13/12/2022

UNIT - II - DISCRETE PROBABILITY DISTRIBUTION

→ Frequency distribution can be classified as
(i) Observed frequency distribution
(ii) Theoretical or expected frequency distribution

→ Observed frequency distributions are based on actual observation.

→ If certain hypothesis is assumed, it is sometimes possible to derive mathematically what the frequency distribution of a certain universe should be. Such distribution are called theoritical or expected frequency distiⁿ

→ There are many types of freq. distribution, out of which some are of great
(i) Binomial distribution
(ii) Poisson's distribution

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(i) Binomial Distribution

→ Binomial distribution is a discrete distribution, it is commonly used distribution that has been developed to represent various descrete phenomenon which occur in business, social sciences, natural sciences & medical research.

→ The following should be satisfied for the application of Binomial Distribution

(i) The experiment consist of n identical trials. where n is finite.

(ii) There are only 2 possible outcomes in each trial denoted by S (success) & F (failure).

(iii) The probability of S remains same from trial to trial. The probability of success is denoted by (p) & probability of failure (q) (where $p+q=1$)

(iv) All the traits are independent.

(v) The random binomial variable x is no. of success in n trials.

** → The descrete random variable taking values 0, 1, 2, 3 . . . . n is said to follow ~~the~~ binomial distibution with parameters n & p if its probability mass funcⁿ (PMF) is given by

$$P(X=x) = P(x) = {}^nC_x p^x q^{n-x} \quad (x = 0,1,2 \ldots n)$$
$$P(x) = 0 \quad \text{otherwise}$$

Note :- $\sum_{x=0}^{n} P(x) = \sum_{x=0}^{n} {}^nC_x p^x q^{n-x} = (q+p)^n = 1$

* Mean of Binomial Distribution

$$\text{Mean} = \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=0}^{n} x\, {}^nC_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=1}^{n} \frac{\cancel{x}\, n (n-1)!}{\cancel{x}(x-1)!\,(n-x)!} p^x q^{n-x}$$

$$\text{Mean} = np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!} p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} {}^{(n-1)}C_{x-1}\, p^{x-1} q^{(n-1)-(x-1)} = 1$$

$$\left(\because \sum_{x=0}^{n} {}^nC_x p^x q^{n-x} = (q+p)^n = 1\right)$$

$$\boxed{\text{Mean} = np}$$

★ Variance of Binomial Distribution

Since w.K.T

$$V[X] = E[X^2] - [E[X]]^2 \quad (E[X] = \text{mean})$$

consider $E[X^2] = \sum_{x=0}^{n} x^2 p(x)$

$$E[x^2] = \sum_{x=0}^{n} [x(x-1)+x]\, p(x)$$

$$= \sum_{x=0}^{n} x(x-1)p(x) + \sum_{x=0}^{n} x\, p(x)$$

$$= \sum_{x=0}^{n} x(x-1)\, nC_x\, p^x q^{n-x} + \cancel{p}\, np$$

$$\Rightarrow \sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!} p^x q^{n-x} + np$$

$$\Rightarrow p \sum_{x=1}^{n} \cancel{x}(x-1) \frac{n(n-1)!}{\cancel{x}(x-1)!\,[(n-1)-(x-1)]!} p^{x-1} q^{(n-1)-(x-1)} + np$$

$$\Rightarrow np^2 \sum_{x=2}^{n} \frac{\cancel{(x-1)}(n-1)(n-2)!}{\cancel{(x-1)}(x-2)!\,[(n-2)-(x-2)]!} p^{x-2} q^{(n-2)-(x-2)} + np$$

$$E[x^2] \Rightarrow np^2 (n-1) \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,[(n-2)-(x-2)]!} p^{x-2} q^{(n-2)-(x-2)} + np$$

$$E[x^2] = np^2 (n-1) + np$$

$$\cancel{V[x] = np^2 + np - n^2p^2}$$

$V[x] = np\{p - np\}$

$V[x]\ \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$V[x] = np - np^2$

$V[x] = np(1-p)$

$V[x] = npq \quad (p+q=1)$

14/12/2022

□ Moment Generating Funcⁿ (MGF) of Binomial Distribution

→ It is tool used to calculate the higher moment. It is denoted by $M_x(t)$ & is defined as

$$M_x(t) = \sum e^{tx} p(x) \rightarrow DRV$$

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx \rightarrow CRV$$

□ MGF of Binomial Distribution

→ The MGF of a random variable x is denoted by $M_x(t)$ & is defined as $\sum_{x=0}^{n} e^{tx} p(x)$

