambda}$
$6x$	$\frac{e^{-\lambda x}}{\lambda^2}$
6	$-e^{-\lambda x}/\lambda^3$
0	$e^{-\lambda x}/\lambda^4$

$$k\left[-\frac{x^3 e^{-\lambda x}}{\lambda} - \frac{3x^2 e^{-\lambda x}}{\lambda^2} - \frac{6x e^{-\lambda x}}{\lambda^3} - \frac{6e^{-\lambda x}}{\lambda^4}\right]_0^\infty - \frac{4}{\lambda^2}$$

$$\sigma^2 = \lambda^2\left(0 - \left(-\frac{6}{\lambda^4}\right)\right) - \frac{4}{\lambda^2}$$

$$\sigma^2 = \frac{6}{\lambda^4} - \frac{4}{\lambda^2}$$

$$\boxed{\sigma^2 = \frac{2}{\lambda^2}}$$

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29/11/2022

Q If $f(x) = \begin{cases} \frac{1}{2}\sin x, & 0<x<\pi \\ 0, & \text{elsewhere} \end{cases}$ is a pdf

find mean & variance

Sol Mean $= \mu = \int_{-\infty}^{\infty} x f(x)\, dx$

$$\mu = \int_0^{\pi} x \cdot \left(\frac{1}{2}\sin x\right) dx$$

$$\frac{1}{2}\int_0^{\pi} x \sin x\, dx$$

x	$\sin x$
1	$-\cos x$
0	$-\sin x$

$$\frac{1}{2}\left[-x\cos x - \sin x\right]_0^{\pi}$$

$$\frac{1}{2}\left[+\pi - 0 - 0 - 0\right]$$

$$\boxed{\mu = \frac{\pi}{2}}$$

Variance $\sigma^2 = \int_0^{\pi} x^2 f(x)\, dx$

$$\frac{1}{2}\int_0^{\pi} x^2 \sin x\, dx$$

x^2	$\sin x$
$2x$	$-\cos x$
2	$-\sin x$
	$\cos x$

$$\frac{1}{2}\left[-x^2\cos x - 2x\sin x \pm 2\cos x\right]_0^{\pi} - \frac{\pi}{2}0$$

$$\frac{1}{2}\left[+\pi^2 + \cancel{2} - 0 - 0 + \cancel{2} - \left(\frac{\pi}{2}\right)^2\right]$$

$$\cancel{\frac{\pi^2}{2} - \frac{\pi}{4}} \qquad \frac{1}{2}(\pi^2 - 4) = \frac{\pi^2 - 4}{2} - \frac{\pi^2}{4}$$

$$4 \cdot \cancel{\frac{2\pi^2 - \pi}{4}} \qquad \frac{\pi^2}{4} \qquad \frac{2\pi^2 - 8 - \pi^2}{4}$$

$$\frac{2x^2 - 18}{\cancel{1}58} \Rightarrow \frac{x^2}{4} - 2$$

Mathematical Expectations

→ Special Constants which serve to give a quantitative description of Random variable

→ Let X be a DRV. Let $x_1, x_2, x_3, \ldots x_n$ denote possible values of X & let $P_1, P_2 \ldots P_n$ denote the corresponding probabilities such that $\sum P_i = 1$ then mathematical expectation is denoted by $E[X]$ & is defined as

$$\boxed{\sum_{i=1}^{n} x_i P_i}$$

→ Let X be a CRV having $f(x)$ as its pdf then mathematical expectation of X is defined as

$$\boxed{E(X) = \int_{-\infty}^{\infty} x f(x)\, dx}$$

∴ Mathematical expectation is also called as expectation or mean of probability

Property of ~~Mat~~ Mathematical expectation

① $E[C] = C$

② $E[CX] = CE[X]$

③ $E[C_1X + C_2] = C_1E[X] + C_2$

① If X is a random variable & C is any real number $E[C] = C$

→ If X is a DRV then $E[X] = \sum_{i=1}^{n} x_i p_i$

Here $E[C] = \sum_{i=1}^{n} C p_i \Rightarrow C \sum_{i=1}^{n} p_i = C(1) = C$

→ If X is CRV then $E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$

$$E[C] = \int_{-\infty}^{\infty} C f(x)\,dx = C\int_{-\infty}^{\infty} f(x)\,dx$$

$$\Rightarrow C(1) = \underline{\underline{C}}$$

② $E[CX] = CE[X]$

① If X is DRV then $E[X] = \sum_{i=1} x_i p_i$

$$E[CX] = \sum_{i=1}^{n} C x_i p_i = C \sum_{i=1}^{n} x_i p_i = CE[X]$$

② If X is a CRV then $E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$

$$E[CX] = \int_{-\infty}^{\infty} C x f(x)\,dx \Rightarrow C\int_{-\infty}^{\infty} x f(x)\,dx = C \cdot E[X]$$

