Assignment - 1

1 a) List and explain the components of finite state automata

ans Finite state automata consists of the following

Q = Finite set of states

Σ = Set of input symbols

q_0 = Initial state

F = Set of final states

δ = Transition function

1b) Summarise the closure properties of regular language

ans Union :- If L_1 and if L_2 are two regular languages, their union $L_1 \cup L_2$ will also be regular

Eg :- $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$

$L_3 = L_1 \cup L_2 = \{a^n \cup b^n \mid n \geq 0\}$ is also regular

Intersection :- If L_1 and if L_2 are two regular languages their intersection $L_1 \cap L_2$ will also be regular

Eg :- $L_1 = \{a^m b^n \mid n \geq 0$ and $m \geq 0\}$ and

$L_2 = \{a^m b^n \cup b^n a^m \mid n \geq 0 \;\&\; m \geq 0\}$

$L_3 = L_1 \cap L_2 \; \{a^m b^n \mid n \geq 0 \;\&\; m \geq 0\}$

is also regular.

<u>Concatenation</u> :- If L_1 and L_2 are two regular languages their concatenation $L_1 \cdot L_2$ will also be regular

example

$L_1 = \{a^n \mid n \geq 0\} \;\&\; L_2 = \{b^n \mid n \geq 0\}$

$L_3 = L_1 \cdot L_2 \; \{a^m \cdot b^n \mid m \geq 0$ and $n \geq 0\}$

is also regular.

<u>Complement</u> :- If $L(G)$ is regular Language its complement $L'(G)$ will also be regular complement of a Language can be found by substracting which are in $L(G)$ from all possible strings

$L(G) = \{a^n \mid n > 3\}$

$L'(G) = \{a^n \mid n <= 3\}$

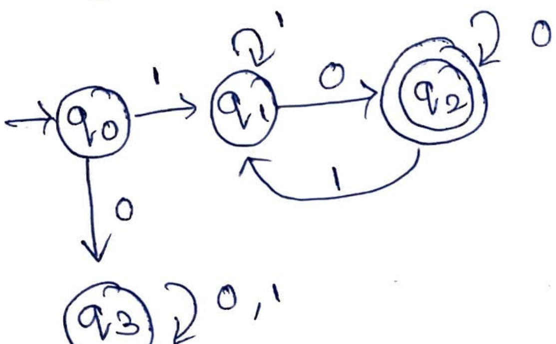
<u>Kleene closure</u> :- If L_1 is a regular Language is ~~KC~~ Kleene closure L_1^* will also be regular