$$= \sum_{x=0}^{n} e^{tx}\, {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x (pe^t)^x q^{n-x}$$

$$M_x(t) = (pe^t + q)^n = (q + pe^t)^n$$

Q → A fair coin is tossed 6 times find the probability of getting 4 heads

Sol p = probability of getting head

q = probability of not getting a head &

given that $n=6$

& $x = 4$

∴ we have $P(x) = nC_x p^x q^{n-x}$

$P(4) = 6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{6-4}$

$P(4) = 6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2$

$P(4) = 6C_4 \left(\frac{1}{2}\right)^6$

$P(4) = \frac{6!}{4!\,(6-4)!} \left(\frac{1}{2}\right)^6$

∴ $P(4) = 0.234375$

Q 10 coins are thrown simultaneously find the probability of getting atleast 7 head

P = probability of getting head $= 1/2$

$q = 1/2$

$n = 10$

$x \geq 7$ (atleast)

wkT $P(x) = nC_x p^x q^{n-x}$

$P(x \geq 7) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7} + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$

$\Rightarrow \frac{10!}{7!\,3!} \left(\frac{1}{2}\right)^{10} + \frac{10!}{8!\,2!} \left(\frac{1}{2}\right)^{10} + \frac{10!}{9!\,1!} \left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10}$

$\boxed{P(x \geq 7) = 0.171875}$

Q → 8 coins are tossed ~~are~~ simultaneously find the probability of getting atleast 6 heads

$P = 1/2$

$q = 1/2$

$n = 8$

$x \geq 6$

$P(x \geq 6) = P(x=6) + P(x=7) + P(x=8)$

$= {}^{8}C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 + {}^{8}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 + {}^{8}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^0$

$= \frac{8!}{6!\,2!} \left(\frac{1}{2}\right)^8 + \frac{8!}{7!\,1!} \left(\frac{1}{2}\right)^8 + 1 \cdot \left(\frac{1}{2}\right)^8$

$P(x \geq 6) = 0.14453125$

Q → A die is thrown 3 times getting 3 or 6 is consider to be success find the probility of getting atleast 2 success

Sol

$p = 1/3$

$q = 2/3$

$n = 3$

$x \geq 2$

$P(x \geq 2) = P(X=2) + P(X=3)$

$= {}^3C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1 + {}^3C_3 \left(\frac{1}{3}\right)^3 (1)$

$= \frac{3!}{2!\,1!} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right) + 1 \left(\frac{1}{3}\right)^3$

$P(x \geq 2) = 0.259259253$

Q → A die is thrown 6 times if getting an even no. is success find the probability of (i) 4 success

(ii) $\leq$ 3 success

(i) $p = 1/2$

$q = 1/2$

$n = 6$

$x = 4$

${}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 \Rightarrow \frac{6!}{4!\,2!} \left(\frac{1}{2}\right)^6$

$= 0.234375$

(ii) $P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= {}^6C_{\cancel{0}}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5 + {}^6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4 + {}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$$

$$= \left(\frac{1}{2}\right)^6\left[\frac{6!}{1! \cancel{\times} 5!} + \frac{6!}{2!4!} + \frac{6!}{3!3!}\right]$$

$P(x \leq 3) = 0.640625$

15-12-2022

Q → ~~A~~ die is tossed thrice A success is getting ~~o~~ 1 or 6 on a toss. find the mean & variance of no. of successes.

$p = 2/6 = 1/3$

$q = 4/6 = 2/3$

$n = 3$

x

mean $= 3\left(\frac{1}{3}\right) = 1$ (np)

variance $= \cancel{3}\left(\frac{1}{\cancel{3}}\right)\left(\frac{2}{3}\right)$ (npq)

$= \frac{2}{3}$

Q → Determine the Binomial distribution for which the mean is 4 & variance is 3

Sol given mean = 4

variance = 3

mean = np

variance = nqp

$$\frac{\text{variance}}{\text{mean}} = \frac{\cancel{np}q}{\cancel{np}} = \frac{3}{4}$$

$$\boxed{q = 3/4}$$

$p + q = 1$

$p = 1 - q$

$p = 1 - 3/4$

$$\boxed{p = 1/4}$$

mean = $np = 4$

$= n\left(\frac{1}{4}\right) = 4$

$= \boxed{n = 16}$

$P(x) = {}^{n}C_x \, p^x \, q^{n-x}$

$$\boxed{P(x) = {}^{16}C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{16-x}}$$

Q ~~Gifra~~ the probability of a defective bolt is $1/8$ find (i) Mean (ii) Variance for the distribution of defective bolts of 640

