

- GELFAND, I.M., GINDIKIN, S.G., AND GRAEV, M.I.  
 1982 Integral geometry in affine and projective spaces, *J. Soviet Math.* **18** (1982), 39–164.
- GELFAND, I.M., GINDIKIN, S.G., AND SHAPIRO, S.J.  
 1979 A local problem of integral geometry in a space of curves, *Funct. Anal. Appl.* **13** (1979), 11–31.
- GELFAND, I.M. AND GRAEV, M.I.  
 1955 Analogue of the Plancherel formula for the classical groups, *Trudy Moscov. Mat. Obshch.* **4** (1955), 375–404.
- 1968a Complexes of straight lines in the space  $\mathbf{C}^n$ , *Funct. Anal. Appl.* **2** (1968), 39–52.
- 1968b Admissible complexes of lines in  $\mathbf{CP}^n$ , *Funct. Anal. Appl.* **3** (1968), 39–52.
- GELFAND, I.M., GRAEV, M.I., AND SHAPIRO, S.J.  
 1969 Differential forms and integral geometry, *Funct. Anal. Appl.* **3** (1969), 24–40.
- GELFAND, I.M., GRAEV, M.I., AND VILENKIN, N.  
 1966 *Generalized Functions, Vol. 5: Integral Geometry and Representation Theory*, Academic Press, New York, 1966.
- GELFAND, I.M. AND SHAPIRO, S.J.  
 1955 Homogeneous functions and their applications, *Uspehi Mat. Nauk* **10** (1955), 3–70.
- GELFAND, I.M. AND SHILOV, G.F.  
 1960 *Verallgemeinerte Funktionen, Vol. I*, German Transl. VEB, Berlin, 1960.
- GINDIKIN, S.G.  
 1975 Invariant generalized functions in homogeneous domains, *Funct. Anal. Appl.* **9** (1975), 50–52.
- 1995 Integral geometry on quadrics, *Amer. Math. Soc. Transl. Ser. 2* **169** (1995), 23–31.
- GLOBEVNIK, J.  
 1992 A support theorem for the X-ray transform, *J. Math. Anal. Appl.* **165** (1992), 284–287.
- 1994 A local support theorem for  $k$ -plane transform in  $\mathbf{R}^n$ , *J. Math. Anal. Appl.* **181** (1994), 455–461.
- GODEMENT, R.  
 1957 Introduction aux travaux de A. Selberg, *Séminaire Bourbaki* **144**, Paris, 1957.
- 1966 The decomposition of  $L^2(G/\Gamma)$  for  $\Gamma = SL(2, \mathbf{Z})$ , *Proc. Symp. Pure Math.* **9** (1966), 211–224.
- GOLDSCHMIDT, H.  
 1990 The Radon transform for symmetric forms on real symmetric spaces, *Contemp. Math.* **113** (1990), 81–96.
- GONCHAROV, A.B.  
 1989 Integral geometry on families of  $k$ -dimensional submanifolds, *Funct. Anal. Appl.* **23** 1989, 11–23.

- GONZALEZ, F.  
 1984 Radon transforms on Grassmann manifolds, thesis, MIT, Cambridge, MA, 1984.  
 1987 Radon transforms on Grassmann manifolds, *J. Funct. Anal.* **71** (1987), 339–362.  
 1988 Bi-invariant differential operators on the Euclidean motion group and applications to generalized Radon transforms, *Ark. Mat.* **26** (1988), 191–204.  
 1990a Bi-invariant differential operators on the complex motion group and the range of the  $d$ -plane transform on  $C^n$ , *Contemp. Math.* **113** (1990), 97–110.  
 1990b Invariant differential operators and the range of the Radon  $d$ -plane transform, *Math. Ann.* **287** (1990), 627–635.  
 1991 On the range of the Radon transform and its dual, *Trans. Amer. Math. Soc.* **327** (1991), 601–619.  
 1994 “Range of Radon transform on Grassmann manifolds,” in: *Proc. Conf. 75 Years of Radon Transform*, Vienna, 1992, International Press, Hong Kong, 1994, 81–91.  
 1999 John’s equation and the plane to line transform on  $\mathbf{R}^3$  (preprint).  
 GONZALEZ, F. AND KAKEHI, T.  
 1999 Radon transforms on affine Grassmann manifolds (preprint).  
 GONZALEZ, F. AND QUINTO, E.T.  
 1994 Support theorems for Radon transforms on higher rank symmetric spaces, *Proc. Amer. Math. Soc.* **122** (1994), 1045–1052.  
 GOODEY, P. AND WEIL, W.  
 1991 Centrally symmetric convex bodies and the spherical Radon transform, preprint, 1991.  
 GRINBERG, E.  
 1985 On images of Radon transforms, *Duke Math. J.* **52** (1985), 939–972.  
 1986 Radon transforms on higher rank Grassmannians, *J. Differential Geom.* **24** (1986), 53–68.  
 1987 Euclidean Radon transforms: Ranges and restrictions, *Contemp. Math.* **63** (1987), 109–134.  
 1992 Aspects of flat Radon transform, *Contemp. Math.* **140** (1992), 73–85.  
 1994 “Integration over minimal spheres in Lie groups and symmetric spaces of compact type,” in: *Proc. Conf. 75 Years of Radon Transform*, Vienna, 1992, International Press, Hong Kong, 1994, 167–174.  
 GRINBERG, E. AND QUINTO, E.T.  
 1998 Morera theorems for complex manifolds, preprint, 1998.  
 GUARIE, D.  
 1992 *Symmetries and Laplacians: Introduction to Harmonic Analysis and Applications*, North Holland, Amsterdam, 1992.