Eg :- $L_1 = (a \cup b)$ $\quad L_1^* = (a \cup b)^*$

2. Draw finite automata that accepts a string which starts with '1' and ends with '0', $\Sigma = \{0, 1\}$

Sol

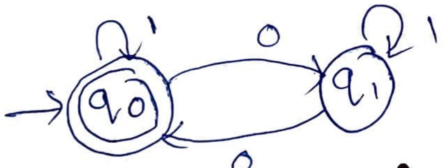
$L = \{10, 1010, 100, \dots\}$



3. Design DFA which accepts even number of 0's over $\{0,1\}$

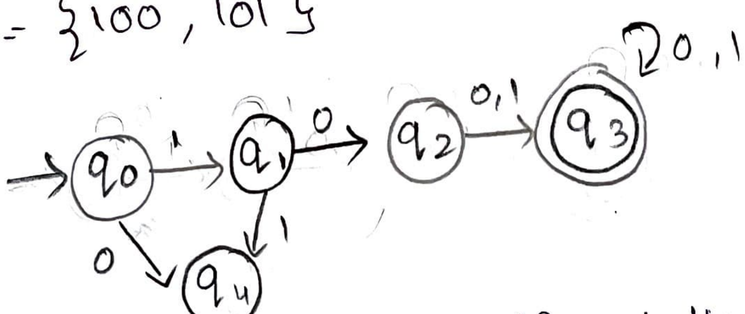
Sol

~~$L = \{0, 00, 0000, \dots\}$~~ $L = \{0, 00, 100, 001, 0000 \dots\}$



HQ) Design DFA which accepts language

$L = \{100, 101\}$

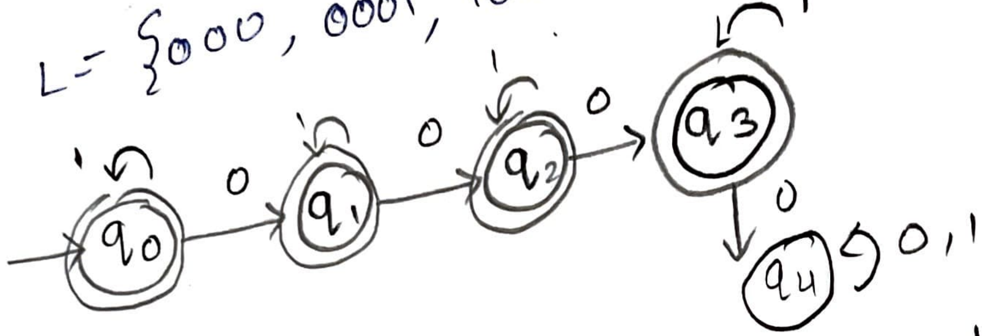


5. Design DFA for the following over $\{0,1\}$

(i) All string containing not more than three 0's

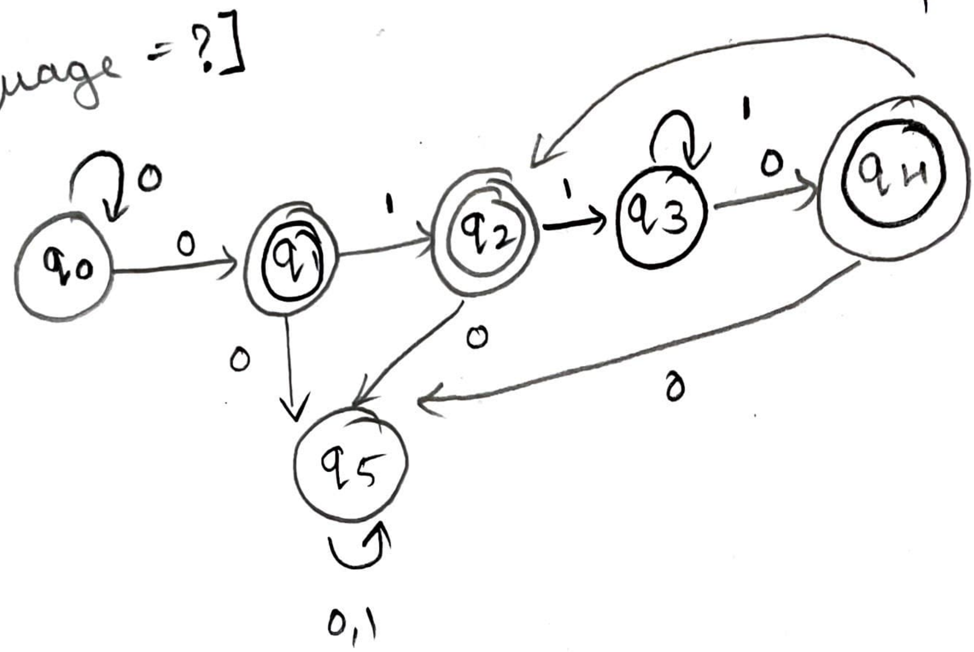
(ii) All string that has at least two occurrences of 1 b/w any two occurences of 0

Sol (i) $L = \{000, 0001, 1000, \ldots\ldots\}$

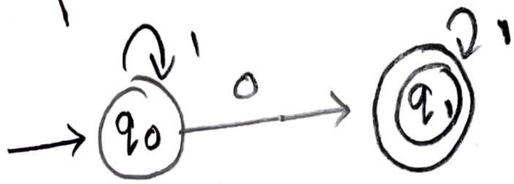


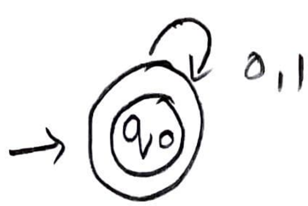
[Language = ?]

(ii)

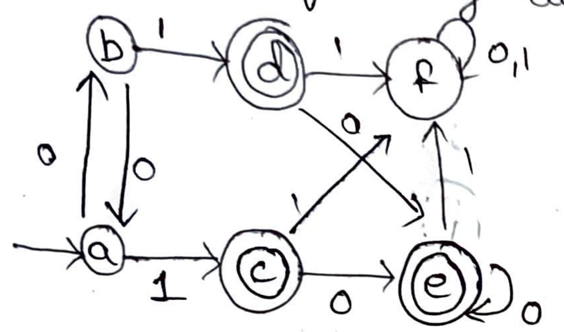


6Q) Construct an NFA equivalent to the regular expression $1^*0+1101$ and $(0+1)^*$





⑦ Minimize the following automata



Sol

Step 1 :- Find zero equivalence

{a, b, f} {c, d, e}

Step 2 :- Find one equivalent {a, b}

$a \xrightarrow{0} b$ $a \xrightarrow{1} c$
$b \xrightarrow{0} a$ $b \xrightarrow{1} d$

∴ a & b are equivalent {a, b for a & f

lets check one equivalence

$a \xrightarrow{0} b$ $a \xrightarrow{1} c$
$f \rightarrow f$ $f \rightarrow f$

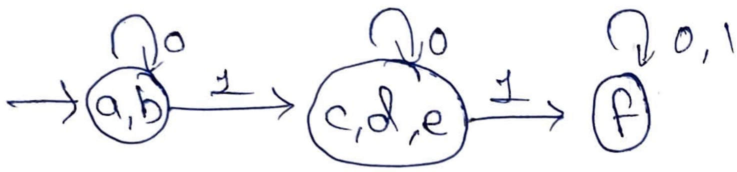
f is not equivalent to a and b
{a, b} {f} , {c, d, e}

Step ③ : Find three equivalence

$a \rightarrow b$ $a \xrightarrow{1} c$
$b \rightarrow a$ $b \xrightarrow{1} d$

a and b are equivalent .