Sol we are given that P is the probability of defective bolt $P = 1/8$

$$P + q = 1$$

$$q = 1 - 1/8$$

$$\boxed{q = 7/8}$$

also given $n = 640$

$$\text{Mean} = np$$

$$= \overset{80}{\cancel{640}}\left(\frac{1}{\cancel{8}}\right)$$

$$\boxed{\text{Mean} = 80}$$

$$\text{Variance} = npq$$

$$= 640\left(\frac{1}{8}\right)\left(\frac{7}{8}\right)$$

$$\boxed{\text{variance} = 70}$$

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Q → Assume that 50% of ~~the~~ all engineering students are good in mathematics. Determine the probability that among 18 engineering students are (i) exaclty 10 $\Rightarrow x=10$

(ii) atleast 10 $\Rightarrow x \geq 10$

(iii) utmost 8 $\Rightarrow x \leq 10$

(iv) atleast 2 & atmost 9 $\Rightarrow 2 \leq x \leq 9$

are good in maths

~~(i) P(x) =~~ $P = 1/2$

$q = 1/2$

$n = 18$

(i) $P(x=10) = {}^nC_x \, p^x q^{n-x}$

$= 18C_{10} \, p^{10} q^{8}$

$= 18C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{8}$

$= \dfrac{18!}{10!(8!)} \left(\frac{1}{2}\right)^{18}$

$= 0.166$

(ii) $P(x \geq 10) = 18C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{8} + 18C_{11} \left(\frac{1}{2}\right)^{11} \left(\frac{1}{2}\right)^{7} +$
$18C_{12} \left(\frac{1}{2}\right)^{12} \left(\frac{1}{2}\right)^{6} + 18C_{13} \left(\frac{1}{2}\right)^{13} \left(\frac{1}{2}\right)^{5} +$
$18C_{14} \left(\frac{1}{2}\right)^{14} \left(1/2\right)^{4} + 18C_{15} \left(\frac{1}{2}\right)^{15} \left(\frac{1}{2}\right)^{3} + 18C_{16}$
$\left(\frac{1}{2}\right)^{16} \left(\frac{1}{2}\right)^{2} + 18C_{17} \left(\frac{1}{2}\right)^{17} \left(\frac{1}{2}\right)^{1} + 18C_{18} \left(\frac{1}{2}\right)^{18} \left(\frac{1}{2}\right)^{0}$

$\left(\frac{1}{2}\right)^{18}\left[102714+4048\right]$

$P(X \geq 10) = 0.40726$

(iii) $P(x \leq 8) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$
$+ P(X=4)+P(X=5)+P(X=6)+P(X=7)$
$+ P(X=8)$

$^{18}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{18} + {}^{18}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{17} + {}^{18}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{16}$
$+ {}^{18}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{15} + {}^{18}C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{14} + {}^{18}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{13}$
$+ {}^{18}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{12} + {}^{18}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{11} + {}^{18}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{10}$

$\Rightarrow \left(\frac{1}{2}\right)^{18}\left[4048+102714\right]$

$P(x \leq 8) = 0.40726$

(iv) $P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(X=6)$
$P(X=7)+P(X=8)+P(X=9)$

Q → Fit a binomial distribution to the following frequency distribution

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4

Here n = no. of trials $= 6$

N = Total frequency $\Sigma f_i = 13+25+52+58+32+16+4$

$= 200$

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{0(13)+1(25)+2(52)+3(58)+4(32)+5(16)+6(4)}{200}$$

Mean $= 2.675$

Mean of B.D $= np$

$\therefore 2.675 = 6(p)$

$$p = \frac{2.675}{6}$$

$\boxed{p = 0.4458333}$

$\boxed{q = 1-p = 0.5541667}$

Hence the B.D to be fitted is given by

$N(p+q)^n$

$N(p+q)^n = 200(0.445833 + 0.5541)^6$

$= 200\big[\,{}^6C_0(0.4458)^0(0.5541)^6 + {}^6C_1(0.4458)^1(0.5541)^5$

$+ {}^6C_2(0.4458)^2(0.5541)^4 + {}^6C_3(0.4458)^3(0.5541)^3$

$+ {}^6C_4(0.4458)^4(0.5541)^2 + {}^6C_5(0.4458)^5(0.5541)^1$

$+ {}^6C_6 (0.4458)^6 (0.5541)^0]$

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	4
E.f	6	28	57				

16/12/22

Q ~~A~~ set of 5 similar coins is tossed 320 times & the result is as follows

~~No~~

No. of heads	0	1	2	3	4	5
frequency	6	27	72	112	71	32

Using χ^2 test, ~~test~~ the hypothesis that the data follow binomial distribution or not.