- GUILLEMIN, V.  
 1976 Radon transform on Zoll surfaces, *Adv. in Math.* **22** (1976), 85–99.  
 1985 The integral geometry of line complexes and a theorem of Gelfand-Graev, *Astérisque No. Hors Série* (1985), 135–149.  
 1987 Perspectives in integral geometry, *Contemp. Math.* **63** (1987), 135–150.
- GUILLEMIN, V. AND STERNBERG, S.  
 1977 *Geometric Asymptotics*, Math. Surveys, American Mathematical Society, Providence, RI, 1977.  
 1979 Some problems in integral geometry and some related problems in microlocal analysis, *Amer. J. Math.* **101** (1979), 915–955.
- GÜNTHER, P.  
 1988 *Huygens' Principle and Hyperbolic Equations*, Academic Press, Boston, 1988.  
 1991 Huygens' Principle and Hadamard's conjecture, *Math. Intelligencer* **13** (1991), 56–63.
- HALPERIN, I.  
 1962 The product of projection operators, *Acta Sci. Math. (Szeged)* **23** (1962), 96–99.
- HAMAKER, C. AND SOLMON, D.C.  
 1978 The angles between the null spaces of X-rays, *J. Anal. Appl.* **62** (1978), 1–23.
- HARINCK, P.  
 1998 Formule d'inversion des intégrales orbitales et formule de Plancherel, *J. Funct. Anal.* **153** (1998), 52–107.
- HARISH-CHANDRA  
 1957 A formula for semisimple Lie groups, *Amer. J. Math.* **79** (1957), 733–760.  
 1958 Spherical functions on a semisimple Lie group I, *Amer. J. Math.* **80** (1958), 241–310.
- HELGASON, S.  
 1959 Differential Operators on homogeneous spaces, *Acta Math.* **102** (1959), 239–299.  
 1961 Some remarks on the exponential mapping for an affine connection, *Math. Scand.* **9** (1961), 129–146.  
 1963 Duality and Radon transforms for symmetric spaces, *Amer. J. Math.* **85** (1963), 667–692.  
 1964 A duality in integral geometry: some generalizations of the Radon transform, *Bull. Amer. Math. Soc.* **70** (1964), 435–446.  
 1965a The Radon transform on Euclidean spaces, compact two-point homogeneous spaces and Grassmann manifolds, *Acta Math.* **113** (1965), 153–180.  
 1965b Radon-Fourier transforms on symmetric spaces and related group representation, *Bull. Amer. Math. Soc.* **71** (1965), 757–763.  
 1966a “A duality in integral geometry on symmetric spaces,” in: *Proc. U.S.-Japan Seminar in Differential Geometry*, Kyoto, 1965, Nippon Hyronsha, Tokyo, 1966, 37–56.

- 1966b Totally geodesic spheres in compact symmetric spaces, *Math. Ann.* **165** (1966), 309–317.
- 1970 A duality for symmetric spaces with applications to group representations, *Adv. in Math.* **5** (1970), 1–154.
- 1972 “Harmonic analysis in the non-Euclidean disk,” in: *Proc. International Conf. on Harmonic Analysis*, University of Maryland, College Park, MD, 1971, Lecture Notes in Math. No. 266, Springer-Verlag, New York, 1972.
- 1973 The surjectivity of invariant differential operators on symmetric spaces, *Ann. of Math.* **98** (1973), 451–480.
- 1978 [DS] *Differential Geometry, Lie Groups and Symmetric Spaces*, Academic Press, New York, 1978.
- 1980a A duality for symmetric spaces with applications to group representations III: Tangent space analysis, *Adv. in Math.* **30** (1980), 297–323.
- 1980b Support of Radon transforms, *Adv. in Math.* **38** (1980), 91–100.
- 1980c *The Radon Transform*, Birkhäuser, Basel and Boston, 1980.
- 1980d “The X-ray transform on a symmetric space,” in: *Proc. Conf. on Differential Geometry and Global Analysis*, Berlin, 1979, Lecture Notes in Math. No. 838, Springer-Verlag, New York, 1980.
- 1981 *Topics in Harmonic Analysis on Homogeneous Spaces*, Birkhäuser, Basel and Boston, 1981.
- 1983a “Ranges of Radon transforms,” AMS Short Course on Computerized Tomography, January, 1982, in: *Proc. Symp. on Applied Mathematics*, American Mathematical Society, Providence, RI, 1983.
- 1983b “The range of the Radon transform on symmetric spaces,” in: *Proc. Conf. on Representation Theory of Reductive Lie Groups*, Park City, Utah, 1982, P. Trombi, ed., Birkhäuser, Basel and Boston, 1983, 145–151.
- 1983c “Operational properties of the Radon transform with applications,” in: *Proc. Conf. on Differential Geometry with Applications*, Nové Mesto, Czechoslovakia, 1983, 59–75.
- 1984 [GGA] *Groups and Geometric Analysis: Integral Geometry, Invariant Differential Operators and Spherical Functions*, Academic Press, New York, 1984. Now published by American Mathematical Society, Providence, R.I., 2000.
- 1990 The totally geodesic Radon transform on constant curvature spaces, *Contemp. Math.* **113** (1990), 141–149.
- 1992 The flat horocycle transform for a symmetric space, *Adv. in Math.* **91** (1992), 232–251.
- 1994a “Radon transforms for double fibrations: Examples and viewpoints,” in: *Proc. Conf. 75 Years of Radon Transform*, Vienna, 1992, International Press, Hong Kong, 1994, 163–179.
- 1994b *Geometric Analysis on Symmetric Spaces*, Math. Surveys Monographs No. 39, American Mathematical Society, Providence, RI, 1994.

