UNIT - III

Friction (Static friction only)

Syllabus

Concept of friction; its uses; laws of friction; various types of friction (block, block-on-block - horizontal and inclined surface -, screw, ladder, wedge) presented in a unified approach (anchored to FBDs) to solve friction problems.

Belt friction (static) treated separately.

Introduction

It has been established since long that all surfaces of the bodies are never perfectly smooth.

It has been observed that whenever, even a very smooth surface is viewed under a microscope, it is found to have some roughness and irregularities, which may not be detected by an ordinary touch.

Two types:

1. Static friction. 2. Dynamic friction.

Co-eff. of friction $(\mu) = \dfrac{\text{Frictional Force } (F_r)}{\text{Normal Reaction } (N_R)}$

STATIC FRICTION

It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.

$\mu_s > \mu_k$

DYNAMIC FRICTION

It is the friction experienced by a body when it is in motion. It is also called kinetic friction.

The dynamic friction is of the following two types :

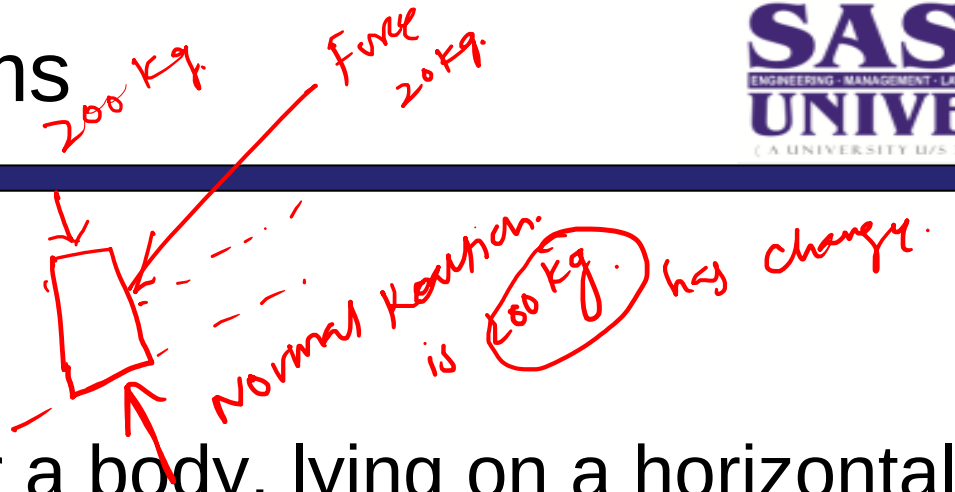
1. Sliding friction. It is the friction, experienced by a body when it slides over another body.

2. Rolling friction. It is the friction, experienced by a body when it rolls over another body

LIMITING FRICTION

It has been observed that when a body, lying over another body, is gently pushed, it does not move because of the frictional force, which prevents the motion. It shows that the force of the hand is being exactly balanced by the force of friction, acting in the opposite direction. If we again push the body, a little harder, it is still found to be in equilibrium. It shows that the force of friction has increased itself so as to become equal and opposite to the applied force.

NORMAL REACTION



It has been experienced that whenever a body, lying on a horizontal or an inclined surface, is in equilibrium, its weight acts vertically downwards through its centre of gravity.

The surface, in turn, exerts an upward reaction on the body. This reaction, which is taken to act perpendicular to the plane, is called normal reaction and is, generally, denoted by R. It will be interesting to know that the term ‘normal reaction’ is very important in the field of friction, as the force of friction is directly proportional to it.

- ANGLE OF FRICTION

Consider a body of weight W resting on an inclined plane as shown in Fig. We know that the body is in equilibrium under the action of the following forces :

1. Weight (W) of the body, acting vertically downwards,
2. Friction force (F) acting upwards along the plane, and
3. Normal reaction (R) acting at right angles to the plane.

Let the angle of inclination (α) be gradually increased, till the body just starts sliding down the plane. This angle of inclined plane, at which a body just begins to slide down the plane, is called the angle of friction. This is also equal to the angle, which the normal reaction makes with the vertical.

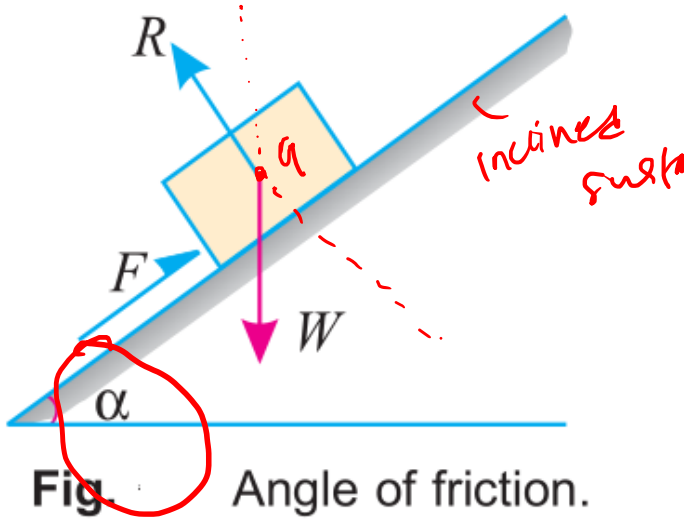


Fig. Angle of friction.

CO-EFFICIENT OF FRICTION

It is the ratio of limiting friction to the normal reaction, between the two bodies, and is generally denoted by μ.

Mathematically, coefficient of friction,

$$\mu = \frac{F}{R} = \tan \phi \qquad \text{or} \qquad F = \mu R$$

$\tan \phi = \mu$

where ϕ = Angle of friction,

F = Limiting friction, and

R = Normal reaction between the two bodies.

LAWS OF STATIC FRICTION

Following are the laws of static friction :

1. The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.
2. The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces. Mathematically :

$$\frac{F}{R} = \text{Constant}$$

where F = Limiting friction, and

R = Normal reaction.

is not dependent of surface of

4. The force of friction is independent of the area of contact between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE

We know that a body, lying on a rough horizontal plane will remain in equilibrium. But whenever a force is applied on it, the body will tend to move in the direction of the force.

In such cases, equilibrium of the body is studied first by resolving the forces horizontally and then vertically.

Now the value of the force of friction is obtained from the relation :

$$F = \mu R$$

where μ = Coefficient of friction, and

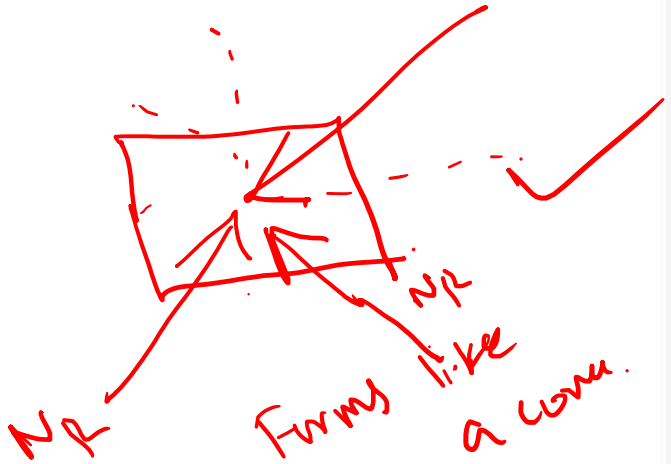
R = Normal reaction.

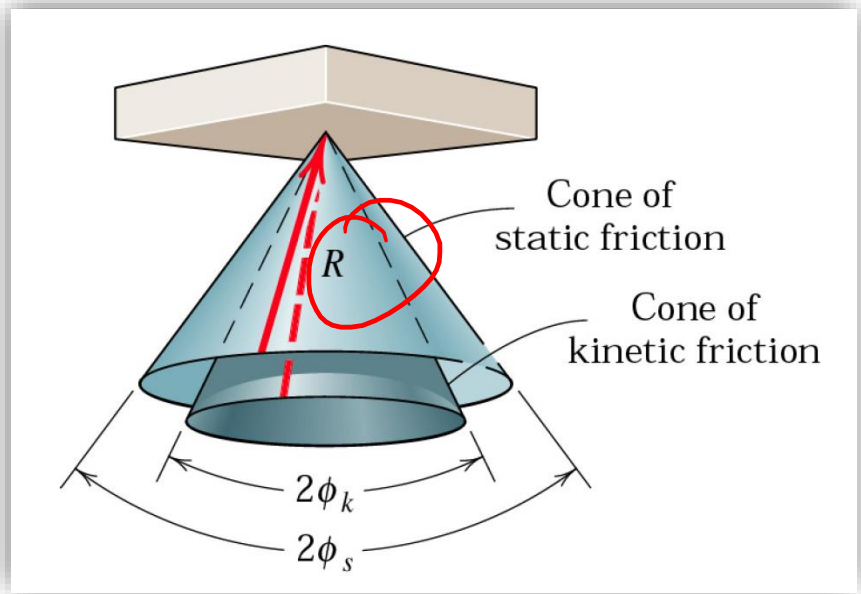
$$\mu = \frac{F_f}{R}$$

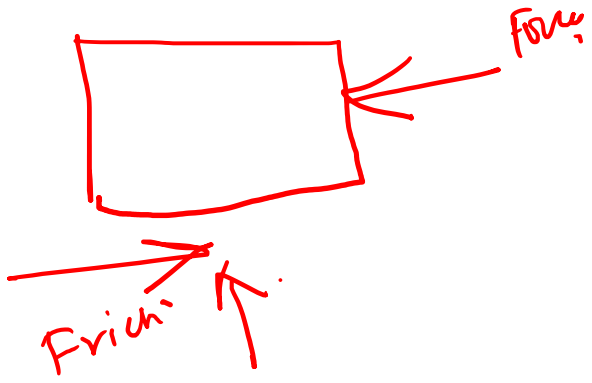
Cone of friction

When the application of force is being revolved to complete a revolution over a body, the resultant (R) of the normal reaction (N) and limiting frictional force (N) about the normal forms a cone with the subtended angle of two times of angle of friction ϕ is called as cone of friction.

The concept of cone of friction is applied to the conical friction clutches.



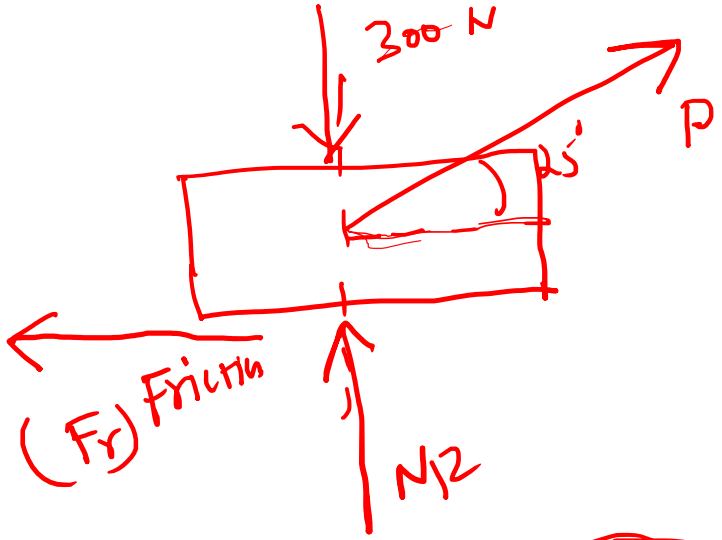




Ex-1

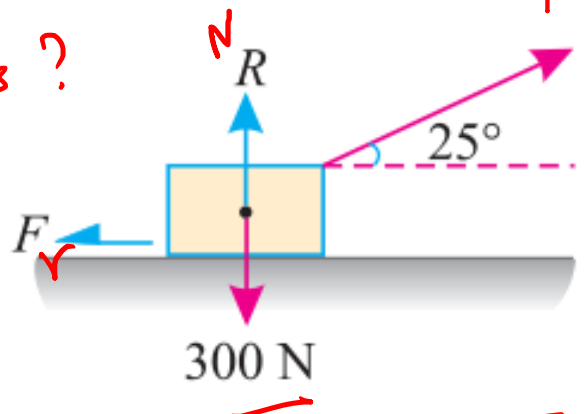
A body of weight 300 N is lying on a rough horizontal plane having a coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25 ° with the horizontal.

Given



$\Sigma F_{Horizontal\ Force}$?

$\Sigma F_{vertical\ Forces}$?



Unknowns

$P,\ F_r,\ N_R$

$\mu = 0.3$

$\mu = \frac{F_r}{N_R}$

$F_r = 0.3\ N/2$

$\Sigma F_x = 0$

$-0.3N_R + P\cos 25^\circ = 0$ — (1)

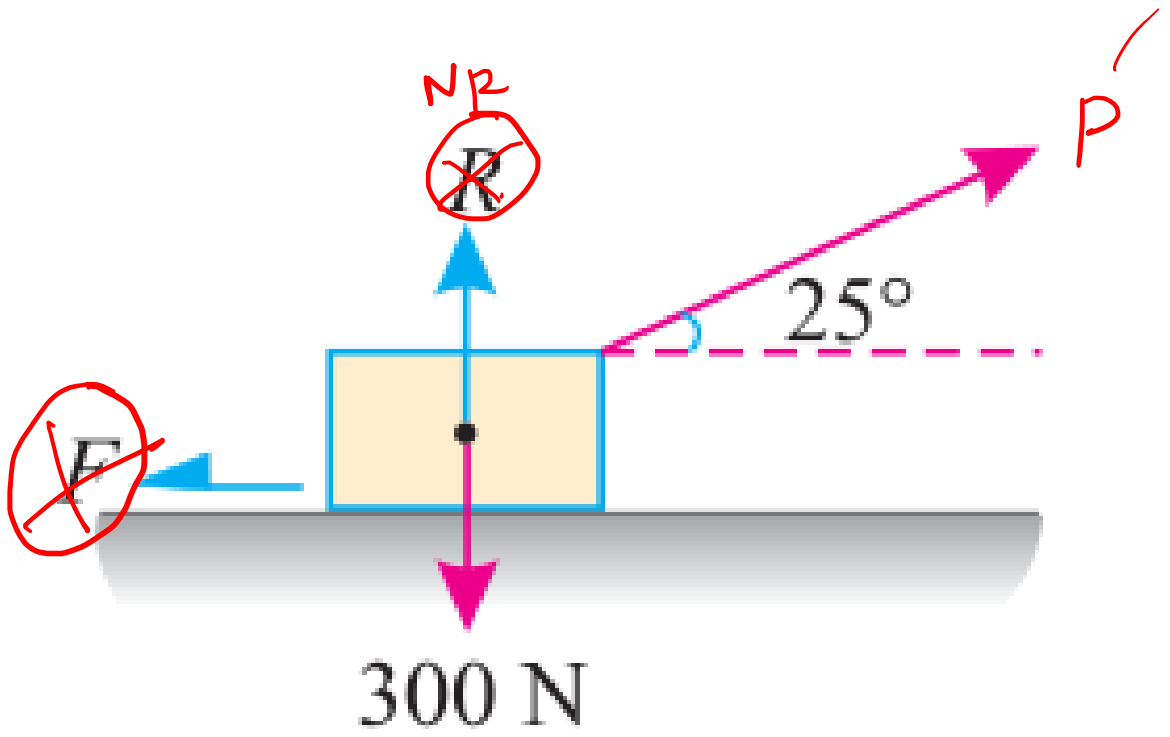
$\Sigma F_y = 0$

$N_R + P\sin 25^\circ = 300$ — (2)

Solving (1) & (2)

$N_R = 263.19\ N$

Magnitude of Force $\Big\}$ $P = 87.12\ N$



EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE

Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium

Consider: Angle of repose = angle of Frict^n (inclined). φ not μ not } lim.

