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Notational Conventions

Algebra As usual, \mathbf{R} and \mathbb{C} denote the fields of real and complex numbers, respectively, and \mathbb{Z} the ring of integers. Let

$$\mathbf{R}^+ = \{t \in \mathbf{R} : t \geq 0\}, \quad \mathbb{Z}^+ = \mathbb{Z} \cap \mathbf{R}^+.$$

If $\alpha \in \mathbb{C}$, $\operatorname{Re} \alpha$ denotes the real part of α , $\operatorname{Im} \alpha$ its imaginary part, $|\alpha|$ its modulus.

If G is a group, $A \subset G$ a subset and $g \in G$ an element, we put

$$A^g = \{gag^{-1} : a \in A\}, \quad g^A = \{aga^{-1} : a \in A\}.$$

The group of real matrices leaving invariant the quadratic form

$$x_1^2 + \cdots + x_p^2 - x_{p+1}^2 - \cdots - x_{p+q}^2$$

is denoted by $\mathbf{O}(p, q)$. We put $\mathbf{O}(n) = \mathbf{O}(o, n) = \mathbf{O}(n, o)$, and write $\mathbf{U}(n)$ for the group of $n \times n$ unitary matrices. The group of isometries of Euclidean n -space \mathbf{R}^n is denoted by $M(n)$.

Geometry The $(n-1)$ -dimensional unit sphere in \mathbf{R}^n is denoted by \mathbf{S}^{n-1} , Ω_n denotes its area. The n -dimensional manifold of hyperplanes in \mathbf{R}^n is denoted by \mathbf{P}^n . If $0 < d < n$ the manifold of d -dimensional planes in \mathbf{R}^n is denoted by $G(d, n)$; we put $G_{d,n} = \{\sigma \in G(d, n) : o \in \sigma\}$. In a metric space, $B_r(x)$ denotes the open ball with center x and radius r ; $S_r(x)$ denotes the corresponding sphere. For \mathbf{P}^n we use the notation $\beta_A(0)$ for the set of hyperplanes $\xi \subset \mathbf{R}^n$ of distance $< A$ from 0, σ_A for the set of hyperplanes of distance $= A$. The hyperbolic n -space is denoted by \mathbf{H}^n and the n -sphere by \mathbf{S}^n .

Analysis If X is a topological space, $C(X)$ (resp. $C_c(X)$) denotes the sphere of complex-valued continuous functions (resp. of compact support). If X is a manifold, we denote:

$$\begin{aligned} C^m(X) &= \left\{ \begin{array}{l} \text{complex-valued } m\text{-times continuously} \\ \text{differentiable functions on } X \end{array} \right\} \\ \mathcal{C}^\infty(X) &= \mathcal{E}(X) = \cap_{m>0} C^m(X). \\ \mathcal{C}_c^\infty(X) &= \mathcal{D}(X) = C_c(X) \cap \mathcal{C}^\infty(X). \\ \mathcal{D}'(X) &= \{\text{distributions on } X\}. \\ \mathcal{E}'(X) &= \{\text{distributions on } X \text{ of compact support}\}. \\ \mathcal{D}_A(X) &= \{f \in \mathcal{D}(X) : \text{support } f \subset A\}. \\ \mathcal{S}(\mathbf{R}^n) &= \{\text{rapidly decreasing functions on } \mathbf{R}^n\}. \\ \mathcal{S}'(\mathbf{R}^n) &= \{\text{tempered distributions on } \mathbf{R}^n\}. \end{aligned}$$

The subspaces $\mathcal{D}_H, \mathcal{S}_H, \mathcal{S}^*, \mathcal{S}_o$ of \mathcal{S} are defined pages in Ch. I, §§1–2.

While the functions considered are usually assumed to be complex-valued, we occasionally use the notation above for spaces of real-valued functions.

The Radon transform and its dual are denoted by $f \rightarrow \widehat{f}$, $\varphi \rightarrow \check{\varphi}$, the Fourier transform by $f \rightarrow \widetilde{f}$ and the Hilbert transform by \mathcal{H} .

I^α , I_-^λ , I_o^λ and I_+^λ denote Riesz potentials and their generalizations. M^r the mean value operator and orbital integral, L the Laplacian on \mathbf{R}^n and the Laplace-Beltrami operator on a pseudo-Riemannian manifold. The operators \square and Λ operate on certain function spaces on \mathbf{P}^n ; \square is also used for the Laplace-Beltrami operator on a Lorentzian manifold, and Λ is also used for other differential operators.

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Errata and Addenda in S. Helgason: The Radon Transform 2nd Edition (First Printing)

These have been entered in the downloadable version.

Page and line in $\begin{cases} \text{above} \\ \text{below} \end{cases}$	Instead of:	Read:
3 ²	omit “ φ on $\mathbb{P} \dots$, functions ”	
5 ₁₀	R^n	\mathbb{P}^n
11 ₁₂	$\pi^{-\frac{1}{2}}$	$\pi^{\frac{1}{2}}$
12 ⁴	(i)	(ii)
16 ₁₁	$dk)$	$)dk$
17 ₁₄	replaced by $f(x) = 0(x ^{-n})$	dropped
17 ₁₃	with	for all lines with
25 ₁₈	$f * \varphi$	$f \times \varphi$
26 ₁₄	\neq	$=$
26 ¹²	$\text{supp}(\widehat{S}) \subset S_R(0)$	$\text{supp}(S) \subset S_R(0)$
28 ⁸	ψ_n	ψ
34 ⁹	(63)	(64)
35 ³	(68)	(69)
35 ⁷	i, j	$i \neq j$
35 ⁶ , 35 ₁ , 36 ² , 36 ⁹	$\partial_{2,1}, \partial_{2,d+1}$	$\partial_{1,1}, \partial_{1,d+1}$
36 ⁹ , 36 ¹² , 36 ¹⁴	$\partial_{2,n}$	$\partial_{1,n}$
39 ⁷	$-\xi_1 + \xi_2 - \eta_1$	$-\xi_1 + \xi_2 + \eta_1$
40 ⁷	z^2	$(z^2 + 1)$
40 ₃	xy	xy^2
42 ₅	$f_1(0)$	$f_1(x)$
42 ₂	$h(\langle x, w \rangle)$	$h(\langle x, w \rangle + t)$

