PageRank Algorithm

MIE 1624 - Introduction to Data Science and Analytics

Group 2:
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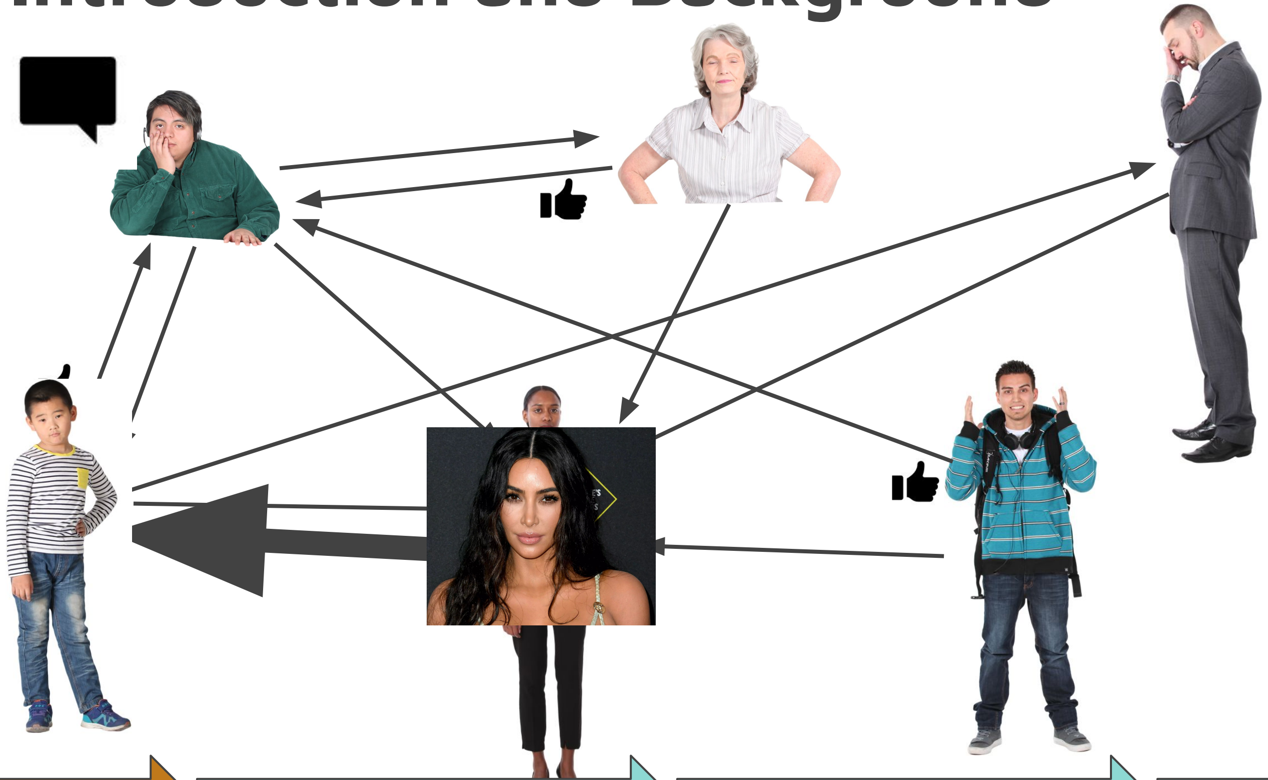
Agenda

1. Introduction and Background (Monica)
2. Mathematics (Arshdeep and Charles)
3. Implementation (Sugumar and Luke)
4. Variants (Yuan and Qisheng)