Let W = Weight of the body,

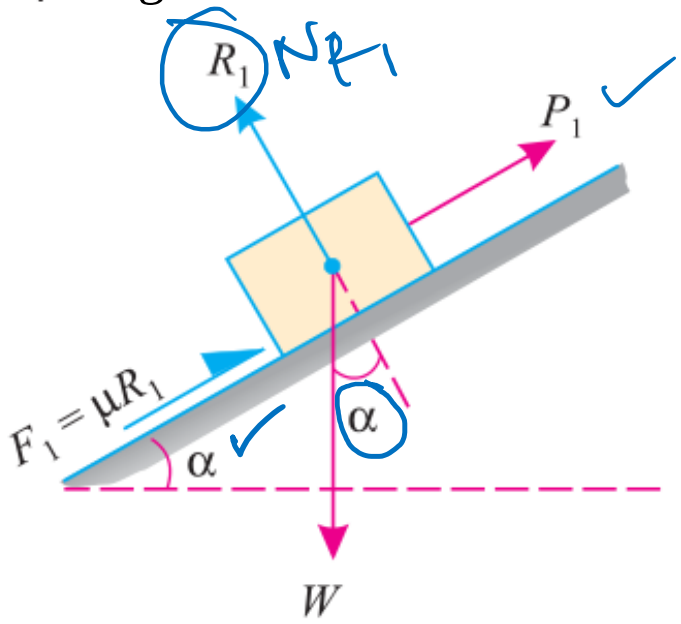
α = Angle, which the inclined plane makes with the horizontal,

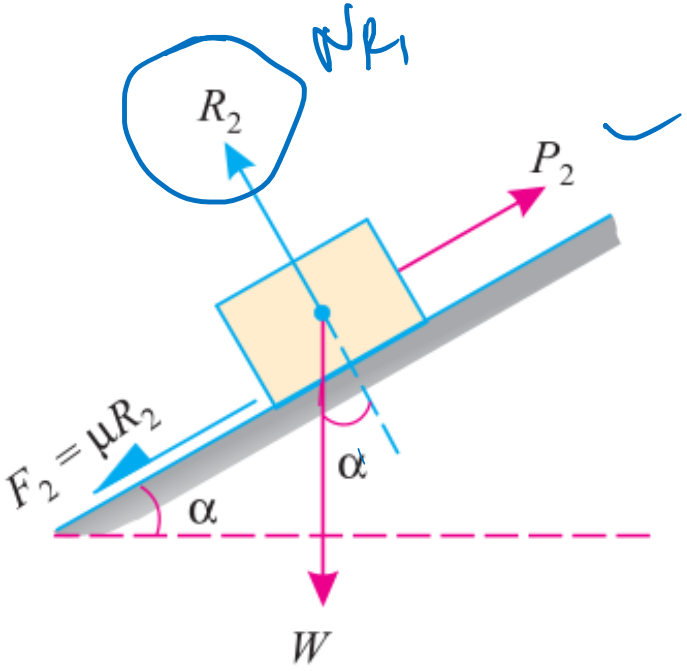
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

φ = Angle of friction, such that μ = tan φ.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases





Ex- 2b

- Determine the frictional force developed on the block shown in fig.

(1) When P = 40N

(2) P=80 N if coefficient static is 0.3 and ~~kinematic is 0.25~~

(3) Also find the value P , when block is about to move

if case ①

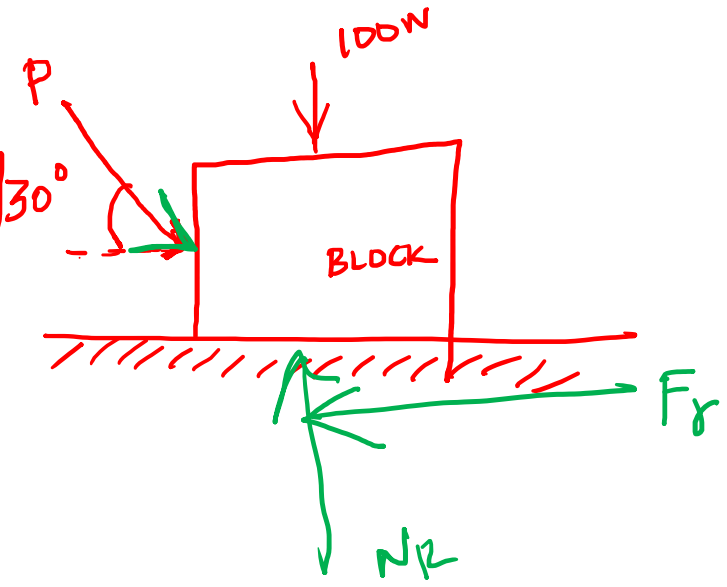
$\Sigma H = 0$

$40\cos 30^\circ - F_r = 0$ So, $F_r = 34.64\,N$

$\Sigma F_y = 0$

$-100 + N_R - 40\sin 30^\circ = 0$

$N_R = 120\,N$



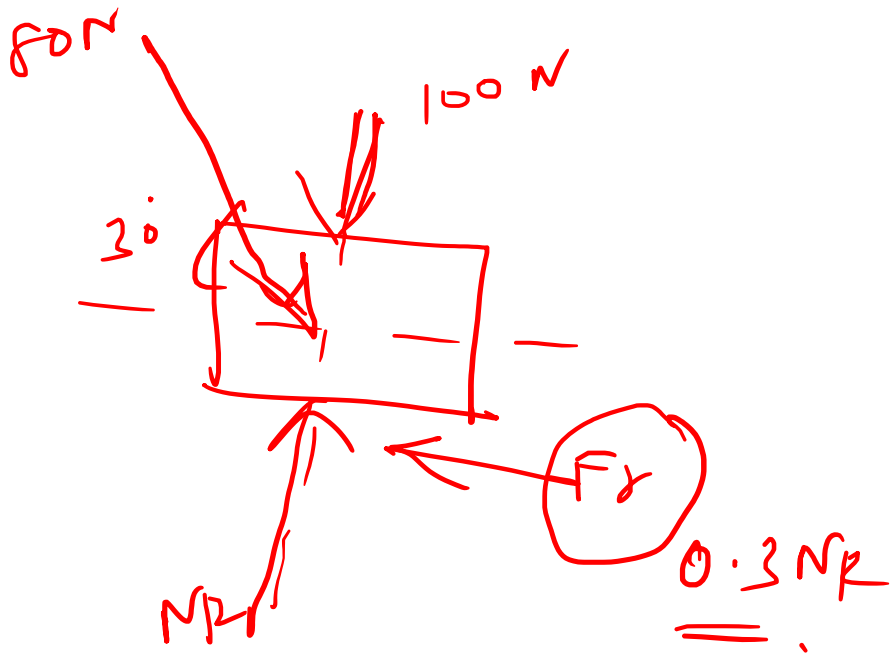
$\Sigma F_y = 0$

$-80 \sin 30^\circ - 100 + N_R = 0$

$N_R = 140\ N$

So Friction $F_r = 0.3 \times 140$

$= 42\ N$

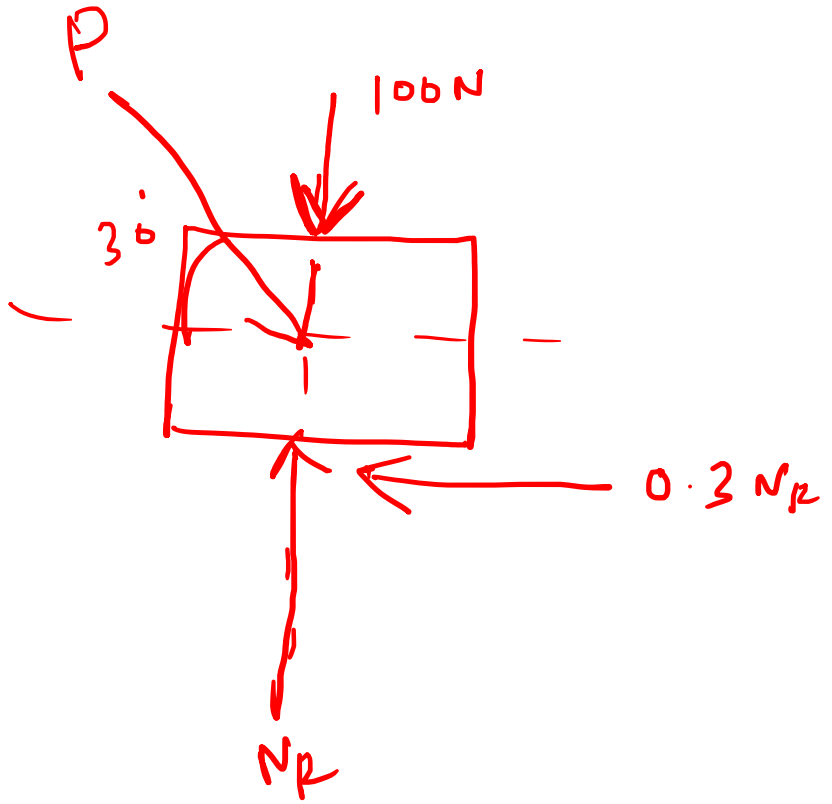


$\Sigma H = 0$

$$P\cos 30^\circ - 0.3N_R = 0 \quad \text{— (1)}$$

$\Sigma F_y = 0$

$$-P\sin 30^\circ + N_R = 100 \quad \text{— (2)}$$



Solving (1) & (2).

$$P = 41.89\ N$$

$$N_R = 120.9\ N$$

limiting Friction $= F_r = 0.3 \times 120.9$

$= 36.27\ N$

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E.M

Friction

Laws:

(i) Frictional Force develope opposite to motion.

(ii) Frictional Resistance $F_r = \tan \phi$

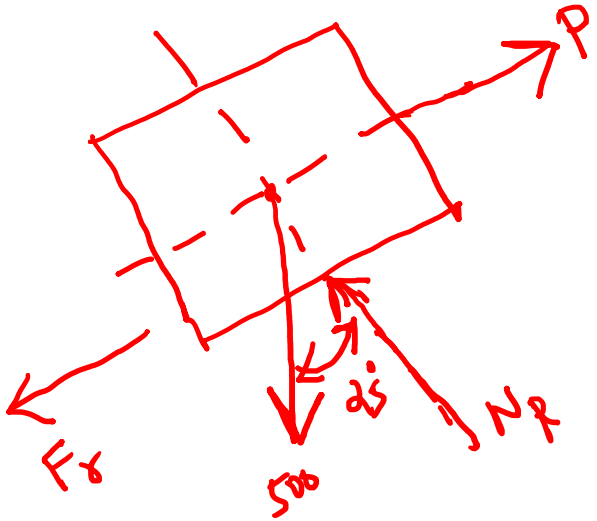
ϕ – angle of friction.

(iii) Co-eff of friction $\mu = \frac{F_r}{N_R}$

(Frictional Resistance No dependence surface area)

Example-3

- A body of weight 500 N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is ~~supported by an effort~~ Force (P) parallel to the plane as shown in Fig. Determine the force 'P' and limiting friction F



$\Sigma F_x = 0$ & $\Sigma F_y = 0$

Co-eff of Friction

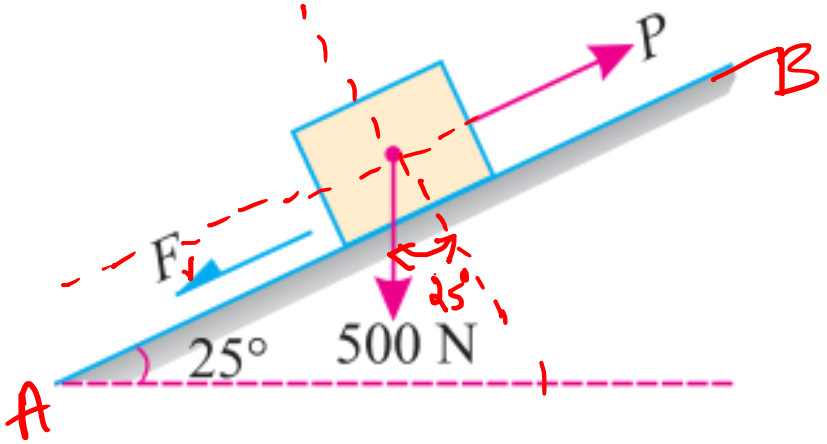
$\mu = \tan\phi$

$= \tan 25°$

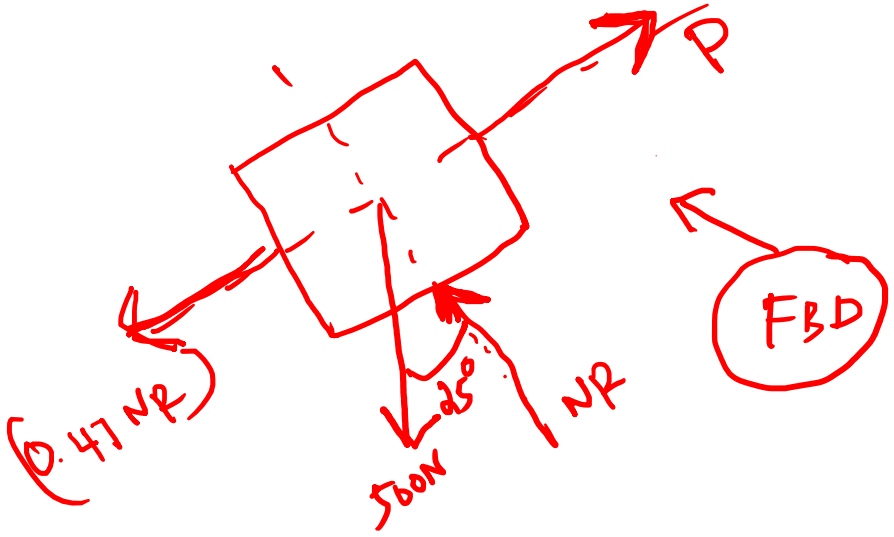
$\boxed{\mu = 0.47}$

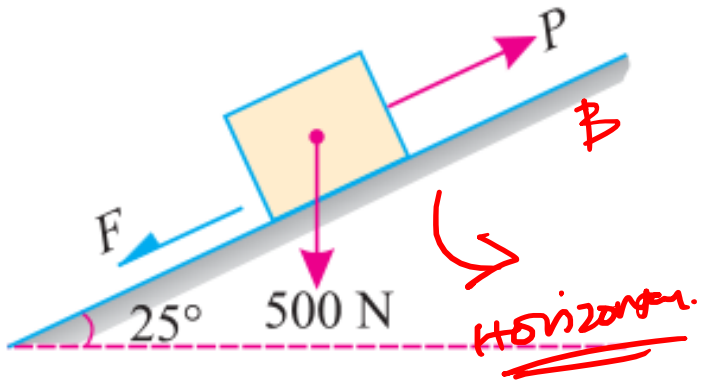
$\mu = \frac{F_r}{N_R}$

So, $\boxed{F_r = 0.47 N_R}$



Ex-3





$\Sigma F_x = 0$
(Plane)

$-0.47N_R - 500\cos 65^\circ + P = 0$

$-0.47N_R + P = 211.31$ — (1)

$\Sigma F_y = 0$

$-500\sin 65 + N_R = 0$

$\boxed{N_R = 453.15\ N}$

Sub N_R to eqn (1)

$\boxed{P = 424.29\ N}$

$\boxed{F_r = 0.47 N_R \Rightarrow 212.9\ N}$ // Ans

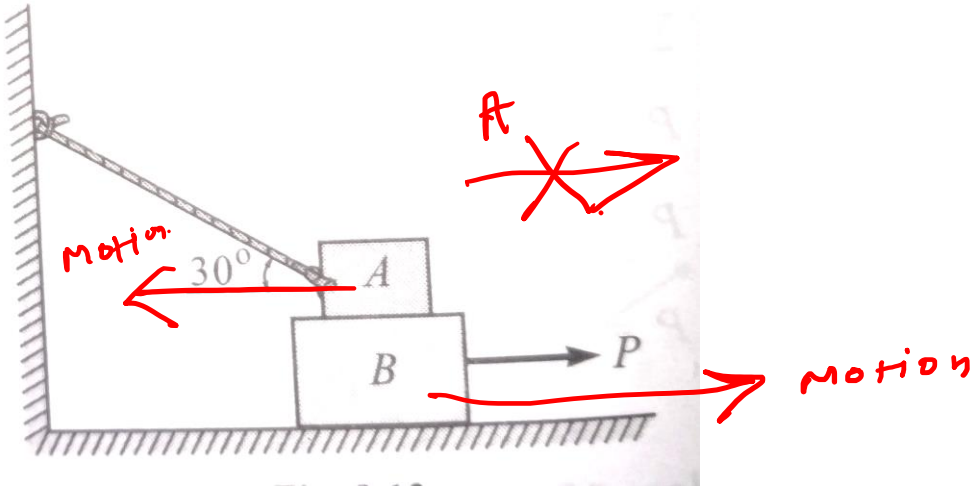
Block on block Ex-4

- Question

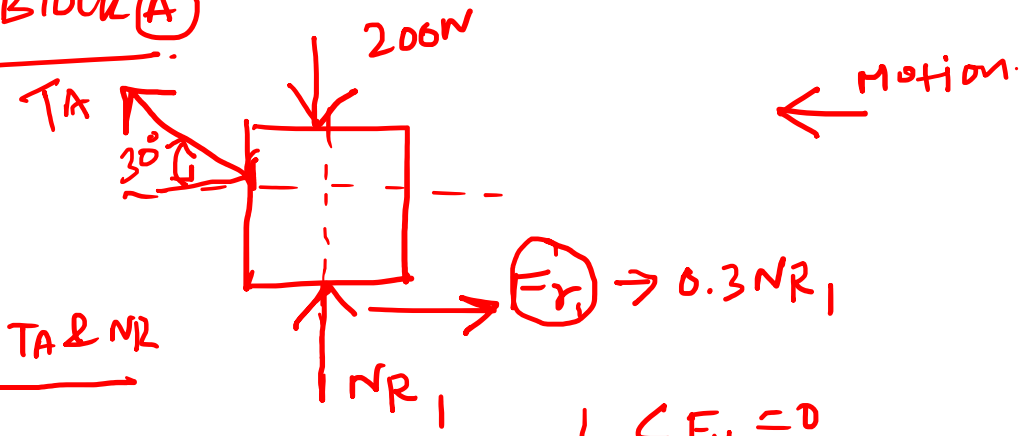
300N 200N

A block of weight A = 20 kg rests on a horizontal surface and supports another block of weight B = 30 kg on top of it as shown in figure (given below). Block weight A is attached to a vertical wall by an inclined string. Find the force 'P' applied to the lower block, that will be necessary to cause the slipping to impend. The coefficient of friction between blocks are 0.3 and between block to floor is ~~0.25~~. 0.3

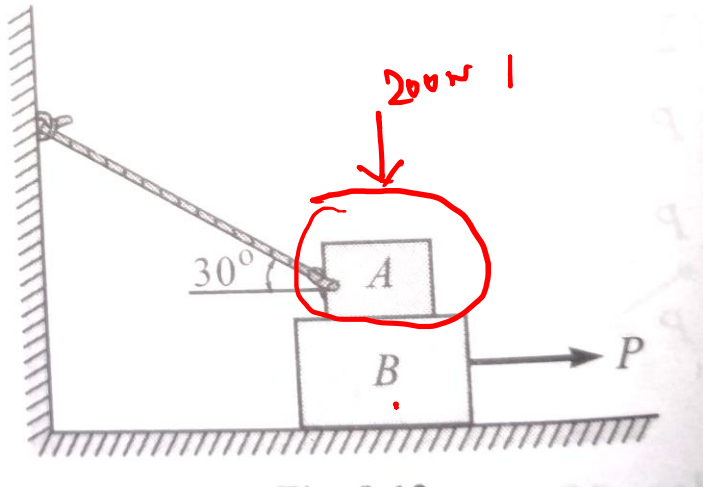
Direction of Motion



FBD ① BLOCK (A)



← Motion.



