Available online at www.sciencedirect.com

ScienceDirect

Procedia - Social and Behavioral Sciences 96 (2013) 451 – 460

Procedia
Social and Behavioral Sciences

13th COTA International Conference of Transportation Professionals (CICTP 2013)

Finding Reliable Shortest Path In Stochastic Time-Dependent Network

Yiyong Pan[a]* Lu Sun[a, b] Minli Ge[a]

[a]*School of Transportation Engineering, Southeast University*
Nanjing, 210096, China
[b]*Department of Civil Engineering, The Catholic University of America*
Washington, DC 20064, USA

Abstract

This paper addresses the problem of finding reliable a priori shortest path to maximize the probability of arriving on time in a stochastic and time-dependent network. Optimal solutions to the problem can be obtained from finding non-dominated paths, which are defined based on first-order stochastic dominance. We formulate the problem of finding non-dominated paths as a general dynamic programming problem because Bellman's principle of optimality can be applied to construct non-dominated paths. A label-correcting algorithm is designed to find optimal paths based on the new property for which Bellman's Principle holds. Numerical results are provided using a small network.

Keywords: stochastic time-dependent network; shortest path; dynamic programming; stochastic dominance; label-correcting.

1. Introduction

Traffic networks are subject to a large degree of uncertainty due to demand fluctuations and supply degradations (Schrank and Lomax, 2009). This results in unstable link travel times with the unreliability. In order to model travelers' route choice decisions in a transportation system, a stochastic time-dependent (STD) network is reasonable to model such uncertainties, where link travel times are not only random variables but also whose probability distribution functions (PDF) vary with time (Hall, 1986; Fu, 1998; Miller-Hooks, 2001; Nielsen, 2003; Gao and Chabini, 2006; Gao and He, 2012).

Stochastic shortest path problem (SSPP) is intended to provide a priori optimal path (as a single solution) or adaptive en-route path trees (as a set of solutions) in stochastic network. Either way, the optimality condition for routing may be defined differently due to various objective functions, including (1) minimizing the expected

* Corresponding author. Tel.: +86-25-8379-2619.
E-mail address: uoupanyg@gmail.com

doi:10.1016/j.sbspro.2013.08.053

travel time (Hall, 1986; Fu, 1998; Miller-Hooks, 2001; Nielsen, 2003; Yang and Miller-Hooks, 2004; Gao and Chabini, 2006; Gao and He, 2012), (2) maximizing the probability of travellers arriving at their destination within a given travel time budget (Frank, 1969; Fan et al. 2005a; Fan and Nie, 2006; Wu and Nie, 2011; Samaranayake et al. 2012) and (3) minimizing travel time budget required to ensure a pre-specified on-time arrival probability (Chen and Ji, 2005; Nie and Wu, 2009a; Chen et al. 2011) . The above three kinds of objective functions can be summarized as two kinds of problems: (a priori or adaptive) least expected time path problem (LETPP) (case 1) and (a priori or adaptive) reliable shortest path problem (RSPP) (case 2 and 3). A priori (or adaptive) least expected travel time path is not desirable to a risk-averse traveler because it may have a high variance in travel time. Meanwhile, travelers may care about the probability of arriving on destination within a given time budget if they cannot know precisely when they can arrive their destinations. A large of empirical studies showed that travelers are interested in not only travel time saving but also reducing travel time variability (Abdel-Aty et al. 1995; Lam, 2000; Brownstone et al. 2003; Wu and Nie, 2011).

In this paper, we focus on the reliable shortest path problem (RSPP) which aims to find a priori paths of maximizing the probability of arriving on time in an STD network. The contributions of this paper can be elaborated from the following aspects: (1) Extending reliable shortest path problem from static and stochastic network to dynamic and stochastic network; (2) An introduction of non-dominated path in STD network, and proof that non-dominated path satisfy Bellman's Principle; (3) An exact label-correcting algorithm is designed to find non-dominated paths as an extension of Algorithm SPOTAR-LC in Nie and Wu (2009a).

The remainder of this paper is organized as follows. In Section 2, the STD network and reliable path is defined. Non-dominated path is defined and its property is provided in Section 3. A label-correcting algorithm is presented in Section 4, and computational tests are conducted in Section 5. In Section 6, conclusions are made.

