

Computation Table:

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	2.5	4	8	16	2.5	5.2
3	3.2	9	27	81	7.5	22.5
4	4.3	16	64	256	25.2	100.8
		$\Sigma x^2 = 30$	$\Sigma x^3 = 100$	$\Sigma x^4 = 354$	$\Sigma xy = 37.1$	$\Sigma x^2y = 130.3$

Substitute the values we get,

$$12.9 = 5a + 10b + 30c \quad \text{--- } ①$$

$$37.1 = 10a + 30b + 100c \quad \text{--- } ②$$

$$130.3 = 30a + 100b + 354c \quad \text{--- } ③$$

Solving ①, ② & ③ we get,

$$a = 1.42, b = -1.07, c = 0.55$$

Substitute the value in Eq ① =

$$y = a + bx + cx^2$$

$$y = 1.42 - 1.07x + 0.55x^2$$

Fit a parabola of Second degree of following data.

x	1	2	3	4
y	1.7	1.8	2.3	3.2

Table							
x	y	x^2	x^3	x^4	xy	x^2y	x^4y
1	1.7	1	1	1	1.7	1.7	1.7
2	1.8	4	8	16	3.6	7.2	
3	2.3	9	27	81	6.9	20.7	
4	3.2	16	64	256	12.8	51.2	
		$\Sigma x^2 = 30$	$\Sigma x^3 = 100$	$\Sigma x^4 = 354$	$\Sigma xy = 37.1$	$\Sigma x^2y = 130.3$	$\Sigma x^4y = 80.8$

→ Substitute the values

$$9 = 9a + 10b + 30c \quad \text{--- } ①$$

$$25 = 9a + 30b + 100c \quad \text{--- } ②$$

$$80.8 = 30a + 100b + 354c \quad \text{--- } ③$$

$$a = 0.258 \quad b = 1.164 \quad c = -0.122$$

a

t

d

e

	x	2	4	6	8	10
d	y	3.07	12.85	31.47	57.38	91.29
e						

Sol: Consider $y = a + bx + cx^2$ whose normal equations are

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Computation table

x	y	x^2	x^3	x^4	xy	x^2y
2	3.07	4	8	16	6.14	12.28
4	12.85	16	64	256	51.4	205.6
6	31.47	36	216	1296	188.82	1132.92
8	57.38	64	512	4096	459.04	3672.32
10	91.29	100	1000	10000	912.9	9129
$\Sigma x = 30$	$\Sigma y = 196.06$	$\Sigma x^2 = 220$	$\Sigma x^3 = 1800$	$\Sigma x^4 = 15664$	$\Sigma xy = 1618.3$	$\Sigma x^2y = 1152.12$

→ Substitute the values-

$$196.06 = 30a + 220b + 220c$$

$$912.9 = 30a + 220b + 1800c$$

$$1152.12 = 220a + 1800b + 15664c$$

$$a = 1164.606 \quad b = -471.96 \quad c = 38.78$$

	x	-3	-1	1	3
	y	15	5	1	5

	x	-3	0	2	4
	y	3	1	1	3

	x	-1	1	2	3	5
	y	8	0	5	16	56.

	x	-1	0	1	2	3	4
	y	9	7	7	9	13	49.

Consider $y = ax + bx^2 + cx^3$ whose normal equation are

$$\sum y = na + b\sum x + c\sum x^2 \quad (1)$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 \quad (2)$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4 \quad (3)$$

Computation table

x	y	x^2	x^3	x^4	xy	x^2y
-3	15	9	-27	81	-45	135
-1	5	1	-1	1	-5	5
1	1	1	1	1	1	1
3	5	9	27	81	15	45

$$\begin{aligned} \sum x = 0 & \quad \sum y = 26 \\ \sum x^2 = 20 & \quad \sum x^3 = 0 \\ \sum x^4 = 164 & \quad \sum xy = -34 \\ \sum x^2y = 186 & \end{aligned}$$

Substitute their values:

$$26 = 4a + 6b + 20c \quad \text{--- } ①$$

$$-34 = 8a + 12b + 16c \quad \text{--- } ②$$

$$186 = 20a + 18b + 164c \quad \text{--- } ③$$

By solving the above ③ Equations

$$a = 2.125 \quad b = -1.7 \quad c = 0.875$$

Substitute their values in Eq ①.

$$= 2.125a - 1.7b + 0.875c^2$$

15	x	-3	0	2	4
	y	3	1	1	3.