Two unknown T_A & N_R

$\Sigma F_x = 0$

$-T_A \cos 30° + 0.3N_{R_1} = 0$

$-0.87T_A + 0.3N_{R_1} = 0$ — ①

$\Sigma F_y = 0$

$T_A \sin 30° + N_{R_1} - 200 = 0$

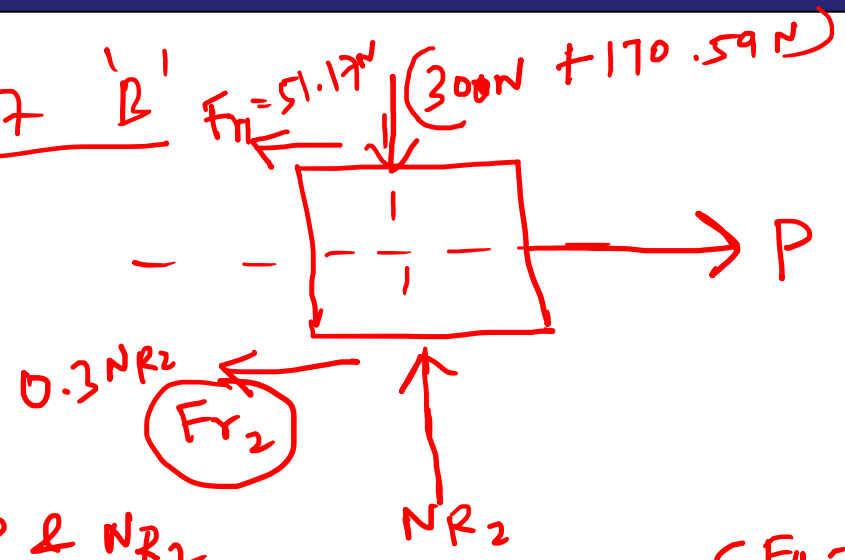
$0.5 + N_{R_1} = 200$ — ②

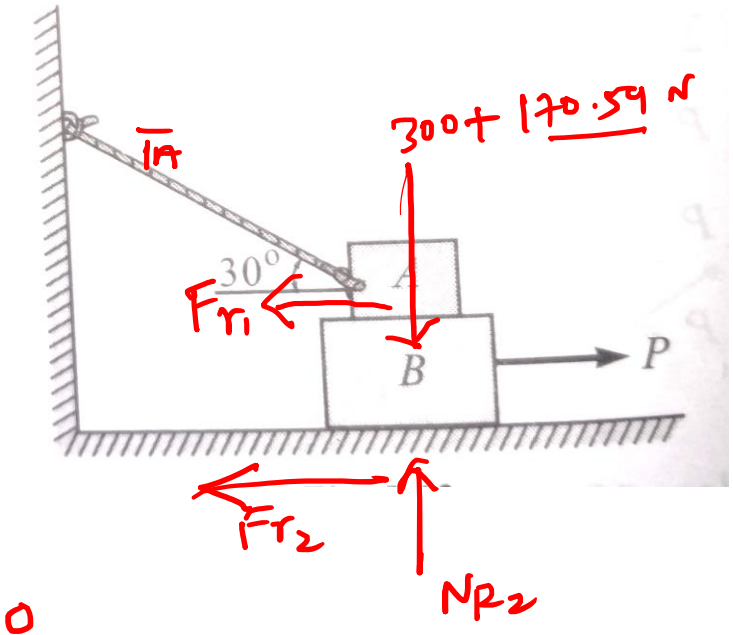
Solving eqn ① & ②

$T_A = 58.8N$ & $N_{R_1} = 170.59\ N$

so, $F_{r_1} = 0.3 \times N_R = 51.17N$

FBD of 'B'





Unknown → P & N_{R_2}

$\Sigma F_x = 0$

$-51.17 - 0.3 N_{R_2} + P = 0$

$-0.3 N_{R_2} + P = 51.17$ — (1)

$\Sigma F_y = 0$

$N_{R_2} - 470.59 = 0$

$N_{R_2} = 470.59 N$

Sub N_{R_2} in eqn (1)

$-0.3(470.59) + P = 51.17$

$P = 192.35 N$

E. Mech

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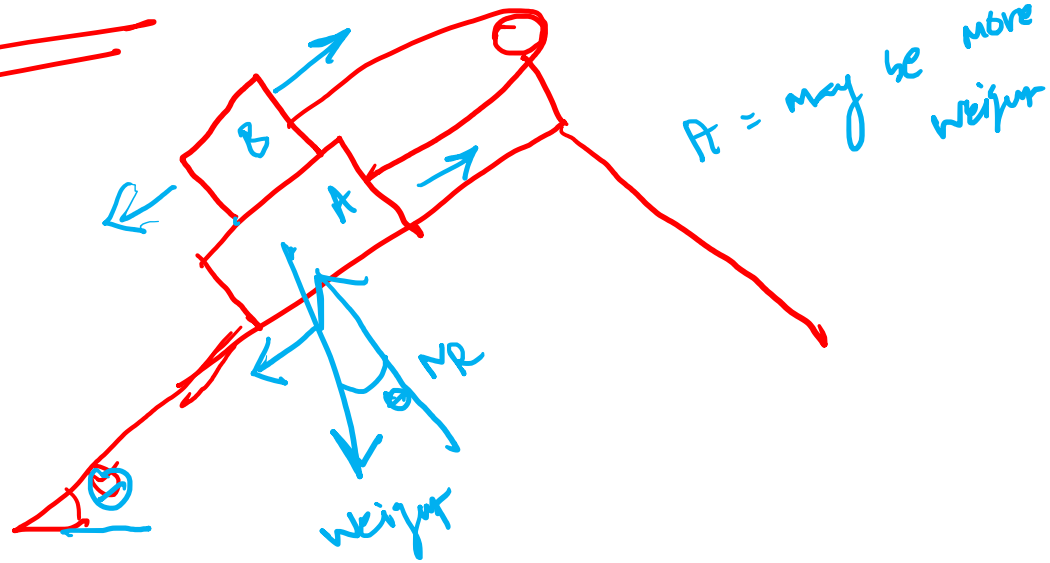
(i) Direction motion.

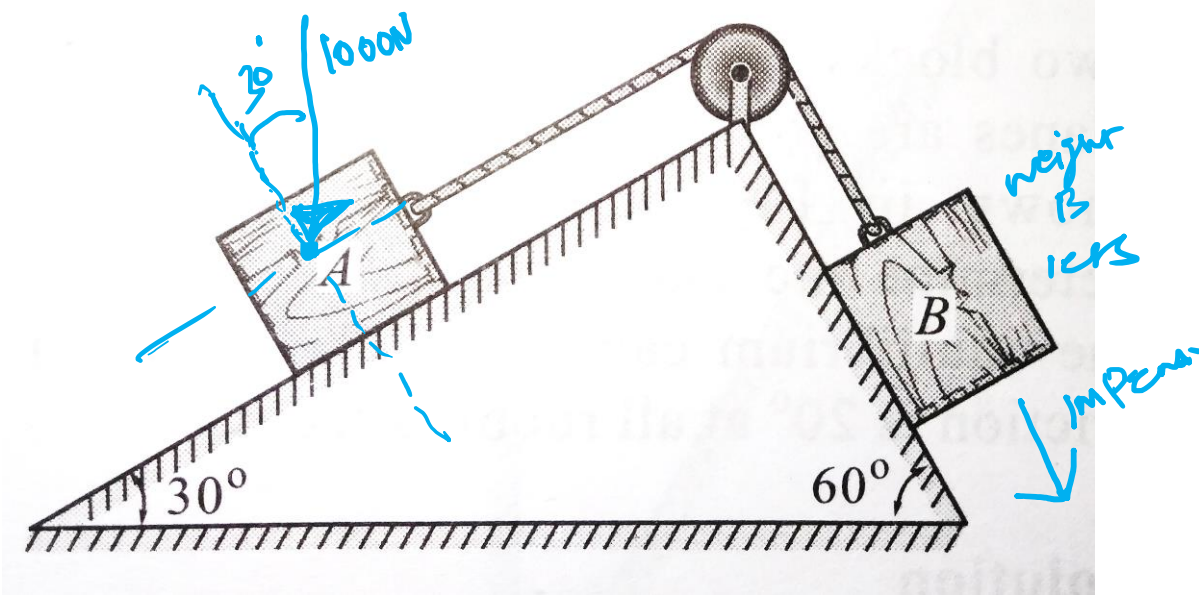


Ex-5

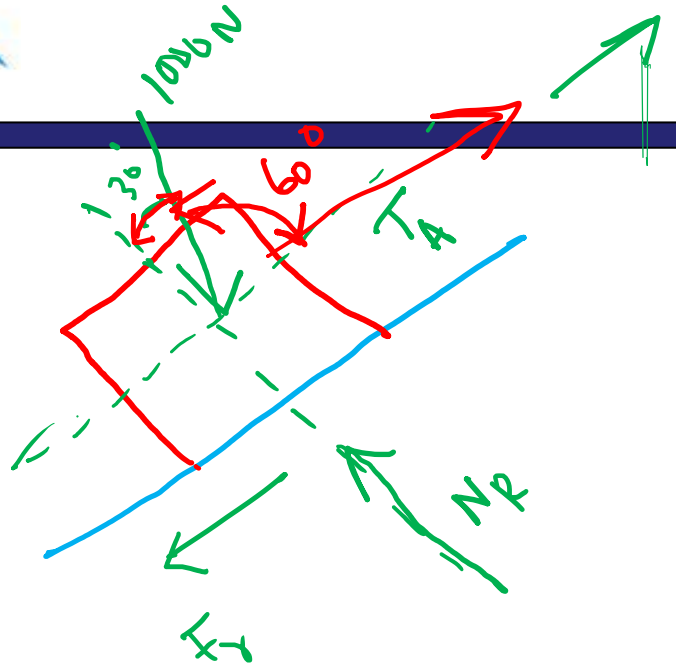
- Question

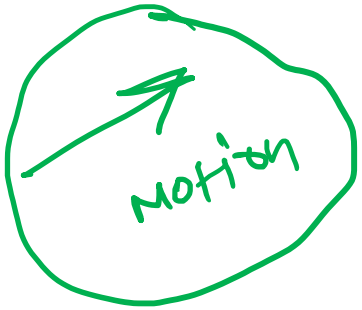
(Follow part -2 only)

Two blocks are placed inclined planes as shown in fig. The block A weights 1000N. Determine minimum weight of the block B for maintaining the equilibrium of the system. Assume that the blocks are connected by an inextensible string passing over a frictionless pulley coefficient of friction 0.25 for all surfaces

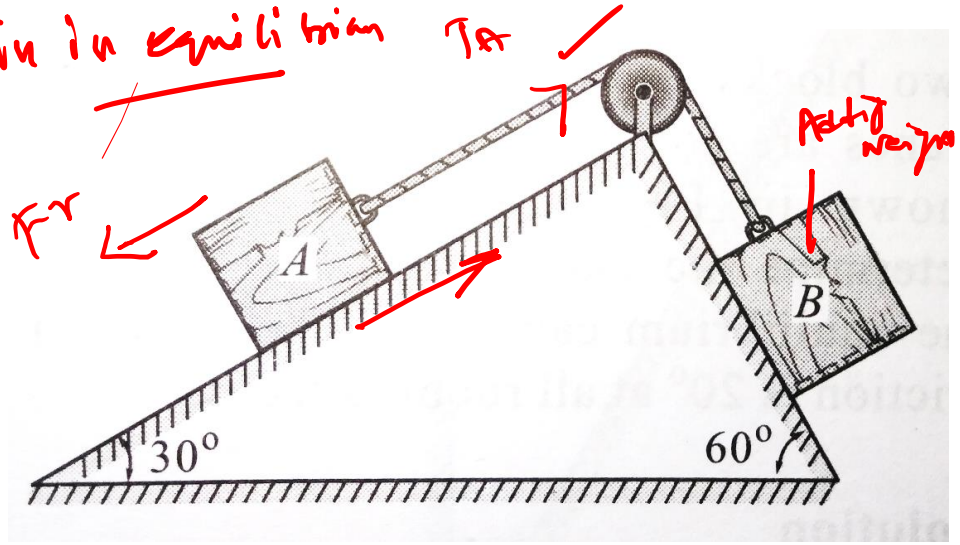


Post ①





Maintain in equilibrium T_A



$(0.25N_R) = F_r$

$\Sigma F_x = 0$

$$-1000\cos 60^\circ - 0.25N_R + T_A = 0$$

$$-0.25N_R + T_A = 500 \quad \text{—— ①}$$

$\Sigma F_y = 0$

$$-1000\sin 60^\circ + N_R = 0$$

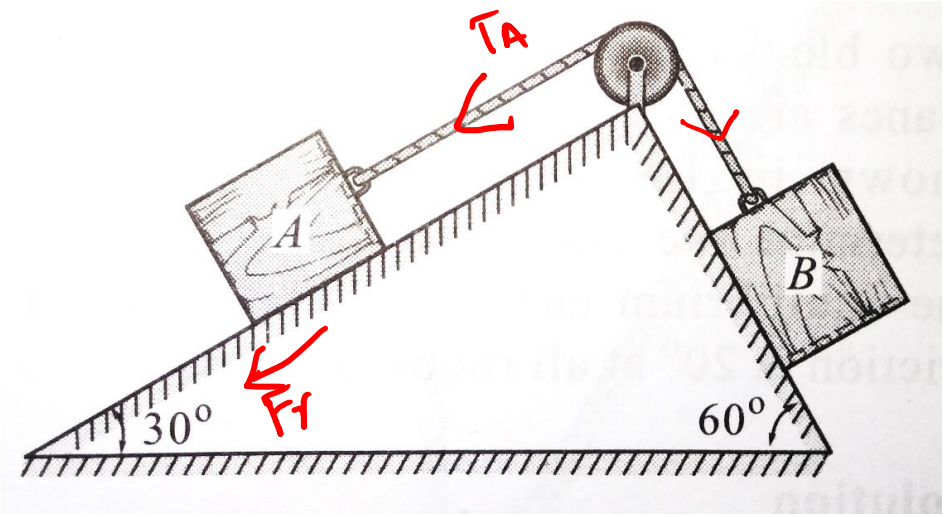
$$N_R = 866.02\ N$$

Sub N_R in ① so,

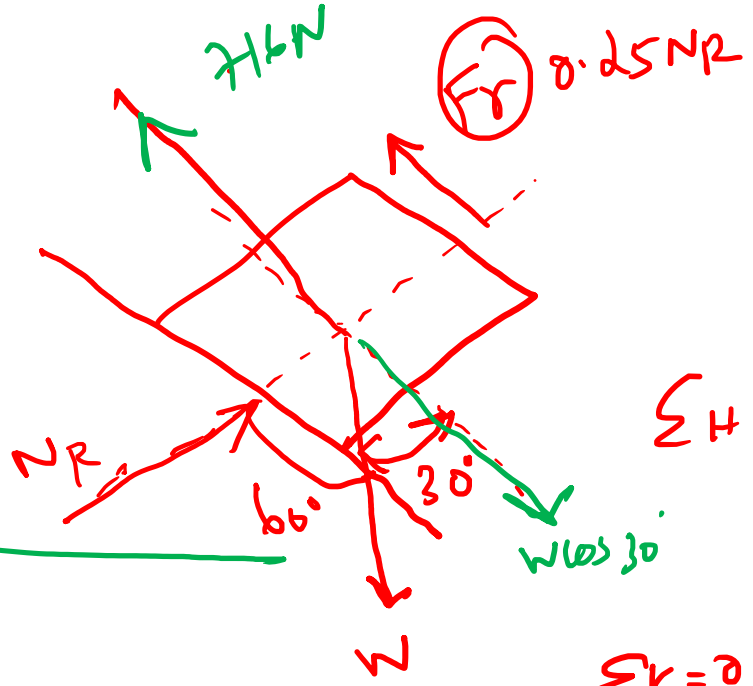
$$\boxed{T_A = 716.5}$$

Part ①

$$\boxed{T_A = 716\ N}$$



Considering Block 'B'