2. Problem statement

Consider an STD network $G=(N,A,T,X)$ consisting of a set of nodes N ($|N|=n$), a set of links A ($|A|=m$). Each node *i* has a set of successor nodes $\Phi(i)=\{j,(i,j)\in A\}$ and a set of predecessor nodes $\Phi^{-1}(i)=\{k,(k,i)\in A\}$. Link travel times are random variables and the probability distributions of link travel times are dependent on the time. Consequently, the travel time on these types of links can be modelled as a continuous-time stochastic process. Denote $\{X_{ij}(t), t\in T=[0,\infty)\}$ as a stochastic process of travel time on link *ij,* where $X_{ij}(t)$ is the travel time of link *ij* for vehicles entering link *ij* at time *t*. For each time instance *t*, $X_{ij}(t)$ is considered as a continuous random variable with probability density function (PDF) denoted by $f_{ij}(t,x)$. Furthermore, it is assumed that the travel times on individual links at a particular point in time are statistically independent.

All paths that connect starting node $o\in N$ and destination node $d\in N$ form a set K^{od}. The travel time along path $\lambda^{od}\in K^{od}$ is also random variable and can be modeled as a continuous-time stochastic process. Denote $\{S_\lambda^{od}(t), t\in T=[0,\infty)\}$ as a stochastic process of travel time along path λ^{od}, where $S_\lambda^{od}(t)$ is the travel time along path λ^{od} for vehicles out of starting node *o* at time t. Let $U_\lambda^{od}(t,x)=P\left(S_\lambda^{od}(t)\le x\right)$ be the cumulative distribution function (CDF) of path travel time $S_\lambda^{od}(t)$.

The most reliable path in a static and stochastic network defined by Frank (1969) can be extended to a STD network with respect to the pre-specified departure time *t* and travel time budget *b* as below:

Definition 1. (*(t,b)*-reliable path). Given a departure time t and travel time budget b, a path $\lambda^{od} \in K^{od}$ is *(t,b)*-reliable path if and only if $U_{\lambda}^{od}(t,b) \geq U_{l}^{od}(t,b), \forall\, l^{od} \in K^{od}$. A set of *(t,b)*-reliable path between o and d is denoted as $\Gamma^{od}(t,b)$.

Proposition 1. Subpaths of *(t,b)*-reliable path may not be *(t,b)*-reliable path.

Proof. Suppose $\lambda^{jd} \in K^{jd}$ is not be *(t,b)*-reliable path, we need to show that $\lambda^{id} = ij \oplus \lambda^{jd} \in K^{id}$ can be *(t,b)*-reliable path, where $\lambda^{id} = ij \oplus \lambda^{jd}$ show that λ^{id} is an extension of λ^{jd} along the link ij. The relationship between the travel times of a path and its subpath is given as follows:

$$S_{\lambda}^{id}(t) = X_{ij}(t) + S_{\lambda}^{jd}(t + X_{ij}(t)) \tag{1}$$

Thus

$$\begin{aligned} U_{\lambda}^{id}(t,b) &= P\big(X_{ij}(t) + S_{\lambda}^{jd}(t + X_{ij}(t)) \leq b\big) \\ &= \int_0^b f_{ij}(t,x) \times U_{\lambda}^{jd}(t+x, b-x)dx \end{aligned} \tag{2}$$

That λ^{id} be *(t,b)*-reliable path implied $U_{\lambda}^{id}(t,b) \geq U_{l}^{id}(t,b), \forall l^{id}$.

$$U_{\lambda}^{id}(t,b) = \int_0^b f_{ij}(t,x) \times U_{\lambda}^{jd}(t+x, b-x)dx \tag{3a}$$

$$U_{l}^{id}(t,b) = \int_0^b f_{ij}(t,x) \times U_{l}^{jd}(t+x, b-x)dx \tag{3b}$$

So

$$\begin{aligned} &U_{\lambda}^{id}(t,b) - U_{l}^{id}(t,b) \\ &= \int_0^b f_{ij}(t,x) \times \left[U_{\lambda}^{jd}(t+x, b-x) - U_{l}^{jd}(t+x, b-x)\right]dx \geq 0 \end{aligned} \tag{4}$$

Without loss of generality, let us assume that only one path $l^{jd} \in K^{jd}$ exists such that $U_{\lambda}^{jd}(t,b) < U_{l}^{jd}(t,b)$ (thus λ^{jd} is not *(t,b)*-reliable path). However, this assumption does not necessarily invalidate the above inequality. #

Let us define *(t,b)*-reliable shortest path problem (*(t,b)*-RSPP):

$$U^{od}(t,b) = \max\left\{U_{l}^{od}(t,b), \forall l^{od} \in K^{od}\right\}, \tag{5a}$$

$$u^{od}(t,b) = \arg\max\left\{U_{l}^{od}(t,b), \forall l^{od} \in K^{od}\right\}. \tag{5b}$$

The Proposition 1 means that we cannot to solve *(t,b)*-reliable shortest path problem by conventional dynamic programming because that Bellman's principle does not hold. We proceed to show how *(t,b)*-reliable path may be obtained without enumerating all paths.