③ If X is DRV then $E[X] = \sum_{i=1}^{n} x_i p_i$

$$E[c_1 X + c_2] = \sum_{i=1}^{n} (c_1 x_i + c_2) p_i \Rightarrow \sum_{i=1}^{n} c_1 x_i p_i + \sum_{i=1}^{n} c_2 p_i \Rightarrow c_1 \sum_{i=1}^{n} x_i p_i + c_2 \sum_{i=1}^{n} p_i \Rightarrow c_1 E[X] + c_2 (1)$$

$$E[c_1 X + c_2] = c_1 E[X] + c_2$$

If x is CRV then $E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$

$$E[c_1 X + c_2] = \int_{-\infty}^{\infty} (c_1 x + c_2) f(x)\, dx \Rightarrow \int_{-\infty}^{\infty} c_1 x f(x)\, dx + \int_{-\infty}^{\infty} c_2 f(x)\, dx \Rightarrow c_1 \int_{-\infty}^{\infty} x f_x\, dx + c_2 \int_{-\infty}^{\infty} f(x)\, dx$$

$$\Rightarrow \underline{c_1 E[X] + c_2}$$

□ **Variance**

Let X be a DRV having Probability func^n^ $P(X = x)$ then the variance of X is ~~defined~~ as denoted by $V(X)$ and is defined as

$$V(X) = E[(X - \mu)^2]$$
$$= E[(X - E[X])^2]$$
$$= E[X^2 + (E[X])^2 - 2X E[X]]$$
$$= E[X^2] + (E[X])^2 - 2E[X]E[X]]$$

$$V[X] = E[X^2] - (E[X])^2$$

□ Properties of Variance

① $V[a]=0$

② $V[ax] = a^2 V[x]$

③ $V[x+b] = V[x]$

④ $V[ax+b] = a^2 V[x]$

① $V[a] = 0$

w.k.T

$$V[x] = E[x^2] - \{E[x]\}^2$$

$$V[a] = E[a^2] - \{E(a)\}^2$$

$$= a^2 - a^2$$

$$\boxed{V[a] = 0}$$

② $V[ax] = a^2 V[x]$

$$V[x] = E[x^2] - \{E[x]\}^2$$

$$V[ax] = E[(ax)^2] - \{E[ax]\}^2$$

$$V[ax] = a^2 E[x^2] - a^2 \{E[x]\}^2$$

$$\cancel{a^2 \{E[x]} \quad a^2 \left[E[x^2] - \{E[x]\}^2\right]$$

$$\boxed{V[ax] = a^2 V[x]}$$

③ $V[X+b] = E[(X+b)^2] - E[(X+b)]^2$

$= E[(X^2+b^2+2Xb)] -$

① A R.V X has the following probability distribution

X	1	2	3	4	5
P(X)	c	c	3c	c^2+c	$6c^2$

find (i) the value of c (ii) $E[4X+1]$
(iii) $V[4x+1]$ (iv) $P(X<3]$ (v) $P(1<X<4)$