$\Sigma H = 0$

$-0.25N_R - 716 + W\cos 30° = 0$

$-0.25N_R + 0.87W = 716$ — ①

$\Sigma V = 0$

$N_R - W\sin 30° = 0$

$N_R = 0.5W$

Sub W in ①

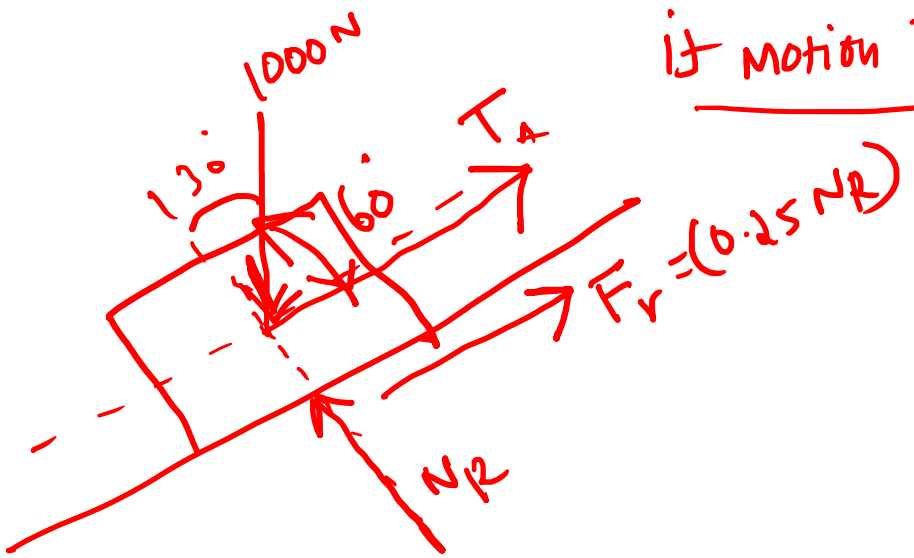
$0.125W + 0.87W = 716$

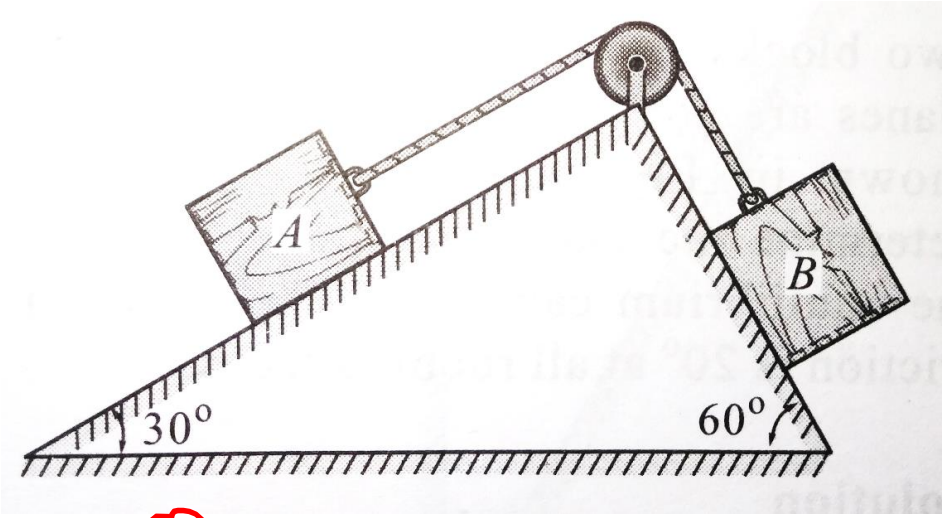
$0.995W = 716$

$$\boxed{W = 719.59N}$$

Part - 2

if motion in downwards of A)





$\Sigma F_x = 0$ $\quad T_A + 0.25 N_R - 1000 \cos 60° = 0$

$T_A + 0.25 N_R = 500 \text{ N}$ — (1)

$\Sigma F_y = 0$ $\quad -1000 \sin 60° + N_R = 0$

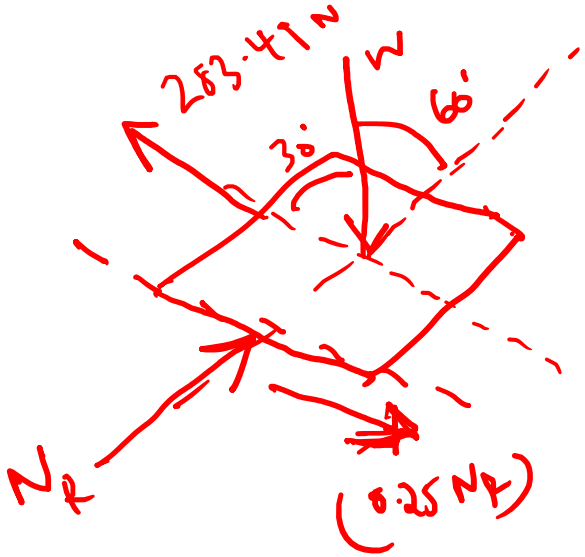
$N_R = 866 \text{ N}$

Sub N_R in (1)

$T_A = 283.49 \text{ N}$

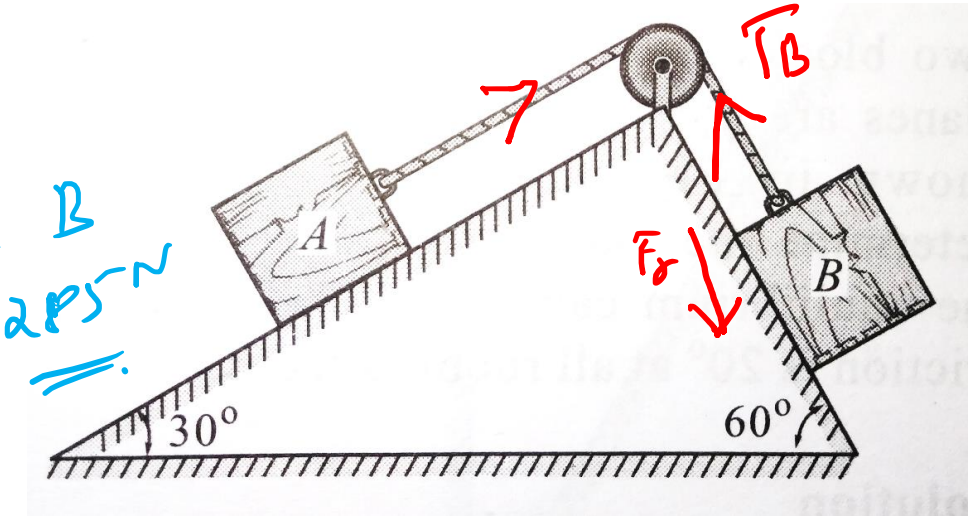
Part – 2

$T_A = T_B = 283.49\,N$



Minimum weight needed for block B

$= 284.9 \simeq 285\,N$



$\Sigma H = 0$

$0.25 N_R - 283.49 + W\cos 30° = 0$

$0.25 N_R + 0.87 W = 283.49$ — (2)

$\Sigma F_y = 0$

$-W\sin 30° + N_R = 0$

$\boxed{N_R = 0.5W}$

Sub N_R in eqn (2)

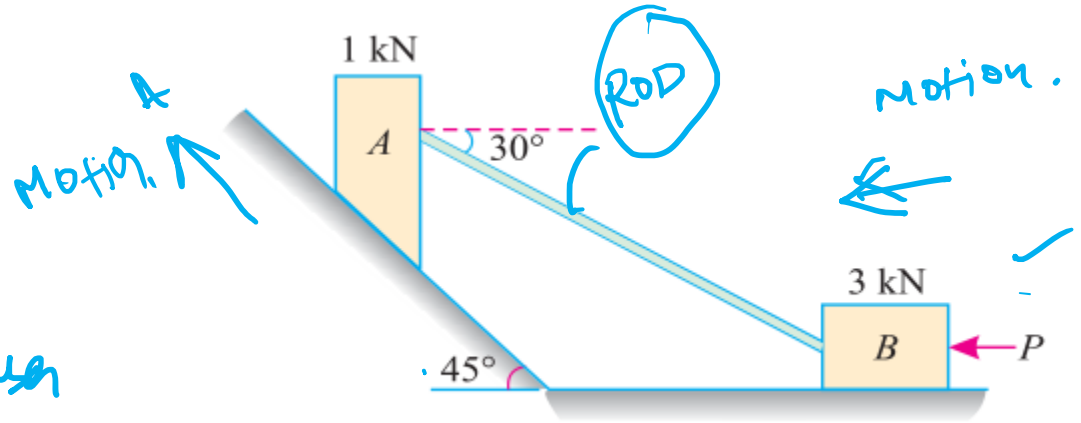
$W = \dfrac{283.49}{0.955} = 284.9\,N$

~~Ex-6~~ HW.

Question: A block (A) weighing 1 kN rests on a rough inclined plane whose inclination to the horizontal is 45°. This block is connected to another block (B) weighing 3 kN rests on a rough horizontal plane by a weightless rigid bar inclined at an angle of 30° to the horizontal as shown in Fig. Find horizontal force (P) required to be applied to the block (B) just to move the block (A) in upward direction. Assume angle of limiting friction as 15° at all surface where there is sliding.

$\mu = \tan 15°$

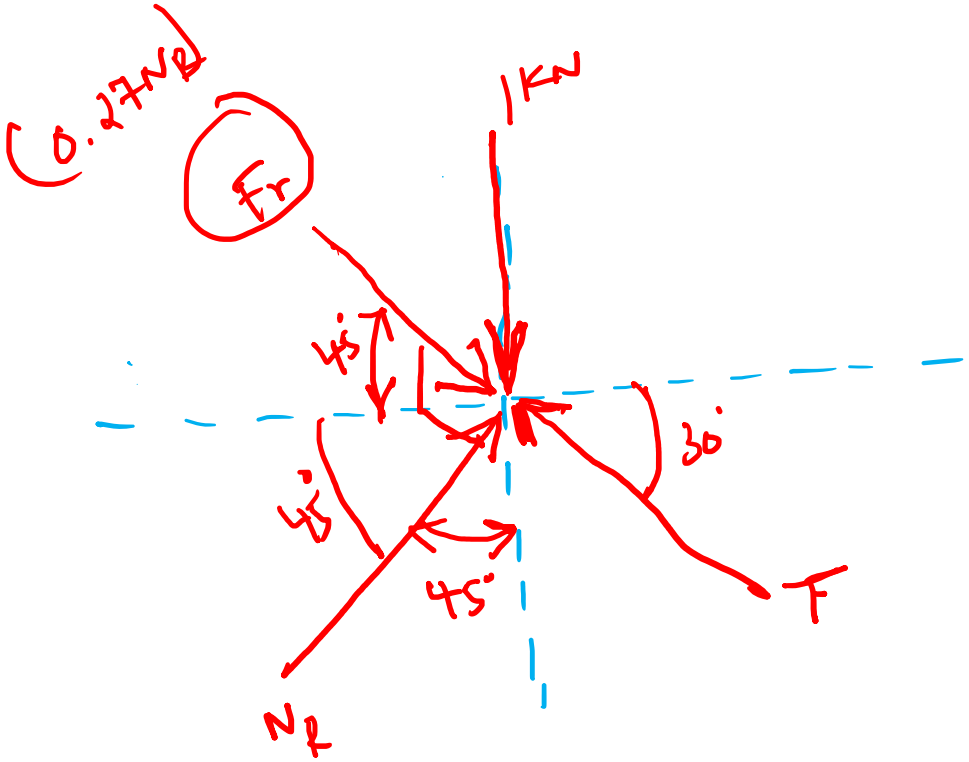
$= 0.27$

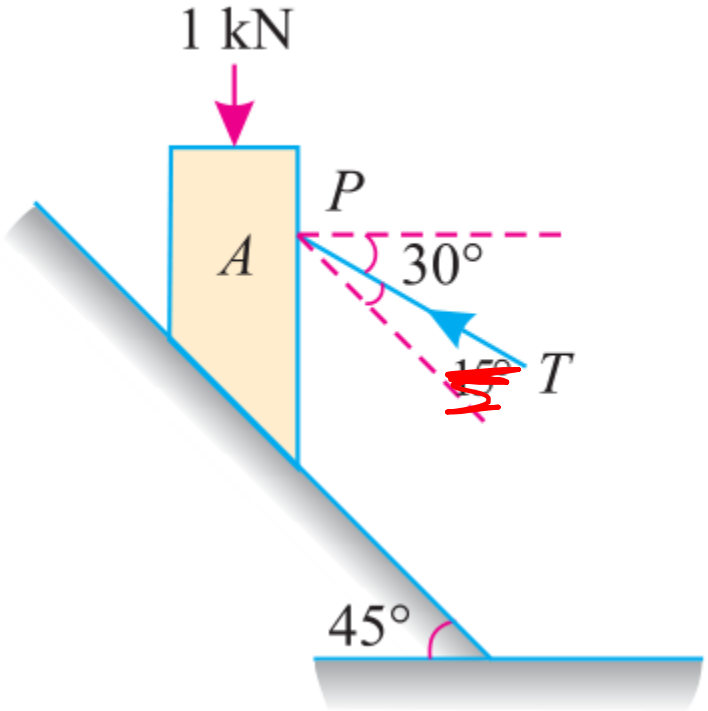


Rod is a comp. member

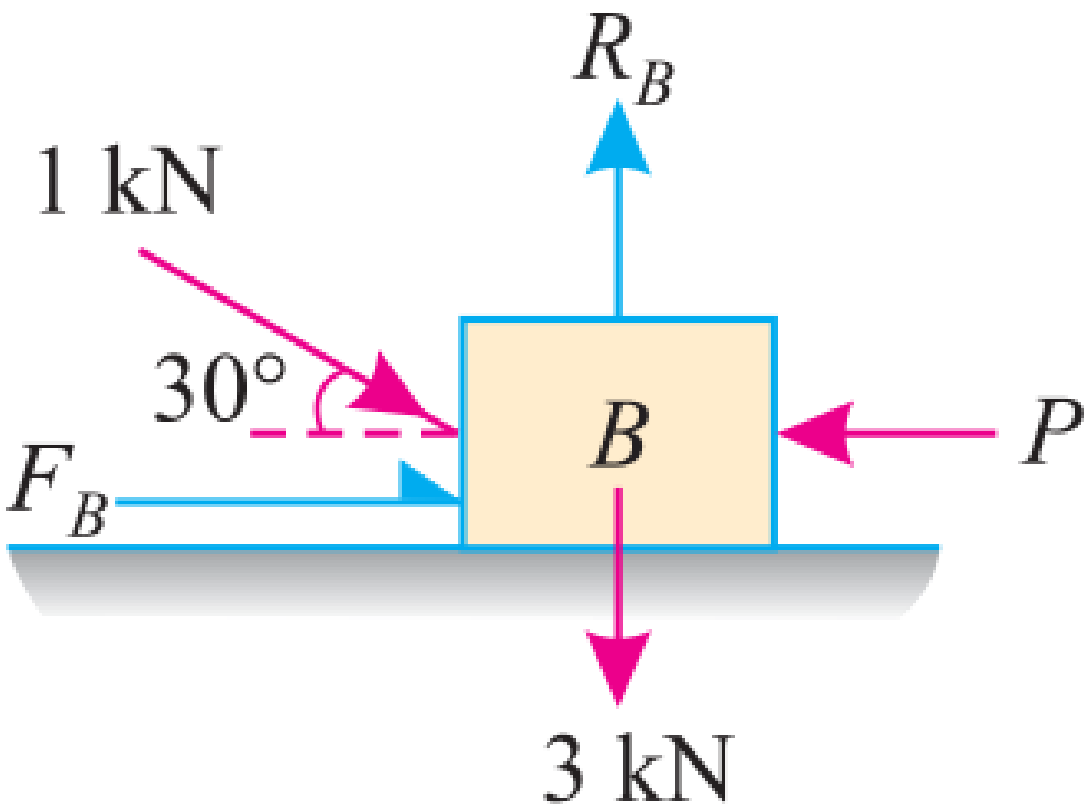
~~Ex-6~~ HW

FBD of A



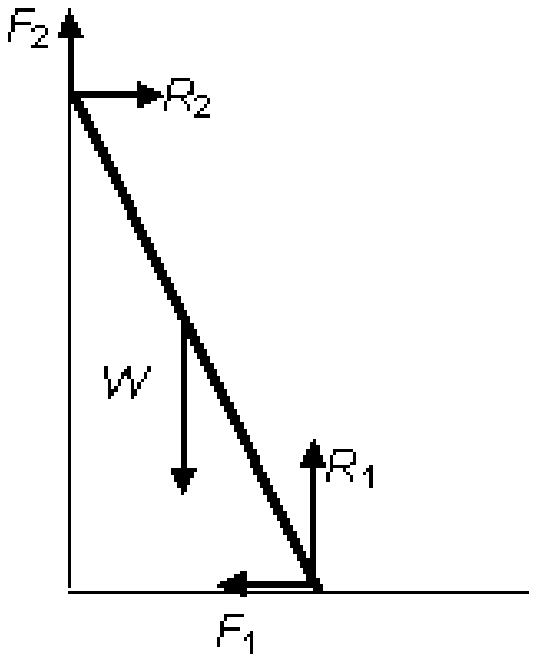


HW



Ladder friction

- **LADDER FRICTION** If a **ladder** is placed against a rough horizontal floor and a vertical wall (smooth or roughly) then **ladder** is subjected to non-concurrent force system. **Ladder** can be analyzed by applying conditions for equilibrium of Non-concurrent force system.