→ n = No. of trials = 5

$N = \sum f_i = 6 + 27 + 72 + 112 + 71 + 32 = 320$

$$\text{Mean} = \frac{(0\times 6) + (1\times 27) + 2(72) + 3(112) + 4(71) + 5(32)}{320}$$

$= 2.971875$

Mean of B.D = np

$2.971875 = 5(P)$

$P = \frac{2.971875}{5}$

$P = 0.594375$

$q = 1 - p = 0.405625$

Here B.D to be fited is given by $N(p+q)^n$

$\Rightarrow 320 \Big[{}^5C_0 (0.594375)^0 (0.405625)^5 + {}^5C_1 (0.594375)$

$(0.405625)^4 + {}^5C_2 (0.594375)^2 (0.405625)^3 +$

${}^5C_3 (0.594375)^3 (0.405625)^2 + {}^5C_4 (0.594375)^4($

$0.405625)^1 + {}^5C_5 (0.594375)^5 (0.405625)^0 \Big]$

	0	1	2	3	4	5
	6	27	72	112	71	32
	4	26	75	111	81	24

$$\chi^2 = \frac{(6-4)^2}{4} + \frac{(27-26)^2}{26} + \frac{(2-75)^2}{75} + \frac{(108)^2}{111}$$

$$+ \frac{77^2}{81} + \frac{19^2}{24}$$

$\chi^2_{cal} = 5.068$ $\qquad$ $\chi^2_{cal} < \chi^2_{tab}$

$\chi^2_{tab} = 11.070$ $\qquad$ the fit is good

Q. Fit a binomial distribution to the following data

Dec 19
Oct 21

(2)

x	0	1	2	3	4	5
f	2	14	20	34	22	8

(3)

x	0	1	2	3	4
f	8	34	69	43	6

(4)

x	0	1	2	3	4	5	6	7
f	7	6	19	35	30	23	7	1

(5) A dice is thrown 102 times & the following distribution is obtained

x	~~0~~	1	2	3	4	5	6
~~f~~	~~15~~	~~25~~	~~16~~	~~20~~	~~12~~		
f		15	25	16	20	12	14

can we conclude that all the faces equally likely to occur test at 5% of significance ($\chi^2_{tab} = 11.07$)

~~(3)~~

(3) $n = 4$

$N = \Sigma f_i = 160$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = 2.03125$$

Mean of B.D $= np$

$2.03125 = 4p$

$p = 0.5078125$

$q = 0.4921875$

expected frequency $N(P+q)^n$

$160(0.5078125 + 0.4921875)^4$

$160\left[{}^4C_0 (0.5078125)^0 (0.4921875)^4 + {}^4C_1 (0.5078125)^1 (0.4921875)^3 + {}^4C_2 (0.5078125)^2 (0.4921875)^2 + {}^4C_3 (0.5078125)^3 (0.4921875)^1 + {}^4C_4 (0.5078125)^4 (0.4921875)^0 \right]$

x	0	1	2	3	4
f	8	34	69	43	6
expected freq	9	39	60	41	11

② $n = 5$

$N = 100$

$\text{mean} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = 2.84$

Mean of B.D $= np$

$2.84 = 5 \times P$

$P = 0.568$

$q = 0.432$

expected frequency $= N(P+q)^n$

$= 100(0.568 + 0.432)^5$

$100\left[{}^5C_0 (0.568)^0 (0.432)^5 + {}^5C_1 (0.568)^1 (0.432)^4 + {}^5C_2 (0.568)^2 (0.432)^3 + {}^5C_3 (0.568)^3 (0.568)^2 + 5C_4 (0.568)^4 (0.432)^1\right.$

$$+ {}^5C_5 (0.568)^5 (0.432)^0 \Big]$$

x	0	1	2	3	4	5
f	2	14	20	34	22	8
expected frequency	2	10	26	34	22	6

20/12/2021

Poisson's Distribution (P.D)

→ A type of probability distribution that is useful in ~~finding~~ describing no. of events that will occur in specific period of time, area ~~of~~ or volume is poisson's distribution

→ The following ~~are~~ examples of RV for which the Poisson's distribution provides good model

1. No. of car accidents in a year on a road
2. No. of earthquakes in a year
3. No. of printing mistakes in each page of a book

Cond^n under which Poisson's Distribution is used.