Sol: Normal Equation $a + b\sum x + c\sum x^2$

$$\sum y = na + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Computation table

x	y	x^2	x^3	x^4	xy	x^2y
-3	3	9	-27	81	-9	27
0	1	0	0	0	0	0
2	1	4	8	16	2	4
4	3	16	64	256	12	48
$\Sigma x = 3$	$\Sigma y = 8$	$\Sigma x^2 = 29$	$\Sigma x^3 = 95$	$\Sigma x^4 = 353$	$\Sigma xy = 5$	$\Sigma x^2y = 79$

Substitute their values.

$$8 = 4a + 3b + 29c \quad \text{--- } ①$$

$$5 = 29a + 29b + 45c \quad \text{--- } ②$$

$$79 = 29a + 45b + 353c \quad \text{--- } ③$$

By Solving the ①, ②, ③ Equations

$$a = 0.850 \quad b = -0.192 \quad c = 0.178.$$

Substitute the values in $a + bx + cx^2$
 $0.850 + 0.192x + 0.178x^2$

x	-1	1	2	3	5
y	8	0	5	16	56

Normal Equation $a + bx + cx^2$

$$\sum y = a\sum x + b\sum x^2 + c\sum x^4$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

Computation table.

x	y	x^2	x^3	x^4	xy	x^2y
-1	8	1	-1	1	-8	-8
1	0	1	1	1	0	0
2	5	4	8	16	20	10
3	16	9	27	81	144	48
5	56	25	125	625	1400	280
		$\sum x^2 = 40$	$\sum x^3 = 160$	$\sum x^4 = 729$	$\sum xy = 1573$	$\sum x^2y = 330$

Substitute their values.

$$85 = 5a + 10b + 40c \quad \text{--- } ①$$

$$330 = 10a + 40b + 160c \quad \text{--- } ②$$

$$1573 = 40a + 160b + 724c \quad \text{--- } ③$$

By solving eq. ①, ②, ③.

$$a = 1 \quad b = -0.097 \quad c = 0.011$$

\Rightarrow Exponential curve :-

$$i) y = ae^{bx}$$

+ Taking log on both side

$$\log_e y = \log_e a + \log_e b x$$

$$\log_e y = \log_e a + \log_e e^{bx}$$

$$\log_e y = \log_e a + b \log_e e$$

$$\log_e y = \log_e a + b x \quad \begin{matrix} \text{it represent} \\ y = A + Bx \end{matrix}$$

$$\text{let } Y = \log_e y$$

$\begin{matrix} \text{it represent} \\ \text{st. line} \end{matrix}$

$$A = \log_e a$$

$$bx = Bx$$

$$\Sigma Y = nA + B\Sigma x$$

$$\Sigma XY = A\Sigma x^2 + B\Sigma x^2$$

$$\boxed{b=B}$$

$$A = \log_e a$$

$$a = e^A$$

$$y = ae^{bx}$$

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\Rightarrow Fit an exponential form of the curve $y = ae^{bx}$

x	2	4	6	8
y	25	38	56	84

$$\text{Sol: Consider } y = ae^{bx}$$

Transforming the exponential equation $y = ae^{bx}$ by taking log on both sides we get

$$\log_e y = \log_e a + \log_e e^{bx}$$

$$\log_e y = \log_e a + b \log_e e$$

$$\log_e y = \log_e a + b \log_e x$$

$$\text{if we replace } \log_e y = Y, \log_e a = A \text{ & } b = B$$

$$\text{then } Y = A + Bx$$

Computation table

x	y	$Y = \log_e y$	x^2	xy
2	25	3.2188	4	6.4376
4	38	3.6375	16	14.35
6	56	4.0253	36	24.1518
8	84	4.4308	64	35.4464
		$\Sigma Y = 15.3124$	$\Sigma x^2 = 120$	$\Sigma xy = 80.5858$