W — weight of ladder.

F_1 & F_2 → Friction.

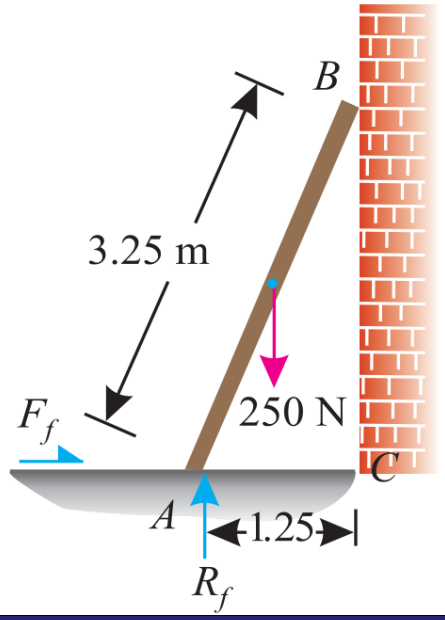
R_1 & R_2 → Reaction.

Man climbs the ladder.

Ex -7 (ladder friction)

A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between the ladder and floor is 0.3. What is the frictional force acting on the ladder at the point of contact between the ladder and the floor? Show that the ladder will remain in equilibrium in this position.

length of ladder = 3.25 m



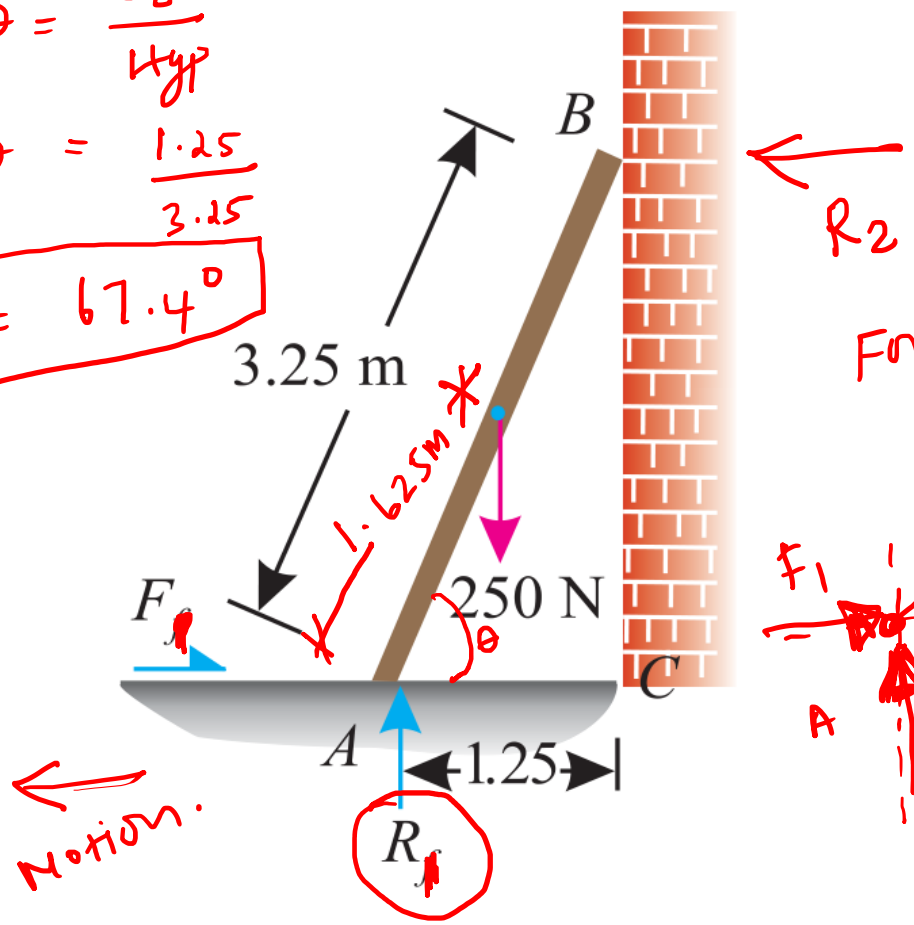
Ex-7

Unknowns → R_2, F_1, R_1

$\cos\theta = \frac{adj}{Hyp}$

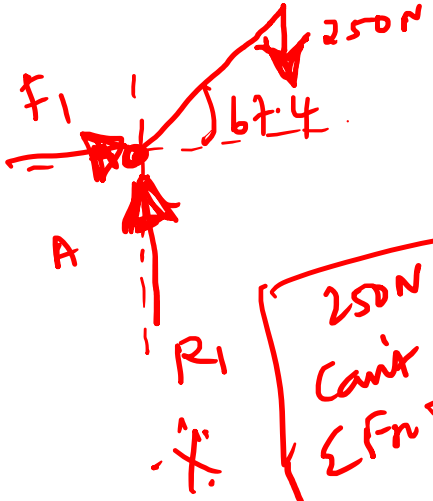
$\cos\theta = \frac{1.25}{3.25}$

$\theta = 67.4^\circ$



R_2

Force action at 'A'



Force action at 'B'

250N can't include $\Sigma F_x = 0, \Sigma F_y = 0$

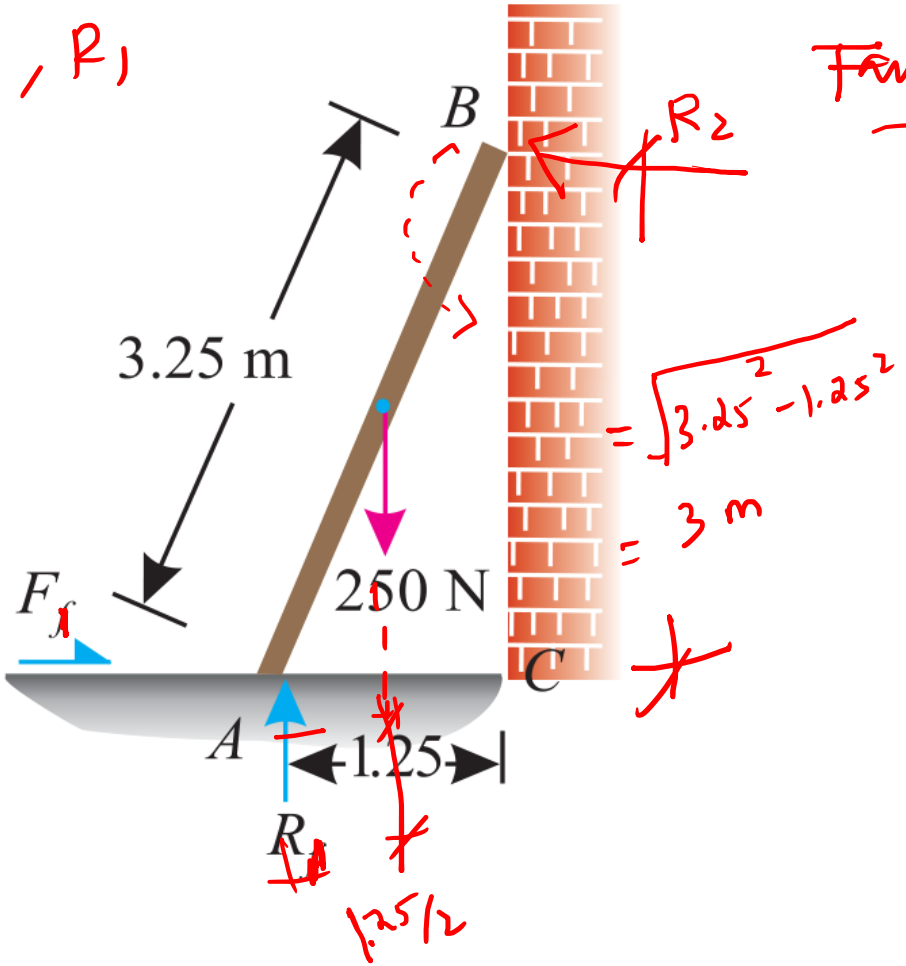
We can't do with horizontal & vertical equilibrium.

Ex-7

Possible way to use $\Sigma M = 0$ equilibrium conditions

unknown

R_2, F_1, R_1



Taking moment about 'A'

so that unknowns (R_1 & F_1) will vanish.

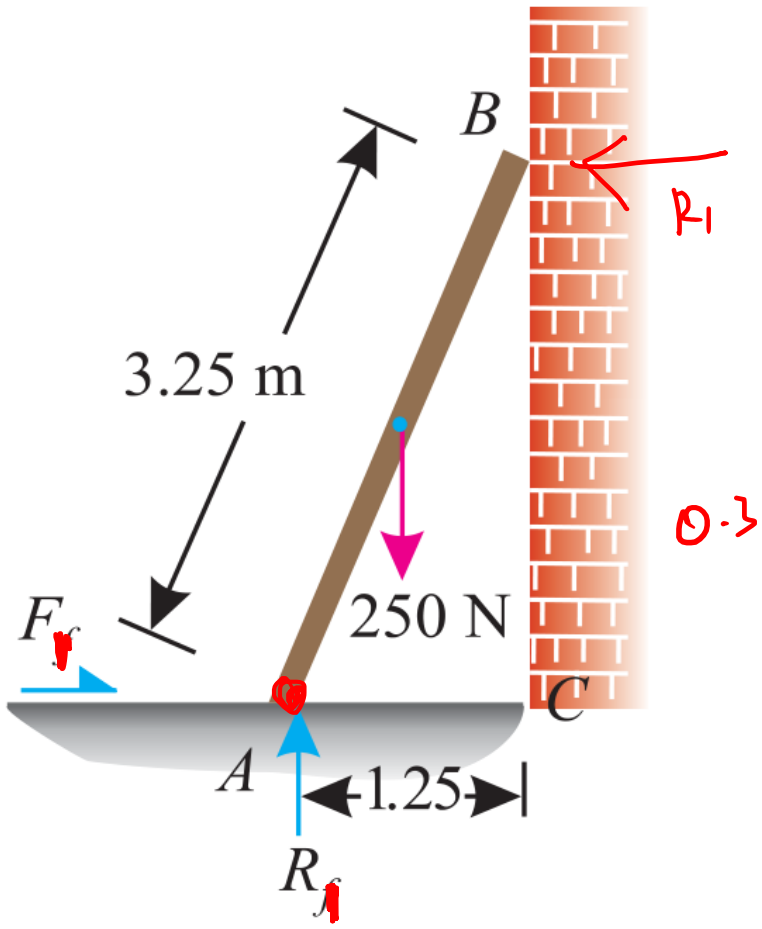
R_2 - unknown. $\Sigma M_A = 0$

$$-(R_2 \times 3) + 250 \times 0.625 = 0$$

$$R_2 = \frac{156.25}{3} = 52.08 \text{ N}$$

$$\boxed{R_2 = 52.08 \text{ N}}$$

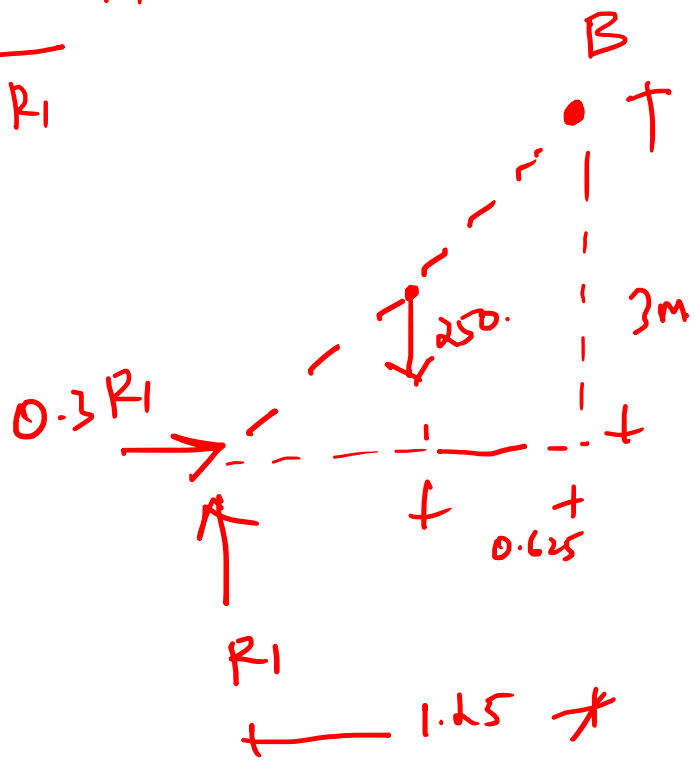
Ex-7



Unknown R_1 & F_1

$F_1 = 0.3 R_1$

$\mu = \frac{F_1}{R_1}$; $\mu = 0.3$ @ Floor.



