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FINANCIAL ENGINEERING I - COMPUTATIONAL PROJECT

PROBLEM 1

PART 1. Find lowest-cost bond portfolio that covers the stream of liabilities as per table below:

Year	**1**	**2**	**3**	**4**	**5**	**6**
Obligation	500	200	800	400	700	900

1.1.1 Mathematical Formulation

Notation:
- s_j: spot rate from present to year j, where $j = 1, \ldots, 6$
- $f_{j,k}$: forward rate between years j and k
- y_j: obligation (liability) in year j
- p_i: price of bond i, where $i = 1, \ldots, 13$
- c_{ij}: cash flow from bond i in year j

We need to calculate the following forward rates: $f_{1,2},\ f_{2,3},\ f_{3,4},\ f_{4,5},\ f_{5,6}$, using:

$$f_{j,k} = \left[\frac{(1+s_k)^k}{(1+s_j)^j}\right]^{\frac{1}{k-j}} - 1$$

Therefore: $f_{1,2} = 2.0\%,\ f_{2,3} = 3.0\%,\ f_{3,4} = 4.0\%,\ f_{4,5} = 5.0\%,\ f_{5,6} = 6.0\%$

Decision Variables:
- x_i: quantity of bond i held in the portfolio
- z_j: carry forward amount from years j to $j+1$

Objective Function:

general form: (total bond portfolio cost)

$$\min \sum_{i=1}^{13} (p_i \cdot x_i) = \min(p_1 x_1 + \cdots + p_{13} x_{13})$$

$$\min(108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 + 92.9x_7 + 110x_8 + 104x_9 + 101x_{10} + 107x_{11} + 102x_{12} + 95.2x_{13})$$

Subject to (constraints):

general form: (yearly obligation constraints)

$$\sum_{i=1}^{13} (c_{ij} \cdot x_i) + (1 + f_{j-1,j}) z_{j-1} - z_j \geq y_j,\ \forall j = 1, \ldots, 6, \text{ and } z_0,\ z_6 = 0$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 10x_{11} + 7x_{12} + 100x_{13} - z_1 \geq 500$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 10x_8 + 8x_9 + 6x_{10} + 110x_{11} + 107x_{12} + (1.02)z_1 - z_2 \geq 200$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 6x_6 + 5x_7 + 110x_8 + 108x_9 + 106x_{10} + (1.03)z_2 - z_3 \geq 800$$

$$10x_1 + 7x_2 + 8x_3 + 6x_4 + 7x_5 + 106x_6 + 105x_7 + (1.04)z_3 - z_4 \geq 400$$

$$10x_1 + 7x_2 + 8x_3 + 106x_4 + 107x_5 + (1.05)z_4 - z_5 \geq 700$$

$$100x_1 + 107x_2 + 108x_3 + (1.06)z_5 \geq 900$$

general form: (bond qty and carryover amount domain constraints)

$$x_i, z_j \geq 0, \qquad \forall i = 1, \ldots, 13 \text{ and } j = 1, \ldots, 6$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8,\ x_9,\ x_{10},\ x_{11},\ x_{12},\ x_{13} \geq 0$$

$$z_1,\ z_2,\ z_3,\ z_4,\ z_5 \geq 0$$

1.1.2 Matrix Formulation

Objective function vector form: $f^{\mathbf{T}} \cdot \mathbf{x}$

$$f^{\mathbf{T}} = \begin{bmatrix} 108 & 94 & 99 & 92.7 & 96.6 & 95.9 & 92.9 & 110 & 104 & 101 & 107 & 102 & 95.2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{x}^{\mathbf{T}} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} & z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix}$$