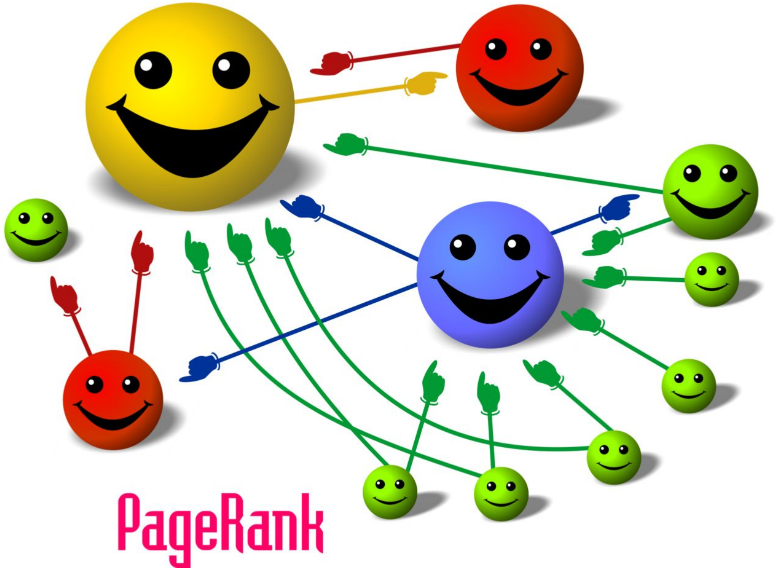
Introduction and Background



Introduction

https://www.businessinsider.in/entertainment/news/people-are-dragging-kim-kardashian-for-describing-life-as-paradise-on-a-day-the-us-recorded-a-record-number-of-covid-19-deaths/articleshow/80376504.cms

Introduction and Background

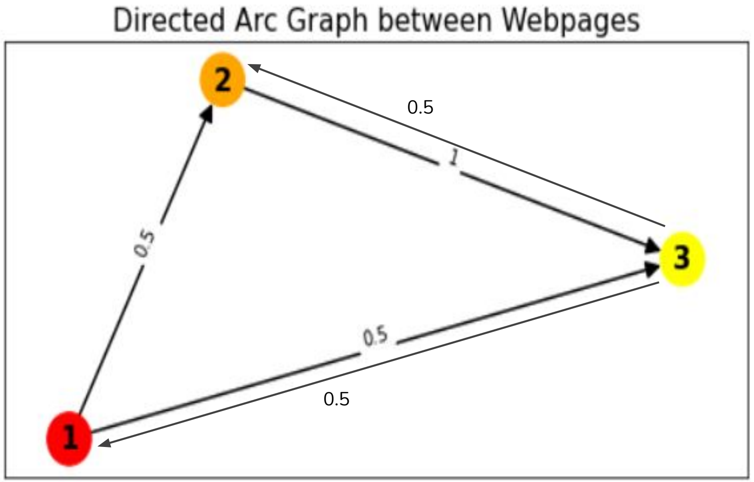


Introduction

https://www.theverge.com/2019/12/4/20994361/google-alphabet-larry-page-sergey-brin-sundar-pichai-co-founders-ceo-timeline
https://en.wikipedia.org/wiki/PageRank

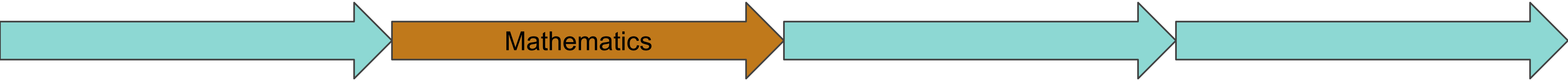
Simplified Algorithm

Divide the score of a page by the # of outgoing links, and equally assign to its destinations

