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Unit 1

DESIGN CONCEPTS

I NOTES

① Digital Hardware

Logic circuits are used to build computer hardware and similar products. These circuits are called digital hardware because of the manner in which it represents and processes information.

② Moore's Law

Gordon Moore, the chairman of intel observed that integrated circuit technology was progressing at an astounding rate doubling the number of transistors that could be placed on a chip every 1½ to 2 years.

The advent of ICs made it possible to place a number of transistors and thus an entire circuit on a single chip.

ICs are placed on semiconductor wafers and the wafer is cut to produce individual chips which are then placed in a special type of chip package.

(c) Types of Chips

i) STANDARD CHIPS

These realize commonly used logic functions (usually involved in fewer than 100 transistors) and performs simple functions. Its functionality is fixed and cannot be changed.

ii) Programmable Logic Device (PLD):-

These contain circuits that can be configured by the user to implement a wide range of different logic circuits. They include a collection of programmable switches which help in configuring the chip in different ways. They can usually be programmed multiple times.

iii) Custom Design CHIPS

Designing a chip from the beginning is also possible ie the chip is first designed and then it is fabricated using appropriate technology.

Valuable chip area is saved and speed also improves due to the absence of switches when compared to PLDs.

These are intended for specific applications and are sometimes called application specific integrated circuits (ASIC).

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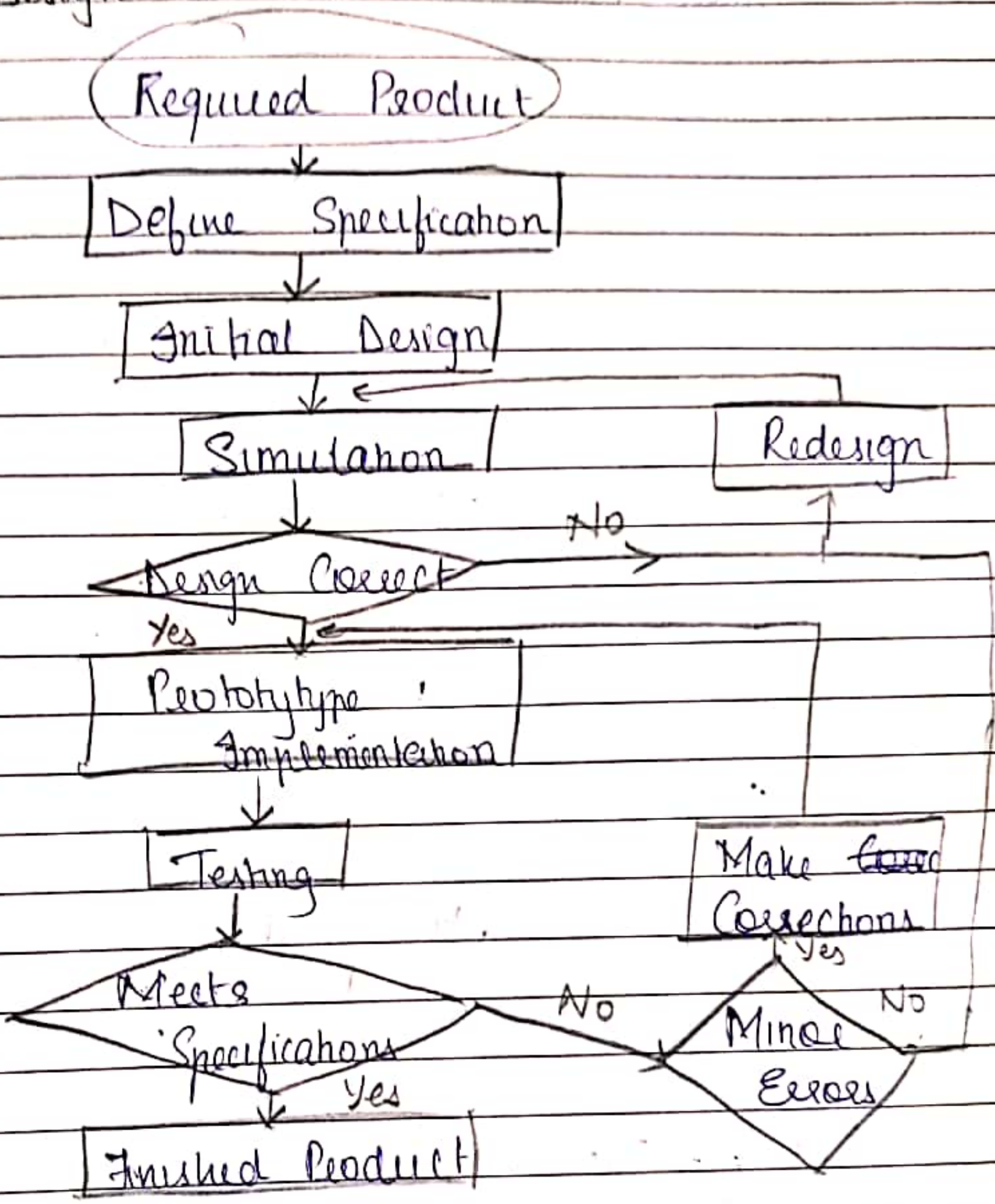
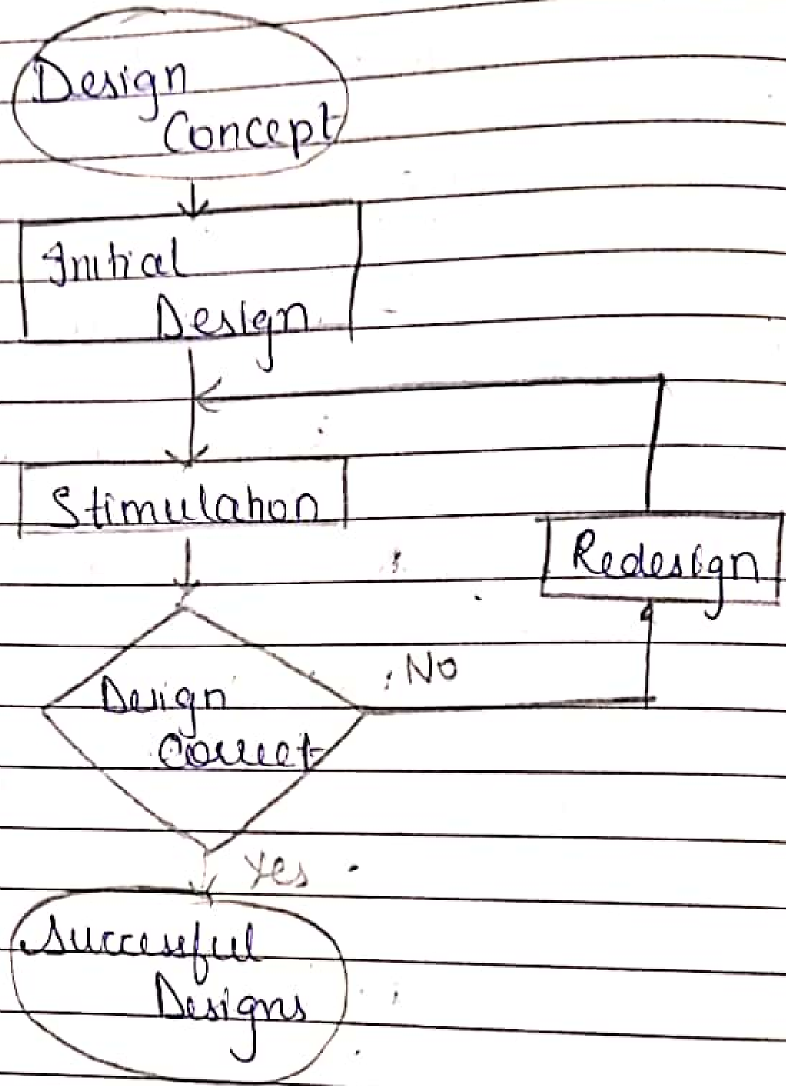
④ General Design



5 Digital Hardware Design

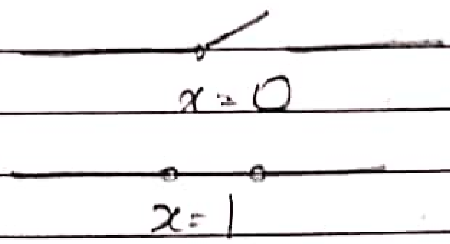


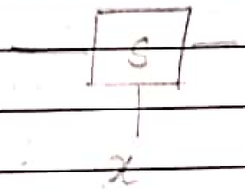
6 Logic Circuits

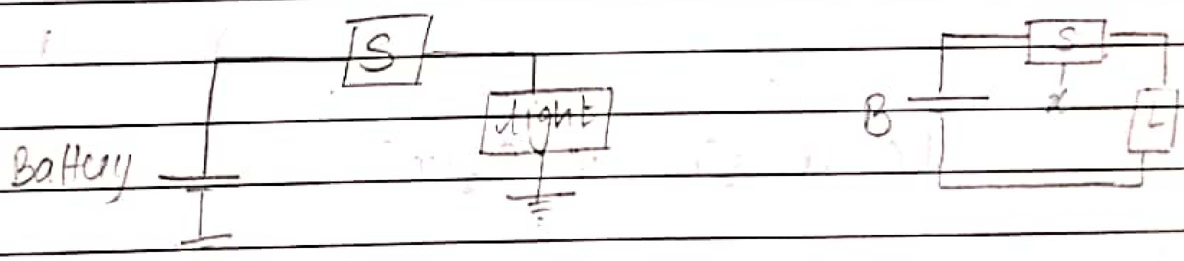
Logic circuits perform digital operations and are usually implemented as electronic circuits where the signal values are restricted. In binary logic circuits there are only two values - zero and one.

7 Variables and Functions

The simplest binary element is a switch that has two stages: on and off. If the switch is controlled by an input variable x then when $x=0$ the switch is open and when $x=1$, the switch is closed.