3. Non-dominated path problem

Definition 2. (*(t,b)*-non-dominated path). Given a departure time t and travel time budget b, a path $\lambda^{od} \in K^{od}$ is *(t,b)*-non-dominated path if and only if $\exists$ no path $l^{od} \in K^{od}$ such that:

(1) $U_{l}^{od}(t+y, b-y) \geq U_{\lambda}^{od}(t+y, b-y), \forall y \in [0,b]$, and

(2) $\exists\, y_0 \in (0,b)$ such that $U_{l}^{od}(t+y_0, b-y_0) > U_{\lambda}^{od}(t+y_0, b-y_0)$.

A set of *(t,b)*-non-dominated path between *o* and *d* is denoted as $\Lambda^{od}(t,b)$.

Proposition 2. Subpaths of *(t,b)*-non-dominated path must be *(t,b)*-non-dominated path.

Proof. Let $\lambda^{id} \in K^{id}$ is *(t,b)*-non-dominated path, supposing one of its subpaths $\lambda^{jd} \in K^{jd}$ is not *(t,b)*-non-dominated path. There exists a path $l^{jd} \in K^{jd}$ such that $U_l^{jd}(t+x+y,b-x-y) \geq U_\lambda^{jd}(t+x+y,b-x-y),\ \forall x+y \in [0,b]$ and $\exists\, x_0 + y_0 \in (0,b)$ such that $U_l^{jd}(t+x_0+y_0,b-x_0-y_0) > U_\lambda^{jd}(t+x_0+y_0,b-x_0-y_0)$. Recalling for $\forall y \in [0,b]$,

$$U_\lambda^{id}(t+y,b-y) = \int_0^{b-y} f_{ij}(t+y,x) \times U_\lambda^{jd}(t+y+x,b-y-x)dx \tag{6a}$$

$$U_l^{id}(t+y,b-y) = \int_0^{b-y} f_{ij}(t+y,x) \times U_l^{jd}(t+y+x,b-y-x)dx \tag{6b}$$

$$\begin{aligned} &U_\lambda^{id}(t+y,b-y) - U_l^{id}(t+y,b-y) \\ &= \int_0^{b-y} f_{ij}(t+y,x) \times \left[U_\lambda^{jd}(t+y+x,b-y-x) - U_l^{jd}(t+y+x,b-y-x)\right]dx \leq 0 \end{aligned} \tag{6c}$$

Moreover, the above inequality holds strictly for some $y_0 \in (0,b)$. This implies that is $\lambda^{id} \in K^{id}$ is not *(t,b)*-non-dominated path because of the existence of $l^{id} \in K^{id}$, contradiction. #

Let us define *(t,b)*-non-dominated path problem ((*t,b*)-NDPP):

$$\overline{U}^{od}(t,b) = \max\left\{U_l^{od}(t,b),\ \forall l^{od} \in \Lambda^{od}(t,b)\right\}, \tag{7a}$$

$$\overline{u}^{od}(t,b) = \arg\max\left\{U_l^{od}(t,b),\ \forall l^{od} \in \Lambda^{od}(t,b)\right\}. \tag{7b}$$

The principle of optimality assured by Proposition 2 can be utilized to search for *(t,b)*-non-dominated path, just like in conventional dynamic programming. The difference is that we are likely to be left with a set of *(t,b)*-non-dominated paths, and dealing with such a set that usually incurs extra computational overhead.

The *(t,b)*-non-dominated path problem is formulated as: find $\Lambda^{id}(t,b), \forall i \in N$ such that:

$$\Lambda^{id}(t,b) = \underset{i \in PDS(j)}{optimal} \left\{\lambda^{id} = ij \oplus \lambda^{jd} \,\middle|\, \lambda^{jd} \in \Lambda^{jd}(t,b)\right\} \tag{8a}$$

$$\Lambda^{dd}(t,b) = 0^{dd}, \tag{8b}$$

$$\overline{U}^{id}(t,b) = \max_{\lambda^{jd} \in \Lambda^{jd}(t,b)} \int_0^b f_{ij}(t,x) \times U_\lambda^{jd}(t+x,b-x)dx, \quad \forall i \neq d \tag{8c}$$

$$\overline{u}^{id}(t,b) = \arg\max_{\lambda^{jd} \in \Lambda^{jd}(t,b)} \int_0^b f_{ij}(t,x) \times U_\lambda^{jd}(t+x,b-x)dx, \quad \forall i \neq d \tag{8d}$$

$$\overline{U}^{dd}(t,b) = 1 \quad , \quad \overline{u}^{dd}(t,b) = 0^{dd} \tag{8e}$$

Where $optimal$ means selecting *(t,b)*-non-dominated paths.