using the Normal Equation

$$15.3124 = 4A + 20B$$

$$80.5858 = 20A + 120B$$

Solving for A and B we get

$$A = 2.82215$$

$$B = 0.20119$$

$$a = \text{antilog } A = 16.8109$$

$$b = B = 0.20119$$

$$\text{Hence } y = ae^{bx}$$

a 2, fit an exponential curve of the form $y = ae^{bx}$

x	1	2	3	4
y	11	35	105	

Computation table:

x	y	$\log_{10} y$	x^2	xy
1	4	1.3862	1	1.3862
2	11	2.3978	4	4.7956
3	35	3.5553	9	10.6659
4	105	4.6051	16	18.4204

$$\sum y = 150 \quad \sum x = 11.9444 \quad \sum x^2 = 30$$

$$\sum xy = 150$$

$$\sum x^3 = 10$$

$$\Sigma y = nA + B\Sigma x$$

$$\Sigma xy = A\Sigma x + B\Sigma x^2$$

$$n \cdot 9.4444 = 4n + 10B \quad \text{--- } \textcircled{1}$$

$$35.2681 = 10A + 30B \quad \text{--- } \textcircled{2}$$

Solving $\textcircled{1}$ & $\textcircled{2}$

$$A = 0.28255$$

$$B = 1.03142$$

Transforming the exponential equation
 $y = ab^x$

Taking log on both sides to base 10

$$\log_{10} y = \log_{10} a + \log_{10} b^x$$

~~$\log_{10} y = \log_{10} a + x \log_{10} b$~~

$$\text{If we replace } \log_{10} y = Y, \log_{10} a = \log_{10} b = B$$

$$\text{then } Y = A + BX.$$

$$\text{1. } Y = nA + B\Sigma x$$

$$\text{2. } XY = A\Sigma x + B\Sigma x^2$$

$$A = e^B = e^{0.28255} = 1.3265$$

$$B = 1.03142$$

b) Fit an exponential curve of the form $y = ab^x$

x	1	2	3	4
y	4	11	35	100

Computation table

Sol:

	x	y	$y = \log_{10} y$	x^2	xy
1	1	4	0.6020	1	0.6020
2	2	11	1.0413	4	2.0826
3	3	35	1.5440	9	4.632
4	4	100	2	16	8

$\Sigma x = 10$ $\Sigma y = 150$ $\Sigma xy = 5.1873$ $\Sigma x^2 = 30$ $\Sigma y^2 = 15.3166$

using a Normal Equation :

$$5.1873 = 4A + 10B$$

$$15.3166 = 10A + 30B$$

Solving eq ① and ②

$$A = 0.12265$$

$$B = 0.46967$$

a - anti log $A = 1.3263$

b - " $B = 2.948$

$$y = ab^x$$

$$y = (1.3263)(2.948)^x$$

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fit an Exponential form of the curve $y = a^x$

x 61 26 7 26

y 350 400 500 600

Sol Consider $y = a^x$

$$\log_{10} y = \log_{10} a^x$$

$$\log_{10} y = \log_{10} a + \log_{10} x^b$$

$$\log_{10} y = \log_{10} a + b \log_{10} x$$

$$Y = A + Bx$$

x	y	$x = \log_{10} x$	$Y = \log_{10} y$	x^2	xy
61	350	1.7853	2.544	3.1872	4.5418
26	400	1.4149	2.602	2.0019	3.6815
7	500	0.8450	2.698	0.7140	2.2805
26	600	1.4149	2.778	2.0019	3.9305
26	840	1.850	$\Sigma Y = 10.6230$	$\Sigma x^2 = 10.6230$	$\Sigma xy = 14.4345$

$$\Sigma Y = nA + B\Sigma x$$

$$\Sigma Y = nB + B\Sigma x^2$$

$$= 10.6230 = 4A + 5.460B$$

$$= 14.4345 = 5.4601A + 7.9050B$$

$$A = 2.8556$$

$$B = -0.1464$$

$$B = b = -0.1464$$

$$A = \log_{10} a \rightarrow a = 10^A$$

$$a = 10^{2.8556} = 717.134$$

Chi-Square Test (χ^2 Test) 24

→ 5th unit

χ^2 Test is defined as

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequencies

E_i = expected frequencies.

χ^2 -Test is used to check the significance of the discrepancy between theory and experiment. It enables us to find the deviation of the experiment from theory is just by chance or it is really due to the inadequacy of the theory to fit the observed data.