→ The cond^n for the applicability of P.D are the same as those for B.D. The addition requirement is that the probability of success is very small. The cond^{n}s are :-

1. The random variable X is descrete
2. The no. of trials is indefinitely very large ~~that~~ ie $n \to \infty$
3. The probability of success in a trial is very small P ~~is~~ is close to 0.
4. The product ~~&~~ of n & p is const ie $\lambda = np$

Note :- A R.V which counts the no. of success in an experiment is called poisson variable.

→ Poisson's Distribution is D.P.D of D.R.V which has no upper bound. It is defined for non negative values of X.

$$\boxed{P(X=x) = P(x,\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}} \quad , x = 0,1,2,\dots$$

Mean of Poisson's Distribution

$$\text{Mean} = \sum_{x=0}^{\infty} x\, p(x)$$

$$= \sum_{x=0}^{\infty} \frac{x e^{-\lambda}\lambda^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{x\,\lambda^{x}}{x!}$$

put $x-1 = m$

$x = m+1$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\cancel{x}\,\lambda^{x}}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x}}{(x-1)!}$$

$$\text{Mean} = e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^{m+1}}{m!}$$

$$= e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^{m}\cdot\lambda}{m!}$$

$$= \lambda e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^{m}}{m!}$$

$$= \lambda e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots\right]$$

$$\text{Mean} = \lambda e^{-\lambda} e^{\lambda}$$

$$\boxed{\text{Mean} = \lambda}$$

Variance of Poisson's Distribution

$$V[X] = E[X^2] - \left[E[X]\right]^2$$

$$E[X^2]$$

$$E[X^2] = \sum_{x=0}^{\infty} x^2 p(x)$$

$$\sum_{x=0}^{\infty} [x(x-1)+x]\, p(x)$$

$$\sum_{x=0}^{\infty} x(x-1)p(x) + \sum_{x=0}^{\infty} x\, p(x) \quad \lambda$$

$$\sum_{x=0}^{\infty} x(x-1)\frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} x\frac{e^{-\lambda}\lambda^x}{x!}$$

$$e^{-\lambda}\sum_{x=2}^{\infty}\frac{\cancel{x(x-1)}\,\lambda^x}{\cancel{x(x-1)}(x-2)} + \sum_{x=1}^{\infty}\cancel{x}\frac{e^{-\lambda}\lambda^x}{\cancel{x}(x-1)}$$

$$e^{-\lambda}\sum_{m=0}^{\infty}\frac{\lambda^{m+2}}{m} + e^{-\lambda}\sum_{m=-1}^{\infty}\frac{\lambda^{m+2}}{m+1}$$

$x-1 = m$

$x = m+1$

$x-2 = m-2$

$x-2 = m$

$x-1+1 = m+1$

$x+1 = m+1$

$x = m+2$

$$\Rightarrow e^{-\lambda}\sum_{x=2}^{\infty}\frac{\cancel{x(x-1)}\,\lambda^x}{\cancel{x(x-1)}(x-2)!} + \lambda$$

$x-2 = m$

$m = 2$

$$\Rightarrow e^{-\lambda}\sum_{x=2}^{\infty}\frac{\lambda^x}{(x-2)!} + \lambda$$

x	2	∞
m	0	∞

$$e^{-\lambda} \sum_{m=0}^{\infty} \frac{\lambda^{m+2}}{m!} + \lambda$$

$$= e^{-\lambda} \lambda^2 \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right] + \lambda$$

$$= \cancel{e^{-\lambda}} \lambda^2 \cancel{e^{\lambda}} + \lambda$$

$$E[x^2] = \lambda^2 + \lambda$$

$$V[x] = E[x^2] - \{E[x]\}^2$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$$

$$\boxed{V[x] = \lambda}$$

MGF of Poisson's Distribution

$$M_x(t) = \sum_{x=0}^{n} e^{tx} p(x)$$

$$= \sum_{x=0}^{n} e^{tx} \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda} \left[1 + \frac{\lambda e^t}{1!} + \frac{(\lambda e^t)^2}{2!} + \cdots\right]$$

$$M_x(t) = e^{-\lambda} e^{\lambda e^t}$$

$$= e^{\lambda(e^t - 1)}$$

21/12/2022

① $P(x=1)\cdot\frac{3}{2} = P(x=3)$

find $P(x\le 3)$

$P(\cancel{2}\ \cancel{0}\le x \le 5)$

Sol W.K.T $P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$P(x=1)\frac{3}{2} = P(x=3)$

$\frac{e^{-\lambda}\cdot\lambda}{1!}\frac{3}{2} = \frac{e^{-\lambda}\lambda^{3\,2}}{3!}$