Taking moment about 'B'

$-(0.3R_1 \times 3) + (R_1 \times 1.25)$

$-(250 \times 0.625) = 0$

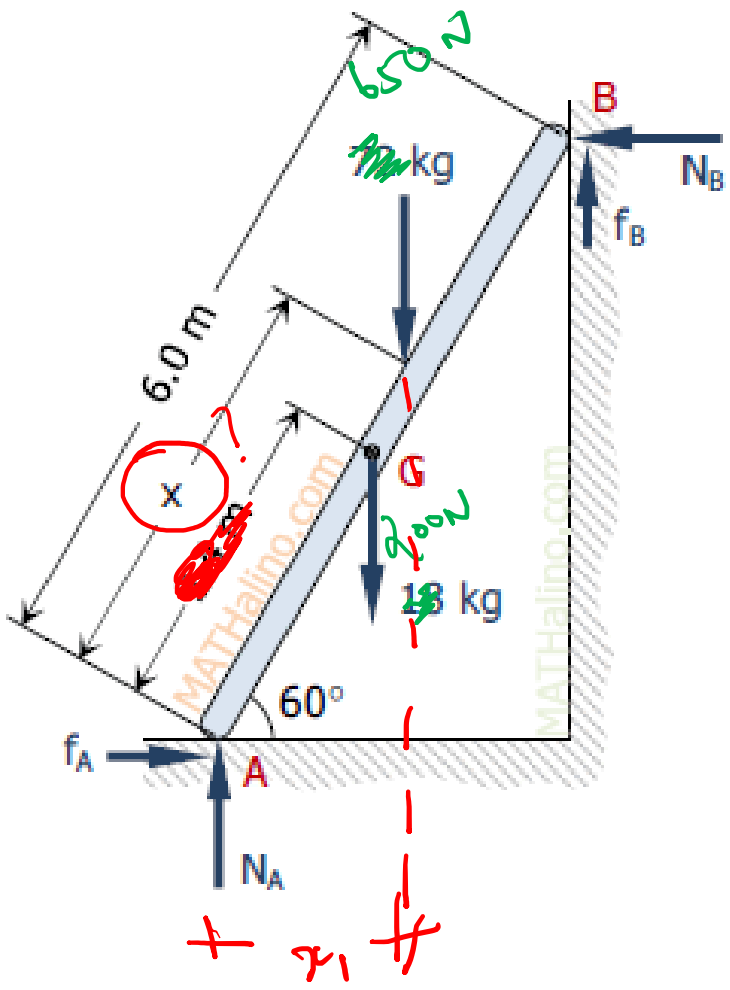
$-0.9R_1 + 1.25R_1 - 156.25 = 0$

$0.35R_1 = 156.25$

$\boxed{R_1 = 446.4\ N}$

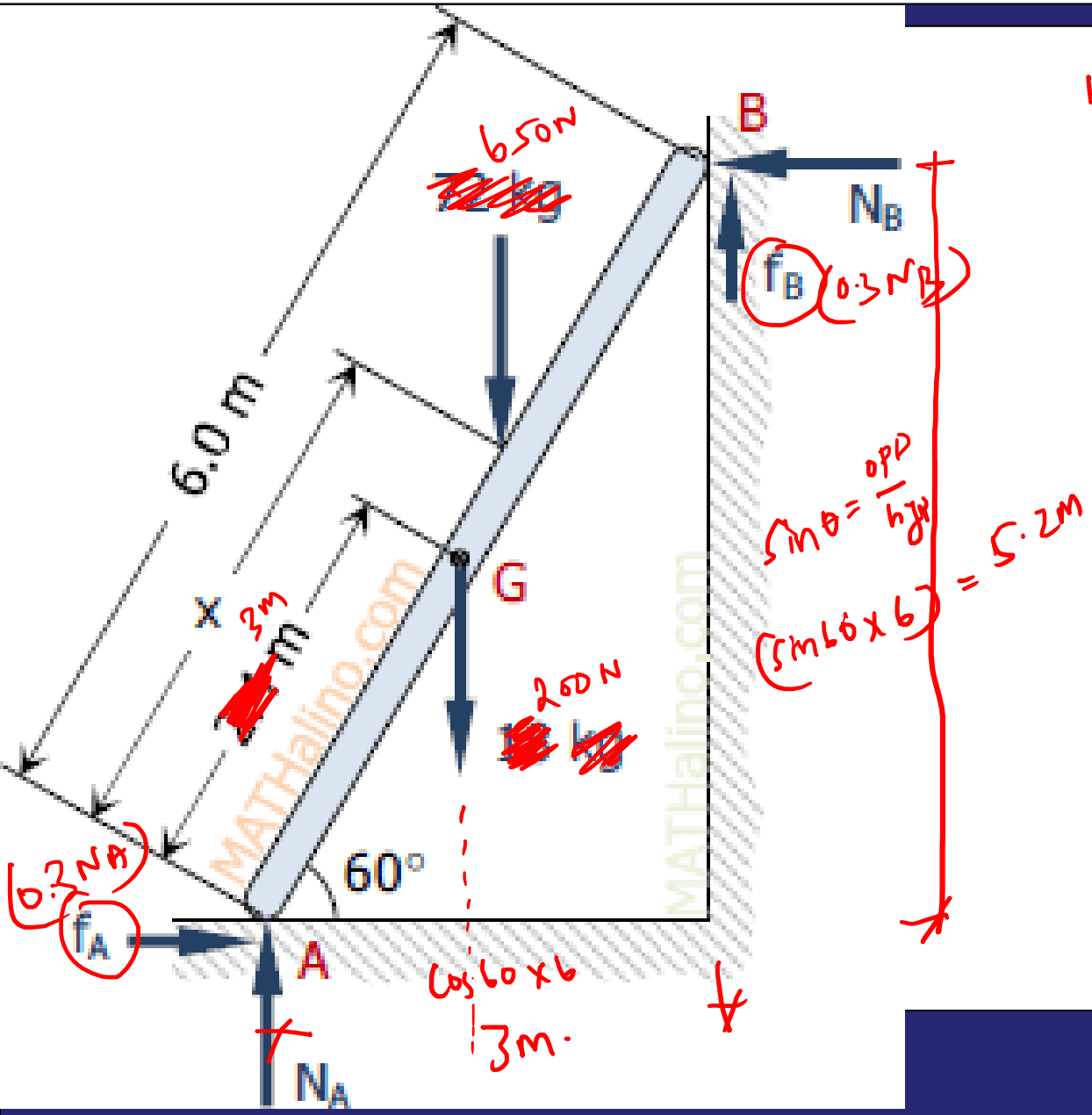
$\boxed{F_{f_1} = 133.93\ N}$

Ex-8 ladder



- The ladder has supported by vertical wall and floor as shown in figure, the coefficient of friction 0.3 for all surfaces, weight of ladder 200N and weight of man 650N,
- (i) find distance in ladder. the man can climb at ladder without ladder slip
- (ii) determine the unknown forces which is in the ladder edges at A and B

Ex-8



Letting them

Moment about A ; $\Sigma M_A = 0$

$$-(N_B \times 5.2) - (0.3 N_B \times 3) + (650 \times x) + (200 \times 1.5) = 0.$$

Solving.

$$-6.1 N_B + 650x = -300$$

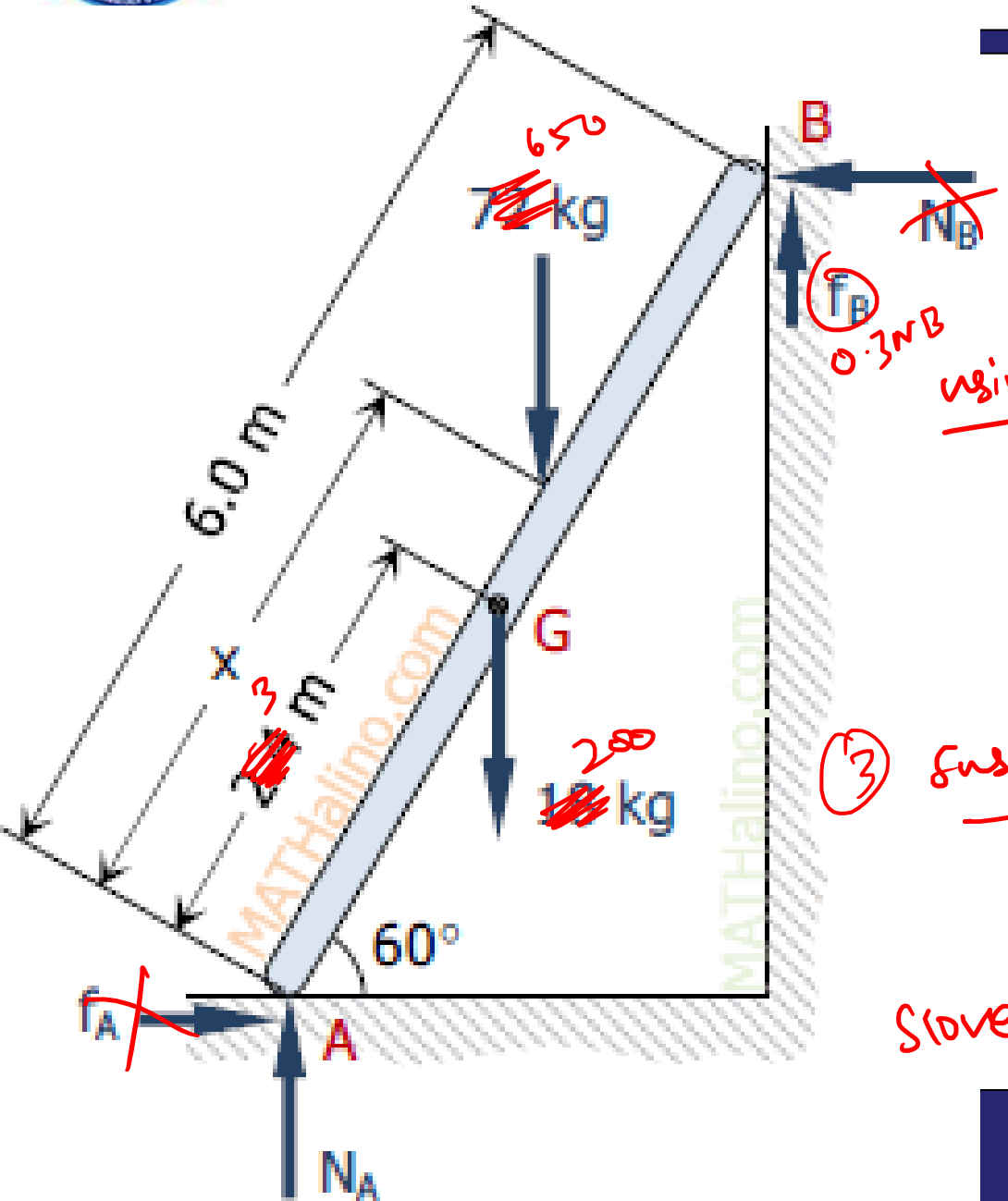
$$6.1 N_B - 650x = 300 \quad \text{—(1)}$$

Moment about 'B' $\Sigma M_B = 0.$

$$-(0.3 N_A \times 5.2) + (N_A \times 3) - (200 \times 1.5) - (650 \times x) = 0$$

$$1.44 N_A - 650x = 300 \quad \text{—(2)}$$

Ex-8



$6.1 N_B - 650x = 200$ — (1)

$1.44 N_A - 650x = 300$ — (2)

using $\Sigma V = 0$.

$N_A + (0.3 N_B) - 200 - 650 = 0$

$N_A + 0.3 N_B = 850$

$N_A = 850 - 0.3 N_B$ — (3)

(3) sub N_A in (2)

$1.44(850 - 0.3 N_B) - 650x = 300$ —

$-0.432 N_B - 650x = -924$ — (4)

Slove eqn (1) & (4)

$\boxed{N_B = 187.38}$ $\boxed{x = 1.296 \text{ M}}$

Sub N_B in eqn (1)

$$N_A = 850 - 0.3\,N_B$$

$$= 850 - 0.3(187.39)$$

$$\boxed{N_A = 793.78\ N}$$

$$\boxed{F_A = 0.3 \times 793.78 = 238.13\ N.}$$

$$\boxed{\begin{aligned} N_B &= 187.39\ N \\ F_B &= 56.22\ N \end{aligned}}$$

$$\cos 60^\circ = \frac{1.29}{x}.$$

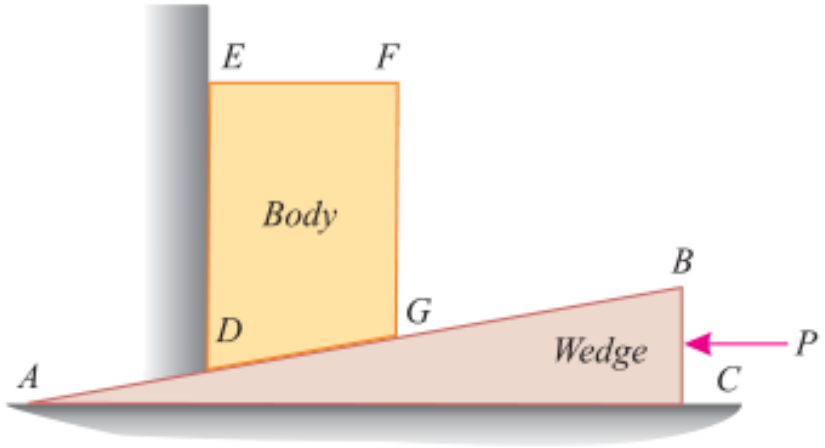
$$x = \frac{1.29}{\cos 60^\circ}$$

$$\boxed{x = 2.58\ m}$$

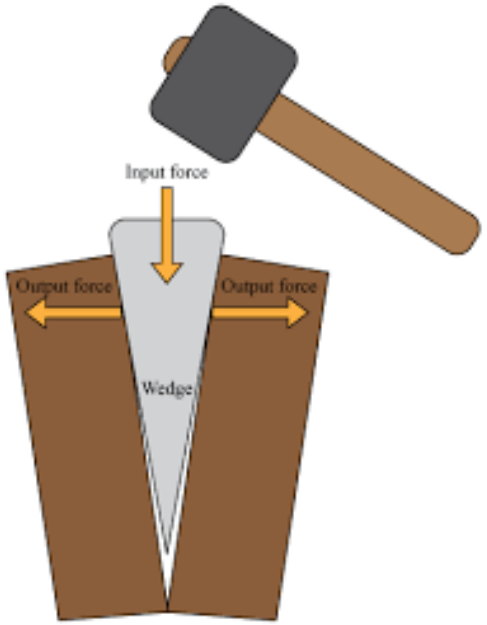
If the man climbs at the lad

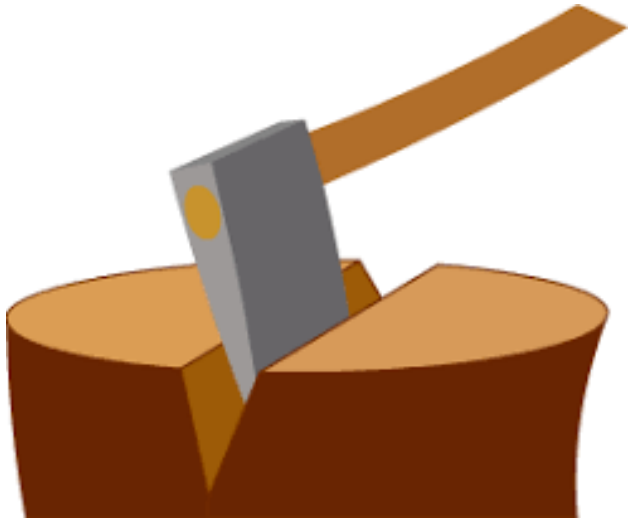
WEDGE FRICTION

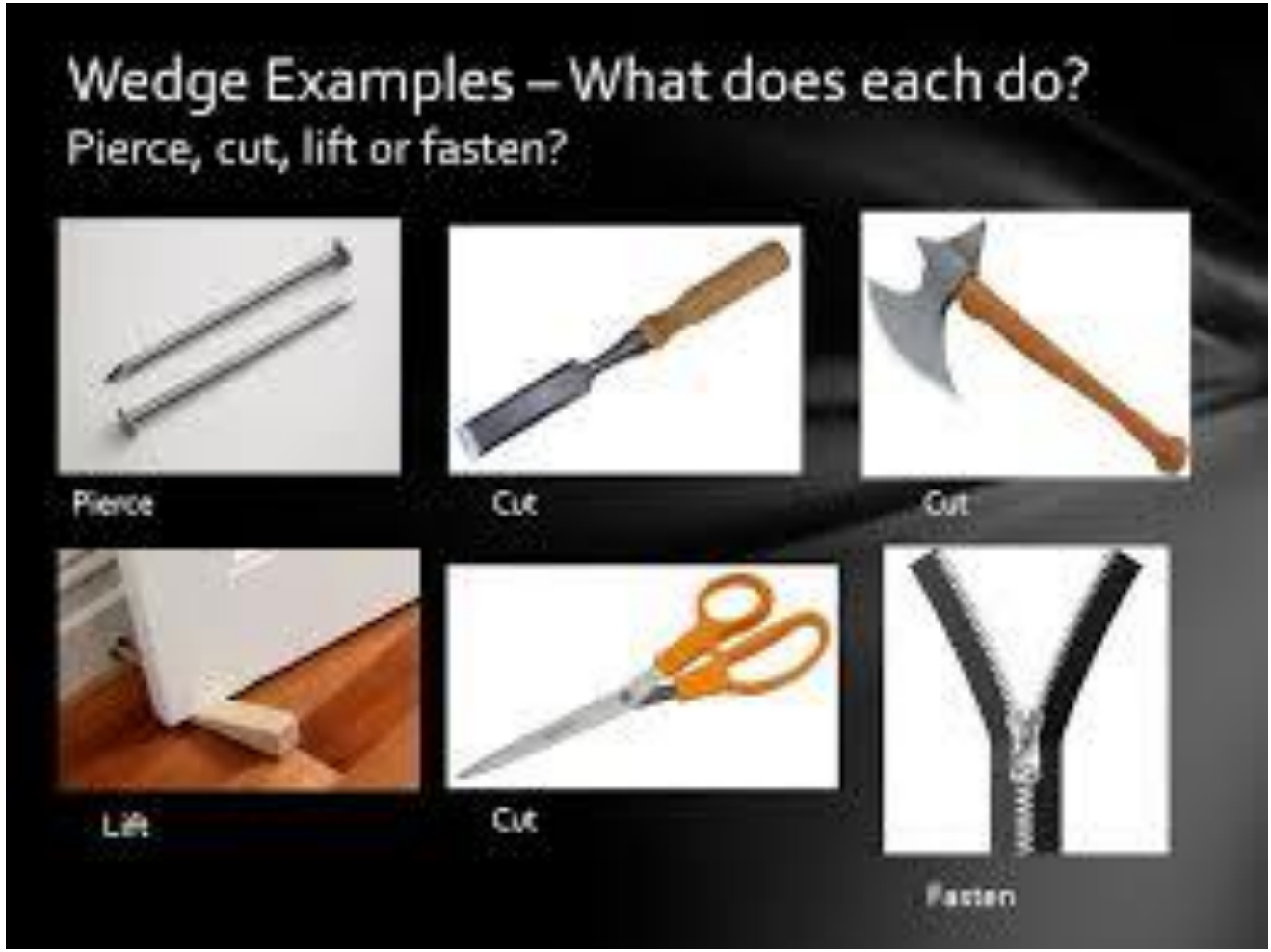
- **A wedge is, usually, of a triangular or trapezoidal in cross-section. It is, generally, used for slight adjustments in the position of a body i.e., for tightening fits or keys for shafts.**
- **Sometimes, a wedge is also used for lifting heavy weights as shown in Fig**



