SEARCH SPACE PARTITIONING USING CONVEX HULL AND CONCAVITY FEATURES FOR FAST MEDICAL IMAGE RETRIEVAL

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ABSTRACT

A new approach is presented for partitioning an image database by classifying and indexing the convex hull shapes and the number of region concavities. The result is a significant increase in image search and retrieval speed. The convex hull is first determined using a novel and efficient approach based on the geometrical heat differential equation. A set of seven templates is available to approximate the boundary as observed from a particular viewpoint. The convex hull is then represented by a vector of three such shape template indices as observed from three viewpoints. This result enables the regions in the image database to be divided into 344 convex hull classes. Concavity information, obtained using a boundary support parameterization, further partitions the database. A given query must now be compared only to shapes of a few classes, hence searching is much faster. Both theoretical background and practical results are discussed.

1. INTRODUCTION

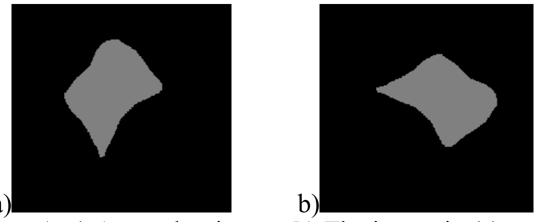
Interest in the potential of digital image databases has increased enormously over the last few years in internet, medical, and environmental applications. In particular, modern medical records systems include large collections of various types of biomedical imagery. Medical specialists and others are exploiting the opportunities offered by the ability to access and manipulate digital X-ray, magnetic resonance, and computed tomography images. Content-based retrieval techniques have the potential to significantly increase the utility of existing medical image databases. The process of using extracted image features to locate a desired image very quickly from a large and dynamic collection is a challenging problem, one that is widely recognized as an area of active research [8]. To efficiently retrieve a medical image according to its content, a medical image retrieval system must be capable of solving the following problems:

- Image segmentation into anatomically distinct regions [14, 20, 21];
- Classification into feature categories by which the image content can be indexed [13];
- Image indexing by features, such as shape [1, 4];
- Image retrieval by matching a query image or feature vector with images or feature vectors in the image database [10, 11].

For an extensive database of images, search space partitioning and indexing techniques can facilitate a significant computational decrease of the query process. It can be accomplished by reducing the size of the search space that a given query must traverse, thus proportionately reducing the total computation load of feature comparison and matching. Moreover, matching a query shape against every shape in the entire database may be infeasible even with a powerful computer platform and a very fast search engine. This paper proposes a new approach capable of partitioning the search space by classifying and indexing extracted shape features.

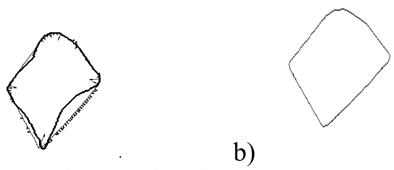
A large number of shape matching methods can be found in the literature. Some of them employ elastic deformation of templates [5, 16], comparison of directional histograms [2], or shocks and skeletal representations of object shape [9, 19]. A fuzzy logic similarity measurement is described by Gadi [6]. Lisani [12] uses differential equations to develop a similarity metric based on the area of shape concavities. Lee [11] proposed an approach that combines polygon curve representation with Fourier descriptors for shape-based retrieval in images of vertebrae.

This paper considers the search space partitioning problem [15] in detail. A solution approach is developed employing the local shape features of convex hull and the number of concave boundary segments. To rapidly determine the convex hull of a region, we employ the geometric heat differential equation (GHDE). We show using consistencies [18] and three directions of observation that 344 models are sufficient to approximate the whole set of convex objects. This result allows us to divide an image database into 344 different classes and to use a three component vector for indexing. Further, we subdivide each class employing the number of concave boundary segments. In order to extract concavity information, we use the boundary support function of each image region. To validate the theoretical concepts, we performed experiments using digitized X-ray vertebrae images where the noise is cleaned and the image contrast is enhanced such that the boundaries are well defined.



a) b)

Figure 1. a) A vertebra image. **b)** The image in (a) rotated by 90 deg.



a) b)

Figure 2. a) The vertebra shown in Fig. 1a together with its convex hull. **b)** convex hull alone.