$\lambda^2 = 9$

$\lambda = \pm 3$

$\lambda = -3, 3$

$\boxed{\lambda = 3}$

(i) $P(x\le 3) = e^{-3}\left[1+3+\frac{9}{2}+\frac{27}{6}\right]$

$= 0.6473$

(ii) $P(2\le x\le 5) = P(x=2)+P(x=3)+P(x=4)+$

$P(x=5)$

$= \frac{e^{-3}(3)^2}{2!} + \frac{e^{-3}(3)^3}{3!} + \frac{e^{-3}(3)^4}{4!} + \frac{e^{-3}(3)^5}{5!}$

$= 0.7169$

② $P(x=2) = \frac{2}{3} P(x=1)$

find (i) $P(x=0)$

(ii) $P(x=1)$

③ $P(x=2) = \frac{2}{3} P(x=1)$

find (i) $P(x=0)$

(ii) $P(1 \leq x \leq 2)$

④ $P(x=1) = 24 \{P(x=3)$

(i) $P(x=0)$

(ii) $P(x \geq 1)$

given $P(x=1) = 24 \{P(x=3)$

$$\frac{\cancel{e^{-\lambda}} \cancel{\lambda^1}}{1!} = 24 \frac{\cancel{e^{-\lambda}} \lambda^{\cancel{3}\,2}}{3!}$$

$$\lambda^2 = \frac{1}{4}$$

$$\lambda = \pm \frac{1}{2}$$

$\times$ $\lambda = -1/2$

✓ $\boxed{\lambda = 1/2}$

(i) $P(x=0) = \frac{e^{-\lambda} \lambda^0}{0!}$

$$= \frac{e^{-1/2} (1/2)^0}{0}$$

$$= e^{-1/2} = 0.6065$$

(ii) $P(x \geq 1) = 1 - P(x=0)$

$= 1 - 0.6065$

$= 0.3935$

7m

① Fit a poisson's Distribution to the following data

x	0	1	2	3	4	5
f	40	30	20	15	10	5

w.k.T $P(x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$

$N = \Sigma f_i = 40 + 30 + 20 + 15 + 10 + 5 = 120$

$\text{Mean} = \lambda = \dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{0(40) + 1(30) + 2(20) + 3(15) + 4(10) + 5(5)}{120}$

$\boxed{\lambda = 1.5}$

$E.F = N f(x)$

$E.F = f(x) = N P(x)$

if $x = 0$ $\quad P(x=0) \overset{Nf(x)=}{=} \dfrac{e^{-1.5}(1.5)^0}{0!} = 0.223$

$120(0.223) = 27$

$x = 1 = P(x=1) \overset{Nf(x)=}{=} 120\left(\dfrac{e^{-1.5}(1.5)^1}{1!}\right) = 40$

$x = 2 = P(x=2) \overset{Nf(x)}{=} 120\left(\dfrac{e^{-1.5}(1.5)^2}{2!}\right) = 30$

$x = 3 = P(x=3) \overset{Nf(x)=}{=} 120\left(\frac{e^{-1.5}(1.5)^3}{3!}\right)$

$= 15$

$x = 4 = P(x=4) \overset{Nf(x)=}{=} 120\left(\frac{e^{-1.5}(1.5)^4}{4!}\right)$

$= 6$

$x = 5 = P(x=5) \overset{Nf(x)=}{=} 120\left(\frac{e^{-1.5}(1.5)^5}{5!}\right)$

$= 2$

x	0	1	2	3.	4	5
f	40	30	20	15.	10	5
E.f	27	40	30	15	6	2

(2)

x	0	1	2	3	4	5	6
f	103	143	98	42	8	4	2

$N = \Sigma f_i = 103 + 143 + 98 + 42 + 8 + 4 + 2 = 400$

$$\text{Mean} = \lambda = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{0(103) + 1(143) + 2(98) + 3(42) + 4(8) + 5(4) + 6(2)}{400}$$