- HERGLOTZ, G.  
 1931 *Mechanik der Kontinua*, Lecture notes, University of Göttingen, Göttingen, Germany, 1931.
- HERTLE, A.  
 1982 A characterization of Fourier and Radon transforms on Euclidean spaces, *Trans. Amer. Math. Soc.* **273** (1982), 595–608.  
 1983 Continuity of the Radon transform and its inverse on Euclidean space, *Math. Z.* **184** (1983), 165–192.  
 1984 On the range of the Radon transform and its dual, *Math. Ann.* **267** (1984), 91–99.
- HILGERT, J.  
 1994 “Radon transform on half planes via group theory,” in: *Noncompact Lie Groups and Some of Their Applications*, Kluwer Academic Publishers, Norwell, MA, 1994, 55–67.
- HÖRMANDER, L.  
 1963 *Linear Partial Differential Operators*, Springer-Verlag, Berlin and New York, 1963.  
 1983 *The Analysis of Linear Partial Differential Operators I, II*, Springer-Verlag, Berlin and New York, 1983.
- HOUNSFIELD, G.N.  
 1973 Computerized transverse axial scanning tomography, *British J. Radiology* **46** (1973), 1016–1022.
- HU, M.-C.  
 1973 Determination of the conical distributions for rank one symmetric spaces, Thesis, MIT, Cambridge, MA, 1973.  
 1975 Conical distributions for rank one symmetric spaces, *Bull. Amer. Math. Soc.* **81** (1975), 98–100.
- ISHIKAWA, S.  
 1997 The range characterization of the totally geodesic Radon transform on the real hyperbolic space, *Duke Math. J.* **90** (1997), 149–203.
- JOHN, F.  
 1934 Bestimmung einer Funktion aus ihren Integralen über gewisse Mannigfaltigkeiten, *Math. Ann.* **109** (1934), 488–520.  
 1935 Anhängigkeit zwischen den Flächenintegralen einer stetigen Funktion, *Math. Ann.* **111** (1935), 541–559.  
 1938 The ultrahyperbolic differential equation with 4 independent variables, *Duke Math. J.* **4** (1938), 300–322.  
 1955 *Plane Waves and Spherical Means*, Wiley-Interscience, New York, 1955.
- KAKEHI, T.  
 1992 Range characterization of Radon transforms on complex projective spaces, *J. Math. Kyoto Univ.* **32** (1992), 387–399.  
 1993 Range characterization of Radon transforms on  $\mathbf{S}^n$  and  $\mathbf{P}^n\mathbf{R}$ , *J. Math. Kyoto Univ.* **33** (1993), 315–228.  
 1995 Range characterization of Radon transforms on quaternion projective spaces, *Math. Ann.* **301** (1995), 613–625.

- 1998 Integral geometry on Grassmann manifolds and calculus of invariant differential operators, preprint, 1998.
- KAKEHI, T. AND TSUKAMOTO, C.  
1993 Characterization of images of Radon transforms, *Adv. Stud. Pure Math.* **22** (1993), 101–116.
- KATSEVICH, A.I.  
1997 Range of the Radon transform on functions which do not decay fast at infinity, *SIAM J. Math. Anal.* **28** (1997), 852–866.
- KOLK, J. AND VARADARAJAN, V.S.  
1992 Lorentz invariant distributions supported on the forward light cone, *Compositio Math.* **81** (1992), 61–106.
- KOORNWINDER, T.H.  
1975 A new proof of the Paley-Wiener theorem for the Jacobi transform, *Ark. Mat.* **13** (1975), 145–149.
- KUCHMENT, P.A. AND LVIN, S.Y.  
1990 Paley-Wiener theorem for exponential Radon transform, *Acta Appl. Math.* **18** (1990), 251–260.
- KURUSA, A.  
1991a A characterization of the Radon transform's range by a system of PDE's, *J. Math. Anal. Appl.* **161** (1991), 218–226.  
1991b The Radon transform on hyperbolic spaces, *Geom. Dedicata* **40** (1991), 325–339.  
1994 Support theorems for the totally geodesic Radon transform on constant curvature spaces, *Proc. Amer. Math. Soc.* **122** (1994), 429–435.
- LAX, P. AND PHILLIPS, R.S.,  
1967 *Scattering Theory*, Academic Press, New York, 1967.  
1979 Translation representations for the solution of the non-Euclidean wave equation, *Comm. Pure Appl. Math.* **32** (1979), 617–667.  
1982 A local Paley-Wiener theorem for the Radon transform of  $L^2$  functions in a non-Euclidean setting, *Comm. Pure Appl. Math.* **35** (1982), 531–554.
- LICHNEROWICZ, A. AND WALKER, A.G.  
1945 Sur les espaces Riemanniens harmoniques de type hyperbolique normal, *C. R. Acad. Sci. Paris* **221** (1945), 394–396.
- LISSIANOI, S. AND PONOMAREV, I.  
1997 On the inversion of the geodesic Radon transform on the hyperbolic plane, *Inverse Problems* **13** (1997), 1053–1062.
- LUDWIG, D.  
1966 The Radon transform on Euclidean space, *Comm. Pure Appl. Math.* **23** (1966), 49–81.
- MADYCH, W.R. AND SOLMON, D.C.  
1988 A range theorem for the Radon transform, *Proc. Amer. Math. Soc.* **104** (1988), 79–85.
- MATSUMOTO, H.  
1971 Quelques remarques sur les espaces riemanniens isotropes, *C. R. Acad. Sci. Paris* **272** (1971), 316–319.