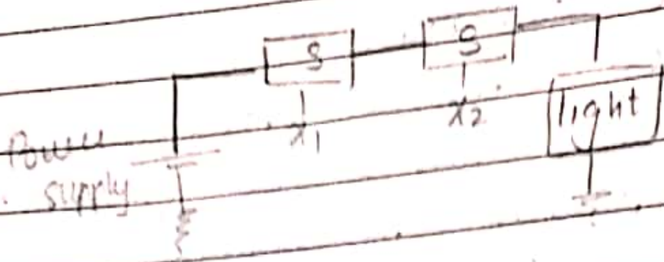


$$L(x) = x$$

x is an input variable and $L(x)$ is a logic function

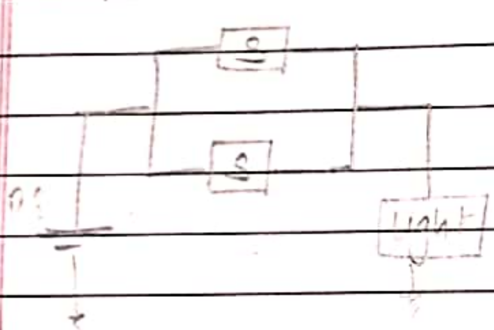
8 Using two switches to control a light

▷ SERIES (AND)



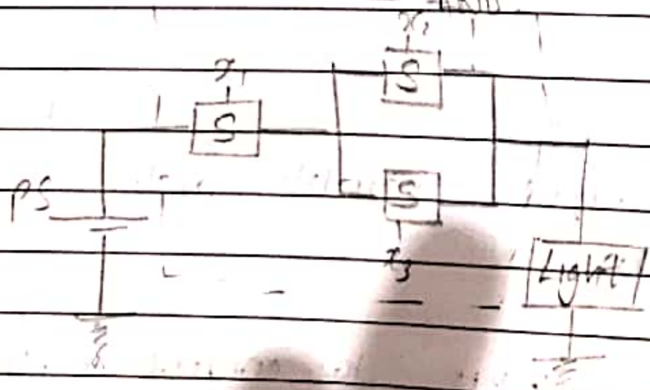
x_1	x_2	$L(x_1,x_2)$
0	0	0
0	1	0
1	0	0
1	1	1

ii) PARALLEL (OR)

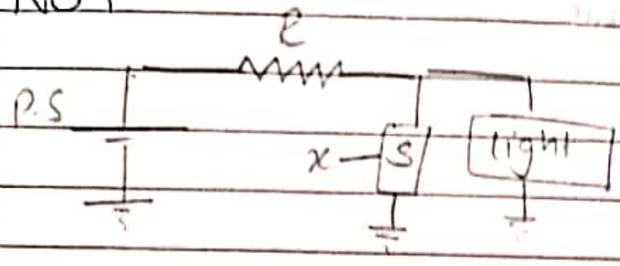


x_1	x_2	$L(x_1,x_2)$
0	0	0
0	1	1
1	0	1
1	1	1

$$L(x_1,x_2,x_3) = x_1 \underset{AND}{\cdot} (x_2 \overset{OR}{+} x_3)$$



iii) NOT



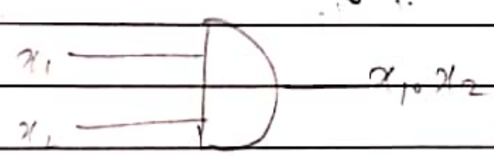
x	$L(x)$
0	1
1	0

(9) Logic Gates and Networks

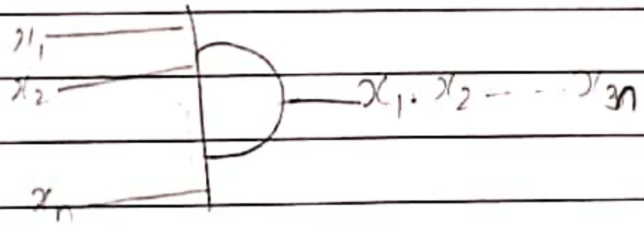
A logic gate has one or more inputs and one output that is a function of its inputs. The logic operations are implemented electronically with transistors resulting in a circuit element that is called a logic gate.

i) AND

2 input AND gate

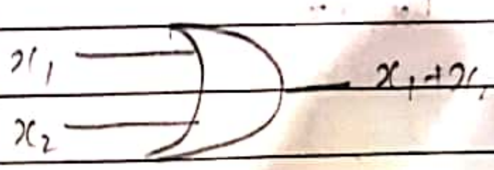


Multi-input AND gate

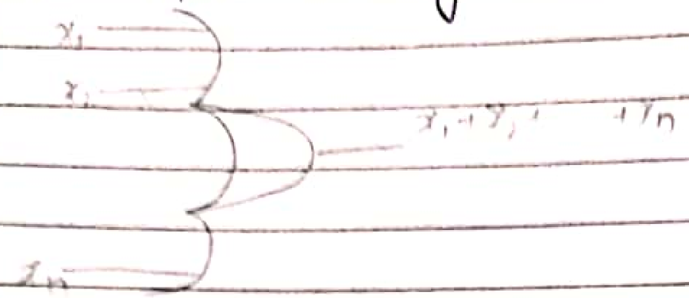


ii) OR

2 input OR gate



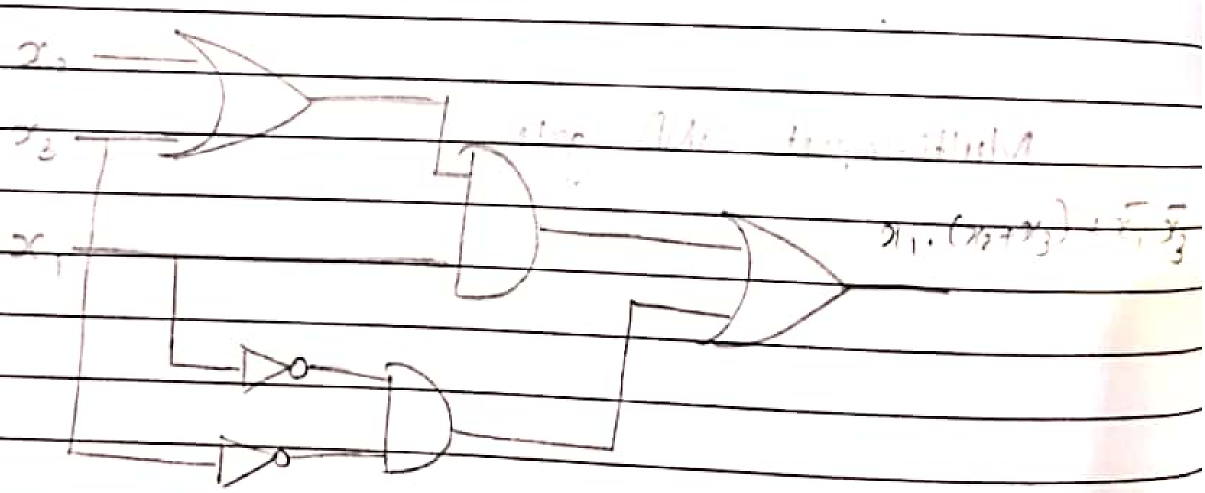
Multi-input OR gate



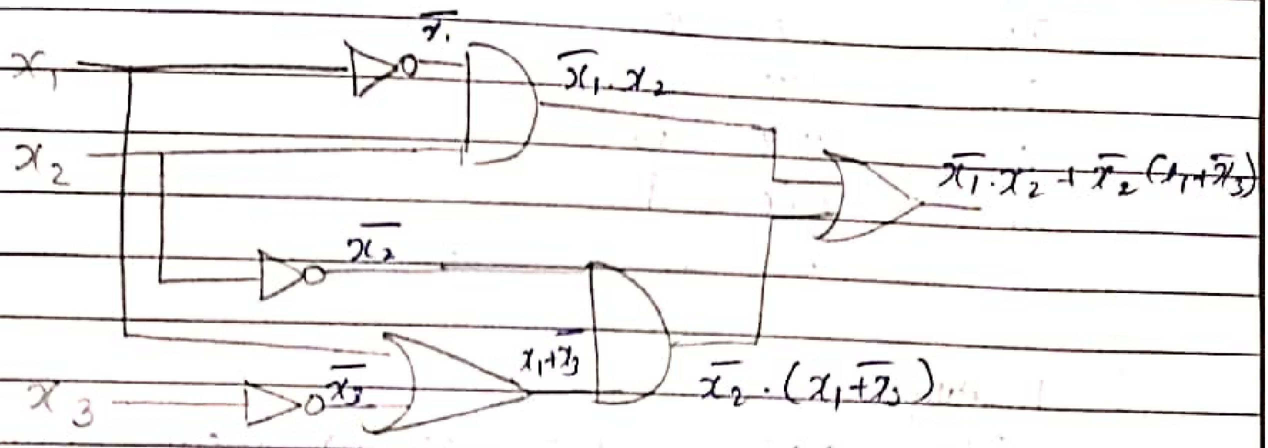
ii) NOT

OR, AND and NOT gates are known as basic gates

Q1. Implement $x_1 \cdot (x_2 + x_3) + \bar{x_1}\,\bar{x_3}$



Q2. $y(x_1, x_2, x_3) = \bar{x_1} \cdot x_2 + \bar{x_2} \cdot (x_1 + \bar{x_3})$



I. NOTES

① A network of gates is called a logic ~~gate~~ network

② Logic Network Analysis
George Boole gave a scheme for logical thought and reasoning which was further refined and developed into Boolean Algebra

I) AXIOMS OF BOOLEAN ALGEBRA

a. 1a = $0.0 = 0$
b. 1b $1+1 = 1$
c. 2a $1.1 = 1$
d. 2b $0+0 = 0$
e 3a $0.1 = 1.0 = 0$
f 3b $0+1 = 1+0 = 1$
g 4a If $x=0$ then $\bar{x} = 1$
h 4b If $x=1$ then $\bar{x} = 0$

II) SINGLE VARIABLE THEOREMS

a 5a $x.0 = 0$
b 5b $x+1 = 1$
c 6a $x.1 = x$
d 6b $x+0 = x$

7a $x.x = x$
7b $x+x = x$
8a $x.\bar{x} = 0$
8b $x+\bar{x} = 1$
9 $\bar{\bar{x}} = x$

imp

(3) Duality

The axioms above are in pairs. Given a logic expression, its dual is obtained by replacing all plus operators with dots operators and vice versa and replacing all zeroes by ones and vice versa. The dual of any true statement in boolean algebra is also true. This reflects the principle of duality. Duality implies that at least 2 different ways exist to express any given logic expression

i> 2 AND 3 VARIABLE PROPERTIES

a 10a $x.y = y.x$
b 10b $x+y = y+x$ } Commutative Property

c 11a $x.(y.z) = (x.y).z$
d 11b $x+(y+z) = (x+y)+z$ } Associative Property

e 12a $x.(y+z) = x.y + x.z$
f 12b $x+(y.z) = (x+y).(x+z)$ } Distributive Property

g 13a $x + x.y = x$
h 13b $x.(x+y) = x$ } Absorption

i 14a $x.y + x.\bar{y} = x$
j 14b $(x+y).(x+\bar{y}) = x$ } Combining

k 15a $\overline{x.y} = \bar{x}+\bar{y}$
l 15b $\overline{x+y} = \bar{x}.\bar{y}$ } De Morgan's Theorem

m 16a $x+\bar{x}y = x+y$

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n | 16b | $x \cdot (\bar{x}+y) = x \cdot y$
o | 17a | $xy + yz + \bar{x}z = xy + \bar{x}z$
p | 17b | $(x+y) \cdot (y+z) \cdot (\bar{x}+z) = (x+y) \cdot (\bar{x}+z)$