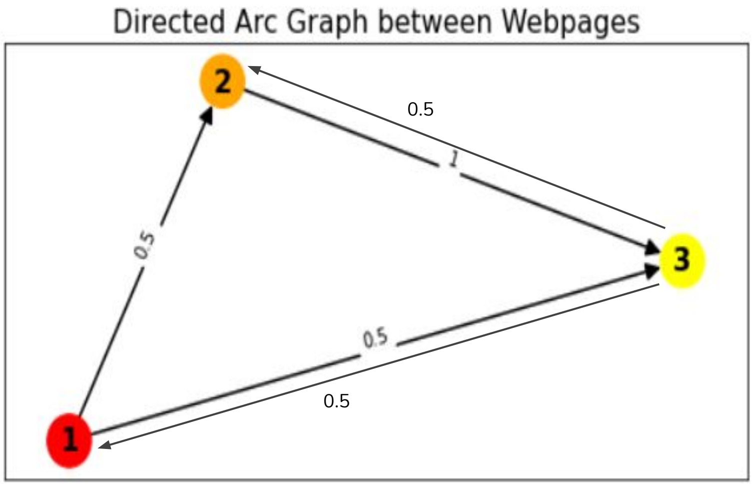


$$PR(p_i) = \sum_{p_j \in W(p_i)} \frac{PR(p_j)}{L(p_j)}$$

- Pi - individual webpages
- PR(Pi) - PageRank of Pi
- W(Pi) - set of pages that link to Pi
- L(Pi) - number of outbound links of Pi



Markov Chain/Stochastic Matrix



i\j	1	2	3
1	0	0	0.5
2	0.5	0	0.5
3	0.5	1	0

Probability Matrix: $$M_{ij} = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \end{bmatrix}$$

Mij is a pattern of Markov Chain/Stochastic Matrix

$$M\vec{x} = \lambda\vec{x}$$

M = Matrix
x = Eigenvector
λ = Eigenvalue

Mathematics

Eigenvalue and Eigenvector

Perron-Frobenius Theorem: If M is a positive, column stochastic matrix, then:

1. Eigenvalue equals to 1.
2. For the eigenvalue 1 there exists a unique eigenvector with the sum of its entries equal to 1.

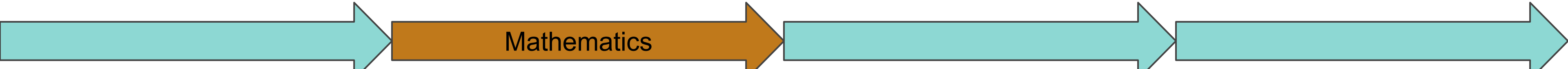
Simplified Algorithm:

$$R = M \times R$$

$$M_{ij} = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \end{bmatrix}$$

$$R_0 = \begin{bmatrix} PR(p_1) \\ PR(p_2) \\ PR(p_3) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

The simplified algorithm could be solved with M as a Markov Chain / Stochastic matrix. R is the eigenvector, where the eigenvalue is 1.



http://pi.math.cornell.edu/~mec/Winter2009/RalucaRemus/Lecture3/lecture3.html

Power Method

Power Method Convergence Theorem: Let M be a positive, column stochastic n × n matrix and R be a probabilistic eigenvector corresponding to the eigenvalue 1 and with all entries equal to 1/n. Then the sequence R, MR, ..., M^kR converges to the vector R^*.

Starting Eigenvector Ro :

$$R_0 = \begin{bmatrix} PR(p_1) \\ PR(p_2) \\ PR(p_3) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

Adjacency Matrix Mij :

$$M_{ij} = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 1 & 0 \end{bmatrix}$$

$$R_1 = M_{ij} \times R_0 = \begin{bmatrix} 0.167 \\ 0.333 \\ 0.5 \end{bmatrix}$$

$$R_2 = {M_{ij}}^2 \times R_0 = \begin{bmatrix} 0.25 \\ 0.333 \\ 0.417 \end{bmatrix}$$

$$R_3 = {M_{ij}}^3 \times R_0 = \begin{bmatrix} 0.208 \\ 0.333 \\ 0.458 \end{bmatrix}$$