4. Solution Algorithm

The solution algorithm to the *(t,b)*-NDPP is designed to find all *(t,b)*-non-dominated paths for each node *i* and thus will find *(t,b)*-reliable path for given departure time *t* and time budget *b* for each node *i*. While *(t,b)*-NDPP may be solved in its continuous form, we only consider the discrete version when discussing solution algorithms because of the following two reasons. First, the solution algorithm to the *(t,b)*-NDPP consists of two main building blocks: the evaluation of the convolution integral in the recursive equation (2), and the iterative construction of *(t,b)*-non-dominated path sets through dynamic programming. Evaluating convolution integrals usually calls for discrete numerical procedures even if the probability density function has a continuous form. Second, probability distributions from real-world applications are often available in discrete forms as probability density function. Also, any continuous stochastic process may be appropriately discretized as described below.

Let $b = L\varphi$ be the traval time budget *b*, where φ is the length of the time unit. For any link ij , the probability mass function $P_{ij}(t, x)$ of $X_{ij}(t)$ may be obtained from its probability density function $f_{ij}(t, x)$ as follows:

$$P_{ij}(t, x) = \begin{cases} \int_x^{x+\varphi} f_{ij}(t, w)dw, & x = 0, \varphi, 2\varphi, \cdots, (L-1)\varphi \\ \int_x^{\infty} f_{ij}(t, w)dw, & x = L\varphi \\ 0, & otherwise \end{cases} \tag{9}$$

The above definition assumes that supporting points of any $X_{ij}(t)$ must take a non-negative multiple of φ. This assumption helps simplify the evaluation of convolution integral. The equation is discretized as:

$$U_{\lambda}^{id}(t+y, b-y) = \sum_{x=0}^{b-y} P_{ij}(t+y, x) \times U_{\lambda}^{jd}(t+x+y, b-x-y), \tag{10}$$
$$\forall y = 0, \varphi, 2\varphi, \cdots, L\varphi = b$$

The solution algorithm to the *(t,b)*-NDPP is based on the Algorithm SPOTAR-LC proposed by Nie and Wu (2009a). The major difference between the two algorithms is that the solution algorithm to the *(t,b)*-NDPP works in a stochastic time-dependent (STD) network while Algorithm SPOTAR-LC works in a static and stochastic network. Note that, Procedure ND-CHECK is adapted from Procedure LR-CHECK (Nie and Wu, 2009a) in order to reduce the amount of effort required to check dominance. The difference between the two algorithms is that Procedure LR-CHECK check the local-reliable path (Nie and Wu, 2009a) while Procedure ND-CHECK check *(t,b)*-non-dominated path.

The algorithm maintains a set of *(t,b)*- non-dominated paths for each node *i*, denoted as $\Lambda_{id}(t,b)$. $\omega(\cdot)$ is a subpath operator used to track paths. That is, $\omega(\lambda^{id}) = \lambda^{jd}$ such that $\lambda^{id} = ij \oplus \lambda^{jd}$. A list of candidate paths is denoted by using *Q*, the path λ^{jd} is selected from *Q* at each iteration of the algorithm and constracts new path $\lambda^{id} = ij \oplus \lambda^{jd}$ and set $Q = Q / k^{jd}$. If λ^{id} is *(t,b)*- non-dominated path at node *i*, set $Q = Q \cup \lambda^{id}$ and $\Lambda_{id}(t,b) = \Lambda_{id}(t,b) \cup \lambda^{id}$. Upon termination, $Q = \phi$ and all $\Lambda_{id}(t,b), \forall i \in N$ contains only *(t,b)*-non-dominated paths. So the *(t,b)*-optimal reliable path can be obtained from checking *(t,b)*-non-dominated paths. With these definitions, a description of the algorithm now follows.

Algorithm ND-path

Inputs: STD network $G = (N, A, T, C)$, destination *d*, department time t out of starting node $i\ (\forall i \neq d)$ and time budget *b*.

Return: The sets of *(t,b)*-non-dominated path and *(t,b)*-reliable path at starting node $i\ (\forall i \neq d)$.

Step 0. Initialization.