Consensus Theorem

(2) Proof of Theorems by Perfect Induction

i) 15a $\overline{x \cdot y} = \bar{x} + \bar{y}$

x	y	$\overline{x \cdot y}$	$\bar{x}+\bar{y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

ii) 17a ~~x.y = y.x~~ $xy + yz + \bar{x}z = xy + \bar{x}z$

x	y	z	LHS	RHS
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	1	1
1	1	1	1	1

iii) 14 b $(x+y) \cdot (x+\bar{y}) = x$ (Distributive)

$$LHS = (x+y) \cdot (x+\bar{y}) = x \cdot x + y \cdot x + x \cdot \bar{y} + y \cdot \bar{y}$$
$$= x + x \cdot y + x \cdot \bar{y} + 0$$
$$= x(1 + y + \bar{y})$$
$$= x(1) = x \quad = RHS$$

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Q3.) $x_1\bar{x}_3 + x_2x_3 + \bar{x}_2\bar{x}_3 = (x_1+\bar{x}_2+x_3)(x_1+x_2+\bar{x}_3)(\bar{x}_1+x_2+\bar{x}_3)$

RHS. $(x_1+\bar{x}_2+x_3)(x_2+\bar{x}_3)$ (combining theorem)

$= x_1x_2 + x_1\bar{x}_3 + \bar{x}_2x_2 + \bar{x}_2\bar{x}_3 + x_2x_3 + x_3\bar{x}_3$

$= x_1x_2 + x_1\bar{x}_3 + \bar{x}_2\bar{x}_3 + x_2x_3$ $(x, \bar{x} = 0)$

$= \bar{x}_2\bar{x}_3 + x_1\bar{x}_3 + x_2x_3$ (Consensus Theorem)

$= x_1\bar{x}_3 + x_2x_3 + \bar{x}_2\bar{x}_3$ (Commutative Theorem)

Q4. Prove that $(x_1+x_3)(\bar{x}_1+\bar{x}_3) = x_1.\bar{x}_3 + \bar{x}_1.x_3$

LHS $(x_1+x_3).(\bar{x}_1+\bar{x}_3)$

$x_1\bar{x}_1 + x_1\bar{x}_3 + x_3\bar{x}_1 + x_3\bar{x}_3$ (Distributive Theorem)

$0 + x_1\bar{x}_3 + x_3\bar{x}_1 + 0$ $(x.\bar{x} = 0)$

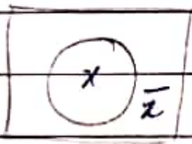
$x_1.\bar{x}_3 + x_3.\bar{x}_1$

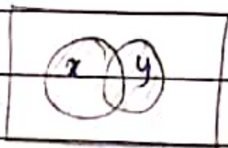
$x_1.\bar{x}_3 + \bar{x}_1.x_3$ (Commutative Method)

$= RHS$

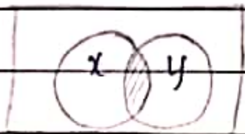
III NOTES

① Venn Diagrams

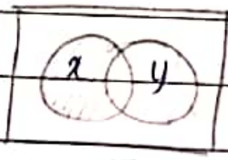




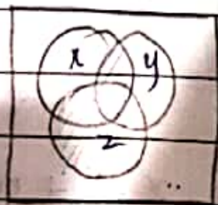
$x+y$



$x \cdot y$



$x \cdot \bar{y}$



$x \cdot y + z$

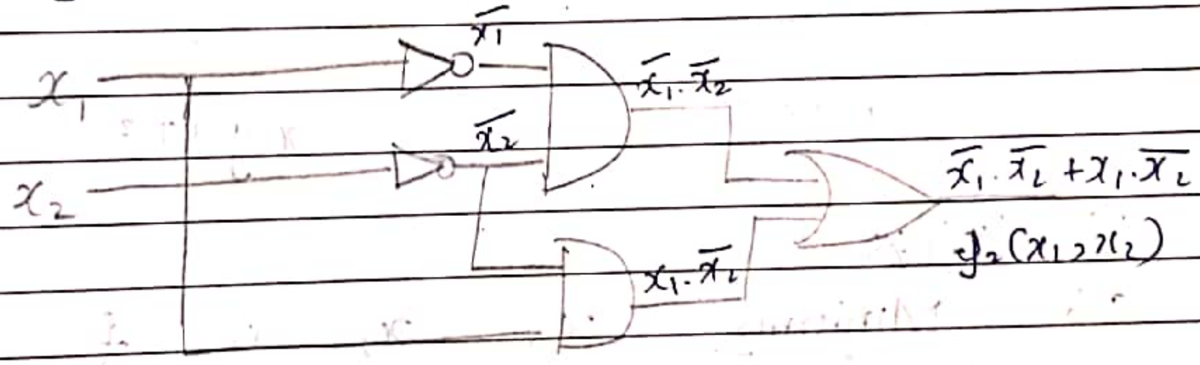
② Synthesis using AND, OR and NOT.

For designing a logic circuit that implements a truth table a product term is created that has a value of 1 for each evaluation for which the output function equals a 1. Then the logical sum is taken.

Eg:-

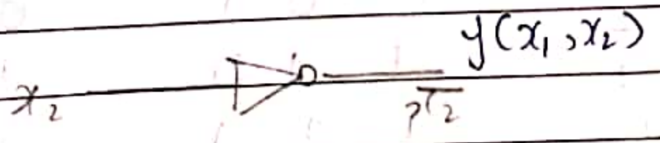
x_1	x_2	f_1	f_2
0	0	0	1
0	1	0	0
1	0	0	1
1	1	1	0

$$f_2(x_1, x_2) = \bar{x}_1 . \bar{x}_2 + x_1 \bar{x}_2$$



Canonical form of products

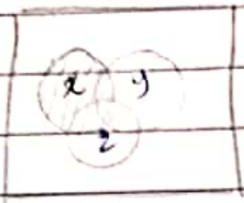
$$f_2(x_1, x_2) = \bar{x}_2(\bar{x}_1 + x_1)$$
$$= \bar{x}_2(1)$$
$$= \bar{x}_2$$



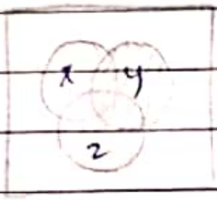
Q5 Verification of the distributive properties

$x.(y+z) = x.y + x.z$

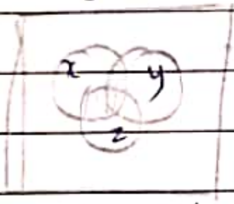
LHS



x

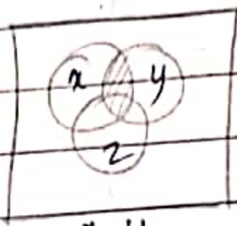


$y+z$

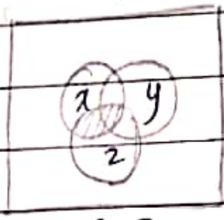


$x.(y+z)$

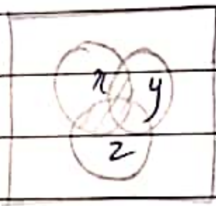
RHS



$x.y$



$x.z$



$x.y + x.z$

~~NOTES~~

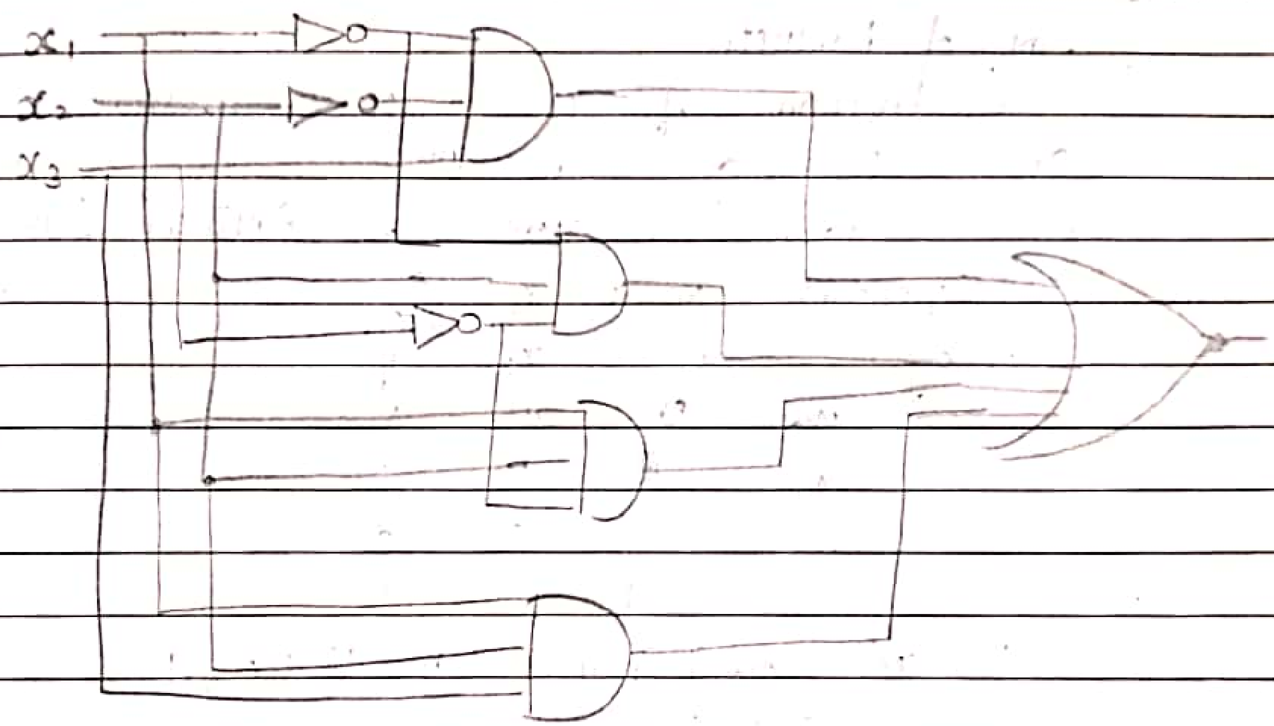
Q6

	Minterms	x_1	x_2	x_3	f
m_0	$\bar{x}_1\bar{x}_2x_3$	0	0	0	0
m_1	$\bar{x}_1\bar{x}_2x_3$	0	0	1	1
m_2	$\bar{x}_1x_2\bar{x}_3$	0	1	0	1
m_3	$\bar{x}_1x_2x_3$	0	1	1	0
m_4	$x_1\bar{x}_2\bar{x}_3$	1	0	0	0
m_5	$x_1\bar{x}_2x_3$	1	0	1	0
m_6	$x_1x_2x_3$	1	1	0	1
m_7	$x_1x_2x_3$	1	1	1	1