Inequality constraints vector form: $\mathbf{A} \cdot \mathbf{x} \leq \mathbf{b}$

$$A = \begin{bmatrix} -10 & -7 & -8 & -6 & -7 & -6 & -5 & -10 & -8 & -6 & -10 & -7 & -100 & 1 & 0 & 0 & 0 & 0 \\ -10 & -7 & -8 & -6 & -7 & -6 & -5 & -10 & -8 & -6 & -110 & -107 & 0 & -1.02 & 1 & 0 & 0 & 0 \\ -10 & -7 & -8 & -6 & -7 & -6 & -5 & -110 & -108 & -106 & 0 & 0 & 0 & 0 & -1.03 & 1 & 0 & 0 \\ -10 & -7 & -8 & -6 & -7 & -106 & -105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.04 & 1 & 0 \\ -10 & -7 & -8 & -106 & -107 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.05 & 1 \\ -100 & -107 & -108 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.06 \end{bmatrix}$$

$$\mathbf{b}^{\mathbf{T}} = \begin{bmatrix} -500 & -200 & -800 & -400 & -700 & -900 \end{bmatrix}$$

Domain constraints vector form: $\mathbf{lb} \leq \mathbf{x}$

$$\mathbf{lb} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1.1.3 Matlab Solution

The full MATLAB code is located in the **Appendix**. Here are the results from ***linprog***:

Quantity of bond and carry over amounts:

$$\mathbf{x}^{\mathbf{T}} = \begin{bmatrix} 0 & 8.4112 & 0 & 0 & 5.9918 & 2.8224 & 0 & 0 & 6.3171 & 0 & 0.2883 & 0 & 3.2883 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Objective function value - (total bond portfolio cost): $f_{val} =$ **$2641.00.** Note that there is no carryover between years.

PART 2. Add a constraint that at most 50% of the bond portfolio's value is in B-rated bonds:

1.2.1 Mathematical Formulation

general form: (50% total portfolio value in B-rated bonds constraint)

$$\frac{(p_1x_1+p_2x_2+p_3x_3+p_4x_4+p_5x_5+p_6x_6)}{(p_1x_1+p_2x_2+p_3x_3+p_4x_4+p_5x_5+p_6x_6+p_7x_7+p_8x_8+p_9x_9+p_{10}x_{10}+p_{11}x_{11}+p_{12}x_{12}+p_{13}x_{13})} \leq 0.5$$

$$p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4 + p_5x_5 + p_6x_6 - p_7x_7 - p_8x_8 - p_9x_9 - p_{10}x_{10} - p_{11}x_{11} - p_{12}x_{12} - p_{13}x_{13} \leq 0$$

$$108x_1 + 94x_2 + 99x_3 + 92.7x_4 + 96.6x_5 + 95.9x_6 - 92.9x_7 - 110x_8 - 104x_9 - 101x_{10} - 107x_{11} - 102x_{12} - 95.2x_{13} \leq 0$$

1.2.2 Matrix Formulation

Update Inequality constraint matrix and vector:

$$\mathbf{A} = \begin{bmatrix} -10 & -7 & -8 & -6 & -7 & -6 & -5 & -10 & -8 & -6 & -10 & -7 & -100 & 1 & 0 & 0 & 0 & 0 \\ -10 & -7-8 & -6 & -7 & -6 & -5 & -10 & -8 & -6 & -110 & -107 & 0 & -1.02 & 1 & 0 & 0 & 0 & \\ -10 & -7 & -8 & -6 & -7 & -6 & -5 & -110 & -108 & -106 & 0 & 0 & 0 & 0 & -1.03 & 1 & 0 & 0 \\ -10 & -7 & -8 & -6 & -7 & -106 & -105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.04 & 1 & 0 \\ -10 & -7 & -8 & -106 & -107 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.05 & 1 \\ -100 & -107 & -108 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.06 \\ 108 & 94 & 99 & 92.7 & 96.6 & 95.9 & -92.9 & -110 & -104 & -101 & -107 & -102 & -95.2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{b^T} = \begin{bmatrix} -500 & -200 & -800 & -400 & -700 & -900 & 0 \end{bmatrix}$$