- MELROSE, R.B.  
 1995 *Geometric Scattering Theory*, Cambridge University Press, London and New York, 1995.
- MICHEL, L.  
 1972 Sur certains tenseurs symétriques des projectifs réels, *J. Math. Pures Appl.* **51** (1972), 275–293.  
 1973 Problèmes d’analyse géométrique liés à la conjecture de Blaschke, *Bull. Soc. Math. France* **101** (1973), 17–69.
- NAGANO, T.  
 1959 Homogeneous sphere bundles and the isotropic Riemannian manifolds, *Nagoya Math. J.* **15** (1959), 29–55.
- NATTERER, F.  
 1986 *The Mathematics of Computerized Tomography*, John Wiley, New York, 1986.
- NIEVERGELT, Y.  
 1986 Elementary inversion of Radon’s transform, *SIAM Rev.* **28** (1986), 79–84.
- ORLOFF, J.  
 1985 Limit formulas and Riesz potentials for orbital integrals on symmetric spaces, thesis, MIT, Cambridge, MA, 1985.  
 1987 “Orbital integrals on symmetric spaces,” in: *Non-Commutative Harmonic Analysis and Lie Groups*, Lecture Notes in Math. No. 1243, Springer-Verlag, Berlin and New York, 1987, 198–219.  
 1990a Invariant Radon transforms on a symmetric space, *Contemp. Math.* **113** (1990), 233–242.  
 1990b Invariant Radon transforms on a symmetric space, *Trans. Amer. Math. Soc.* **318** (1990), 581–600.
- ORTNER, N.  
 1980 Faltung hypersingularer Integraloperatoren, *Math. Ann.* **248** (1980), 19–46.
- PALAMODOV, V. AND DENISJUK, A.  
 1988 Inversion de la transformation de Radon d’après des données incomplètes, *C. R. Acad. Sci. Paris Sér. I Math.* **307** (1988), 181–183.
- PALEY, R. AND WIENER, N.  
 1934 *Fourier Transforms in the Complex Domain*, American Mathematical Society, Providence, RI, 1934.
- PETROV, E.F.  
 1977 A Paley-Wiener theorem for a Radon complex, *Izv. Vyssh. Uchebn. Zaved. Math.* **3** (1977), 66–77.
- POISSON, S.D.  
 1820 *Nouveaux Mémoires de l’Académie des Sciences*, Vol. III, 1820.
- PRUDNIKOV, A.P., BRYCHKOV, YU. A., AND MARICHEV, O.I.  
 1990 *Integrals and Series*, Vol. I–V, Gordon and Breach, New York, 1990.

- QUINTO, F.T.  
 1978 On the locality and invertibility of the Radon transform, thesis, MIT, Cambridge, MA, 1978.  
 1980 The dependence of the generalized Radon transform on defining measures, *Trans. Amer. Math. Soc.* **257** (1980), 331–346.  
 1981 Topological restrictions on double fibrations and Radon transforms, *Proc. Amer. Math. Soc.* **81** (1981), 570–574.  
 1982 Null spaces and ranges for the classical and spherical Radon transforms, *J. Math. Anal. Appl.* **90** (1982), 405–420.  
 1983 The invertibility of rotation invariant Radon transforms, *J. Math. Anal. Appl.* **91** (1983), 510–521; erratum, *J. Math. Anal. Appl.* **94** (1983), 602–603.  
 1987 Injectivity of rotation invariant Radon transforms on complex hyperplanes in  $\mathbb{C}^n$ , *Contemp. Math.* **63** (1987), 245–260.  
 1992 A note on flat Radon transforms, *Contemp. Math.* **140** (1992), 115–121.  
 1993a Real analytic Radon transforms on rank one symmetric spaces, *Proc. Amer. Math. Soc.* **117** (1993), 179–186.  
 1993b Pompeiu transforms on geodesic spheres in real analytic manifolds, *Israel J. Math.* **84** (1993), 353–363.  
 1994a “Radon transforms satisfying the Bolker assumption,” in: *Proc. Conf. 75 Years of Radon Transform*, Vienna, 1992, International Press, Hong Kong, 1994, 231–244.  
 1994b Radon Transform on Curves in the Plane, in: *Lectures in Appl. Math.* No. 30, American Mathematical Society, Providence, RI, 1994.
- RADON, J.  
 1917 Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten, *Ber. Verh. Sächs. Akad. Wiss. Leipzig. Math. Nat. Kl.* **69** (1917), 262–277.
- RAMM, A.G.  
 1995 Radon transform on distributions, *Proc. Japan Acad. Ser. A Math. Sci.* **71** (1995), 205–206.
- RAMM, A. AND KATSEVICH, A.I.  
 1996 *The Radon transform and Local Tomography*, CRC Press, Boca Raton, FL, 1996.
- RENARD, D.  
 1997 Formule d’inversion des intégrales orbitales tordues sur les groupes de Lie réductifs réels, *J. Funct. Anal.* **147** (1997), 164–236.
- RICHTER, F.  
 1986a Differential Operatoren auf Euclidischen  $k$ -Ebenräumen und Radon Transformationen, Dissertation, Humboldt Universität, Berlin, 1986.  
 1986b *On the  $k$ -Dimensional Radon Transform of Rapidly Decreasing Functions*, Lecture Notes in Math. No. 1209, Springer-Verlag, Berlin and New York, 1986.  
 1990 On fundamental differential operators and the  $p$ -plane transform, *Ann. Global Anal. Geom.* **8** (1990), 61–75.