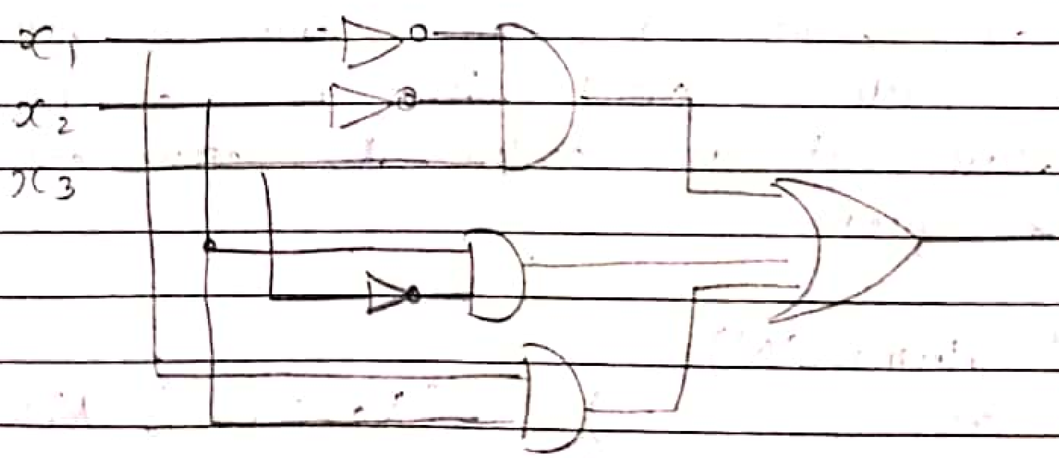
$$f(x_1, x_2, x_3) = \bar{x}_1 \cdot \bar{x}_2 \cdot x_3 + \bar{x}_1\, x_2 \cdot \bar{x}_3$$
$$x_1\, x_2\, x_3$$

$$= \bar{x}_1 \cdot \bar{x}_2 \cdot x_3 + (\bar{x}_1 + x_1)\, x_2 \bar{x}_3 + x_1 x_2 (x_3 + \bar{x}_3)$$

$$y(x_1, x_2, x_3) = \bar{x}_1 \bar{x}_2 x_3 + x_2 \bar{x}_3 + x_1 x_2$$



Canonical Sum of Products



Sum of Products

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IV NOTES

① Minterms (m):-

For a function of n variables a product term in which each of the variables appear once (in complemented or uncomplimented form), is called a minterm.

② Sum of Products

A function f can be represented by an expression ie the sum of minterms where each minterm is ANDed with the corresponding value of f.

Eg:-

	x_1	x_2	f
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

$$f(x_1, x_2) = m_0 \cdot 1 + m_1 \cdot 1 + m_2 \cdot 0 + m_3 \cdot 1$$
$$= m_0 + m_1 + m_3$$

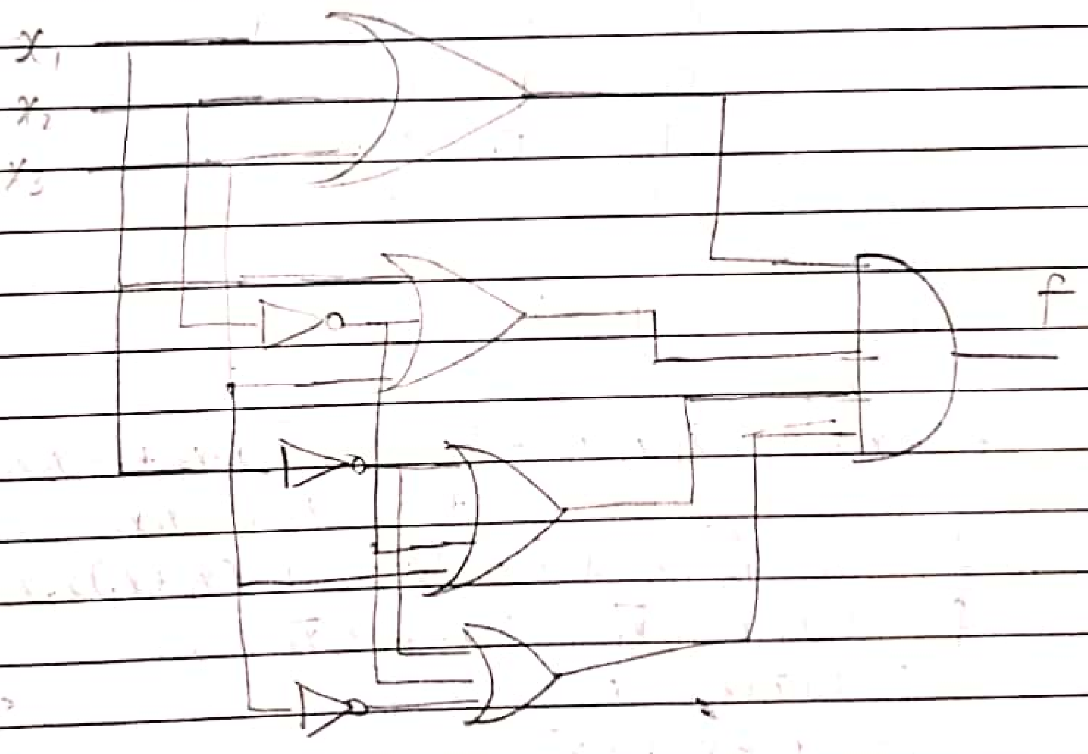
A logic expression that consist of product terms (AND terms) that are summed (OR) is said to be sum of products (SOP). If each product term is a minterm then it is called the canonical sum of products.

③ Maxterms (M):-

Eg:- $f(x_1\, x_2\, x_3) = \Pi(M_0, M_2, M_6, M_7)$

	Maxterms	x_1	x_2	x_3	f
M_0	$x_1 + x_2 + x_3$	0	0	0	0
M_1	$\bar{x}_1 + x_2 + \bar{x}_3$	0	0	1	1
M_2	$x_1 + \bar{x}_2 + x_3$	0	1	0	0
M_3	$x_1 + \bar{x}_2 + \bar{x}_3$	0	1	1	1
M_4	$\bar{x}_1 + x_2 + x_3$	1	0	0	1
M_5	$\bar{x}_1 + x_2 + \bar{x}_3$	1	0	1	1
M_6	$\bar{x}_1 + \bar{x}_2 + x_3$	1	1	0	0
M_7	$\bar{x}_1 + \bar{x}_2 + \bar{x}_3$	1	1	1	0

$$f(x_1, x_2, x_3) = (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + x_3)(\bar{x}_1 + \bar{x}_2 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)$$



Canonical forms

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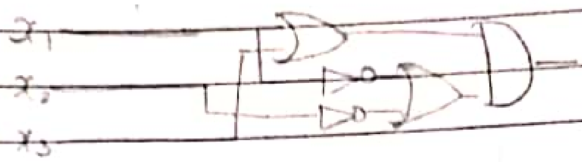
Q8.

$f(x_1, x_2, x_3) = (x_1 + x_3).(\bar{x}_1 + \bar{x}_2)$



Product of sums

Q7. $f(x_1 x_2 x_3) = \sum_m (1, 2, 3, 4, 5, 6, 7)$

S T $f(x_1 x_2 x_3) \equiv \cancel{x_1} + x_2 + x_3$

Minterm	x_1	x_2	x_3	f
m_0	0	0	0	0
m_1	0	0	1	1
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	1
m_5	1	0	1	1
m_6	1	1	0	1
m_7	1	1	1	1

$f(x_1 x_2 x_3) = \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3$

$f = (\bar{x}_1 + x_1)\bar{x}_2 x_3 + (\bar{x}_1 + x_1) x_2 \bar{x}_3 + (\bar{x}_1 + x_1) x_2 x_3 + x_1 \bar{x}_3 (\bar{x}_2 + x_2)$

$f = \bar{x}_2 x_3 + x_2 \bar{x}_3 + x_2 x_3 + x_1 \bar{x}_3$

$= x_3(\bar{x}_2 + x_2) + x_2(\bar{x}_3 + x_3) + x_1 \bar{x}_3$

$= x_3 + x_2 + x_1 \bar{x}_3$

$= x_2 + (x_3 + x_1 \bar{x}_3)$

$: x_2 + [(x_3 + x_1)(x_3 + \bar{x}_3)]$

$= x_2 + x_3 + x_1$

$a + a = a$

$a + a' = 1$

$(a + bc) = (a+b)(a+c)$

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Q8. $f(x_1, x_2, x_3) = \Pi M(0,1,2,3,4,5,6)$
S.T $f(x_1, x_2, x_3) = x_1 \cdot x_2 \cdot x_3$

M	x_1	x_2	x_3	f
M_0	0	0	0	0
M_1	0	0	1	0
M_2	0	1	0	0
M_3	0	1	1	0
M_4	1	0	0	0
M_5	1	0	1	0
M_6	1	1	0	0
M_7	1	1	1	1