2. CONVEX HULL MODEL

In the past decade, applications of differential equations and variational methods in boundary detection have experienced significant growth [3]. A disadvantage of these methods is that they are relatively slow in converging towards concave portions of embedded object boundaries within an image. To overcome this disadvantage, we employ the following model based on the GHDE and designed to converge to the convex hull of the image region:

$$\frac{\partial C}{\partial t} = P\frac{d\mathbf{T}}{ds} - s\mathbf{T} \tag{1}$$

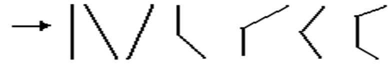
In the above equation, C represents a smooth convex curve parameterized by $r(t,s) = \langle x(t,s), y(t,s)\rangle$ in the domain of [-1,1]x[-1,1]. The family is parameterized by $t \in [0,\infty)$ and $s \in [0,\infty)$ parameterizes the curve. Our approach uses d**T**/ds instead of k**N** because the former requires less computation. By **T** we denote the tangent vector, s denotes the length of the arc segment, k is the curvature, and **N** is the inward normal vector used to guide the curve shortening toward the convex hull.

In Eq. 1, $P = P(I(x,y), \mathbf{N})$ is a penalty function of the image $I(x,y)$ and the normal vector **N**. The function P and the vector s**T** are designed to stop curve evolution when the convex hull is reached. The vector s**T** relates to the curve C and stabilizes the process of convergence.

Based on the model of Eq. 1, we developed a curve evolution algorithm capable of determining the convex hull of an image region, as shown in Figs. 1 and 2:

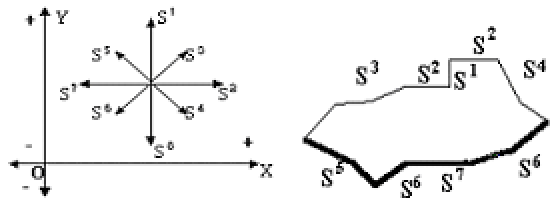
$$r^{j+1} = r^j + \delta^j \Delta\mathbf{T}^j P^j - s^j \mathbf{T}^j \tag{2}$$

The parameter δ^j represents the discrete time step. A tool based on Mathematica is developed to test the capabilities of the algorithm of Eq. 2. Experiments are performed using vertebrae and other medical subjects of different sizes. A literature survey shows that the computation speed of most convex hull algorithms depends on the number of pixels in the region's convex hull. The number of arithmetic operations in our convex hull algorithm is $60jm$, which is independent of the number of pixels in the region. The number of vectors used is m and j is the number of time steps.



1 2 3 4 5 6 7

Figure 3. Seven boundary segment templates that can be observed from a single view direction, shown at left.



a) b)

Figure 4. **a)** The plane points satisfy 8 regularities. **b)** Visible edges from below, shown as thick, are described by $Rg = 6{,}7{,}6{,}5$ and approximated by boundary template 6.