- RIESZ, M.  
 1949 L'integrale de Riemann-Liouville et le problème de Cauchy, *Acta Math.* **81** (1949), 1–223.
- ROUVIÈRE, F.  
 1983 Sur la transformation d'Abel de groupes des Lie semisimples de rang un, *Ann. Scuola Norm. Sup. Pisa* **10** (1983), 263–290.  
 1994 *Transformations de Radon*, Lecture notes, Université de Nice, Nice, France, 1994.  
 1999 *Inverting Radon transforms: the group-theoretic approach*, preprint, 1999.
- RUBIN, B.  
 1998a Inversion of  $k$ -plane transform via continuous wavelet transforms, *J. Math. Anal. Appl.* **220** (1998), 187–203.  
 1998b Inversion of fractional integrals related to the spherical Radon transform, *J. Funct. Anal.* **157** (1998), 470–487.
- SCHIMMING, R. AND SCHLICHTKRULL, H.  
 1994 Helmholtz operators on harmonic manifolds, *Acta Math.* **173** (1994), 235–258.
- SCHWARTZ, L.  
 1966 *Théories des Distributions*, 2nd Ed., Hermann, Paris, 1966.
- SEKERIS, A.  
 1993 A theorem on the support for the Radon transform in a complex space, *Math. Notes* **54** (1993), 975–976.
- SELBERG, A.  
 1963 “Discontinuous groups and harmonic analysis,” in: *Proc. International Congress Math.*, Stockholm, 1962, 177–189, Almqvist & Wiksells, Uppsala, 1963.
- SEMYANISTY, V.I.  
 1960 On some integral transforms in Euclidean space, *Soviet Math. Dokl.* **1** (1960), 1114–1117.  
 1961 Homogeneous functions and some problems of integral geometry in spaces of constant curvature, *Soviet Math. Dokl.* **2** (1961), 59–62.
- SHAHSHAHANI, M. AND SITARAM, A.  
 1987 The Pompeiu problem in exterior domains in symmetric spaces, *Contemp. Math.* **63** (1987), 267–277.
- SHEPP, L.A., ET AL.  
 1983 “AMS Short Courses on Computerized Tomography,” Cincinnati, OH, January, 1982, in: *Proc. Sympos. Appl. Math.* **27**, American Mathematical Society, Providence, RI, 1983.
- SHEPP, L.A. AND KRUSKAL, J.B.  
 1978 Computerized tomography: The new medical X-ray technology, *Amer. Math. Monthly* **85** (1978), 420–438.
- SMITH, K.T. AND SOLMON, D.C.  
 1975 Lower-dimensional integrability of  $L^2$  functions, *J. Math. Anal. Appl.* **51** (1975), 539–549.

- SMITH, K.T., SOLMON, D.C., AND WAGNER S.L.  
 1977 Practical and mathematical aspects of the problem of reconstructing objects from radiographs, *Bull. Amer. Math. Soc.* **83** (1977), 1227–1270. Addendum *ibid* **84** (1978), p. 691.
- SOLMON, D.C.  
 1976 The X-ray transform, *J. Math. Anal. Appl.* **56** (1976), 61–83.  
 1987 Asymptotic formulas for the dual Radon transform, *Math. Z.* **195** (1987), 321–343.
- STRICHARTZ, R.S.  
 1981  $L^p$ : Estimates for Radon transforms in Euclidean and non-Euclidean spaces, *Duke Math. J.* **48** (1981), 699–727.  
 1992 Radon inversion-variation on a theme, *Amer. Math. Monthly* **89** (1982), 377–384 and 420–425.
- SYMEONIDIS, E.  
 1999 On the image of a generalized  $d$ -plane transform on  $\mathbf{R}^n$ , *J.Lie Theory* **9** (1999), 39–68.
- SZABO, Z.I.  
 1991 A short topological proof for the symmetry of two-point homogeneous spaces, *Invent. Math.* **106** (1991), 61–64.
- TITS, J.  
 1955 Sur certains classes d’espaces homogènes de groupes de Lie, *Acad. Roy. Belg. Cl. Sci. Mém. Collect.* **29** (1955), No. 3.
- TRÈVES, F.  
 1963 Equations aux dérivées partielles inhomogènes a coefficients constants dépendent de parametres, *Ann. Inst. Fourier (Grenoble)* **13** (1963), 123–138.
- VOLCHKOV, V.V.  
 1997 Theorems on injectivity of the Radon transform on spheres, *Dokl. Akad. Nauk* **354** (1997), 298–300.
- WANG, H.C.  
 1952 Two-point homogeneous spaces, *Ann. of Math.* **55** (1952), 177–191.
- WEISS, B.  
 1967 Measures which vanish on half spaces, *Proc. Amer. Math. Soc.* **18** (1967), 123–126.
- WHITTAKER, E.T. AND WATSON, G.N.  
 1927 *A Course of Modern Analysis*, Cambridge University Press, London, 1927.
- WIEGERINCK, J.J.O.O.  
 1985 A support theorem for the Radon transform on  $\mathbf{R}^n$ , *Nederl. Akad. Wetensch. Proc. A* **88** (1985), 87–93.
- WOLF, J.A.  
 1967 *Spaces of Constant Curvature*, McGraw–Hill, New York, 1967.

- ZALCMAN, F.  
1980 Offbeat integral geometry, *Amer. Math. Monthly* **87** (1980), 161–175.  
1982 Uniqueness and nonuniqueness for the Radon transforms, *Bull. London Math. Soc.* **14** (1982), 241–245.
- ZHU, F-L.  
1996 Sur la transformation de Radon de  $M_{2,n}(\mathbf{H})$ , *Bull. Sci. Math.* **120** (1996), 99–128.