⇒ The minimized automata is



⑧ Convert the following NFA to DFA

	0	1
→p	p, r	q
q	r, s	p
*r	p, s	r
*s	q, r	∅

Sol) Corresponding DFA will be

	0	1
p	pr	q
q	rs	p
pr	psr	qr
rs	pqr	r
qr	psr	pr
r	ps	r
ps	pqr	q
psr	pqsr	qr
pqr	psr	qpr

qpr	psr	qpr
psr	pqsr	pqr

9. Write and explain the properties of regular sets

Sol 1. The union of two regular set is regular

$\Rightarrow$ $RE_1 = a(aa)^*$ and $RE_2 = (aa)^*$

So $L_1 = \{a, aaa, aaaaa \ldots\}$ (String of odd Length excluding Null)

and

$L_2 = \{\varepsilon, aa, aaaa, aaaaaa \ldots\}$ (String of even length

$L_1 \cup L_2 = \{\varepsilon, a, aa, aaa, aaaa, aaaaa, \ldots\}$ including (String of all possible lengths including NULL) NULL)

~~RE~~ $RE(L_1 \cup L_2) = a^*$

2. The intersection of two regular sets is regular

$\Rightarrow$ $RE_1 = a(a^*)$ and $RE_2 = (aa)^*$

$L_1 = \{a, aa, aaa, aaaa, \ldots\}$

$L_2 = \{\varepsilon, aa, aaaa, aaaaaa, \ldots\}$

$L_1 \cap L_2 = \{aa, aaaa, aaaaaa \ldots\}$

$RE(L_1 \cap L_2) = aa(aa)^*$

3. The complement of a regular set is regular

$RE = (aa)^*$

So

$L = \{\varepsilon, aa, aaaa, aaaaaa, \ldots\}$

complement of L is all the string that is not in L

So

$L' = \{a, aaa, aaaaa, \ldots\}$

$RE(L') = a(aa)^*$

4. The difference of two regular set is regular

$RE = a(a^*)$ and $RE_2 = (aa)^*$

$L_1 = \{a, aa, aaa, aaaa, \ldots\}$

$L_2 = \{\varepsilon, aa, aaa, aaaaaa, \ldots\}$

$L_1 - L_2 = \{a, aaa, aaaaa, aaaaaaa, \ldots\}$

$RE(L_1 - L_2) = a(aa)^*$

5. The Reversal of a regular set is regular

$L = \{01, 10, 11, 10\}$

$RE(L) = 01 + 10 + 11 + 10$

$L^R = \{10, 01, 11, 01\}$

$RE(L^R) = 01 + 10 + 11 + 10$

6. The closure of a regular set is regular

$L = \{a, aaa, aaaaa, \ldots\}$

$RE(L) = a(aa)^*$

$L^* = \{a, aa, aaa, aaaa, aaaaa, \ldots\}$

$RE(L^*) = a(a^*)$

7. The concatenation of two regular set is regular

$RE_1 = (0+1)^*0$ and $RE_2 = 01(0+1)^*$

$L_1 = \{0, 00, 10, 000, 010, \ldots\}$

$L_2 = \{01, 010, 011, \ldots\}$

$L_1L_2 = \{001, 0010, 0011, 0001, \ldots\}$

set of strings containing 001 as a substring which can be represented by an RE - $(0+1)^*001(0+1)^*$.

b) obtain the regular expression to accept strings of a's, b's and c's such that fourth symbol from is a's and ends with 'b'

9b) The regular expression is

$$(a+b+c)^* a (a+b+c)^* (a+b+c)^* b$$

10 b) Discuss the closure properties of Context free Language

ans **Union**

$L_1 = \{a^n b^n c^m \mid n >= 0 \;\&\; m >= 0\}$

$L_2 = \{a^n b^m c^m \mid n >= 0 \text{ and } m >= 0\}$

$L_3 = L_1 \cup L_2 = \{a^n b^n c^m \cup a^n b^m c^m \mid n >= 0, m >= 0\}$

Concatenation

$L_1 = \{a^n b^n \mid n >= 0\}$ and $L_2 = \{c^m d^m \mid m >= 0\}$

$L_3 = L_1 \cdot L_2 = \{a^n b^n c^m d^m \mid n >= 0 \;\&\; m >= 0\}$

Kleene closure

$L_1 = \{a^n b^n \mid n >= 0\}$

$L_1^* = \{a^n b^n \mid n >= 0\}^*$

Intersection and Complementation

$L_1 = \{a^n b^n c^m \mid n >= 0 \;\&\; m >= 0\}$

$L_2 = \{a^m b^n c^n \mid n \geq 0 \;\&\; m >= 0\}$

$L_3 = L_1 \cap L_2$

—x—x—