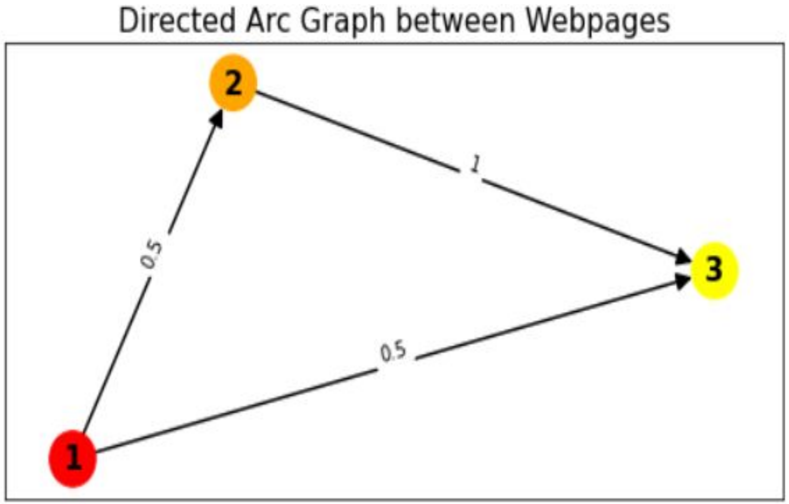
...

$$R_{100} = {M_{ij}}^{100} \times R_0 = \begin{bmatrix} 0.222 \\ 0.333 \\ 0.444 \end{bmatrix}$$

Mathematics

http://pi.math.cornell.edu/~mec/Winter2009/RalucaRemus/Lecture3/lecture3.html

Issue with the Simplified Algorithm



Problem:

- Sink node (webpage 3) with no outgoing flow resulting in lack of balance of the network
- In this scenario, the PageRank of all webpages converges to 0

$$R_0 = \begin{bmatrix} PR(p_1) \\ PR(p_2) \\ PR(p_3) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \qquad M_{ij}' = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & 0 \end{bmatrix}$$

$$R_{100} = (M_{ij}')^{100} \times R_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Oops!!

Mathematics

Enhancing the Simplified Algorithm

Simplified Algorithm:

- Divide the score of a page by the # of outgoing links, and equally assign to its destinations

$$PR(p_i) = \sum_{p_j \in W(p_i)} \frac{PR(p_j)}{L(p_j)}$$

Enhancement - Damping Factor:

- Divide the score of the sink node in the graph by total number of node, and equally assign to each node
- Adopt the same approach for all webpages in the network, but against only a portion of its total score (1 - d)
- Various researches recommend a desired damping factor of 0.85

General Equation:

$$PR(p_i) = \frac{1-d}{N} + d \sum_{p_j \in W(p_i)} \frac{PR(p_j)}{L(p_j)}$$

- Pi - individual webpages
- PR(Pi) - PageRank of Pi
- d - damping factor, typically 0.85
- N - total number of pages
- W(Pi) - set of pages that link to Pi
- L(Pi) - number of outbound links of Pi

Mathematics

Damping in Power Method

Solution of the Google co-founders - Page and Brin

General Equation:

$$PR(p_i) = \frac{1-d}{N} + d \sum_{p_j \in W(p_i)} \frac{PR(p_j)}{L(p_j)}$$

General Equation in Matrix Form:

$$R_{i+1} = \left(\frac{1-d}{N} \times \mathrm{E} + \mathrm{d}M\right) \times R_i$$

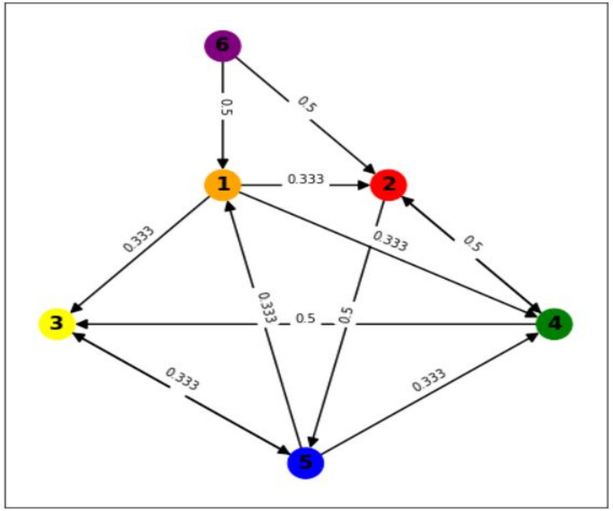
$$M_{damped} = \frac{1-d}{N} \times \mathrm{E} + \mathrm{d}M$$