Step 0.1. Imput department time *t* and time budget *b.*

Step 0.2. Initialize labels:

for all $i \in N \setminus \{d\}$: **do**

$\overline{U}^{id}(t+x,b-x)=0,\ \forall x=0,\varphi,2\varphi,\cdots,M\varphi=b,\ \Lambda_{id}(t,b)=\phi$.

end for

for $i=d:\overline{U}^{dd}(t+x,b-x)=1, U_0^{dd}(t+x,b-x)=1, \forall x=0,\varphi,2\varphi,\cdots,M\varphi=b,\ Q=0^{dd}$.

Step 1. Check candidate paths

If $Q\neq\phi$ **then**

Select the path λ^{jd} from the Q, set $Q=Q/\lambda^{jd}$, go to step 2.

Else

Go to step 3

End if

Step 2. Update labels.

For all $i\in\Phi^{-1}(j)$ (i.e. , $\forall(i,j)\in A$) **do**

Step 2.1. cycle check:

Call procedure CYCLE-CHECK.

If node i is already on path λ^{jd} **then**

go to step 2, take the next link.

End if

Step 2.2. Temporal label creation:

Construct a new path $\lambda^{id}=ij\oplus\lambda^{jd}$, calculate

$$U_\lambda^{id}(t+y,b-y)=\sum_{x=0}^{b-y}P_{ij}(t+y,x)\times U_\lambda^{jd}(t+x+y,b-x-y),\quad \forall y=0,\varphi,2\varphi,\cdots,M\varphi=b$$

Step 2.3. Label comparison:

If $\Lambda_{id}(t,b)\neq\phi$ **then**

Call procedure ND-CHECK.

If λ^{id} is *(t,b)*-non-dominance path, **then**

$Q=Q\cup\lambda^{id},\ \omega(\lambda^{id})=\lambda^{jd}$

End if

Else

$\overline{U}^{id}(t+y,b-y)=U_\lambda^{id}(t+y,b-y),\ \forall y=0,\varphi,2\varphi,\cdots,M\varphi=b$

$\Lambda_{id}(t,b)=\lambda^{id},\omega(\lambda^{id})=\lambda^{jd}$

End if

End for

Go to step 1

Step 3. Stop and find the *(t,b)*-optimal reliable path.

For all $i\in N\setminus\{d\}$**: do**

$$\overline{U}^{id}(t,b)=\max_{\lambda^{id}\in\Lambda^{id}(t,b)}U_\lambda^{id}(t,b),\quad\forall i\in N$$

$$\overline{u}^{id}(t,b)=\arg\max_{\lambda^{id}\in\Lambda^{id}(t,b)}U_\lambda^{id}(t,b),\quad\forall i\in N$$

End for

5. Numerical experiments

The objectives of the computational tests are to verify validity and feasibility of algorithms using a small network. The link travel time $X_{ij}(t)$ is assumed to follow a normal distribution $N(\mu_{ij}(t), \sigma^2{}_{ij}(t))$:

$$f_{ij}(t,x) = \frac{1}{\sigma_{ij}(t)\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu_{ij}(t))^2}{2\sigma_{ij}(t)^2}\right\} \tag{11}$$

Where $\mu_{ij}(t), \sigma_{ij}(t)$ are parameters as a function of time *t*. We note that the mean and variance of the normal distribution are $\mu_{ij}(t)$ and $\sigma^2{}_{ij}(t)$, respectively. The algorithms were coded using matlab and tested on a Windows-XP (64) workstation with two 2.00 GHz Xeon CPUs and 4G RAM.

Fig. 1 shows the topology of a samll network and the probability density functions (PDF) and cumulative distribution functions (CDF) of each link for different departure time. The destination is node 4. Set $\varphi = 0.1$. For the illustrative purpose, only three such periods are defined, the parameters of each link travel time are seted as follows:

$$[\mu_{ij}(t), \sigma_{ij}(t)] = \begin{cases} [40,15], t \in [0,30] \\ [50,12], t \in [30,60] \\ [60,8], \ \ t \in [60,+\infty] \end{cases} . \tag{12}$$

We emphasize that the cumulative distribution functions in this experiment are carefully set to satisfy the stochastic FIFO condition (see Definition 3) and can be seen from the fig. 1.

