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M-3 UNIT-II Assignment

1. Find Mean, Variance and MGF of Binomial Distribution.

Ans: **Mean :**

Binomial distribution is defined as

$$P(x) = nC_x p^x q^{n-x}$$

$$\text{Mean } (\mu) = \sum_{x=0}^{n} x \cdot P(x)$$

$$= \sum_{x=0}^{n} x\, nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \cancel{x} \frac{n!}{\cancel{x}(x-1)!(n-x)!} p^x q^{n-x}$$

$$= n \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)![(n-1)-(x-1)]!} p^x \cdot \frac{p}{p} \cdot q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)![(n-1)-(x-1)]!} p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} n-1C_{x-1}\, p^{x-1} q^{(n-1)-(x-1)}$$

$$\boxed{\text{Mean} = E[X] = np}$$

Variance: $V[X] = E[X^2] - [E[X]]^2$

Consider $E[X^2] = \sum_{x=0}^{n} x^2 p(x)$

$$= \sum_{x=0}^{n} [x(x-1)+x] P(x)$$

$$= \sum_{x=0}^{n} x(x-1) P(x) + \sum_{x=0}^{n} x \cdot P(x)$$

$$= \sum_{x=0}^{n} x(x-1) n C_x p^x q^{n-x} + np \quad (\because np = \sum_{x=0}^{n} x \cdot P(x))$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x} + np$$

$$= \sum_{x=0}^{n} \frac{x(x-1)\, n(n-1)!}{x(x-1)![(n-1)-(x-1)]!} p^x \frac{p}{p} q^{(n-1)-(x-1)} + np$$

$$E[X^2] = np \sum_{x=1}^{n} \frac{(x-1)(n-1)!}{(x-1)![(n-1)-(x-1)]!} p^{x-1} q^{(n-1)-(x-1)} + np$$

$$= np \sum_{x=1}^{n} \frac{(x-1)(n-1)(n-2)!}{(x-1)(x-2)![(n-2)-(x-2)]!} p^{x-1} \cdot \frac{p}{p} \cdot q^{(n-2)-(x-2)} + np$$

$$= np^2 \sum_{x=2}^{n} \frac{(n-1)(n-2)!}{(x-2)![(n-2)-(x-2)]!} p^{x-2} q^{(n-2)-(x-2)} + np$$

$$= np^2(n-1) \sum_{x=2}^{n} (n-2)C_{x-2}\, p^{x-2} q^{(n-2)-(x-2)} + np$$

$$E[X^2] = np^2(n-1) + np \quad (\because \sum_{x=0}^{n} nC_x p^x q^{n-x} = (q+p) = 1)$$

$$= n^2p^2 - np^2 + np$$

$$\therefore V[X] = E[X^2] - [E[X]]^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= np(1-p)$$

$$\boxed{V[X] = npq}$$

Moment Generating function (MGF):

MGF is a tool used to calculate higher moments it is denoted by $M_x(t)$ and is defined as $\sum e^{tx} p(x)$.

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\,dx \longrightarrow \text{CRV}$$

$$M_x(t) = \sum e^{tx} p(x) \longrightarrow \text{DRV}$$

$$M_x(t) = \sum_{x=0}^{n} e^{tx} p(x)$$

$$= \sum_{x=0}^{n} e^{tx}\, nC_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} nC_x (pe^t)^x q^{n-x}$$

$$M_x(t) = (q+pe^t)^n$$

$$\therefore \sum_{x=0}^{n} nC_x (pe^t)^x q^{n-x} = (q+pe^t)^n$$

2. Find Mean, Variance and MGF of poission distribution.

Ans: Mean: poission Distribution is defined as $P(x) = \frac{e^{-\lambda}\lambda^{k}}{x!}$, where $x = 1, 2, 3 \ldots, \infty$

$$\text{Mean} = \sum_{x=0}^{\infty} x \cdot P(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^{k}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{x \cdot \lambda^{k}}{x!} = e^{-\lambda} \sum_{x=1}^{\infty} \frac{\cancel{x} \cdot \lambda^{k}}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{k}}{(x-1)!}$$

Put $x-1 = m \Rightarrow x = m+1$

x	1	∞
m	0	∞

$$= e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^{m+1}}{m!}$$

$$= e^{-\lambda} \cdot \lambda \sum_{m=0}^{\infty} \frac{\lambda^{m}}{m!}$$

$$= e^{-\lambda} \cdot \lambda \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right]$$

$$= e^{-\lambda} \cdot \lambda e^{x} \qquad \left[\because e^{\lambda} = 1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right]$$

$$\boxed{\text{Mean} = \lambda}$$

Variance: Variance $V[X] = E[X^2] - [E[X]]^2$

Consider $E[X^2] = \sum_{x=0}^{\infty} x^2 P(x)$

$$= \sum_{x=0}^{\infty} [x(x-1) + x] P(x)$$

$$= \sum_{x=0}^{\infty} x \cdot (x-1) P(x) + \sum_{x=0}^{\infty} x \cdot P(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^{k}}{x!} + \lambda \qquad \left(\text{Since } \sum_{x=0}^{n} x\, P(x) = \lambda\right)$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\cancel{x}(x-1)}{\cancel{x}(x-1)!} \lambda^{x} + \lambda$$

$$= e^{-\lambda} \sum_{x=2}^{\infty} \frac{\cancel{(x-1)} \lambda^{k}}{\cancel{(x-1)}(x-2)!} + \lambda$$