$(x \cdot x) = x$
$(x+y)(x+\bar{y}) = x$

$$f = (x_1+x_2+x_3)(x_1+x_2+\bar{x}_3)(x_1+\bar{x}_2+x_3)(x_1+\bar{x}_2+\bar{x}_3)(\bar{x}_1+x_2+x_3)(\bar{x}_1+x_2+\bar{x}_3)(\bar{x}_1+\bar{x}_2+x_3)$$

$$= (x_1+x_2)(x_3+\bar{x}_3)(x_1+\bar{x}_2)(x_3+\bar{x}_3)(\bar{x}_1+x_2)(x_3+\bar{x}_3)(\bar{x}_2+x_3)(x_1+\bar{x}_1)$$

$$= (x_1+x_2)(x_1+\bar{x}_2)(\bar{x}_1+x_2)(\bar{x}_2+x_3)$$

$$= (x_1)(x_2+\bar{x}_2)(x_2)(x_1+\bar{x}_1)(\bar{x}_2+x_3)$$

$$= x_1 \cdot x_2 \cdot (\bar{x}_2+x_3)$$

$$= x_1 x_2 \bar{x}_2 + x_1 x_2 x_3$$

$$= x_1 x_2 x_3$$

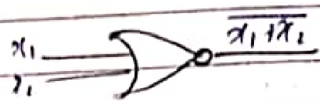
I) NOTES
① Universal Gates
~~NAND~~ Zand

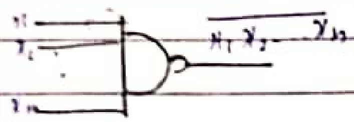
D) NAND & NOR LOGIC GATES
NAND & NOR gates are obtained by complimenting the output generated by

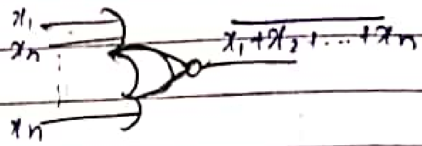
AND & OR gate respectively

NAND

NOR



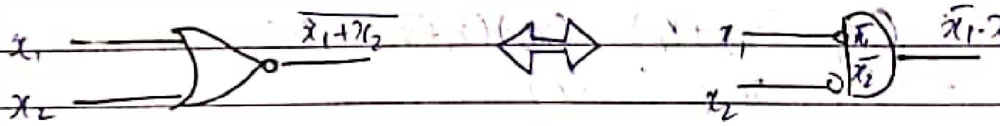




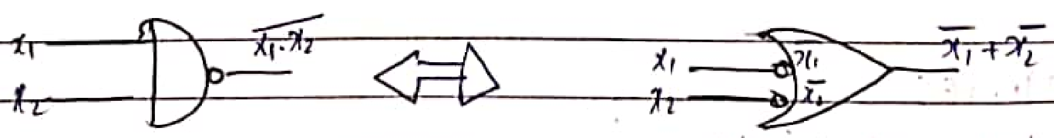
x_1	x_2	$\overline{x_1 \cdot x_2}$	$\bar{x}_1 \cdot \bar{x}_2$	$\overline{x_1+x_2}$	$\bar{x}_1+\bar{x}_2$
0	0	1	1	1	1
0	1	1	0	0	1
1	0	1	0	0	1
1	1	0	0	0	0

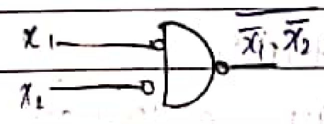
De Morgan's Rules

$\Rightarrow \overline{x_1+x_2} = \bar{x}_1 \cdot \bar{x}_2$



$\Rightarrow \overline{x_1 \cdot x_2} = \bar{x}_1 + \bar{x}_2$



$\Rightarrow$ 

$= (\bar{x}_1)' + (\bar{x}_2)' = x_1 + x_2$

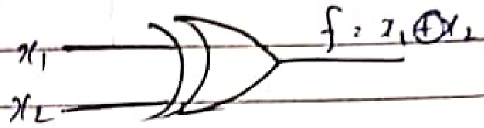
⇒ 

$x_1 . x_2$

2) EXCLUSIVE GATES

a. Exclusive OR

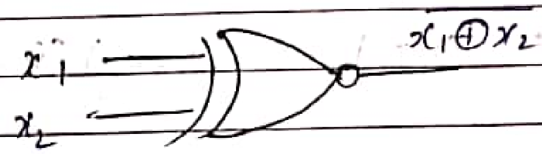
x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0



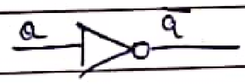
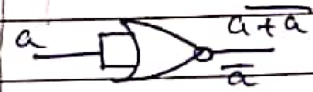
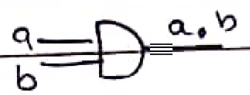
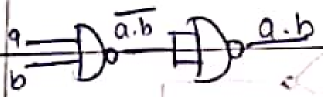
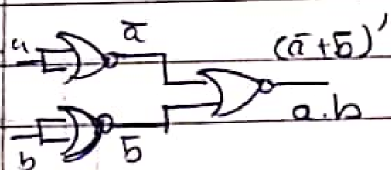
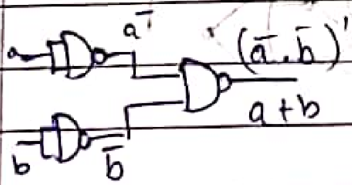
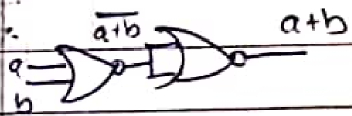
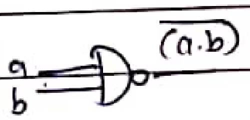
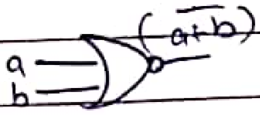
b. Exclusive NOR

x_1	x_2	f
0	0	1
0	1	0
1	0	0
1	1	1

$f = \overline{x_1 \oplus x_2}$

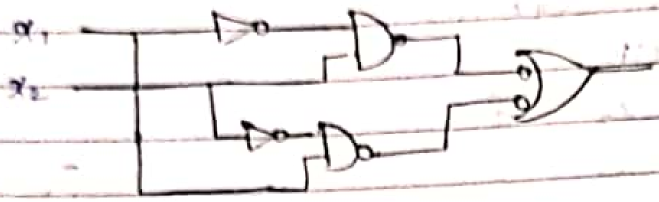


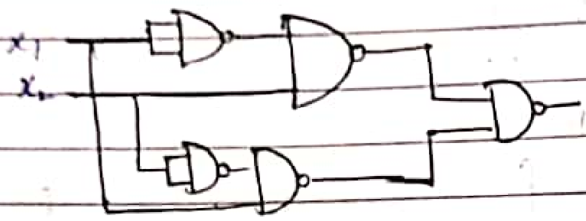
3)-

	Gate	NAND	NOR
NOT			
AND			
OR			
NAND			
NOR			

Q 11

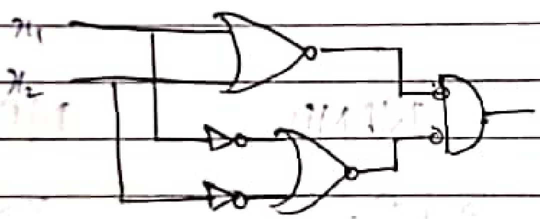
Q9 $f(x_1, x_2)_{SOP} = \bar{x}_1 x_2 + x_1 \bar{x}_2$

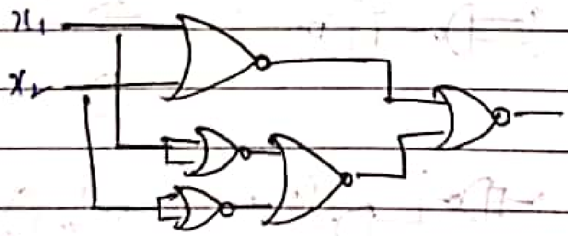




(NAND)

Q10 $f(x_1, x_2)_{POS} = (x_1 + x_2)(\bar{x}_1 + \bar{x}_2)$





(NOR)

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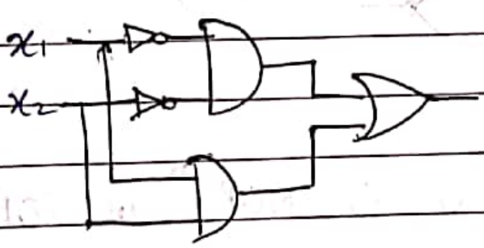
Q 11 $f(x_1, x_2, x_3) = \Sigma(0,1,6,7)$

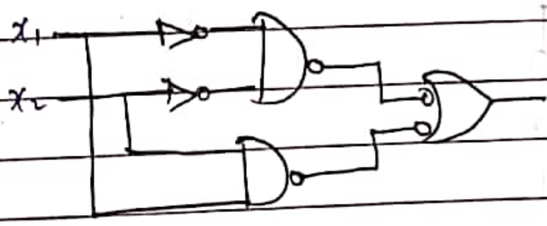
x_1	x_2	x_3	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$f(x_1, x_2, x_3)_{SOP} = \underline{\bar{x}_1\bar{x}_2\bar{x}_3} + \underline{\bar{x}_1\bar{x}_2 x_3} + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3$$

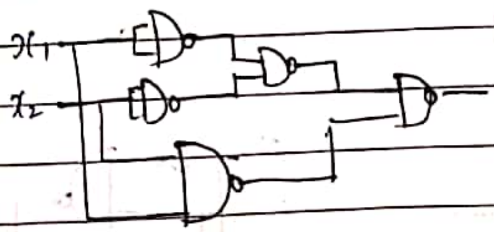
$$= \bar{x}_1\bar{x}_2 + x_1 x_2$$



(Basic)



(Bubbles)



(NAND)

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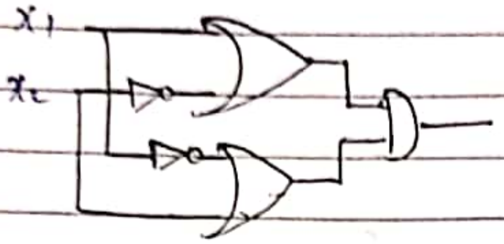
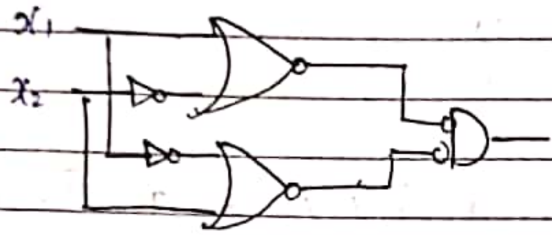
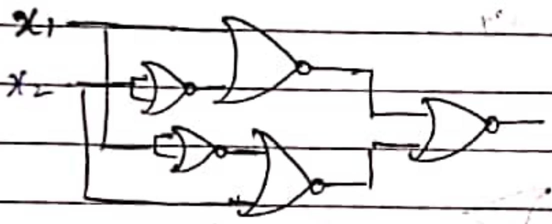
$$f(x_1, x_2, x_3)_{POS} = (x_1 + \bar{x}_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + x_3)(\bar{x}_1 + x_2 + \bar{x}_3)$$
$$= (x_1 + \bar{x}_2)(\bar{x}_1 + x_2)$$