3. SEARCH SPACE PARTITIONING

Once the convex hull is defined, we then utilize a theoretical result published in [7], which states that three observation directions chosen 120 degrees apart are sufficient to observe the entire boundary of a convex object. Let us define the vector (a_1,a_2,a_3), which represents three directions of observation 120 degrees apart. The boundary as viewed from direction a_i can be approximated and classified into one of the seven boundary templates listed in Fig. 3. Each of these templates is described by regularities and consistencies defined in [17, 18]. A regularity is a straight segment in one of eight possible quantized orientations (Fig. 4a). A consistency is a connected set of regularities (Fig. 4b). Furthermore, each consistency decomposes into four finite numerical sequences (FNS): Rg – sequence of regularities; Rp - the number of consecutive repetitions of each Rg element; An - the angles of Rg elements relative to the x axis; and Le - the lengths of the regularities from Rg. It follows that a maximum of seven FNS $Rg_{i_1}, Rg_{i_2}, \ldots, Rg_{i_7}$ are sufficient to approximate the boundary that can be observed from the direction a_i. Thus each a_i for i=1,2,3 takes at most seven values. Therefore only 343 vectors (a_1,a_2,a_3) are sufficient to describe the boundaries of all convex objects except the parallelogram [7]. Consequently, there are 344 possible models of convex boundaries, which allows one to partition the regions in the image database into 344 convex hull classes.

The practical application requires preprocessing of the regions in order to compute their convex hulls. A given convex boundary can often be represented by several convex hull codes, based on the direction of view. To handle this, we propose determining all rotational variants of a given region and indexing their convex hull classes in the image database. Region searching can now be done from a single query vector having arbitrary view direction. This idea forces the index to grow to accommodate multiple listings for each region, but this is constrained by the fact that the number of rotational variants tends to be small. Overall, the search space is reduced by roughly two orders of magnitude.

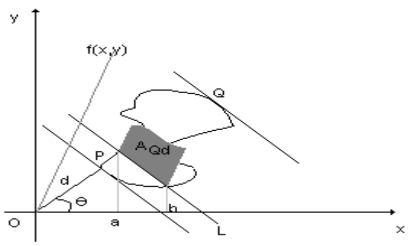


Figure 5. A compact shape Sh, two tangent and one secant line.

4. REGION BOUNDARY SUPPORT

Suppose $B \subseteq R^2$ is a smooth boundary of a medical image region, as shown in Fig. 5. Let us consider an angle θ and denote the tangent line to B at the point P by:

$$x\cos\theta + y\sin\theta = d \qquad (3).$$

Then there exists a point Q such that its tangent line is:

$$x\cos(\theta+\pi) + y\sin(\theta+\pi) = -d \qquad (4).$$

Let us denote the boundary support by S=$\{(\theta,d) \in [0,2\pi) \times R | l(\theta,d)\ \}$. Assuming B is connected, the boundary of S is made up of two curves $g(\theta)$ = d and $g(\theta+\pi)$ = -d, where g is a 2π periodic function [17]. To compute the number of concave segments, the use of $g(\theta)$ is sufficient.

Applying the above concepts to the vertebra image given in Fig. 1a, one determines the corresponding support curve, $g(\theta)$, given in Fig. 6. Furthermore, it follows from the definition of a tangent line that if a boundary contains a concavity, then there are at least two boundary points with a common tangent line. A concave section of a smooth boundary must have at least two inflection points, at which the tangent lines transition between the region's exterior and its interior near this point of tangency. We therefore conclude that the tangent lines to a concavity form a loop on the curve, $g(\theta)$. For example, each vertebra given in Fig. 1 has three concavities and each support curve shown in Fig. 6 contains three loops. The convex hull of the vertebra shown in Fig. 2b is transformed in Fig. 7 to a support curve without loops. Also, it follows from the formulated concept that $g(\theta)$ is invariant with respect to plane rotation and choice of origin point. In fact, a plane rotation of an image region corresponds to a translation of $g(\theta)$ in the support space. Figure 1 illustrates two different orientations of a vertebra region, while the corresponding curves $g(\theta)$ are shown in Fig. 6. As we may observe, a clockwise rotation of 90 degrees in Fig. 1 corresponds to a translation of $\pi/2$ to the left in Fig. 6.