1.2.3 Matlab Solution

The full MATLAB code is located in the **Appendix**. Here are the results from ***linprog***:

Quantity of bond and carry over amounts:

$$\mathbf{x^T} = \begin{bmatrix} 0 & 8.4112 & 0 & 0 & 5.5027 & 0 & 3.3566 & 0 & 6.3502 & 0 & 0.3183 & 0 & 3.3183 & 0 & 0 & 0 & \$49.84 & 0 \end{bmatrix}$$

Objective function value - (total bond portfolio cost): $f_{val} =$ **$2644.40.** Note there is a small carry over from year 4 to year 5.

PART 3. Repeat part 2 but with at most 25% of the bond portfolio's value in B-rated bonds:

1.3.1 Mathematical Formulation

general form: (25% total portfolio in B-rated bonds)

$$\frac{(p_1x_1+p_2x_2+p_3x_3+p_4x_4+p_5x_5+p_6x_6)}{(p_1x_1+p_2x_2+p_3x_3+p_4x_4+p_5x_5+p_6x_6+p_7x_7+p_8x_8+p_9x_9+p_{10}x_{10}+p_{11}x_{11}+p_{12}x_{12}+p_{13}x_{13})} \leq 0.25$$

$$3p_1x_1 + 3p_2x_2 + 3p_3x_3 + 3p_4x_4 + 3p_5x_5 + 3p_6x_6 - p_7x_7 - p_8x_8 - p_9x_9 - p_{10}x_{10} - p_{11}x_{11} - p_{12}x_{12} - p_{13}x_{13} \leq 0$$

$$324x_1 + 282x_2 + 297x_3 + 278.1x_4 + 289.8x_5 + 287.7x_6 - 92.9x_7 - 110x_8 - 104x_9 - 101x_{10} - 107x_{11} - 102x_{12} - 95.2x_{13} \leq 0$$

1.3.2 Matrix Formulation

Update Inequality constraint matrix and vector:

$$\mathbf{A} = \begin{bmatrix} -10 & -7 & -8 & -6 & -7 & -6 & -5 & -10 & -8 & -6 & -10 & -7 & -100 & 1 & 0 & 0 & 0 & 0 \\ -10 & -7-8 & -6 & -7 & -6 & -5 & -10 & -8 & -6 & -110 & -107 & 0 & -1.02 & 1 & 0 & 0 & 0 & \\ -10 & -7 & -8 & -6 & -7 & -6 & -5 & -110 & -108 & -106 & 0 & 0 & 0 & 0 & -1.03 & 1 & 0 & 0 \\ -10 & -7 & -8 & -6 & -7 & -106 & -105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.04 & 1 & 0 \\ -10 & -7 & -8 & -106 & -107 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.05 & 1 \\ -100 & -107 & -108 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.06 \\ 324 & 282 & 297 & 278.1 & 289.8 & 287.7 & -92.9 & -110 & -104 & -101 & -107 & -102 & -95.2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{b^T} = \begin{bmatrix} -500 & -200 & -800 & -400 & -700 & -900 & 0 \end{bmatrix}$$

1.3.3 Matlab Solution

The full MATLAB code is located in the **Appendix**. Here are the results from ***linprog***:

Quantity of bond and carry over amounts:

$$\mathbf{x^T} = \begin{bmatrix} 0 \; 7.1271 \; 0 \; 0 \; 0 \; 0 \; 10.4068 \; 0 \; 6.4637 \; 0 \; 0.4215 \; 0 \; 3.4215 \; 0 \; 0 \; 0 \; \$742.60 \; \$129.62 \end{bmatrix}$$

Objective function value - (total bond portfolio cost): $f_{val} =$ **$2679.80.** Note there is a carry over from year 4 to 5 and year 5 to 6.

1.4 Summary

The cost of the three portfolios are summarized in the table below. Portfolio 1 is the least expensive and portfolio 3 is the most expensive. However, portfolio 1 has a much higher quantity of B-rated (riskier) bonds.