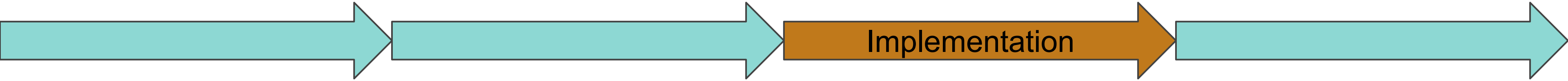
$$\Longrightarrow \quad R_{i+1} = M_{damped} \times R_i$$

, where ERi=1, |R|=1, and all entries of E are 1

Implementation

Code walkthrough of the Python implementation





Weighted PageRank

- Proposed by Wenpu Xing and Ali Ghorbani in 2004.
- Allows distribution of the page rank according to the importance or the popularity of the webpage
- Assigns a weight value to each edge
- If each edge has the same weight, this is identical to the original PageRank algorithm.

Variants

https://www.ijser.org/researchpaper/A-Literature-Survey-on-Page-Rank-Algorithm.pdf
https://pdfs.semanticscholar.org/70f8/d27954ce7ef49e351aca6d4b6b368cfef1c7.pdf

Weighted PageRank

$$PR(u) = (1-d) + d\sum_{v \in B(u)} \frac{PR(v)}{N_v} W^{in}(v,u) W^{out}(v,u)$$

$W^{in}(v,u) = \frac{I_u}{\sum_{p \in R(v)} I_p}$ where:

- I_u and I_p are number of inlinks of pages u and p, respectively
- $R(v)$ is the set of pages pointed by v

and

$W^{out}(v,u) = \frac{O_u}{\sum_{p \in R(v)} O_p}$ where:

- O_u and O_p are number of outlinks of pages u and p, respectively
- $R(v)$ is the set of pages pointed by v

Advantages:

- Takes into account the importance of both the inlinks and outlinks of the pages
- Distributes rank scores based on the popularity of the pages
- Converges very fast

Limitations:

- Does not consider user access pattern

Variants

https://www.ijser.org/researchpaper/A-Literature-Survey-on-Page-Rank-Algorithm.pdf
https://pdfs.semanticscholar.org/70f8/d27954ce7ef49e351aca6d4b6b368cfef1c7.pdf

PageRank based on Visits of links (VOL)

- Proposed by Gyanendra Kumar, Neelam Duhan, A. K. Sharma in 2011 at International Conference on Computer & Communication Technology (ICCCT)-2011.
- Assigns more value to the outgoing links that are most visited by users

https://www.ijser.org/researchpaper/A-Literature-Survey-on-Page-Rank-Algorithm.pdf
https://pdfs.semanticscholar.org/70f8/d27954ce7ef49e351aca6d4b6b368cfef1c7.pdf

PageRank based on Visits of links (VOL)

$$PR(u) = (1-d) + d \sum_{v \in B(u)} \frac{PR(v)L_u}{TL(v)}$$

- $B(u)$ is the set of pages pointing to u
- L_u is the number of visits of links which are pointing from v to u
- $TL(v)$ is the total number of visits of all links from v

Advantages:

- Displays most valuable pages on the top of the result list based on user browsing behaviour

Limitation:

- Converges slower compared to weighted PageRank

https://www.ijser.org/researchpaper/A-Literature-Survey-on-Page-Rank-Algorithm.pdf
https://pdfs.semanticscholar.org/70f8/d27954ce7ef49e351aca6d4b6b368cfef1c7.pdf

Variants

- Weighted PageRank
- PageRank based on visits of links (vol)
- Weighted PageRank based on visits of links
- Personalized PageRank
- Personalized Weighted PageRank
- Topic sensitive PageRank
- Ratio based Weighted PageRank
- etc.

Variants

https://www.ijser.org/researchpaper/A-Literature-Survey-on-Page-Rank-Algorithm.pdf
https://docs.oracle.com/cd/E56133_01/2.4.0/reference/algorithms/pagerank.html

Questions?