(Basic)



(Bubble)



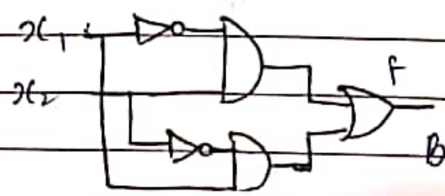
(NOR)

Q12 Implement Ex-OR or Ex-NOR in SOP & POS

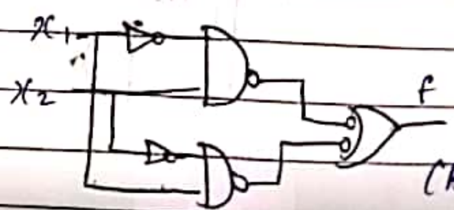
Exclusive OR

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0

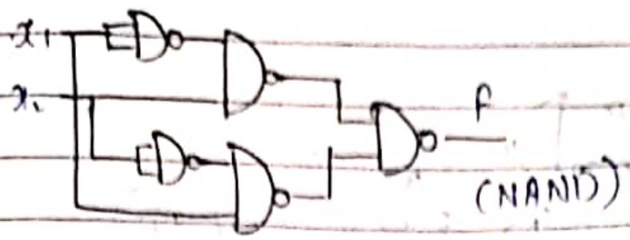
$$f(x_1, x_2)_{SOP} = \bar{x}_1 . x_2 + x_1 \bar{x}_2$$



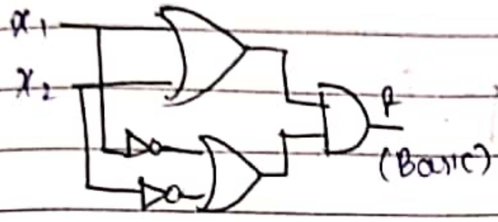
Basic

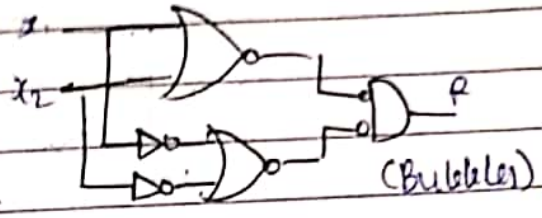


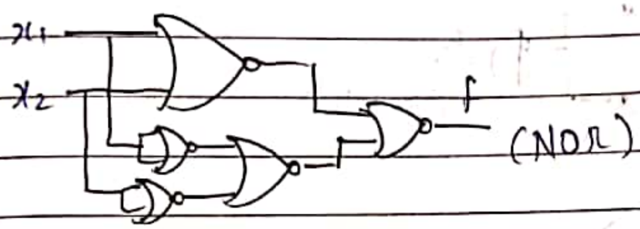
(Bubble)



$f(x_1, x_2)_{POS} = (x_1 + x_2)(\bar{x}_1 + \bar{x}_2)$



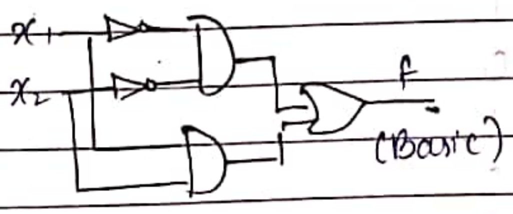


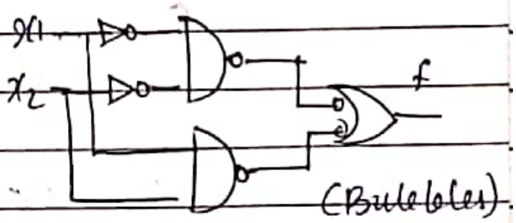


Exclusive NOR

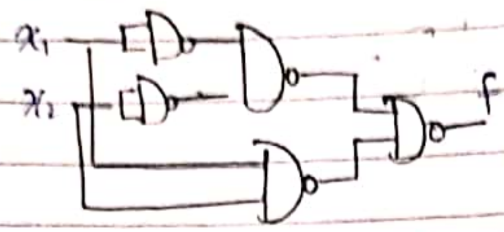
x_1	x_2	f
0	0	1
0	1	0
1	0	0
1	1	1

$f(x_1, x_2)_{SOP} = \bar{x}_1 \cdot \bar{x}_2 + x_1 x_2$



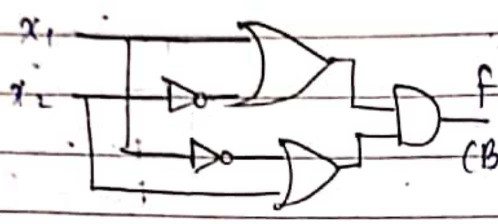


~~$f(x_1\ x_2)$~~

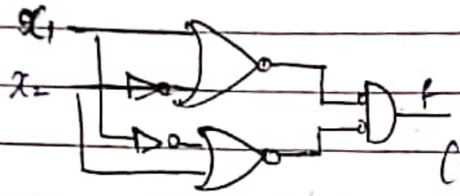


(NAND)

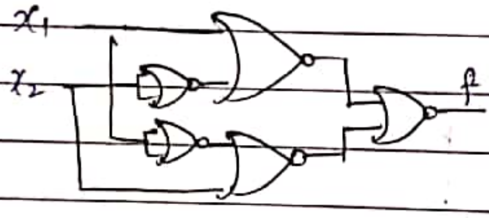
$f(x_1, x_2)_{POS} = (x_1 + \bar{x}_2)(\bar{x}_1 + x_2)$



(Basic)



(Bubble)



(NOR)

Q13 Design Examples

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

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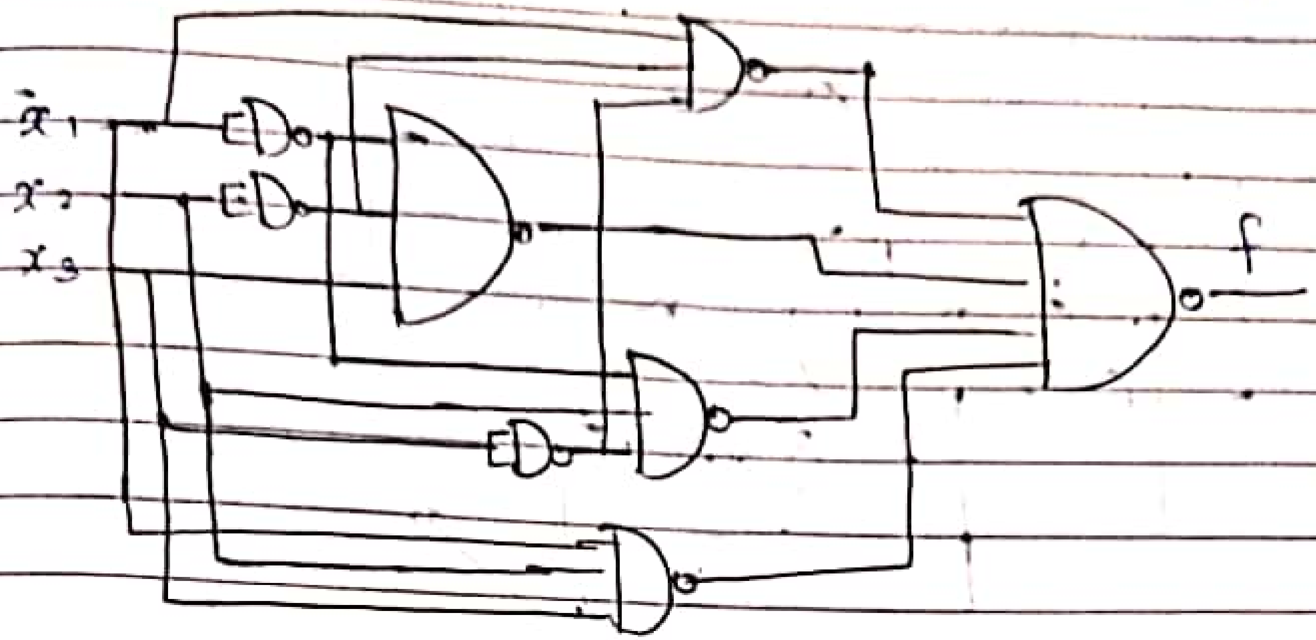
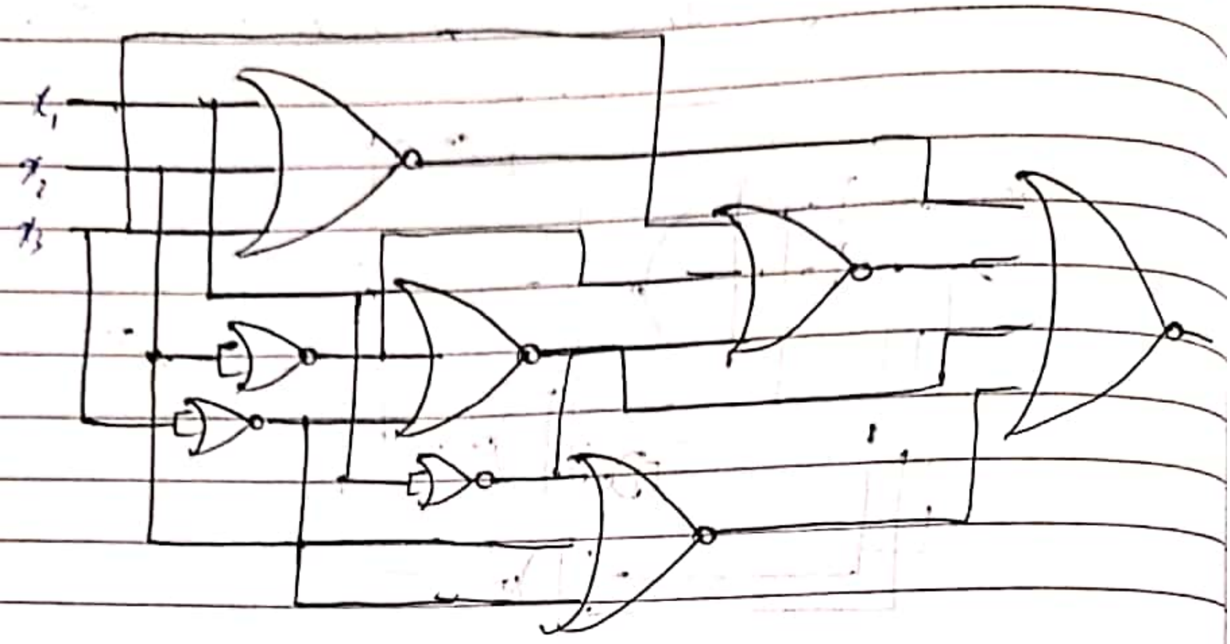
$$f(x_1, x_2, x_3)_{sop} = \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$$



$$f(x_1, x_2, x_3)_{pos} = (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)$$
$$(\bar{x}_1 + \bar{x}_2 + x_3)$$

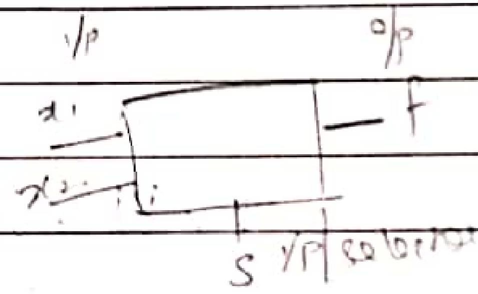

x_1
x_2
x_3

Multiplexer Circuit

This circuit enables us to choose data from exactly one of a number of possible sources.