	Description	**Total Portfolio Cost**
Portfolio 1	No additional constraints	$2641.00
Portfolio 2	At most 50% value in B-rated bonds	$2644.40
Portfolio 3	At most 25% value in B-rated bonds	$2679.80

PROBLEM 2

PART 1A. Use Yahoo Finance to get monthly adjusted closing prices of SPY, GOVT and EEMV between Jan 2014 and Jan 2021. Compute each assets: (1) expected returns, (2) standard deviations, and (3) covariances between assets

2.1A.1 Mathematical Formulation

Notation:
i: asset reference, where: SPY = 1, GOVT = 2 and EEMV =3
$\bar{r}_i$: arithmetic mean return of asset i
μ_i: geometric mean return of asset i
σ_i: standard deviation of asset i returns
σ_{ij}: covariance between asset i and asset j, where $i \neq j$

$(\sigma_i)^2 = \sigma_{ii}$: variance of asset i

2.1A.2 Matrix Representation and Determined Values

Values have been determined as per the MATLAB code **Appendix**.

Arithmetic mean return:

$$\mathbf{\bar{r}} = \begin{vmatrix} r_1 \\ r_2 \\ r_3 \end{vmatrix} = \begin{vmatrix} 0.0114 \\ 0.0023 \\ 0.0046 \end{vmatrix}$$

Geometric mean return:

$$\mu = \begin{vmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{vmatrix} = \begin{vmatrix} 0.01058 \\ 0.00224 \\ 0.00390 \end{vmatrix}$$

Standard deviation of returns:

$$\sigma = \begin{vmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{vmatrix} = \begin{vmatrix} 0.0412 \\ 0.0115 \\ 0.0380 \end{vmatrix}$$

Covariance Matrix:

$$\mathbf{H} = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix} = \begin{vmatrix} 1.685e-03 & -1.458e-04 & 1.120e-03 \\ -1.458e-04 & 1.302e-04 & -3.230e-05 \\ 1.120e-03 & -3.230e-05 & 1.430e-03 \end{vmatrix}$$

PART 1B. Use mean-variance optimization to generate efficient frontier of the three assets. Create table with optimal weights (w_i) and portfolio variance (σ_P^2) for each expected return (R).

2.1B.1 Mathematical Formulation

Decision variable:

w_i: weight of asset i of overall portfolio

$$\mathbf{w^T} = \begin{vmatrix} w_1 & w_2 & w_3 \end{vmatrix}$$

Objective function:

general form: (minimize portfolio variance σ_P^2)

$$\min \frac{1}{2}\mathbf{w^T} \cdot \mathbf{H} \cdot \mathbf{w}$$

$$min \begin{vmatrix} w_1 & w_2 & w_3 \end{vmatrix} \cdot \begin{vmatrix} 1.685e-03 & -1.458e-04 & 1.120e-03 \\ -1.458e-04 & 1.302e-04 & -3.230e-05 \\ 1.120e-03 & -3.230e-05 & 1.430e-03 \end{vmatrix} \cdot \begin{vmatrix} w_1 \\ w_2 \\ w_3 \end{vmatrix}$$

Subject to:

general form: (obtain an expected return)

$$\sum_{i=1}^{3}(\mu_i \cdot w_i) = R$$

$$\mu_1 w_1 + \mu_2 w_2 + \mu_3 w_3 = R$$

$$0.01058 w_1 + 0.00224 w_2 + 0.00390 w_3 = R$$

general form: (portfolio weights must sum to 1)