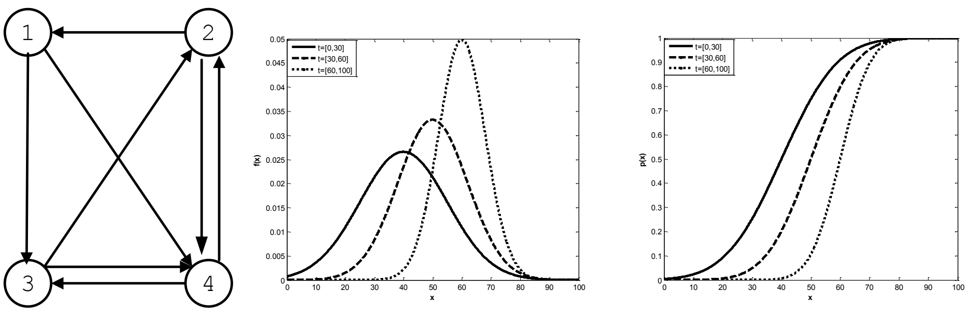

Fig. 1. A samll network , PDF and CDF of each link for different departure time.

Fig. 2 (left) shows $U_\lambda^{14}(10+y,60-y)$ as a function of *y* when giving the departure time *t=10* and time budget *b=60* on three predetermined paths between the OD pairs 1–4: path: 1-4; path: 1-3-4; path: 1-3-2-4. Fig. 2 (right) shows $U_\lambda^{34}(10+y,60-y)$ as a function of *y* when giving the departure time *t=10* and time budget *b=60* on two predetermined paths between the OD pairs 3–4: path: 3-4; path: 3-2-4. Fig. 3 (left) shows $U_\lambda^{14}(10+y,90-y)$ as a function of *y* when giving the departure time *t=10* and time budget *b=90* on three predetermined paths between the OD pairs 1–4: path: 1-4; path: 1-3-4; path: 1-3-2-4. Fig. 3 (right) shows $U_\lambda^{34}(10+y,90-y)$ as a function of *y* when giving the departure time *t=10* and time budget *b=90* on two predetermined paths between the OD pairs 3–4: path: 3-4; path: 3-2-4.

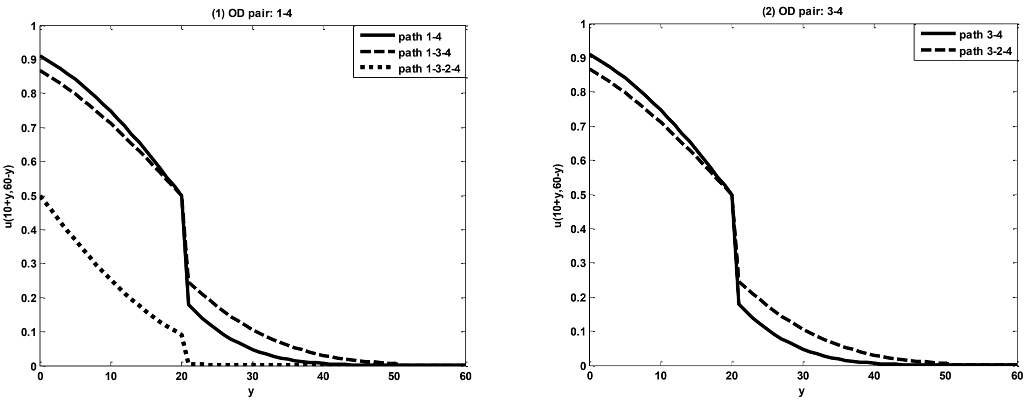

Fig. 2. $U_\lambda^{14}(10+y,60-y)$ and $U_\lambda^{34}(10+y,60-y)$ with respect to *y*

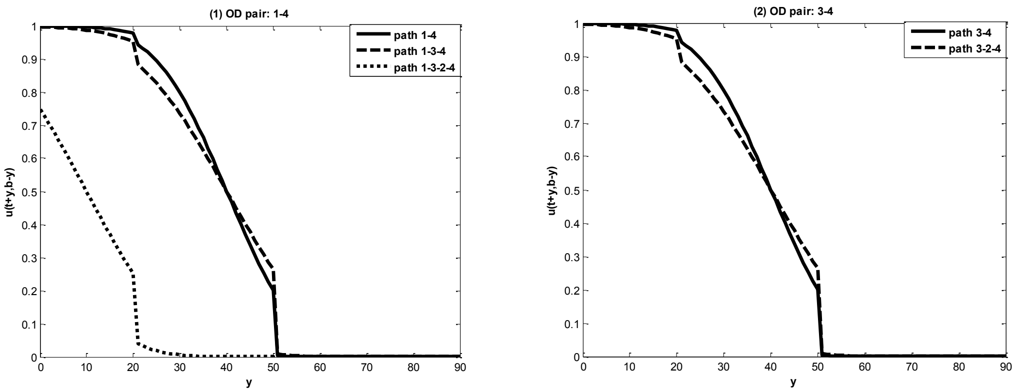

Fig. 3. $U_\lambda^{14}(10+y,90-y)$ and $U_\lambda^{34}(10+y,90-y)$ with respect to *y*