Eg:- Television, radio.

S	x_1	x_2	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1



$S \to 0$ $x_1 \to f$

$S \to 1$ $x_2 \to f$

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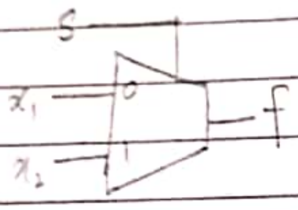
$x_1 \to 0 \quad \bar{x}_1 \to 1 \qquad 1 \cdot 0 \cdot 1$

$$y(s, x_1, x_2) = \bar{s}x_1\bar{x}_2 + \bar{s}x_1x_2 +$$
$$\bar{s}x_1(\bar{x}_2 + x_2)$$
$$s\bar{x}_1x_2 + sx_1x_2$$
$$sx_2(\bar{x}_1 + x_1)$$
$$= \bar{s}x_1 + sx_2$$

SYMBOL OF MULTIPLEXER SYMBOL



s	f
0	x_1
1	x_2

KARNAUGH'S MAP

① This method provides a systematic way of optimising the number of some items needed in the expression. It is made of squares with each square representing a min-term of a function that is to be minimised

② The simplified expression obtained from the map are always in the

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one of the standard forms - SOP or POS

③ 2 Variables K-Map

$x_1 \backslash x_2$	0	1
0	$\bar{x}_1\bar{x}_2$	$\bar{x}_1 x_2$
1	$x_1\bar{x}_2$	$x_1 x_2$

$\bar{x}_1\bar{x}_2$
$x_1\bar{x}_2$
↓
$\bar{x}_2(\bar{x}_1+x_1)$
$\rightarrow \bar{x}_2$

Q. $f(x_1,x_2) = \Sigma(1,3)$

$x_1 \backslash x_2$	0	1
0	0_0	1_1
1	0_2	1_3

↓
x_2

$f(x_1,x_2) = x_2$

[No. of combinations
$\rightarrow 2^n$ n = no. of variables

④ 3 Variables K-Map

$x_1 \backslash x_2x_3$	00	01	11	10
0	m_0 $\bar{x}_1\bar{x}_2\bar{x}_3$	m_1 $\bar{x}_1\bar{x}_2 x_3$	m_3 $\bar{x}_1 x_2 x_3$	m_2 $\bar{x}_1 x_2\bar{x}_3$
1	m_4 $x_1\bar{x}_2\bar{x}_3$	m_5 $x_1\bar{x}_2 x_3$	m_7 $x_1 x_2 x_3$	m_6 $x_1 x_2\bar{x}_3$

[We need to arrange so only 1 change takes place b/w adjacent boxes]

$\overset{2\,1\,0}{011}$
$2^1 \cdot 3$

(★ You can combine boxes only in power of 2 like $2^0=1$, $2^1=2$, $2^2=4$)

(★ 3 V Kmap has 3 adj boxes
$\rightarrow \boxed{3} \boxed{1} \boxed{2} \boxed{0}$]

Q15 $y(x_1, x_2, x_3) = \Sigma(1, 2, 3, 5)$
using K-map

$x_1 \backslash x_2 x_3$	00	01	11	10
0	(0)	1 (1)	1 (3)	1 (2)
1	(4)	1 (5)	(7)	(6)

$\rightarrow \bar{x}_1 x_2$

$\bar{x}_2 x_3$

$$y(x_1, x_2, x_3) = \bar{x}_1 x_2 + \bar{x}_2 x_3$$

⑤ 4 Variable K-Map

$x_1 x_2 \backslash x_3 x_4$	00	01	11	10
00	m_0 $\bar{x}_1 \bar{x}_2 \bar{x}_3 \bar{x}_4$	m_1 $\bar{x}_1 \bar{x}_2 \bar{x}_3 x_4$	m_3 $\bar{x}_1 \bar{x}_2 x_3 x_4$	m_2 $\bar{x}_1 \bar{x}_2 x_3 \bar{x}_4$
01	m_4 $\bar{x}_1 x_2 \bar{x}_3 \bar{x}_4$	m_5 $\bar{x}_1 x_2 \bar{x}_3 x_4$	m_7 $\bar{x}_1 x_2 x_3 x_4$	m_6 $\bar{x}_1 x_2 x_3 \bar{x}_4$
11	m_{12} $x_1 x_2 \bar{x}_3 \bar{x}_4$	m_{13} $x_1 x_2 \bar{x}_3 x_4$	m_{15} $x_1 x_2 x_3 x_4$	m_{14} $x_1 x_2 x_3 \bar{x}_4$
10	m_8 $x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4$	m_9 $x_1 \bar{x}_2 \bar{x}_3 x_4$	m_{11} $x_1 \bar{x}_2 x_3 x_4$	m_{10} $x_1 \bar{x}_2 x_3 \bar{x}_4$

Q16 A four variable logic function that is equal to one if any 3 or all 4 of the variables wxyz are equal to 1. Design a minimum cost SOP circuit that implements this majority function

w	x	y	z	f
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

wx \ yz	00	01	11	10
00	0_{0}	0_{1}	0_{3}	0_{2}
01	0_{4}	0_{5}	1_{7}	0_{6}
11	0_{12}	1_{13}	1_{15}	1_{14}
10	0_{8}	0_{9}	1_{11}	0_{10}

$y(w,x,y,z) = \Sigma(7, 11, 13, 14, 15)$

$= xyz + wyz + wxy + wxz$

Q17 Simplify $f(A,B,C,D) = \Sigma(2,3,6,7,12,13,14)$

AB \ CD	00	01	11	10
00	0_0	0_1	1_3	1_2
01	0_4	0_5	1_7	1_6
11	1_{12}	1_{13}	0_{15}	1_{14}
10	0_8	0_9	0_{11}	0_{10}

$\rightarrow \bar{A}C$

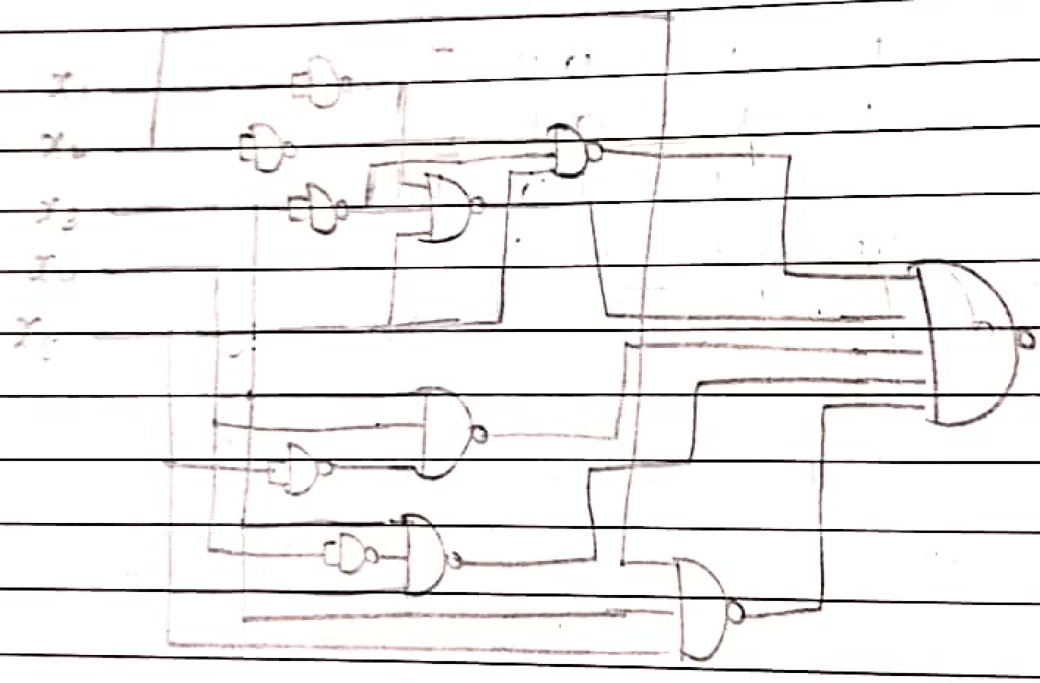
$\rightarrow BC\bar{D}$

$AB\bar{C}$

$f(A,B,C,D) = \bar{A}C + BC\bar{D} + AB\bar{C}$

Q19 — H.W

→ NAND Gate



(c) Prime Implicant

In choosing adjacent squares in a map, it must be ensured that

i) All the minterms in the function are covered when we combine the squares

ii) The no of terms in the expression is minimised

iii) There are no redundant terms (ie minterms that are already covered by other terms).