Practical application

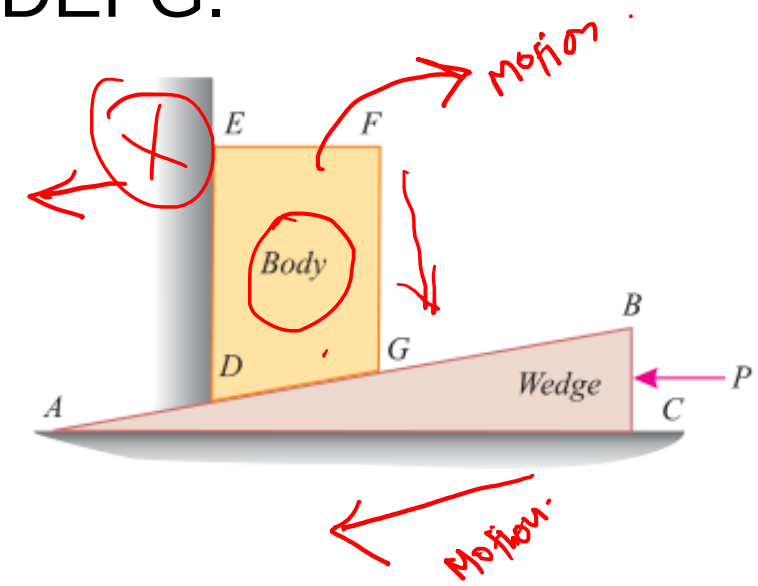


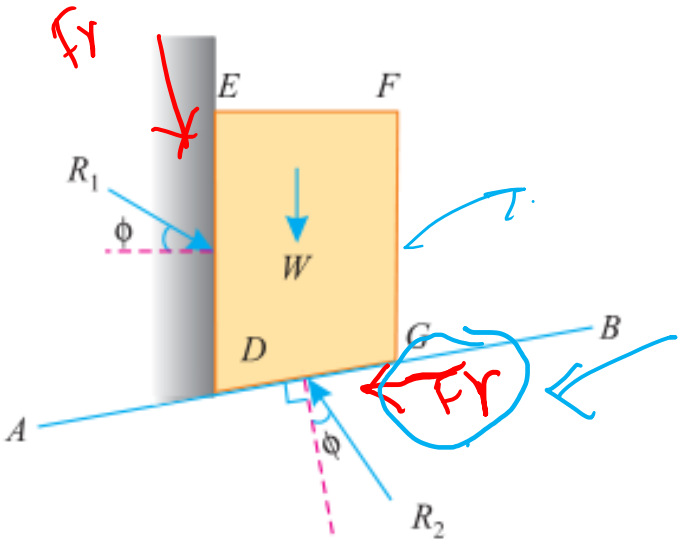




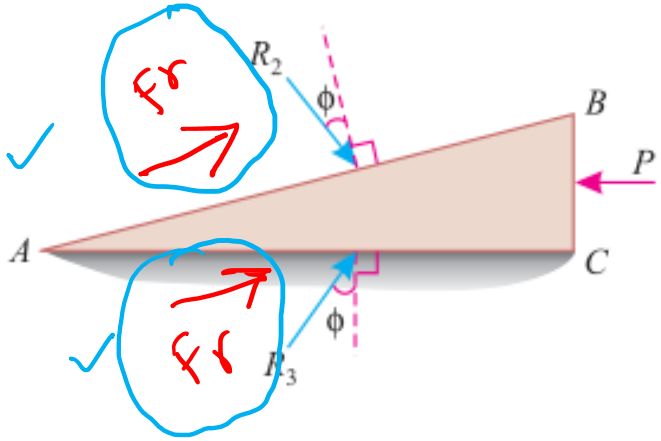
FBD

- It will be interesting to know that the problems on wedges are basically the problems of equilibrium on inclined planes. Thus, these problems may be solved either by the equilibrium method or by applying Lami's theorem. Now consider a wedge ABC, which is used to lift the body DEFG.





(a) Forces on the body *DEFG*



(a) Forces on the wedge *ABC*

Example -9

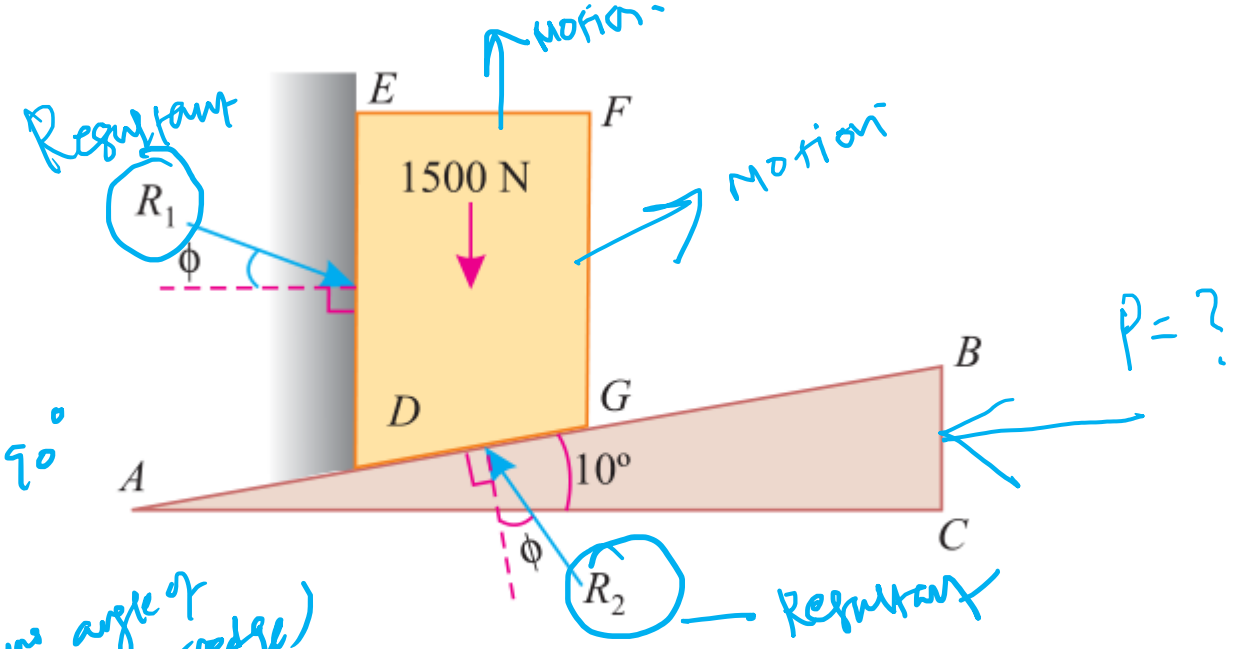
A block weighing 1500 N, overlying a 10° wedge on a horizontal floor and leaning against a vertical wall, is to be raised by applying a horizontal force to the wedge. Assuming the coefficient of friction between all the surface in contact to be 0.3, determine the minimum horizontal force required to raise the block.

FBD ① → top block

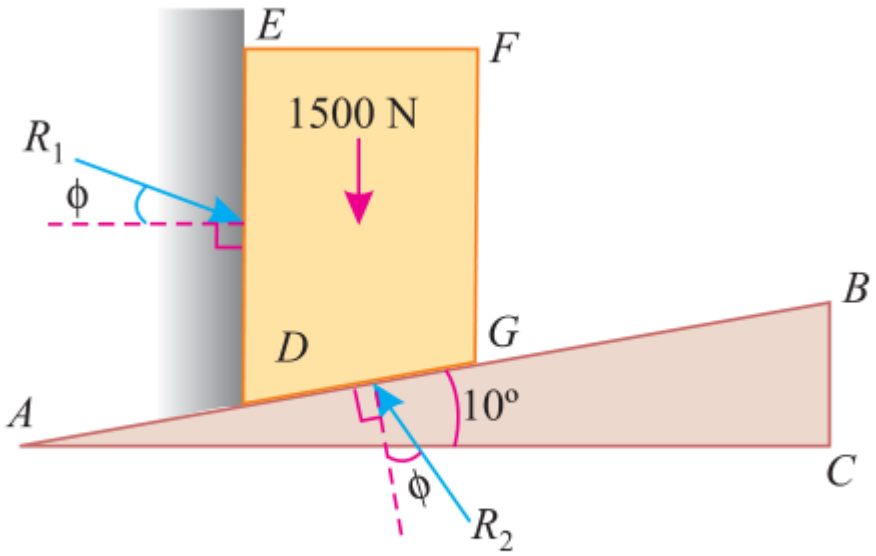
FBD ② → wedge.

NR ⟺ Friction = 90°

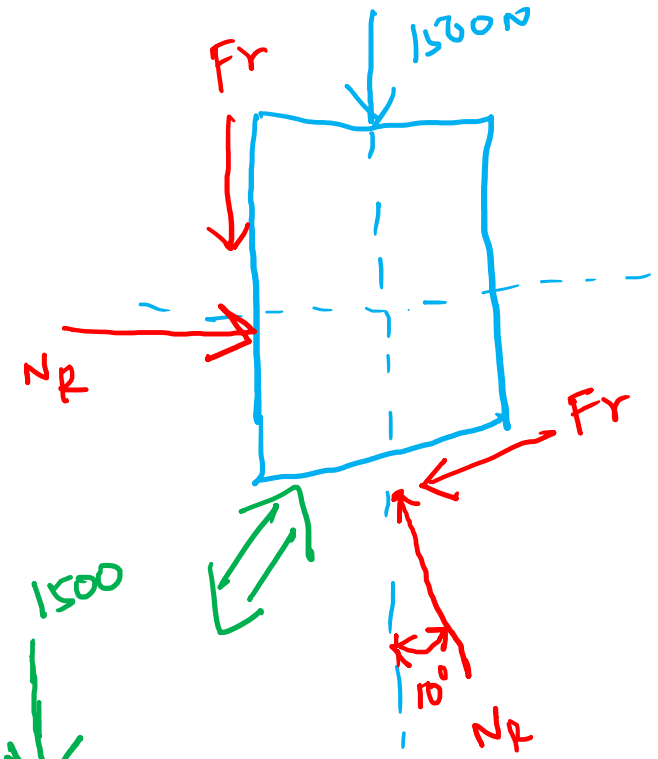
(Normal axis to NR depends angle of wedge)

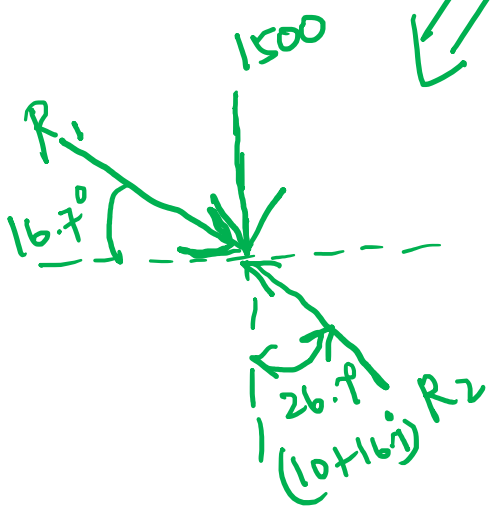


(a) Block *DEFG*



(a) Block *DEFG*





⇒ Resultant is usually occurs b/w Fr & N_R

⇒ The Resultant angle means b/w N_R to Resultant

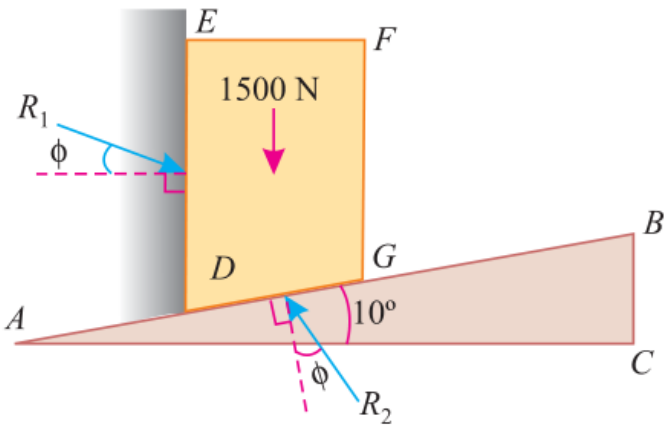
⇒ Resultant May occurs From N_R, Angle of Friction.

⇒ Angle of friction

$\phi = \tan^{-1}\mu$

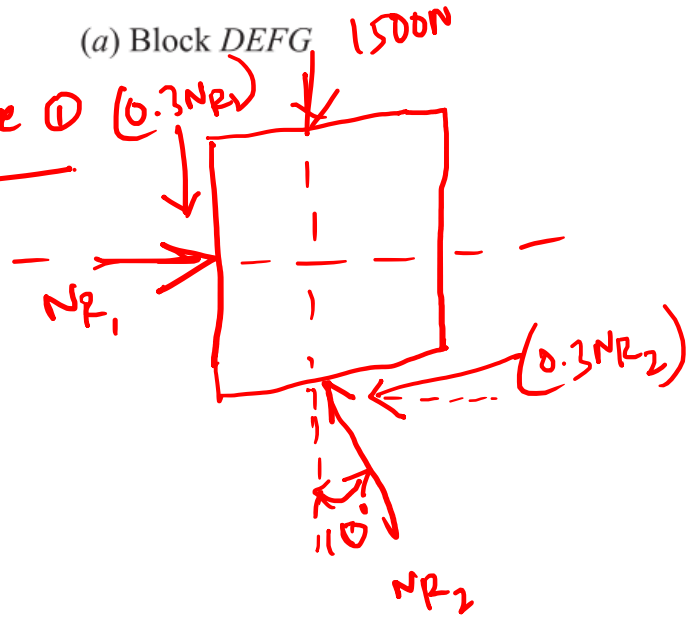
$= \tan^{-1} 0.3$

$= 16.7°$



(a) Block DEFG

Type ①



unknown N_{R_1} & N_{R_2}

$\Sigma F_x = 0$

$N_{R_1} - 0.3N_{R_2}\cos 10° - N_{R_2}\cos 80° = 0$

$N_{R_1} - 0.295\,N_{R_2} - 0.17\,N_{R_2} = 0$

$N_{R_1} - 0.47\,N_{R_2} = 0$ — ①

$\Sigma F_y = 0$

$-0.3N_{R_1} - 1500 + N_{R_2}\sin 80° - 0.3N_{R_2}\sin 10° = 0$

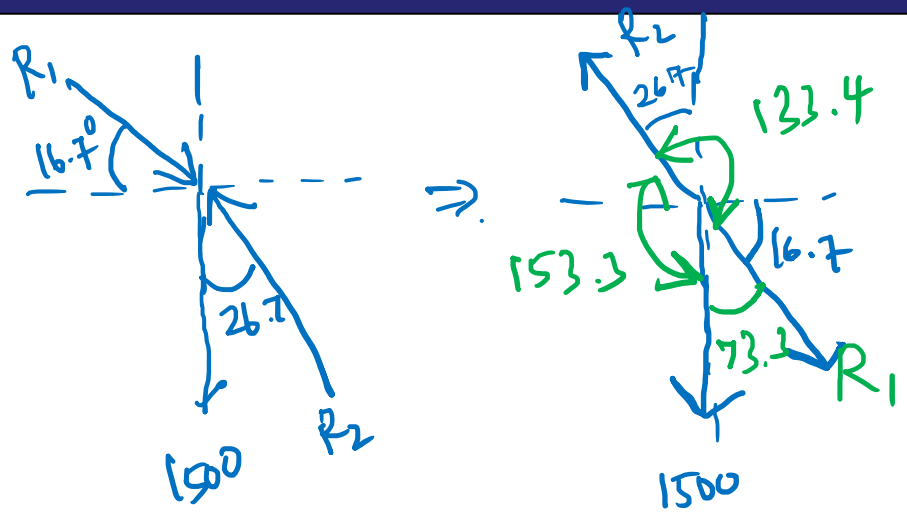
$-0.3N_{R_1} + 0.93\,N_{R_2} = 1500$ — ②