Conditions for using χ^2 -Test

- 1, The total number of observation used in this test must be large
- 2, The test is wholly dependent on the degree of freedom
- 3, Each of observation making up the sample for the χ^2 -test should be independent of each other.
- 4, The observation collected for χ^2 -Test must be based on the method of random Sampling.

b) find the value of χ^2 for the following data and test for goodness of fit.

(5% level of significance)

observed frequency	14	15	18	20	15	10
Expected frequency	17	10	15	25	10	15

$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(14-17)^2}{17} + \frac{(15-10)^2}{10} + \frac{(18-15)^2}{15} + \frac{(20-25)^2}{25} + \frac{(15-10)^2}{10}$$

$$= \frac{9}{17} + \frac{5}{2} + \frac{3}{5} + \frac{25}{25} + \frac{5}{2} + \frac{5}{3}$$

$$= \frac{224.8}{255} = 8.7960$$

Table Value of χ^2 with 5 degree of freedom is 11.07
at 5% (0.05) level of significance

Since $\chi^2_{\text{cal}} < \chi^2_{\text{tab}}$ (i.e.) ($8.7960 < 11.07$)

Therefore we accept null hypothesis and the fit
is good.

→ Find the value of χ^2 for the following data and
For goodness of fit
(1% level of significance)

Observed frequency 9 27 36 18

Expected frequency 15 30 30 15

Sol:

$$\sum_i = \frac{O_i - E_i}{E_i}$$

$$\sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

O_i = observed frequency

E_i = expected

$$= \frac{(9-15)^2}{15} + \frac{(27-30)^2}{30} + \frac{(36-30)^2}{30} + \frac{(18-15)^2}{15}$$

$$= \frac{12}{5} + \frac{3}{10} + \frac{6}{5} + \frac{3}{5}$$

$$= \frac{9}{2} = 4.5$$

Table value of χ^2 with 1 degree of freedom is 11.345

→ T-Test (in)

when the population standard deviation is not known and the size of sample is less than or equal to thirty we use

t-test

To test the significance of mean sample the test statistic

given by

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$$

$$\text{where } \bar{x} = \frac{\sum x_i}{n}$$

n = Size of Sample

$$s = \text{Standard deviation} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

Q) Find the student's t-test for the following variable values.

in a sample of eight -4, -2, 0, 1, 2, 2, 3, 3 Taking mean universe to be zero ($\mu = 0$)

Sol: we have $t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$

$$\text{where } \bar{x} = \frac{\sum x_i}{n}$$

n = Size of Sample.

$$s = \text{Standard deviation} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{-4 - 2 - 2 + 0 + 1 + 2 + 3 + 3}{8}$$

$$\bar{x} = 0.25$$

Computation table

$$x - \bar{x}$$

$$(x - \bar{x})^2$$

$$\alpha = -4 - 0.25 = 4.25$$

$$18 - 0.6625$$

$$-2 + 0.25 = 2.25$$

$$5 - 0.6625$$

$$-2 - 0.25 = 2.25$$

$$0.25$$

$$1.75$$

$$3.0625$$

$$3 + 0.6625$$

$$2.25$$

$$7.5625$$

$$2.75$$

$$7.5625$$

$$\sum_{n=2}^{\infty}$$

$$2^{(n-\bar{n})} = 0$$

$$\sum_{n=2}^{\infty} (x - \bar{x})^2 = 49.1$$

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$S = \sqrt{\frac{49.1}{7}} = 2.659215781$$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$$

$$= \frac{0.25 - 0}{2.659215781} \times \sqrt{8} = 0.265908011$$

$$t_{cal} = 0.265908011$$

$$t_{tab} = 1.895$$

Since $t_{cal} > t_{tab}$

101188, 83, 95, 98, 107 & 100. Do these data support the assumption of a population mean of 160 (5%)

$$\bar{x} = \frac{90+120+110+101+88+93+95+98+107+100}{10}$$

$$\bar{x} = 97.2$$

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Computation table.