Taking into account that $g(\theta)$ is a periodic function with a period 2π, we conclude that the curve S is invariant to both rotation and origin point selection. S is invariant to scaling, as well. We have developed a tool in C++ capable of transforming an image region boundary to a support curve, *S,* according to the theoretical concepts described here.

a)
b)

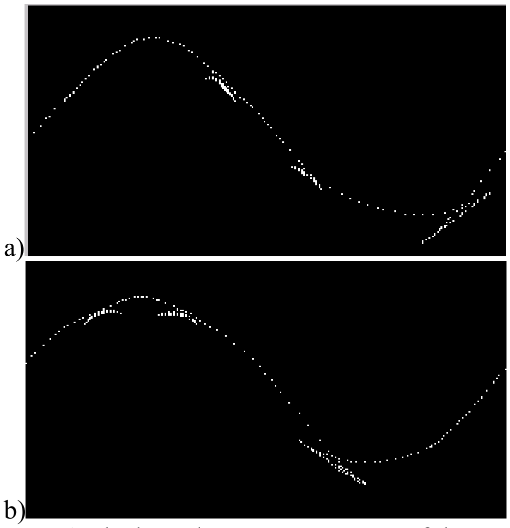


Figure 6. The boundary support curves of the vertebrae shown in Fig 1a and 1b, respectively.

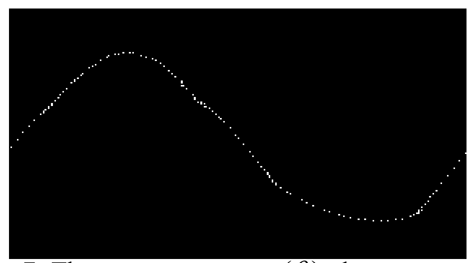


Figure 7. The support curve $g(\theta)$ that corresponds to the convex hull shown in Fig 2b.

5. REGION INDEXING IN DATABASE

In sections 2-4, we developed two approaches capable of extracting the local image features: convex hull and number of concavities. These features allow a partitioning of the database to facilitate efficient image retrieval. The partitioning process has two main steps.

In the first step, the convex hull of each region in the image database is computed. Then each convex hull boundary is described by a FNS with respect to the observation vector (a_1,a_2,a_3), where a_i observes exactly one of the seven approximating templates. All the images whose regions possess the same vector description (Rg_j,Rg_k,Rg_m) (where Rg_i is a FNS for i=j,k,m) of their own convex hulls form a particular class in the database. It follows from section 3 that a maximum of 344 classes can be constructed. Assign to each consistency a number through the mapping $Rg_j \to j$, where j=1,2,3,…,7. Then, each class is indexed by a three component vector (j,k,m) over the numbers 1,…,7.

In the second step, the number of concave segments for each image region is easily determined by counting the number of the loops on the curve $g(\theta)$. Using this property, each class (j,k,m) is subdivided according to the number of concavities. Each sub-class consists of (j,k,m) regions, which possess the same number of concave boundary segments, c. Hence, each sub-class can be indexed by a vector of the form (j,k,m,c), where $c \in [0,\infty)$. Additional concavity information could be used to improve robustness.

6. ADVANTAGES AND FUTURE DIRECTIONS

The primary advantage of our partitioning and image database management approach is the factor of several hundred reduction in the number of image database regions necessary to traverse and compare to the query region. This is very important for extensive databases because matching query shapes against every region conflicts with the goal of high retrieval speed.

A disadvantage of the above approach is the requirement for advance computation of each region's convex hull, rotational variants, and number of concave segments. These are needed to index the sub-classes. But once prepared, it is very fast to search for the class whose region boundaries most closely match the query boundary. Moreover, our convex hull and supporting boundary curve approach allows for the development of computationally inexpensive algorithms. As described in section 2, the number of arithmetic operations taken by the convex hull algorithm is around $60jm$. The calculation complexity of the concavity segments computation algorithm is roughly $O(n)$, where n is the number of boundary pixels.

Work is underway to develop a similarity metric based on local geometric concave and convex features for the content-based image retrieval of medical images.

7. ACKNOWLEDGEMENTS

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