Put $x-2 = m$

$x = m+2$

x	2	∞
m	0	∞

$$= e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^{x}}{(x-2)!} + \lambda$$

$$= e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^{m+2}}{m!} + \lambda$$

$$= e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^{2}\lambda^{m}}{m!} + \lambda$$

$$= e^{-\lambda} \cdot \lambda^{2} \sum_{m=0}^{\infty} \frac{\lambda^{m}}{m!} + \lambda$$

$$= e^{-\lambda} \lambda^{2} [e^{\lambda}] + \lambda \qquad \left[\because \sum_{m=0}^{\infty} \frac{\lambda^{m}}{m!} = e^{\lambda}\right]$$

$= \lambda^2 e^{\lambda-\lambda} + \lambda$

$= \lambda^2 + \lambda$

$E[x^2] = \lambda^2 + \lambda$

$V[x] = E[x^2] - [E[x]]^2$

$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$

$V[x] = \lambda$

$\therefore \boxed{\text{Mean} = \text{Variance of PD} = \lambda}$

MGF of Poission Distribution :

$M_x(t) = \sum e^{tx} p(x)$

$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^x}{x!}$

$= \sum_{x=0}^{\infty} e^{-\lambda} \frac{(\lambda e^t)^x}{x!}$

$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$

$= e^{-\lambda}\left[1 + \frac{\lambda e^t}{1!} + \frac{(\lambda e^t)^2}{2!} + \cdots\right]$

$M_x(t) = e^{-\lambda} \cdot e^{\lambda e^t} = e^{\lambda(e^t-1)}$

$\boxed{M_x(t) = e^{\lambda(e^t-1)}}$

3. Fit a poission Distribution to the following data.

x	0	1	2	3	4
$f(x)$	109	65	22	3	1

Sol: We know that $P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$N = \sum f_i = 109+65+22+3+1$

$= 200$

Mean $= \lambda = \frac{\sum x_i f_i}{\sum f_i} = \frac{0(109)+1(65)+2(22)+3(3)+4(1)}{200}$

$= \frac{65+44+13}{200} = \frac{122}{200} = 0.61$

$P(0) = \frac{e^{-0.61}(0.61)^0}{0!} = 0.5433$

$P(1) = \frac{e^{-0.61}(0.61)^1}{1!} = 0.3314$

$P(2) = \frac{e^{-0.61}(0.61)^2}{2!} = 0.1010$

$P(3) = \frac{e^{-0.61}(0.61)^3}{3!} = 0.0205$

$P(4) = \frac{e^{-0.61}(0.61)^4}{4!} = 0.0031$

Expected frequency is $N P(x)$

$= 200\, P(0) = 200(0.5433) = 108.66$

$= 200\, P(1) = 200(0.3314) = 66.28$

$= 200\, P(2) = 200(0.1010) = 20.2$

$= 200\, P(3) = 200(0.0205) = 4.1$

$= 200\, P(4) = 200(0.0031) = 0.62$

x	0	1	2	3	4
F	109	65	22	3	1
E.F	109	66	20	4	1

Fitting of poission Distribution is good.

4. Fit a Binomial Distribution to the following data.

x	0	1	2	3	4	5
$f(x)$	10	20	30	15	15	10

Sol; We know that $P(x) = nC_x\, p^x q^{n-x}$

$n =$ no. of trials $= 5$

$N = \Sigma f_i = 10+20+30+15+15+10$

$\boxed{N = 100}$

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{0(10)+1(20)+2(30)+3(15)+4(15)+5(10)}{100}$$

$$= \frac{0+20+60+45+60+50}{100}$$

$$= \frac{235}{100} = 2.35$$

Mean of Binomial Distribution $= np$

$2.35 = 5p$

$p = \frac{2.35}{5} = 0.47$

$\boxed{p = 0.47}$

$p+q = 1$

$q = 1-p$

$= 1-0.47$

$\boxed{q = 0.53}$

$\boxed{n = 5}$

Expected frequency is $N(p+q)^n$

$100(0.47+0.53)^5$

$= 100[5C_0(0.47)^0(0.53)^5 + 5C_1(0.47)^1(0.53)^4 + 5C_2(0.47)^2(0.53)^3 + 5C_3(0.47)^3(0.53)^2 + 5C_4(0.47)^4(0.53)^1 + 5C_5(0.47)^5(0.53)^0]$

$= 100\,[0.418 + 0.185 + 0.3288 + 0.2916 + 0.1293 + 0.0229]$

$= 4.18 + 18.54 + 32.88 + 29.16 + 12.93 + 2.29$

$= 99.98$

x	0	1	2	3	4	5
F	10	20	30	15	15	10
Ef	4	19	33	29	13	2

5. The first four moments of a distribution about $\lambda = 4$ are 1, 4, 10, 45 find moments about the mean.

Sol: Given first four moments

$\mu_1' = 1$

$\mu_2' = 4$

$\mu_3' = 10$

$\mu_4' = 45$

$\boxed{\mu_1 = 0}$ (since $\mu_1 = 0$)

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 4 - (1)^2 = 4 - 1 = 3$ $\boxed{\mu_2 = 3}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= 10 - 3(4)(1) + 2(1) = 10 - 12 + 2 = 0$

$\boxed{\mu_3 = 0}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')$

$= 45 - 4(10)(1) + 6(4)(1)^2 - 3(1) = 45 - 40 + 24 - 3$

$\boxed{\mu_4 = 26}$