$$\sum_{i=1}^{3}(w_i) = 1$$

$$w_1 + w_2 + w_3 = 1$$

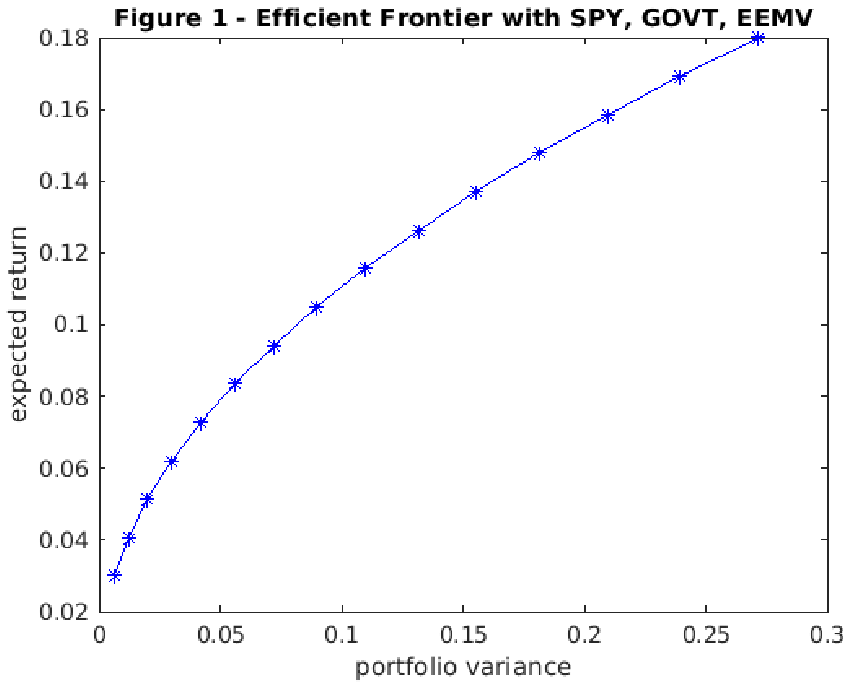
2.1B.2 Matrix Formulation

Equality constraints in Matrix form: $\mathbf{A_{eq}} \cdot \mathbf{w} = \mathbf{b_{eq}}$

$$\begin{vmatrix} 0.01058 & 0.00224 & 0.00390 \\ 1 & 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} w_1 \\ w_2 \\ w_3 \end{vmatrix} = \begin{vmatrix} R \\ 1 \end{vmatrix}$$

2.1B.3 MATLAB Results

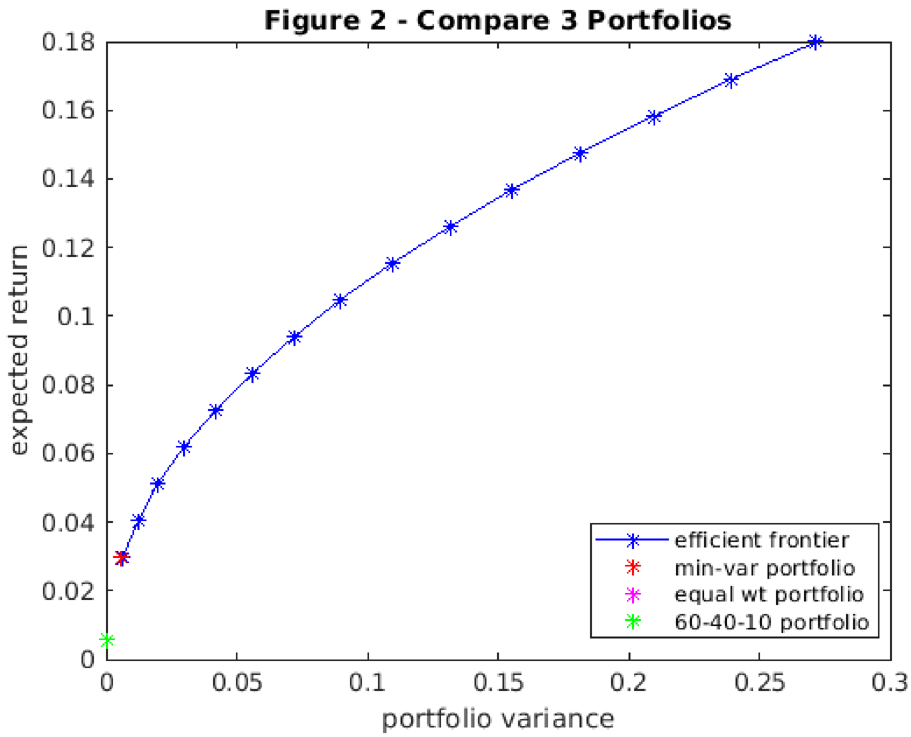
An efficient frontier was obtained in MATLAB as per the code in the Appendix. For the table of expected returns, portfolio variance and weights (w1, w2, w3), pleas refer to the appendix. Below, you will see in Figure 1, the efficient frontier for three assets: SPY, GOVT and EEMV:



PART 1C. Use returns from Feb 21 to determine exp_returns and variance for (1) equal weight portfolio and (2) 60% SPY- 30% GOVT -10% EEMV portfolio. Compare with the minimum variance portfolio obtained on the efficient frontier. The results of this are show in the table below:

Portfolio	**Expected Portfolio Rtn**	**Portfolio Variance**
Min Variance Portfolio	0.030	0.00617
Equal Weight	0.006	0.00055
60% SPY- 40% GOVT- 10% EEMV	0.012	0.00071

I plotted these portfolios on the efficient frontier curve for a graphical view below. The equal weight portfolio did not appear on the graph due to its really low return and variance. Overall, the best portfolio for return is the min-variance portfolio as it is on the efficient frontier.



PART 2A. Repeat PART 1 but with 8 assets now, including CME, BR, CBOE, ICE and ACN:

2.1A.1 Mathematical Formulation

Notation:
i: asset reference: SPY = 1, GOVT = 2, EEMV = 3, CME = 4, BR = 5, CBOE = 6, ICE = 7, and ACN = 8
All other notation remains the same

2.1A.2 Matrix Representation and Determined Values

Values have been determined as per the MATLAB code **Appendix**.

Arithmetic mean return:

$$\bar{\mathbf{r}} = \begin{vmatrix} \bar{r_1} \\ \bar{r_2} \\ \bar{r_3} \\ \bar{r_4} \\ \bar{r_5} \\ \bar{r_6} \\ \bar{r_7} \\ \bar{r_8} \end{vmatrix} = \begin{vmatrix} 0.0114 \\ 0.0023 \\ 0.0046 \\ 0.0164 \\ 0.0195 \\ 0.0106 \\ 0.0139 \\ 0.0167 \end{vmatrix}$$

Geometric mean return:

$$\mu = \begin{vmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \\ \mu_6 \\ \mu_7 \\ \mu_8 \end{vmatrix} = \begin{vmatrix} 0.01058 \\ 0.00224 \\ 0.00390 \\ 0.01494 \\ 0.01781 \\ 0.00836 \\ 0.01254 \\ 0.01519 \end{vmatrix}$$

Covariance Matrix:

Standard deviation of returns:

$$
\mathbf{H} = \begin{vmatrix} \sigma_{11} & \dots & \sigma_{18} \\ \sigma_{21} & \dots & \sigma_{28} \\ \sigma_{31} & \dots & \sigma_{38} \\ \sigma_{41} & \dots & \sigma_{48} \\ \sigma_{51} & \dots & \sigma_{58} \\ \sigma_{61} & \dots & \sigma_{68} \\ \sigma_{71} & \dots & \sigma_{78} \\ \sigma_{81} & \dots & \sigma_{88} \end{vmatrix} \qquad \sigma = \begin{vmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \\ \sigma_7 \\ \sigma_8 \end{vmatrix} = \begin{vmatrix} 0.0412 \\ 0.0115 \\ 0.0380 \\ 0.0553 \\ 0.0588 \\ 0.0669 \\ 0.0530 \\ 0.0558 \end{vmatrix}
$$