x	\bar{x}	$(x - \bar{x})^2$
90	97.2	739.84
120	97.2	519.84
110	97.2	163.84
101	97.2	14.44
88	97.2	84.64
95	97.2	4.44
98	97.2	0.64
107	97.2	64.64
100	97.2	4.64
$\sum(x - \bar{x})^2 = 1833.64$		

$$S = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}}$$

$$N = 27355753$$

$$S = \sqrt{\frac{1833.64}{10}} = S = 427355753$$

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{97.2 - 160}{\frac{203.7344444}{14.27355753}} \times \sqrt{10}$$

$$t = -13.913$$

$$t_{cal} = -13.9132$$

$$t_{tab} = 1.833$$

Since $t_{cal} > t_{tab}$

No, these data does not support assumption of:

Q3:- The height of 10 females of a given location were found to be 66, 63, 58, 64, 57, 64, 66, 60, 60, 62 inches.

It is reasonable to believe that average of height is greater than 60 inches. Test at 5% level of significance

$$\Sigma \text{Sol. } \bar{x} = \frac{66 + 63 + 58 + 64 + 57 + 64 + 66 + 60 + 60 + 62}{10}$$

$$\bar{x} = 62$$

$$x - \bar{x}$$

$$66 - 62 \\ 4$$

$$63 - 62 \\ 1$$

$$58 - 62 \\ -4$$

$$64 - 62 \\ 2$$

$$57 - 62 \\ -5$$

$$64 - 62 \\ 2$$

$$66 - 62 \\ 4$$

$$60 - 62 \\ -2$$

$$60 - 62 \\ -2$$

$$62 - 62 \\ 0$$

$$(x - \bar{x})^2$$

$$16$$

$$1$$

$$16$$

$$4$$

$$25$$

$$4$$

$$16$$

$$4$$

$$4$$

$$0$$

$$S(x) = \sum (x - \bar{x})^2 = 90$$

$$T_{\text{lab}} = 1.833$$

$$T_{\text{ca}} = 2.000$$

$$= 2.000$$

$$t = \frac{\mu - \mu}{\frac{q + 2 - 60}{3.16227766} \times \sqrt{10}} = \frac{\left(\frac{\mu}{\sqrt{n}} \right)}{q + 2 - 60}$$

$$S = \frac{q}{q + 2 - 60} = \sqrt{10} = 3.16227766$$

$$\frac{1-n}{n(n-1)B} \int_0^1$$

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F-Test

The objective of the F-Test is to find whether two independent estimates of population Variance differ significantly or whether the two samples may be regarded as drawn from normal population having the same Variance.

To carry out the test of Significance we calculate F ratio which is defined as

$$F = \frac{s_1^2}{s_2^2} (s_1^2 > s_2^2)$$

$$F = \frac{s_2^2}{s_1^2} (s_2^2 > s_1^2)$$

$$s_1^2 = \frac{\sum (x - \bar{x})^2}{n_1 - 1}$$

$$s_2^2 = \frac{\sum (y - \bar{y})^2}{n_2 - 1}$$

degree of freedom $v_1 = n_1 - 1$
 $v_2 = n_2 - 1$

Q1. Two Samples are drawn from two normal Equations.
 Test whether the two Samples have the same Variance
 at 5% level of significance

					74	76	82	85	87
Sample 1	60	65	71						
Sample 2	61	66	67	85	78	63	85	86	87

Sol let

Sample 1 be 'x'

Sample 2 be = 'y'

$$\text{Calculate } \bar{x} . \quad \frac{\sum x}{n} = \frac{600}{8} = 75$$

$$\bar{y} . \quad \frac{\sum y}{n} = \frac{770}{10} = 77$$

Computation table.

x	$x - \bar{x}$	$(x - \bar{x})^2$	y	$y - \bar{y}$	$(y - \bar{y})^2$
60	-15	225	61	-16	256
65	-10	100	66	-11	121
71	-4	16	67	-10	100
74	-1	1	85	8	64
76	1	1	78	1	1
82	7	49	63	-14	196
85	10	100	85	8	64
87	12	144	86	9	81
$\Sigma x = 600$		$\Sigma (x - \bar{x})^2 = 636$	88	11	121
$\Sigma y = 770$		$\Sigma (y - \bar{y})^2 = 3930$	91	14	196
					$(y - \bar{y})^2 = 1200$