$\lambda = 1.3225$

$EF = N\,P(x)$

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$P(0) = \frac{e^{-1.3225} (1.3225)^{0}}{0!} = 0.2664$$

$$P(1) = \frac{e^{-1.3225} (1.3225)^{1}}{1!} = 0.3524$$

$$P(2) = \frac{e^{-1.3225} (1.3225)^{2}}{2!} = 0.2330$$

$$P(3) = \frac{e^{-1.3225} (1.3225)^{3}}{3!} = 0.1027$$

$$P(4) = \frac{e^{-1.335} (1.3225)^{4}}{4!} = 0.0339$$

$$P(5) = \frac{e^{-1.335} (1.3225)^{5}}{5!} = 0.0089$$

$$P(6) = \frac{e^{-1.335} (1.3225)^{6}}{6!} = 0.0019$$

$F(x) = NP(x)$

$F(0) = 400(0.2664) = 106.56 \sim 107$

$F(1) = 400(0.3524) = 140.96 \sim 141$

$F(2) = 400(0.2330) = 93.2 \sim 93$

$F(3) = 400(0.1027) = 41.08 \sim 41$

$F(4) = 400(0.0339) = 13.56 \sim 14$

$F(5) = 400(0.0089) = 3.56 \sim 3$

$F(6) = 400(0.0019) = 0.76 \sim 1$

x	0	1	2	3	4	5	6
f	103	143	98	42	8	4	2
E.f	107	141	93	41	14	3	1

22/12/2022

Moments, Skewness & Kurtosis

→ Moments of a R.V ~~shas~~ serve to describe a shape of distribution

❐ Moments about Arithmetic Mean (or) Central Moments

(i) Central Moments for individual series

→ Let $\bar{X}$ be the mean of individual series.

→ Let x be the deviation of X from its mean $\bar{X}$ ie $x_i = X_i - \bar{X}$.

→ Let N be the total no. of observation given in the series then we have

$$\boxed{\mu_r = \frac{\Sigma x^r}{N}}$$ where $r = 0, 1, 2 \ldots n$

μ_1 = Mean

μ_2 = variance

μ_3 = Skewness

μ_4 = Kurtosis

(ii) Central Moments for frequency distribution

→ If n observations are given $X_1, X_2 \ldots X_n$ & corresponding frequencies $f_1; f_2 \ldots f_n$ then arithmetic mean of F.D is given by

$$\bar{X} = \frac{\sum x_i f_i}{\sum f_i} = \frac{\sum x_i f_i}{N}$$

→ If $x_i = X_i - \bar{X}$ be the deviation of X from its mean $\bar{x}$ then

$$\boxed{\mu_r = \frac{\sum x_i^r f_i}{N}} \quad r = 0, 1 \ldots n$$

~~Note →~~

Q → find the first four moments for the set of numbers

① 2, 4, 6, 8

Sol $N = 4$

$$\text{Mean} = \bar{X} = \frac{2+4+6+8}{4} = 5$$

$\cancel{\mu_1 = x_2 = -3}$

$\cancel{x_4 = -1}$

$\cancel{x_6 =}$

X	x	x^2	x^3	x^4
2	$2-5=-3$	9	-27	81
4	$4-5=-1$	1	-1	1
6	$6-5=1$	1	1	1
8	$8-5=3$	9	27	81
	$\Sigma x=0$	$\Sigma x^2=20$	$\Sigma x^3=0$	$\Sigma x^4=164$

$$\mu_1 = \frac{\Sigma x}{N} = \frac{0}{4} = 0$$

$$\mu_2 = \frac{\Sigma x^2}{4} = \frac{20}{4} = 5$$

$$\mu_3 = \frac{\Sigma x^3}{4} = \frac{0}{4} = 0$$

$$\mu_4 = \frac{164}{4} = 41$$

② 3, 6, 8, 10, 18

$N = 5$

$$\text{Mean } \bar{x} = \frac{3+6+8+10+18}{5}$$

$$= \frac{45}{5}$$

$$\bar{x} = 9$$

x	x'	x^2	x^3	x^4
3	$3-9=-6$	36	-216	1296
6	$6-9=-3$	9	-27	81
8	$8-9=-1$	1	-1	1
10	$10-9=1$	1	1	1
18	$18-9=9$	81	729	6561
	$\Sigma x=0$	$\Sigma x^2=128$	$\Sigma x^3=486$	$\Sigma x^4=7940$

$$\mu_1 = \frac{\Sigma x}{N} = \frac{0}{9} = 0$$

$$\mu_2 = \frac{\Sigma x^2}{N} = \frac{128}{5} = 25.6$$

$$\mu_3 = \frac{\Sigma x^3}{N} = \frac{486}{5} = 97.2$$

$$\mu_4 = \frac{\Sigma x^4}{N} = \frac{7940}{5} = 1588$$

HW

3 4, 5, 6, 1, 4

Raw Moments (or) Moments about Arbitary origin

→ When actual Mean of distribution is a fraction then it is difficult to calculate the moments, in such case we first compute moment about an arbitary origin A & the convert these moments into the moments about actual mean

→ Raw moments are denoted by μ_r'

& is defined as $\boxed{\mu_r' = \frac{1}{N} \Sigma f_i d_i^r}$

where $d_i = x_i - A$

□ Relationship b/w Moments about Mean in terms of Moments about any point

→ $\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$

$\mu_3 = \mu_3' - 3\mu_2' \mu_1' + 2(\mu_1')^3$

$\mu_4 = \mu_4' - 4\mu_3' \mu_1' + 6\mu_2' (\mu_1')^2 - 3(\mu_1')^4$

Q → Calculate first four moments of the following distribution about the mean.