Q.18 $f(A,B,C,D) = \Sigma(0,2,3,5,7,8,9,10,11,13,15)$

AB \ CD	00	01	11	10	
00	1 (0)	(1)	1 (3)	1 (2)	$\rightarrow A\bar{B}\bar{D}$
01	0 (4)	1 (5)	1 (7)	0 (6)	$\rightarrow CD$
11	0 (12)	1 (13)	1 (15)	0 (14)	$BD \leftarrow$
10	1 (8)	1 (9)	1 (11)	1 (10)	$\rightarrow A\bar{B}$

$$f(A,B,C,D) = A\bar{B}\bar{D} + CD + BD + A\bar{B}$$

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~~$f = \pi(2, 3, \ldots)$ → Take 0s or 2, 3, 4, …~~

~~$f(pos) = \Sigma(3, 6, 9)$~~

~~→ Take 0s~~

(1) A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares in the map. If a minterm in a square is covered by only one prime implicant, that prime implicant is known as an essential prime implicant.

(7) 5 Variable K-Map — not allowed after 4.

x_1x_2 \ $x_3x_4x_5$	000	001	011	010	110	111	101	100
00	m_0	m_1	m_3	m_2	m_6	m_7	m_5	m_4
01	m_8	m_9	m_{11}	m_{10}	m_{14}	m_{15}	m_{13}	m_{12}
11	m_{24}	m_{25}	m_{27}	m_{26}	m_{30}	m_{31}	m_{29}	m_{28}
10	m_{16}	m_{17}	m_{19}	m_{18}	m_{22}	m_{23}	m_{21}	m_{20}

Q.19 $f(x_1 x_2 x_3 x_4 x_5) = \Sigma(1, 3, 5, 6, 9, 11, 13, 14, 15, 17, 19, 21, 22, 29, 30, 31)$.

$\bar{x}_1 \bar{x}_3 x_5$ $\bar{x}_2 x_3 x_4 x_5$

x_1x_2 \ $x_3x_4x_5$	000	001	011	010	110	111	101	100
00	0	1	1	0	1	0	1	0
01	0	1	1	0	1	1	1	0
11	0	0	0	0	1	1	1	0
10	0	1	1	0	1	0	1	0

$x_1 x_2 x_3$

$\bar{x}_2 \bar{x}_3 x_5$ $x_3 x_4 \bar{x}_5$ $x_3 \bar{x}_4 x_5$ $\bar{x}_4 \bar{x}_5$

we can combine boxes again ONLY if an uncombined term can be paired up it

$$f(sop)(x_1,x_2,x_3,x_4,x_5) = \bar{x}_1\bar{x}_3x_5 + \bar{x}_2\bar{x}_3x_5 + x_3x_4\bar{x}_5 + x_2\bar{x}_4x_1 + x_1x_2x_5$$

Conclusion: only in terms of 2^r

$2^0 \to 1$ only 0 is reduced

$2^1 \to 2$ only 1 is reduced

$2^2 \to 4$ only 2 are reduced

$$\cancel{\bar{f}(pos)(x_1,x_2,x_3,x_4,x_5) = (x_4x_5)(x_1x_3x_4x_5) + (\bar{x}_3x_4x_5) + (x_1x_2\bar{x}_3)}$$

$$\bar{f}_{pos}(x_1,x_2,x_3,x_4,x_5) = (\bar{x}_4+\bar{x}_5)(\bar{x}_1+\bar{x}_3+\bar{x}_4+\bar{x}_5)(\bar{x}_1+\bar{x}_2+\bar{x}_3)(\bar{x}_2+\bar{x}_4+\bar{x}_5)$$

Don't Care Conditions

Functions that have unspecified o/p for some i/p combinations are called as incompletely specified functions and the unspecified minterms of a function are called 'Don't Care Conditions'.

Q.20 $f(x_1,x_2,x_3) = \Sigma_m(1,4,7) + \Delta(2,5) / \Sigma_d(2,5)$

$\bar{x}_2x_3$

$x_1 \backslash x_2x_3$	00	01	11	10
0	0 (0)	1 (1)	0 (3)	X (2)
1	1 (4)	X (5)	1 (7)	0 (6)

$x_1\bar{x}_2$ $\quad$ x_1x_3

Δ Don't care are combined IF USEFUL otherwise ignored

$$f(x_1,x_2,x_3) = x_1\bar{x}_2 + x_1x_3 + \bar{x}_2x_3$$

Q 21 $f(x_1, x_2, x_3, x_4) = \Sigma(0,2,8,9,10,15) + D(1,3,6,7)$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1 (0)	X (1)	X (3)	1 (2)
01	0 (4)	0 (5)	X (7)	X (6)
11	0 (12)	0 (13)	1 (15)	0 (14)
10	1 (8)	1 (9)	0 (11)	1 (10)

$f(x_1, x_2, x_3, x_4) = \bar{x}_2\bar{x}_3 + x_2 x_3 x_4 + \cancel{x_1 x_2 x_3 x_4}$
$+ \bar{x}_2\bar{x}_4$

$\bar{f}(x_1, x_2, x_3, x_4) = (\bar{x}_2 + x_3)(x_2 + \bar{x}_3 + \bar{x}_4)(\bar{x}_2 + \bar{x}_3 + x_4)$

Quine-McClusky Method (QM Method)

The QM method can be adopted for minimizing Boolean functions for ~~...~~ larger number of input variables.

① Procedure

i) Arrange the given minterms in ascending order and make groups based on the number of ones in their binary representation

ii) Compare the minterms present in successive groups and form pairs of two minterms differ in a single bit position

iii) Repeat step (ii) with the newly formed terms.

iv) Formulate the prime implicant chart

v) Find essential prime implicants. If a minterm is covered only by one prime implicant, then that is an essential prime implicant.

vi) Reduce the prime implicant chart by removing the row of each essential prime implicant and the columns correspondi to the ~~essential~~ ~~prime~~ ~~implicant~~ minterms covered in essential prime implicants. Repeat step v) for the reduced prime implicant table.

Q.22 $f(x_1\ x_2\ x_3\ x_4\ x_5) = \Sigma(0, 2, 4, 5, 6, 7, 8, 10, 14, 17, 18, 21, 29, 31) + \Sigma_d(11, 20, 22)$

		$x_1\ x_2\ x_3\ x_4\ x_5$		
00000	✓0 -	00000	✓(0,2)	000-0
00010	✓2 -	00010	✓(0,4)	00-00
00100	✓4 -	00100	✓(0,8)	0-000
00101	✓8 -	01000	(2,6)	00-10
00110	✓5	00101	✓(2,10)	0-010
00111	✓6	00110	✓(4,5)	0010-
01000	✓10	01010	✓(4,6)	001-0
01010	✓17	10001	✓(4,20)	-0100
01110 — 01011	✓20	10100	(8,10)	010-0
10001		~~10010~~	(5,7)	001-1
10010 ~~10100~~	✓7	00111	(5,21)	-0101
10101 — 10100	✓14	01110	✓(6,7)	0011-
11101 — 10100	✓11	01011	(6,14)	0-110
11111	✓21	10101	✓(6,22)	-0110
	✓22	10110	(10,11)	0101-
	✓29	11101	✓(10,14)	01-10
	✓31	11111	(17,21)	10-01
			(18,22)	10-10
			(20,21)	1010-
			(20,22)	101-0

2 | 5
2 | 2-1
1-1

2 | 38

00101

~~(same likes are rejected)~~

(21,29) 1-101

(29,31) 111-1

(0,2,4,6) 00--0 $\rightarrow \bar{x}_1\bar{x}_2\bar{x}_5$

(0,2,8,10) 0-0-0 $\rightarrow \bar{x}_1\bar{x}_3\bar{x}_5$

(0,4,2,6) 00--0

(0,8,2,10) 0-0-0

(2,6,10,14) 0--10 $\rightarrow \bar{x}_1 x_4\bar{x}_5$

(2,6,18,22) -0-10 $\rightarrow \bar{x}_2 x_4\bar{x}_5$

(2,10,6,14) 0--10

(2,18,6,22) -0-10

(4,5,6,7) 001-- $\rightarrow \bar{x}_1\bar{x}_2 x_3$

(4,5,20,21) -010- $\rightarrow \bar{x}_2 x_3\bar{x}_4$

(4,6,5,7) 001--

(4,6,20,22) -01-0 $\rightarrow \bar{x}_2 x_3\bar{x}_5$

(4,20,5,21) -010-

(4,20,6,21) -01-0

(10,11) = $\bar{x}_1 x_2\bar{x}_3 x_4$

(17,19) = $x_1\bar{x}_2\bar{x}_3 x_4$

(21,29) = $x_1 x_3\bar{x}_4 x_5$

(29,31) = $x_1 x_2 x_3 x_5$

minterms are included

	0	2	4	5	6	7	8	10	14	17	18	21	29	31
$\bar{x}_1 x_2 \bar{x}_3 x_4$								✓						
$x_1 \bar{x}_2 \bar{x}_3 x_4$										✓		✓		
$x_1 x_3 \bar{x}_4 x_5$												✓	✓	
$x_1 x_2 x_3 x_5$													✓	✓
$\bar{x}_1 \bar{x}_2 \bar{x}_5$	✓	✓	✓		✓									
$\bar{x}_1 \bar{x}_3 \bar{x}_5$	✓	✓					✓	✓						
$\bar{x}_1 x_4 \bar{x}_5$		✓			✓			✓	✓					
$\bar{x}_2 x_4 \bar{x}_5$		✓			✓						✓			
$\bar{x}_1 \bar{x}_2 x_3$			✓	✓	✓	✓								
$\bar{x}_3 x_3 \bar{x}_4$			✓	✓								✓		
$\bar{x}_2 x_3 \bar{x}_5$			✓		✓							X		

* If all minterms are not ticked. Eg: 6, 10 are not ticked.

Reduced Prime Implicant Chart

	6	10
✓ $\bar{x}_1 x_2 \bar{x}_3 x_4$		✓
$x_1 x_2 \bar{x}_3 x_4$		
$\bar{x}_1 \bar{x}_2 \bar{x}_5$	✓	
$\bar{x}_2 x_3 \bar{x}_4$		
$\bar{x}_2 x_3 \bar{x}_5$	✓	

→ As 6 is the only one left, then we will take 1 from the 2 $\bar{x}_1 \bar{x}_2 \bar{x}_5$ OR $\bar{x}_2 x_3 \bar{x}_5$

Also draw another small table.

$$f(x_1, x_2, x_3, x_4, x_5) = x_1 \bar{x}_2 \bar{x}_3 x_4 + x_1 x_2 x_3 x_5 + \bar{x}_1 \bar{x}_3 \bar{x}_5 + \bar{x}_1 x_4 \bar{x}_5 + \bar{x}_2 x_4 \bar{x}_5 + \bar{x}_1 \bar{x}_2 x_3$$