As shown, path: 1-4 and path: 1-3-4 are both *(10, 60)*-non-dominance path and *(10, 90)*-non-dominance path between the OD pairs 1–4, path: 3-4 and path: 3-2-4 are both *(10, 60)*-non-dominance path and *(10, 90)*-non-dominance path between the OD pairs 3–4. Figs. 2-3 demonstrate a jump on the $U_\lambda^{14}(t+y,b-y)$ and $U_\lambda^{34}(t+y,b-y)$ around the *y=20*, which obviously coincides with the boundary between three periods.

6. Concluding remarks

This paper addresses the reliable shortest path problem (RSPP) which aims to find a priori paths of maximizing the probability of arriving on time in a stochastic time-dependent network where link travel times are not only random variables but also whose probability distribution functions vary with time. It is shown that, in such a network, RSPP can not be solved by conventional dynamic programming because that Bellman's principle does not hold. Non-dominated paths are defined based on stochastic dominance. Meanwhile, we show that non-dominated path problem (NDPP) can be formulated and solved using conventional dynamic programming because non-dominated path satisfy Bellman's principle of optimality.

An exact label-correcting is designed to solve NDPP. Numerical experiments verify the correctness of the proposed algorithm ND-path. The algorithm ND-path provides more practical method because travelers are easy to obtain the expected results by setting the prescribed time budget. Hence the algorithm ND-path is more attractive in dealing with reliable shortest path problem (RSPP) in STD network.

Acknowledgements

This study is sponsored in part by the National Science Foundation (NSF) under Grant CMMI-0408390 and NSF CAREER Award CMMI-0644552, by National Science Foundation of China under Grant No. 51050110143 and U1134206, by Jiangsu Natural Science Foundation under Grant No. SBK200910046. The authors are grateful to anonymous reviewers for their insightful comments and constructive suggestions, which help us refine the content and presentation of the original manuscript.

References

Abdel-Aty, M., Kitamura, R., & Jovanis, P. (1995). Exploring route choice behavior using geographical information system-based alternative routes and hypothetical travel time information input. *Transportation Research Record*, 1493, 74–80.

Andreatta, G., Romeo, L. (1988). Stochastic shortest paths with recourse. *Networks* 18 (3), 193–204.

Astarita, V. (1996). A continuous time link model for dynamic network loading based on travel time function. *The Proceedings of the 13th International Symposium on Transportation and Traffic Theory*, 79-103.

Bander, J. and White, C. C. III, (2002). A heuristic search approach for a nonstationary stochastic shortest path problem with terminal cost. *Transportation Science* 36(2):218–230.

Brownstone, D., Ghosh, A., Golob, T. F., Kazimi, C., & Van Amelsfort, D. (2003). Drivers' willingness-to-pay to reduce travel time: Evidence from the San DiegoI-15 congestion pricing project. *Transportation Research*, 37A(4), 373–387.

Chen A, Ji Z, (2005). Path finding under uncertainty. *Journal of Advanced Transportation* , 39(1):19–37.

Chen, B.Y., Lam, W.H.K., Sumalee, A., Li, Q. and Shao, H., (2012), Finding reliable shortest paths in road networks under uncertainty. *Networks & Spatial Economics*.

Cheung, R.K., (1998). A note on the stochastic shortest route problem. *Naval Research Logistics* 25 (4), 729–732.

Fan, Y., Kalaba, R. and Moore, J., (2005a). Arriving on time. *Journal of Optimization Theory and Applications*, 127, 497-513.

Fan, Y., Kalaba, R., Moore, J., (2005b). Shortest paths in stochastic networks with correlated link costs. *Computers and Mathematics with Applications* 49 (9–10), 1549–1564.

Fan YY, Nie Y, (2006). Optimal routing for maximizing the travel time reliability. *Netw Spat Econ* 6:333–344.

Frank, H., (1969). Shortest paths in probabilistic graphs. *Oper Res* 17(4), 583–599.

Franklin, J., A. Daoud, (2011). *Proof in mathematics: an introduction*, Sydney: Kew Books. ISBN 0-646-54509-4. (Ch. 8.)

Fu, L., L.R. Rilett. (1998). Expected shortest paths in dynamic and stochastic traffic networks, *Transportation Research Part B*. 32(7) 499-516.

Fu, L., (2001). An adaptive routing algorithm for in-vehicle route guidance systems with real-time information. *Transportation Research Part B* 35 (8), 749–765.