## Notational Conventions

**Algebra** As usual,  $\mathbf{R}$  and  $\mathbb{C}$  denote the fields of real and complex numbers, respectively, and  $\mathbb{Z}$  the ring of integers. Let

$$\mathbf{R}^+ = \{t \in \mathbf{R} : t \geq 0\}, \quad \mathbb{Z}^+ = \mathbb{Z} \cap \mathbf{R}^+.$$

If  $\alpha \in \mathbb{C}$ ,  $\operatorname{Re} \alpha$  denotes the real part of  $\alpha$ ,  $\operatorname{Im} \alpha$  its imaginary part,  $|\alpha|$  its modulus.

If  $G$  is a group,  $A \subset G$  a subset and  $g \in G$  an element, we put

$$A^g = \{gag^{-1} : a \in A\}, \quad g^A = \{aga^{-1} : a \in A\}.$$

The group of real matrices leaving invariant the quadratic form

$$x_1^2 + \cdots + x_p^2 - x_{p+1}^2 - \cdots - x_{p+q}^2$$

is denoted by  $\mathbf{O}(p, q)$ . We put  $\mathbf{O}(n) = \mathbf{O}(o, n) = \mathbf{O}(n, o)$ , and write  $\mathbf{U}(n)$  for the group of  $n \times n$  unitary matrices. The group of isometries of Euclidean  $n$ -space  $\mathbf{R}^n$  is denoted by  $M(n)$ .

**Geometry** The  $(n-1)$ -dimensional unit sphere in  $\mathbf{R}^n$  is denoted by  $\mathbf{S}^{n-1}$ ,  $\Omega_n$  denotes its area. The  $n$ -dimensional manifold of hyperplanes in  $\mathbf{R}^n$  is denoted by  $\mathbf{P}^n$ . If  $0 < d < n$  the manifold of  $d$ -dimensional planes in  $\mathbf{R}^n$  is denoted by  $G(d, n)$ ; we put  $G_{d,n} = \{\sigma \in G(d, n) : o \in \sigma\}$ . In a metric space,  $B_r(x)$  denotes the open ball with center  $x$  and radius  $r$ ;  $S_r(x)$  denotes the corresponding sphere. For  $\mathbf{P}^n$  we use the notation  $\beta_A(0)$  for the set of hyperplanes  $\xi \subset \mathbf{R}^n$  of distance  $< A$  from 0,  $\sigma_A$  for the set of hyperplanes of distance  $= A$ . The hyperbolic  $n$ -space is denoted by  $\mathbf{H}^n$  and the  $n$ -sphere by  $\mathbf{S}^n$ .

**Analysis** If  $X$  is a topological space,  $C(X)$  (resp.  $C_c(X)$ ) denotes the sphere of complex-valued continuous functions (resp. of compact support). If  $X$  is a manifold, we denote:

$$\begin{aligned} C^m(X) &= \left\{ \begin{array}{l} \text{complex-valued } m\text{-times continuously} \\ \text{differentiable functions on } X \end{array} \right\} \\ \mathcal{C}^\infty(X) &= \mathcal{E}(X) = \cap_{m>0} C^m(X). \\ \mathcal{C}_c^\infty(X) &= \mathcal{D}(X) = C_c(X) \cap \mathcal{C}^\infty(X). \\ \mathcal{D}'(X) &= \{\text{distributions on } X\}. \\ \mathcal{E}'(X) &= \{\text{distributions on } X \text{ of compact support}\}. \\ \mathcal{D}_A(X) &= \{f \in \mathcal{D}(X) : \text{support } f \subset A\}. \\ \mathcal{S}(\mathbf{R}^n) &= \{\text{rapidly decreasing functions on } \mathbf{R}^n\}. \\ \mathcal{S}'(\mathbf{R}^n) &= \{\text{tempered distributions on } \mathbf{R}^n\}. \end{aligned}$$

The subspaces  $\mathcal{D}_H, \mathcal{S}_H, \mathcal{S}^*, \mathcal{S}_o$  of  $\mathcal{S}$  are defined pages in Ch. I, §§1–2.

While the functions considered are usually assumed to be complex-valued, we occasionally use the notation above for spaces of real-valued functions.

The Radon transform and its dual are denoted by  $f \rightarrow \widehat{f}$ ,  $\varphi \rightarrow \check{\varphi}$ , the Fourier transform by  $f \rightarrow \widetilde{f}$  and the Hilbert transform by  $\mathcal{H}$ .

$I^\alpha$ ,  $I_-^\lambda$ ,  $I_o^\lambda$  and  $I_+^\lambda$  denote Riesz potentials and their generalizations.  $M^r$  the mean value operator and orbital integral,  $L$  the Laplacian on  $\mathbf{R}^n$  and the Laplace-Beltrami operator on a pseudo-Riemannian manifold. The operators  $\square$  and  $\Lambda$  operate on certain function spaces on  $\mathbf{P}^n$ ;  $\square$  is also used for the Laplace-Beltrami operator on a Lorentzian manifold, and  $\Lambda$  is also used for other differential operators.