$$
\mathbf{H} = \begin{vmatrix}
1.685e-03 & -1.458e-04 & 1.120e-03 & 8.377e-04 & 1.443e-03 & 9.063e-04 & 1.177e-03 & 1.740e-03 \\
-1.458e-04 & 1.301e-04 & -3.229e-05 & -1.071e-04 & -7.785e-06 & -5.560e-05 & -1.272e-04 & -1.271e-04 \\
1.120e-03 & -3.229e-05 & 1.428e-03 & 2.218e-04 & 9.239e-04 & 3.306e-04 & 3.924e-04 & 9.07e-04 \\
8.377e-04 & -1.071e-04 & 2.218e-04 & 3.019e-03 & 1.079e-03 & 2.137e-03 & 1.811e-03 & 1.03e-03 \\
1.443e-03 & -7.785e-06 & 9.239e-04 & 1.079e-03 & 3.415e-03 & 9.667e-04 & 1.202e-03 & 1.87e-03 \\
9.063e-04 & -5.560e-05 & 3.306e-04 & 2.137e-03 & 9.667e-04 & 4.423e-03 & 1.605e-03 & 1.13e-03 \\
1.177e-03 & -1.272e-04 & 3.924e-04 & 1.811e-03 & 1.202e-03 & 1.605e-03 & 2.776e-03 & 1.51e-03 \\
1.740e-03 & -1.271e-04 & 9.075e-04 & 1.030e-03 & 1.873e-03 & 1.130e-03 & 1.512e-03 & 3.07e-03
\end{vmatrix}
$$

<u>PART 2B</u>. Use mean-variance optimization to generate efficient frontier of the 8 assets. Create table with optimal weights (w_i) and portfolio variance (σ_P^2) for each expected return (R).

<u>2.2B.1 Mathematical Formulation</u>

<u>Decision variable</u>:

w_i: weight of asset i of overall portfolio

$$
\mathbf{w^T} = \begin{vmatrix} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 & \end{vmatrix}
$$

<u>Objective function</u>:

general form: (minimize portfolio variance σ_P^2)

$$
\min \frac{1}{2} \mathbf{w^T} \cdot \mathbf{H} \cdot \mathbf{w}
$$

<u>Subject to:</u>

general form: (obtain an expected return)

$$
\sum_{i=1}^{8} (\mu_i \cdot w_i) = R
$$

$$
\mu_1 w_1 + \mu_2 w_2 + \mu_3 w_3 + \mu_4 w_4 + \mu_5 w_5 + \mu_6 w_6 + \mu_7 w_7 + \mu_8 w_8 \; = R
$$

$$
0.01058 w_1 + 0.00224 w_2 + 0.00390 w_3 + 0.01494 w_4 + 0.01781 w_5 + 0.00836 w_6 + 0.01254 w_7 + 0.01519 w_8 = R
$$

general form: (portfolio weights must sum to 1)

$$\sum_{i=1}^{8}(w_i) = 1$$

$$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 = 1$$

2.1B.2 Matrix Formulation

Equality constraints in Matrix form: $\mathbf{A_{eq}} \cdot \mathbf{w} = \mathbf{b_{eq}}$

$$\mathbf{A_{eq}} = \begin{vmatrix} 0.01058 & 0.00224 & 0.00390 & 0.01494 & 0.01781 & 0.00836 & 0.01254 & 0.01519 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix}$$

$$\mathbf{b_{eq}} = \begin{vmatrix} R \\ 1 \end{vmatrix}$$

2.1B.3 MATLAB Results

Finally, an efficient frontier diagram was produce for the 8-asset portfolio along with the original 3-asset portfolio for comparison. From this diagram, it's clear that the 8-asset portfolio has potential for higher returns and surprisingly, for the same variance as the 3-asset portfolio. I don't believe diversification has anything to do with this since we already contain several ETFs in the 3-asset portfolio. With more time, I would investigate this.