(i) $\Sigma P_i = 1$

$$c+c+3c+c^2+c+6c^2=1$$

$$7c^2+6c-1=0$$

$$c = -1 \ \& \ 1/7$$

$$\boxed{c = 1/7}$$

(ii) $E[4x+1] = 4E[x]+1$

$$E[c_1x+c_2] = c_1E[x]+c_2$$

$$= 4E[x]+1$$

$$E[x] = 1(c)+2(c)+3(3c)+4(c^2+c)+5(6c^2)$$

$$E[x] = 0.14+0.28+1.26+0.56+0.0784+0.588$$

$$E[x] = 2.9761$$

$$4E[x]+1 \Rightarrow 4[2.916]+1 \Rightarrow \underline{12.916}$$

3

(iii) $V[4X+1]$

$= 4^2 V[X]$

$V[X] = E[X^2] - [E[X]]^2$

$E[X^2] = \sum_{i=1}^{n} x_i^2 P_i$

ie $= 1^2(c) + 2^2(c) + 3^2(3c) + 4^2(c^2+c) + 5^2(6c$

$E[X^2] = 10.24$

$V[X] = 10.24 - (2.971)^2$

$V[X] = 1.4131$

(iv) $P(X<3) = P(X=1) + P(X=2)$

$= c + c$

$= 2c$

$= 2\left(\frac{1}{7}\right)$

$= \frac{2}{7} = 0.2857$

(v) $P(1<X<4) = P(X=2) + P(X=3)$

$= c + 3c$

$= 4c$

$= \frac{4}{7} = 0.5714$

③ If X is a RV & the pdf of X is ~~f(x)~~

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

find (i) $E[X]$

(ii) $E[X^2]$

(iii) $E[(X-1)^2]$

Sol (i) $E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$

$$= \int_{0}^{\infty} x f(x)\, dx$$

$$= \int_{0}^{\infty} x e^{-x}\, dx$$

$x \quad e^{-x}$

$1 \quad -e^{-x}$

$0 \quad e^{-x}$

$$= \left[-xe^{-x} - e^{-x} \right]_0^{\infty}$$

$$= [1]$$

$$\boxed{E[X] = 1}$$

(ii) $E[X^2] = \int_{0}^{\infty} x^2 f(x)\, dx$

$x^2 \quad e^{-x}$

$2x \quad -e^{-x}$

$2 \quad e^{-x}$

$0 \quad -e^{-x}$

$$= \left[-x^2 e^{-x} - 2xe^{-x} - 2e^{-x} \right]_0^{\infty}$$

$$= [-(-2)$$

$$= 2$$

(iii) $E[(x-1)^2] = E[x^2+1-2x]$

$E[x^2] + E[1] - 2E[x]$

$= \cancel{2} + 1 - \cancel{2}$

$\boxed{E[(x-1)^2] = 1}$

④ If X is RV defined by pdf as

$$f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(i) $E[x] = \int_0^1 x(3x^2)$

$= \int_0^1 3x^3$

$= 3x^4/4$

$\boxed{E[x] = 3/4}$

(ii) $E[x^2] = \int_0^1 x^2\, 3x^3$

$= \int_0^1 \frac{3x^6}{5}$

$\boxed{E[x^2] = \frac{3}{5}}$

(iii) $E[3X-2] = E[3X] - E[2]$

$= 3E[X] - 2$

$= 3\left(\frac{3}{4}\right) - 2$

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$\frac{9}{4} - 2$

$\frac{9-8}{4}$

$= \frac{1}{4}$

(5)

X	-3	-2	-1	0	1	2	3
P(X)	0.001	0.01	0.1	K	0.1	0.01	0.001

Sol (i) find K

(ii) $E[X]$

(iii) $E[X^2]$

(iv) $V[X]$

(v) $E[2X \mp 2]$

(vi) $V[2X \pm 3]$

(vii) $P(X>3)$

(viii) $P(X \geq 3)$

(ix) $P(X<2)$

(x) $P(X \leq 2)$

Sol (i) $\Sigma p_i = 1$

$0.09 + 0.01 + 0.1 + K + 0.1 + 0.01 + 0.001 = 1$

$K = 0.222 = 1$

$\boxed{K = 0.778}$

(ii) $\sum_{i=1}^{n} x_i P_i$

$-3(0.\cancel{0}01) - 2(\cancel{0}.01) - 1(\cancel{0.1}) + 0(K) + 1(\cancel{0.1}) + 2(\cancel{0}.01) + 3(\cancel{0}.001)$

$\boxed{E[X] = 0}$

(iii) $[9(0.001)+4(0.01)+1(0.1)]\,2$

$$\boxed{E[x^2] = 0.298}$$

(iv) $= V[X] = \sum_{i=1}^{n} x_i^2 p_i - \mu^2$

$= 0.298 - 0$

$$\boxed{V[X] = 0.298}$$

(v) $E[2x \mp 2]$

$2E[X] \mp 2$

$$\boxed{E[2x \mp 2] \mp 2}$$

(vi) $V[2x \pm 3] = 2^2 \{V[X]$

$= 4[0.298)$

$$\boxed{V[2x \pm 3] = 1.192}$$

(vii) $P(X > 3)$

$= 0$

(viii) $P(X \geq 3) = P(X = 3)$

$= 0.001$

(ix) $P(X < 2) = P(X=-3) + P(X=-2) + P(X=-1) + P(X=0) + P(X=1)$

$= 0.001 + 0.01 + 0.1 + 0.778 + 0.1 \Rightarrow 0.989$

(x) $P(X \leq 2) \Rightarrow P(X=2) + P(X=3) \Rightarrow 0.01 + 0.001 \Rightarrow 0.011$