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

Sol → We shall first calculate the moment about the arbitary point A

$A = 4$

$d = x - A \Rightarrow x - 4$

$N = \Sigma f_i = 1+8+28+56+70+56+28+8+1 = 256$

x	f	d	fd	fd^2	fd^3	fd^4
0	1	0-4=-4	-4	16	-64	256
1	8	1-4=-3	-24	72	-216	648
2	28	2-4=-2	-56	112	-224	448
3	56	3-4=-1	-56	56	-56	56
4	70	4-4=0	0	0	0	0
5	56	5-4=1	56	56	56	56
6	28	6-4=2	56	112	224	448
7	8	7-4=3	24	72	216	648
8	1	8-4=4	4	16	64	256
		$\Sigma d=0$	$\Sigma fd=0$	$\Sigma fd^2=512$	$\Sigma fd^3=0$	$\Sigma fd^4=2816$

$$\mu_1' = \frac{\Sigma fd}{N} = \frac{0}{256} = 0$$

$$\mu_2' = \frac{\Sigma fd^2}{N} = \frac{512}{256} = 2$$

$$\mu_3' = \frac{\Sigma fd^3}{N} = \frac{0}{256} = 0$$

$$\mu_4' = \frac{\Sigma fd^4}{N} = \frac{2816}{256} = 11$$

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2 = 2$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 = 0$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4 = 11$$

Note

Skewness $= \beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$

Kurtosis $= \beta_2 = \dfrac{\mu_4}{\mu_2^2}$

23/12/2022

Q Calculate first 4 moments of the following distribution about mean & also evaluate β_1, β_2

x	1	2	3	4	5
f	2	3	5	4	1

$A = 3$

$N = \Sigma f_i = 2+3+5+4+1 = 15$

$d = x - a = x - 3.$

x	f	d	fd	fd^2	fd^3	fd^4
1	2	-2	-4	8	-16	32
2	3	-1	-3	3	-3	3
3	5	0	0	0	0	0
4	4	1	4	4	4	4
5	1	2	2	4	8	16
$\Sigma x = 15$	$\Sigma f = 15$	$\Sigma d = 0$	$\Sigma fd = -1$	$\Sigma fd^2 = 19$	$\Sigma fd^3 = -7$	$\Sigma fd^4 = 55$

$\mu_1' = \frac{\Sigma fd}{N} = \frac{-1}{15} =$

$\mu_2' = \frac{\Sigma fd^2}{N} = \frac{19}{15} =$

$\mu_3' = \frac{\Sigma fd^3}{N} = \frac{-7}{15} =$

$\mu_4' = \frac{\Sigma fd^4}{N} = \frac{55}{15}$

$\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 1.2622$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = \frac{-7}{15} - 3\left(\frac{19}{15}\right)\left(\frac{-1}{15}\right) + 2\left(\frac{-1}{15}\right)^3 = \cancel{0.2074}\ -0.2139$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3\mu_1'^4$

$= \frac{55}{15} - 4\left(\frac{-7}{15}\right)\left(\frac{-1}{15}\right) + 6\left(\frac{19}{15}\right)\left(\frac{-1}{15}\right)^2 - 3\left(\frac{55}{15}\right)^4$

$= -538.68$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{-0.2139^2}{1.2622^3} = 0.02275$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{-538.68}{1.2622^2} = -338.12$

HW
Q →

x	1	2	3	4	5	6	7	8	9
f	1	6	13	25	30	22	9	5	2

Note

$$\mu_r = E[X-\mu]^2$$

$$\mu_r = \sum_{i=1}^{n} [x_i - \mu]^r p_i$$

if $r=0$ $\quad \mu_0 = (1)(1) = 1 \quad \mu_0 = 1$

$$r=1 = \mu_1 = \sum_{i=1}^{n} (x_i - \mu)' p_i = \sum_{i=1}^{n} x_i p_i - \mu \sum_{i=1}^{n} p_i$$

$$= \mu - \mu = 0$$

$$\boxed{\mu_1 = 0}$$