# Index

- Abel's integral equation, 11
- Adjoint space, 119
- Antipodal manifold, 110
- Approximate reconstruction, 48
- Ásgeirsson's mean-value theorem, 39, 158
- Cauchy principal value, 19
- Cauchy problem, 42
- Cayley plane, 113
- Cone
  - backward, 43
  - forward, 43
  - light, 45, 128, 130
  - null, 128
  - retrograde, 130, 134
  - solid, 45
- Conjugacy class, 64
- Conjugate point, 110
- Curvature, 124, 126
- Cusp forms, 81
- Darboux equation, 16, 91, 136
- Delta distribution, 27, 161
- Dirichlet problem, 73
- Distribution, 149
  - convolution, 150
  - derivative, 149
  - Fourier transform of, 151
  - Radon transform of, 23
  - support of, 149
  - tempered, 149
- Divergence theorem, 13
- Double fibration, 57
- Duality, 57
- Dual transform, 2, 76
- Elliptic space, 93
- Exponentially decreasing functions, 118
- Fourier transform, 150
- Fundamental solution, 158
- Funk transform, 65, 66
- Generalized sphere, 64
- Gram determinant, 127
- Grassmann manifold, 29, 76
- Harmonic line function, 40
- Hilbert transform, 18
- Horocycle, 68

- Huygens' principle, 45, 144, 145
- Hyperbolic space, 85, 96
  - Cayley, 117
  - complex, 117
  - quaternion, 117
  - real, 117
- Incident, 55
- Inductive limit, 147
- Invariant differential operators, 3
- Inversion formula, 15, 25, 27, 29, 41, 66, 68, 70, 91, 94, 96–98, 101, 103
- Isometry, 3, 21, 123
- Isotropic, 123, 124
  - geodesic, 129
  - space, 119
  - vector, 128
- John's mean value theorem, 40, 74
- Laplace-Beltrami operator, 123
- Laplacian, 3
- Light cone, 45, 128, 130
- Lorentzian, 123
  - manifold, 123
  - structure, 123
- Mean value operator, 8, 90, 111, 133
- Mean value theorem, 39, 40, 158
- Modular group, 80
- Multiplicity, 111
- Null cone, 128
- Orbital integrals, 64, 128, 133
- Paley-Wiener theorem, 14, 152
- Plancherel formula, 20, 151
- Plane wave, 1
  - normal of, 41
- Plücker coordinates, 38
- Poisson
  - equation, 164
  - integral, 73
  - kernel, 73
- Projective spaces
  - Cayley, 113
  - complex, 113
  - quaternion, 113
  - real, 113
- Property ( $S$ ), 50
- Pseudo-Riemannian
  - manifold, 123
  - structure, 123
- Radial function, 16
- Radiograph, 48
- Radon transform, 2, 60
  - $d$ -dimensional, 28
  - for a double fibration, 59
  - of distributions, 62
  - of measures, 61
- Rapidly decreasing functions, 4, 118
- Residue, 161
- retrograde
  - cone, 134
- Retrograde cone, 130
  - solid, 130
- retrograde cone, 134
- Riesz potential, 161
- Riesz potentials
  - generalized, 137
- Seminorms, 4, 147
- Source, 43
- Spacelike, 128
  - geodesic, 129
  - vectors, 128
- Spherical function, 99
- Spherical slice transform, 107
- Spherical transform, 99
- Support theorem, 2, 9, 85
- Theta series, 81
- Timelike, 128
  - geodesic, 129
  - vectors, 128
- Totally geodesic, 83, 110

- Transversality, 57
- Two-point homogeneous, 83
- Wave, 1, 43
  - incoming, 43
  - outgoing, 43
- Wave equation, 42
- Wave operator, 134
- X-ray, 47
  - reconstruction, 47
  - transform, 28, 37, 68, 99, 118



Errata and Addenda in S. Helgason: The Radon Transform 2<sup>nd</sup> Edition (First Printing)

These have been entered in the downloadable version.

Page and line in $\begin{cases} \text{above} \\ \text{below} \end{cases}$	Instead of:	Read:
3 <sup>2</sup>	omit “ $\varphi$ on $\mathbb{P} \dots$ , functions ”	
5 <sub>10</sub>	$R^n$	$\mathbb{P}^n$
11 <sub>12</sub>	$\pi^{-\frac{1}{2}}$	$\pi^{\frac{1}{2}}$
12 <sup>4</sup>	(i)	(ii)
16 <sub>11</sub>	$dk)$	$)dk$
17 <sub>14</sub>	replaced by $f(x) = 0( x ^{-n})$	dropped
17 <sub>13</sub>	with	for all lines with
25 <sub>18</sub>	$f * \varphi$	$f \times \varphi$
26 <sub>14</sub>	$\neq$	$=$
26 <sup>12</sup>	$\text{supp}(\widehat{S}) \subset S_R(0)$	$\text{supp}(S) \subset S_R(0)$
28 <sup>8</sup>	$\psi_n$	$\psi$
34 <sup>9</sup>	(63)	(64)
35 <sup>3</sup>	(68)	(69)
35 <sup>7</sup>	$i, j$	$i \neq j$
35 <sup>6</sup> , 35 <sub>1</sub> , 36 <sup>2</sup> , 36 <sup>9</sup>	$\partial_{2,1}, \partial_{2,d+1}$	$\partial_{1,1}, \partial_{1,d+1}$
36 <sup>9</sup> , 36 <sup>12</sup> , 36 <sup>14</sup>	$\partial_{2,n}$	$\partial_{1,n}$
39 <sup>7</sup>	$-\xi_1 + \xi_2 - \eta_1$	$-\xi_1 + \xi_2 + \eta_1$
40 <sup>7</sup>	$z^2$	$(z^2 + 1)$
40 <sub>3</sub>	$xy$	$xy^2$
42 <sub>5</sub>	$f_1(0)$	$f_1(x)$
42 <sub>2</sub>	$h(\langle x, w \rangle)$	$h(\langle x, w \rangle + t)$