Gao, S., (2005). Optimal adaptive routing and traffic assignment in stochastic time-dependent networks. Ph.D. Dissertation, Massachusetts Institute of Technology

Gao, S., I. Chabini. (2006). Optimal routing policy problems in stochastic time-dependent networks. *Transportation Research Part B*. 40(2) 93-122.

Gao, S., Huang, H., (2012). Real-time traveler information for optimal adaptive routing in stochastic time-dependent networks.*Transportation Research Part C* 21, 196-213.

Huang, H., Gao, S., (2012). Optimal paths in dynamic networks with dependent random link travel times. *Transportation Research Part B* 46, 579-598.

Lam, T., (2000). The effect of variability of travel time on route and time-of-day choice. Ph.D. Dissertation, Department of Economics, University of California, Irvine.

Maher M, Zhang YL, (2000). Route choice to maximize the probability of arrival within a specified time. In: Bell MGH, Cassir C (eds) Reliability of transport networks. Research Studies Press, Baldock, Hertfordshire, England.

Miller-Hooks, E., (1997). Optimal routing in time-varying, stochastic networks: algorithms and implementation. PhD thesis, The University of Texas, Austin,TX.

Miller-Hooks, E.D., Mahmassani, H.S., (1998). Least possible time paths in stochastic, time-varying networks. *Computers and Operations Research* 25 (2),1107–1125.

Miller-Hooks, E.D., Mahmassani H.S., (2000). Least expected time paths in stochastic, time-varying transportation networks. *Transportation Science* 34:198–215.

Miller-Hooks, E.D., (2001). Adaptive least-expected time paths in stochastic, time-varying transportation and data networks. *Networks* 37(1) 35-52.

Mirchandani, P.B., (1976). Shortest distance and reliability of probabilistic networks. *Computers and Operations Research* 3 (4), 347–355.

Nielsen, L.R., (2003). Route Choice in Stochastic Time-Dependent Networks. Ph.D Dissertation. University of Aarhus, Denmark.

Nie, Y., Wu, X., (2009a). Shortest path problem considering on-time arrival probability. Transportation Research Part B 43 (6), 597–613.

Nie, Y., Wu, X., (2009b). Reliable a priori shortest path problem with limited spatial and temporal dependencies. In: Proceedings of the 18th International Symposium on Transportation and Traffic Theory, pp. 169–196.

Pretolani, D., Nielsen, L.R., Andersen, K.A., Ehrgott, M., (2009). Time-adaptive and history-adaptive multicriterion routing in stochastic, time-dependent networks. *Operations Research Letters* 37, 201–205.

Polychronopoulos, G.H., Tsitsiklis, J.N., (1996). Stochastic shortest path problems with recourse. *Networks* 27 (2), 133–143.

Provan, J.S., (2003). A polynomial-time algorithm to find shortest paths with recourse. *Networks* 41 (2), 115–125.

Samaranayake, S., S. Blandin, A. Bayen. (2012). A tractable class of algorithms for reliable routing in stochastic networks. *Transportation Research Part C* 20, 199-217.

Schrank, D., Lomax, T., (2009). *Annual urban mobility report*. Technical report, Texas Transportation Institute.

Sigal, C.E., Alan, A., Pritsker, B., Solberg, J.J., (1980). The stochastic shortest route problem. *Operations Research* 28 (5), 1122–1129.

Sun L, Yang J, Mahmassani H. (2008). Travel time estimation based on piecewise truncated quadratic speed trajectory. *Transportation Research Part A* 42, 173–86.

Sun L, Yang J, Mahmassani H, Gu W, Kim BJ. (2010). Data mining based Adaptive regression for developing equilibrium static traffic speed-density relationships. *Canadian Journal of Civil Engineering* 37, 1–12.

Sun L, Gu W, Mahmassani H. (2011). Estimation of expected travel time using the method of moment. *Canadian Journal of Civil Engineering* 38, 1–12.

Waller, S.T., Ziliaskopoulos, A.K., (2002). On the online shortest path problem with limited arc cost dependencies. *Networks* 40 (4), 216–227.

Wu X, Nie, Y., (2011). Modeling heterogeneous risk-taking behavior in route choice: a stochastic dominance approach. *Transportation Research Part A* 45, 896–915

Hall, R.W. (1986). The fastest path through a network with random time-dependent travel times. *Transportation Science*. 20(3) 182-188.

Yang, B., E. Miller-Hooks. (2004). Adaptive routing considering delays due to signal operations. *Transportation Research Part B* 38(5) 385–413.