Page and line in $\begin{cases} \text{above} \\ \text{below} \end{cases}$	Instead of:	Read:
44 ⁹	w_{i_k}	$w_{i_k} dw$
44 ₅	\tilde{f}	\tilde{f}_1
58 ¹¹	o	c
62 ⁴	$(\lambda(D)f)^\sim$	$(\lambda(D)f)^\wedge$
67 ⁴	\cosh	\coth
69 ³	$k'a_{t'}, n'a_t N \cdot 0$	$k'a_{t'} n'a_t N \cdot 0$
69 ¹²	-2	-1
69 ₆	$ch s$	$ch^3 s$
97 ⁵	a circle	two circles.
97 ⁷	“a circular arc”	“a pair of circular arcs”
98	$k - 1$	$(k - 1)!$.
101 ¹⁰ , 101 ₂	$f \times \tau$	$\pi^{-1} f \times \tau$
101 ₇	$-4\pi^2$	-4π
103 ³ , 103 ¹²	$(4n + 1) \sum_0^\infty$	$\sum_0^\infty (4n + 1)$
119 ⁶	formula	formula $f = Q(L)((\hat{f})^\sim)$. Here Q is given by
153 ₄	sequences	positive sequences
153 ₁₅	$n + 1$	$-(n + 1)$
153 ⁵	Absolute value signs missing	
156 ₂	(1)	(25)
156 ₅	Interchange P_1 and G_1	
167 ¹³	(60)	(61)
180 ¹²	397	394

44₄ Here one should use the following remark: If $\varphi(\lambda)$ is even, holomorphic on \mathbb{C} and satisfies the exponential type estimate (13) in Theorem 3.3, Ch. V, then the same holds for the function Φ on \mathbb{C}^n given by $\Phi(\zeta) =$

$\Phi(\zeta_1, \dots, \zeta_n) = \varphi(\lambda)$ where $\lambda^2 = \zeta_1^2 + \dots + \zeta_n^2$. To see this put

$$\lambda = \mu + iv, \quad \zeta = \xi + i\eta \quad \mu, \nu \in \mathbf{R}, \quad \xi, \eta \in \mathbf{R}^n.$$

Then

$$\mu^2 - \nu^2 = |\xi|^2 - |\eta|^2, \quad \mu^2 \nu^2 = (\xi \cdot \eta)^2,$$

so

$$|\lambda|^4 = (|\xi|^2 - |\eta|^2)^2 + 4(\xi \cdot \eta)^2$$

and

$$2|\operatorname{Im} \lambda|^2 = |\eta|^2 - |\xi|^2 + [(|\xi|^2 - |\eta|^2)^2 + 4(\xi \cdot \eta)^2]^{1/2}.$$

Since $|(\xi \cdot \eta)| \leq |\xi| |\eta|$ this implies $|\operatorname{Im} \lambda| \leq |\eta|$ so the estimate (13) follows for Φ .

45₁ Note that Part (ii) can also be stated: The solution is outgoing (incoming) if and only if

$$\int_{\pi} f_0 = \int_{H_{\pi}} f_1 \quad \left(\int_{\pi} f_0 = - \int_{H_{\pi}} f_1 \right)$$

for an arbitrary hyperplane $\pi (0 \notin \pi)$ H_{π} being the halfspace with boundary π which does not contain 0.

58¹¹ The subscripts 0 should be c .

102⁴ The function τ is only locally integrable but not integrable. However for λ real $\tau\varphi_{\lambda}$ is integrable and (62) holds by virtue of the proof of (53), p. 100.

102₁₁ The implication (62) & (63) \Rightarrow (60) is justified as follows. Using the decomposition $\tau = \varphi\tau + (1 - \varphi)\tau$ where φ is the characteristic function of a ball $B(0)$ we see that $f \times \tau \in L^2(X)$ for $f \in \mathcal{D}^b(X)$. Since $\sigma \in L^1(X)$ we have $f \times \tau \times \sigma \in L^2(X)$. Now (60) follows since by the Plancherel theorem the spherical transform is injective on L^2 .

155³ From formula (24) below for $j = 0$ and $j = 1$, it is clear that sequences $\delta_1, \delta_2, \dots, M_1, M_2, \dots$ ($\delta_i > 0, M_1 > 0$) exist such that (3) holds for $j = 0, j = 1$. Fix the δ_1 and M_1 . Then the idea is to shrink $\delta_2, \delta_3 \dots$ and $1/M_2, 1/M_3, \dots$ so that by the argument below, (3) holds for $j = 2$, etc.

167¹³ (60) should be (61). It should also be observed as a result of (39) that if $f(x) = O(|x|^{-N})$ then $I^{\lambda}f(x)$ is holomorphic near $\lambda = 0$ and $I^0 = f$.

168₁₁ The idea of a proof of this nature involving a contour like Γ_m appears already in Ehrenpreis [1956], although not correctly carried out in details.