Name -

$$S_1^2 = \frac{\sum (x_i - \bar{x})^2}{n_1 - 1} = \frac{636}{7} = 90.8571$$

$$V_1 = n_1 - 1 = 7$$

$$V_2 = n_2 - 1 = 9$$

$$S_2^2 = \frac{\sum (y_j - \bar{y})^2}{n_2 - 1} = \frac{1200}{9} = 133.333$$

$$F = \frac{S_2^2}{S_1^2} = \frac{133.333}{90.8571} = 1.4675$$

$$F_{tab} = F_{9,7} = 3.68 (T-1)$$

$$F_{cal} = 1.4675$$

Since $F_{tab} > F_{cal}$ (H_0 is accepted)
We conclude that population has some Variance

Q2: The values of two random samples are given

S_1	15	25	16	20	22	24	21	17	19	23	.
S_2	85	81	27	38	26	29	32	34	33	27	29 31

let

Sample 1 be = \bar{x}

2 be = \bar{y}

$$\bar{x} = \frac{202}{10} = 20.2$$

$$\bar{y} = \frac{370}{12} = 30.8333$$

Unit IProbability:

- What is probability
- Probability is the chance that some event will happen.
- It is the ratio of the number of ways a certain event can occur to the number of possible outcomes.

Conditional probability:-

Conditional probability is a measure of the probability of an event given that another event has already occurred. If the event of interest is A and the event B is known or assumed to have occurred, "the conditional probability of A given B", is usually written as $P(A|B)$.

Replacement:

Note: if we replace the marbles in the bag each time, then the chances do not change and the events are independent.

With replacement: the events are independent (the chances don't change)

Without replacement: the events are dependent (the chances change)

Dependent Events

→ Events can also be dependent

→ which means they can be affected by previous Event)

→ Example: Marbles in a Bag

→ 2 blue and 3 red marbles are in a bag.

→ What are the chances of getting a blue marble?

→ The chance is 2 in 5

→ But after taking one out the chances change!

→ So the next time:

→ We got a red marble before, then the chance of a blue marble next is 2 in 4

→ if we got a blue marble before, then the chance of a blue marble next is 1 in 4

→ This is because we are removing marbles from the bag.

→ So the next event depends on what happened in the previous event, and is called dependent.

Independent Events

- Events can be independent
- Each Event is not affected by any other Events.
- Example: Tossing a coin.
what it did in the past will not affect the current toss.
- The chance is simply $\frac{1}{2}$ or 50%, just like ANY toss of the coin
- So Each toss is an independent Event.

Addition of probability :- (2M)

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication theorem on probability

If A and B are any two events in a sample space

then

$$P(A \cap B) = P(A)P(B|A), P(A) > 0$$

$$P(B \cap A) = P(B)P(A|B), P(B) > 0$$

Conditional probability :-

If A and B are any two events in a sample space then
conditional probability is defined as

A given B

$$P(A|B) = P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}, P(B) > 0$$

$$P(B|A) = P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

(Q1)

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, P(A) > 0$$

Note: the probability of an event A when another event B is known to have already happened is called conditional probability

Theorem of total probability (vimp)

Statement: Let B_1 and B_2, \dots, B_n be a set of exhaustive and mutually exclusive events of a sample space S ,

$$P(B_i) \neq 0, i = 1, 2, \dots, n,$$

Let A be any event of S , the probability of

$$P(A) = \sum_{i=1}^n P(B_i A) = \sum_{i=1}^n P(B_i)P(A|B_i)$$

Prove:-
since

Let B_1 and B_2, \dots, B_n be a set of exhaustive and mutually exclusive events of a sample space S .

$$S = \bigcup_{i=1}^n B_i; \quad B_i \cap B_j = \emptyset, \text{ if } i \neq j$$

Let A and B be any event of S .

$$\therefore A \cap B, A \cap B_1, \dots, A \cap B_n$$

$$A =$$

Hence by additive rule of mutually exclusive events we obtain probability

$$P(A) = P[(A \cap B_1) \cup (A \cap B_2) \cup \dots \cup (A \cap B_n)]$$

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \dots + P(A \cap B_n)$$

$$P(A) = P(B_1 \cap A) + P(B_2 \cap A) + \dots + P(B_n \cap A)$$