Page and line in $\begin{cases} \text{above} \\ \text{below} \end{cases}$	Instead of:	Read:
44 <sup>9</sup>	$w_{i_k}$	$w_{i_k} dw$
44 <sub>5</sub>	$\tilde{f}$	$\tilde{f}_1$
58 <sup>11</sup>	$o$	$c$
62 <sup>4</sup>	$(\lambda(D)f)^\sim$	$(\lambda(D)f)^\wedge$
67 <sup>4</sup>	$\cosh$	$\coth$
69 <sup>3</sup>	$k'a_{t'}, n'a_t N \cdot 0$	$k'a_{t'} n'a_t N \cdot 0$
69 <sup>12</sup>	$-2$	$-1$
69 <sub>6</sub>	$ch s$	$ch^3 s$
97 <sup>5</sup>	a circle	two circles.
97 <sup>7</sup>	“a circular arc”	“a pair of circular arcs”
98	$k - 1$	$(k - 1)!$ .
101 <sup>10</sup> , 101 <sub>2</sub>	$f \times \tau$	$\pi^{-1} f \times \tau$
101 <sub>7</sub>	$-4\pi^2$	$-4\pi$
103 <sup>3</sup> , 103 <sup>12</sup>	$(4n + 1) \sum_0^\infty$	$\sum_0^\infty (4n + 1)$
119 <sup>6</sup>	formula	formula $f = Q(L)((\hat{f})^\sim)$ . Here $Q$ is given by
153 <sub>4</sub>	sequences	positive sequences
153 <sub>15</sub>	$n + 1$	$-(n + 1)$
153 <sup>5</sup>	Absolute value signs missing	
156 <sub>2</sub>	(1)	(25)
156 <sub>5</sub>	Interchange $P_1$ and $G_1$	
167 <sup>13</sup>	(60)	(61)
180 <sup>12</sup>	397	394

44<sub>4</sub> Here one should use the following remark: If  $\varphi(\lambda)$  is even, holomorphic on  $\mathbb{C}$  and satisfies the exponential type estimate (13) in Theorem 3.3, Ch. V, then the same holds for the function  $\Phi$  on  $\mathbb{C}^n$  given by  $\Phi(\zeta) =$

$\Phi(\zeta_1, \dots, \zeta_n) = \varphi(\lambda)$  where  $\lambda^2 = \zeta_1^2 + \dots + \zeta_n^2$ . To see this put

$$\lambda = \mu + iv, \quad \zeta = \xi + i\eta \quad \mu, \nu \in \mathbf{R}, \quad \xi, \eta \in \mathbf{R}^n.$$

Then

$$\mu^2 - \nu^2 = |\xi|^2 - |\eta|^2, \quad \mu^2 \nu^2 = (\xi \cdot \eta)^2,$$

so

$$|\lambda|^4 = (|\xi|^2 - |\eta|^2)^2 + 4(\xi \cdot \eta)^2$$

and

$$2|\operatorname{Im} \lambda|^2 = |\eta|^2 - |\xi|^2 + [(|\xi|^2 - |\eta|^2)^2 + 4(\xi \cdot \eta)^2]^{1/2}.$$

Since  $|(\xi \cdot \eta)| \leq |\xi| |\eta|$  this implies  $|\operatorname{Im} \lambda| \leq |\eta|$  so the estimate (13) follows for  $\Phi$ .

45<sub>1</sub> Note that Part (ii) can also be stated: The solution is outgoing (incoming) if and only if

$$\int_{\pi} f_0 = \int_{H_{\pi}} f_1 \quad \left( \int_{\pi} f_0 = - \int_{H_{\pi}} f_1 \right)$$

for an arbitrary hyperplane  $\pi (0 \notin \pi)$   $H_{\pi}$  being the halfspace with boundary  $\pi$  which does not contain 0.

58<sup>11</sup> The subscripts 0 should be  $c$ .

102<sup>4</sup> The function  $\tau$  is only locally integrable but not integrable. However for  $\lambda$  real  $\tau\varphi_{\lambda}$  is integrable and (62) holds by virtue of the proof of (53), p. 100.

102<sub>11</sub> The implication (62) & (63)  $\Rightarrow$  (60) is justified as follows. Using the decomposition  $\tau = \varphi\tau + (1 - \varphi)\tau$  where  $\varphi$  is the characteristic function of a ball  $B(0)$  we see that  $f \times \tau \in L^2(X)$  for  $f \in \mathcal{D}^b(X)$ . Since  $\sigma \in L^1(X)$  we have  $f \times \tau \times \sigma \in L^2(X)$ . Now (60) follows since by the Plancherel theorem the spherical transform is injective on  $L^2$ .

155<sup>3</sup> From formula (24) below for  $j = 0$  and  $j = 1$ , it is clear that sequences  $\delta_1, \delta_2, \dots, M_1, M_2, \dots$  ( $\delta_i > 0, M_1 > 0$ ) exist such that (3) holds for  $j = 0, j = 1$ . Fix the  $\delta_1$  and  $M_1$ . Then the idea is to shrink  $\delta_2, \delta_3 \dots$  and  $1/M_2, 1/M_3, \dots$  so that by the argument below, (3) holds for  $j = 2$ , etc.

167<sup>13</sup> (60) should be (61). It should also be observed as a result of (39) that if  $f(x) = O(|x|^{-N})$  then  $I^{\lambda}f(x)$  is holomorphic near  $\lambda = 0$  and  $I^0 = f$ .

168<sub>11</sub> The idea of a proof of this nature involving a contour like  $\Gamma_m$  appears already in Ehrenpreis [1956], although not correctly carried out in details.