solve ①&②

$N_{R_1} = 893$ N

$N_{R_2} = 1901$ N

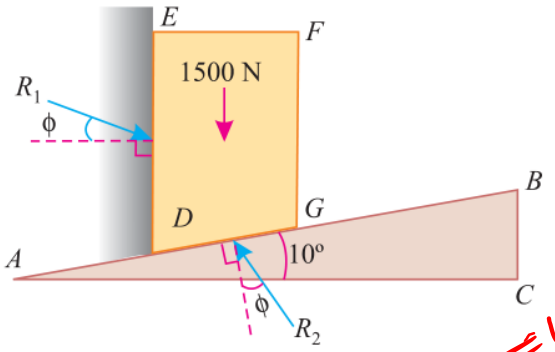
Type-2



$$\frac{R_1}{\sin 153.3} = \frac{R_2}{\sin 73.3} = \frac{1500}{\sin 133.4}$$

$R_1 = 927.6$ N

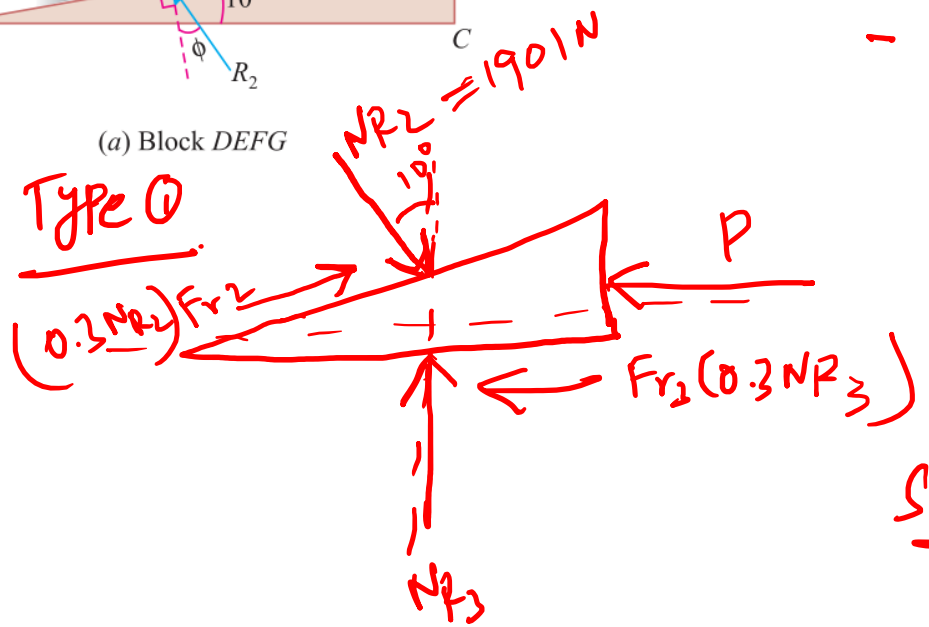
$R_2 = 1977.4$ N



(a) Block DEFG

FBD ② (wedge)

Type ①



$\Sigma F_y = 0$

$-N_{R2}\sin 80^\circ + 0.3N_{R2}\sin 10^\circ + N_{R3} = 0$

$-0.93N_{R2} + N_{R3} = 0$ — ②

Sub $N_{R2} = 1901$

$N_{R3} = 1767.9\ N$

$\Sigma F_x = 0$

$-P - 0.3N_{R3} + 0.3N_{R2}\cos 10^\circ + N_{R2}\cos 80^\circ = 0$

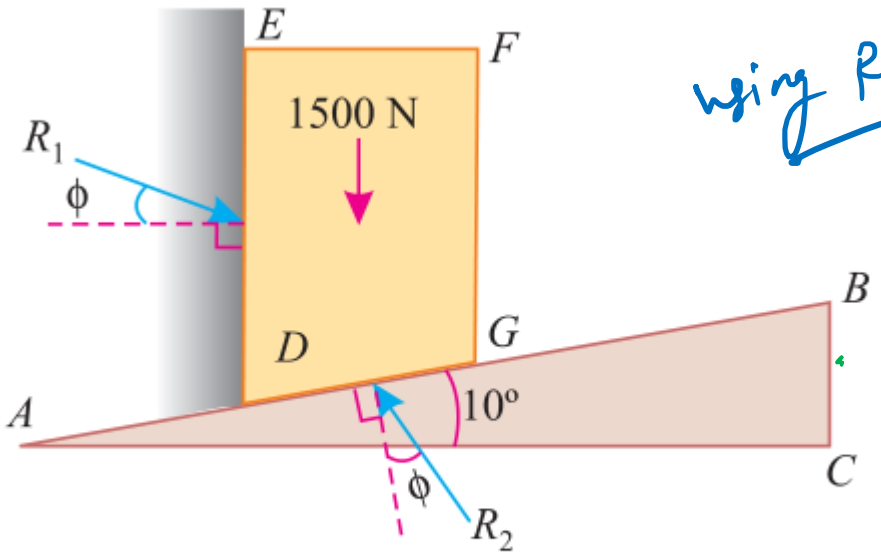
$891.7 - 0.3N_{R3} = P$ — ①

Sub $N_{R3} = 1767.9\ N$ to eqn ①

$891.7 - 0.3(1767.9) = P$

$P = 361.33\ N$

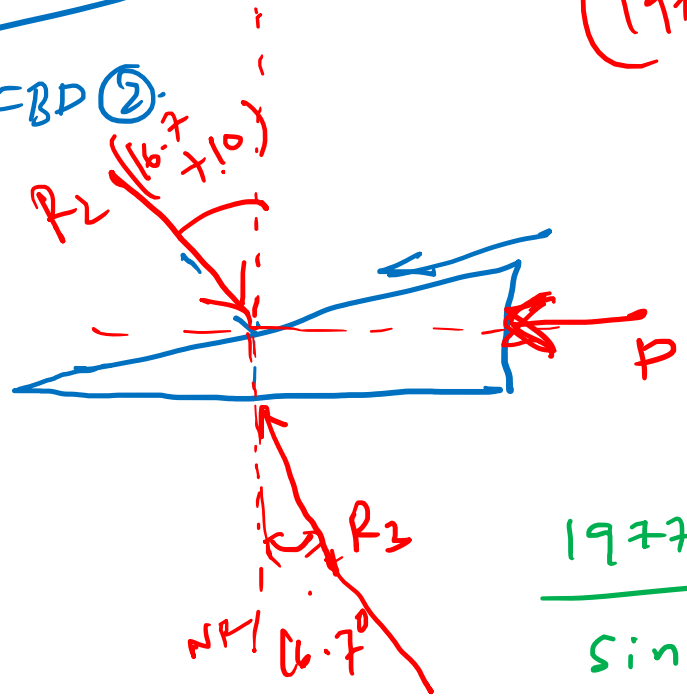
By Type ②

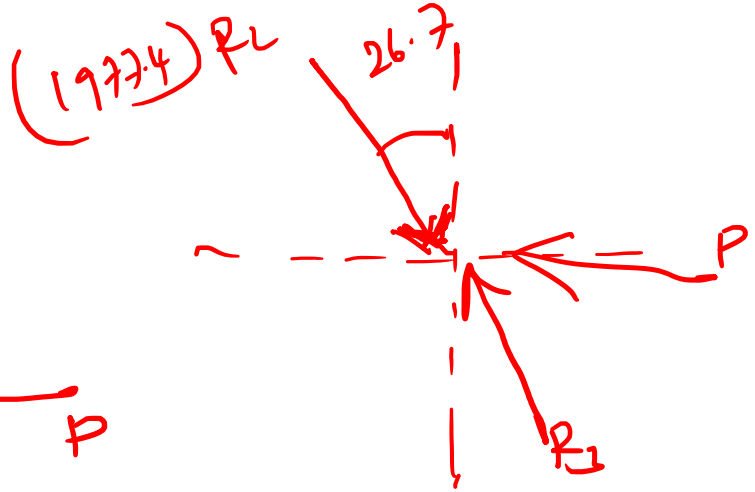


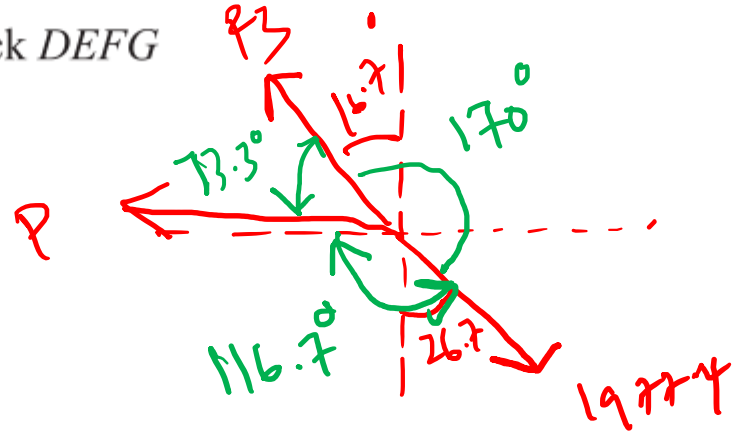
(a) Block *DEFG*

Using Resultant way

FBD ②







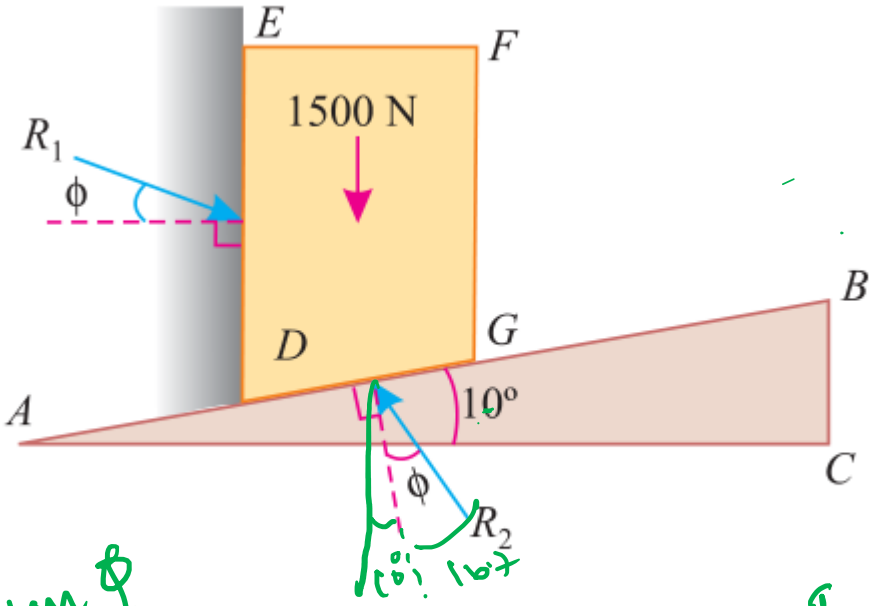
$$\frac{1977.4}{\sin 73.3^\circ} = \frac{R_3}{\sin 116.7} = \frac{P}{\sin 170^\circ}$$

$$\boxed{P = 358.49\ N}$$

Angle of Friction ϕ

$\tan\phi = \mu$

Resultant to Normal Reaction angle is nothing but = angle of Friction. (ϕ)

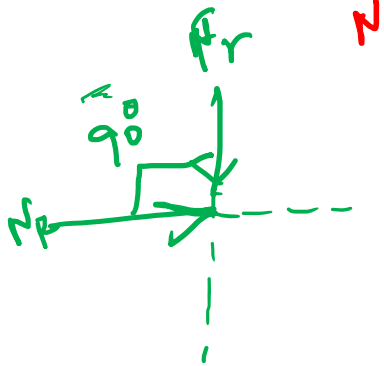


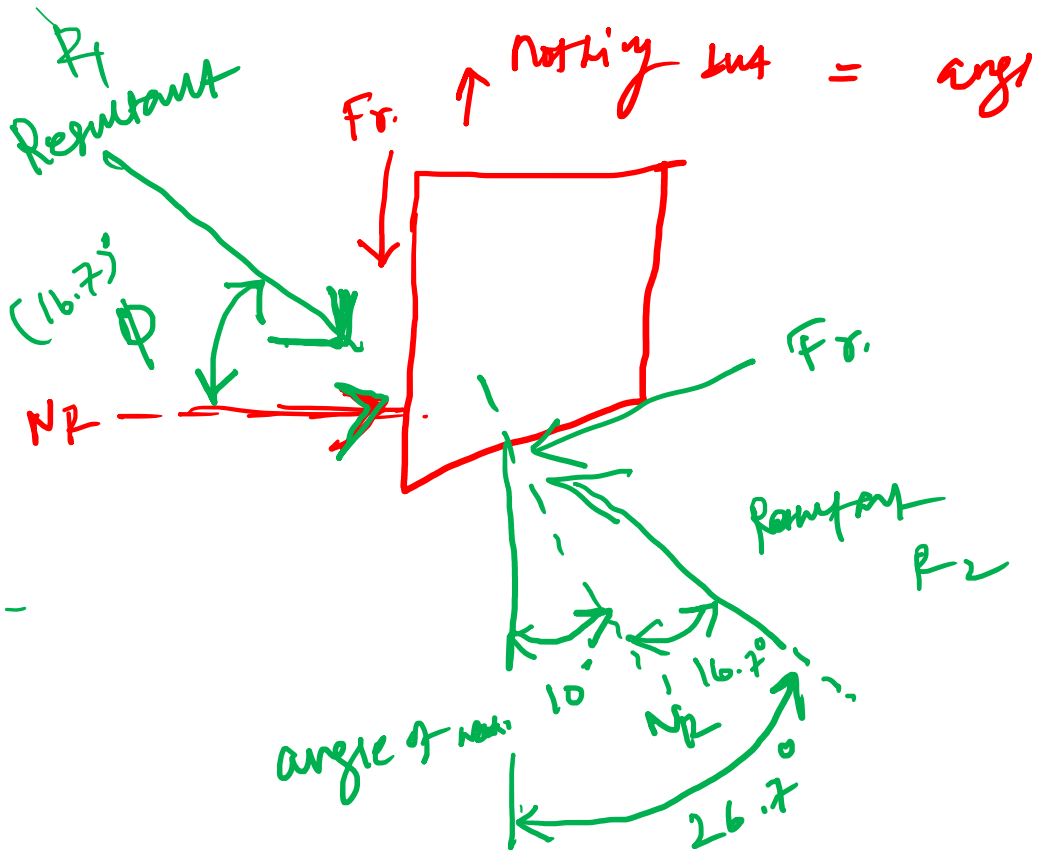
(a) Block DEFG

$\mu = \tan\phi$

$0.3 \tan^{-1} = \phi$

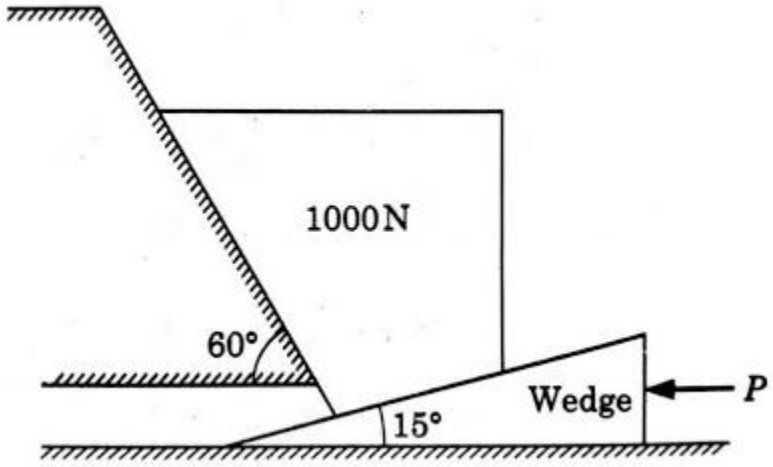
$\phi = 16.7$





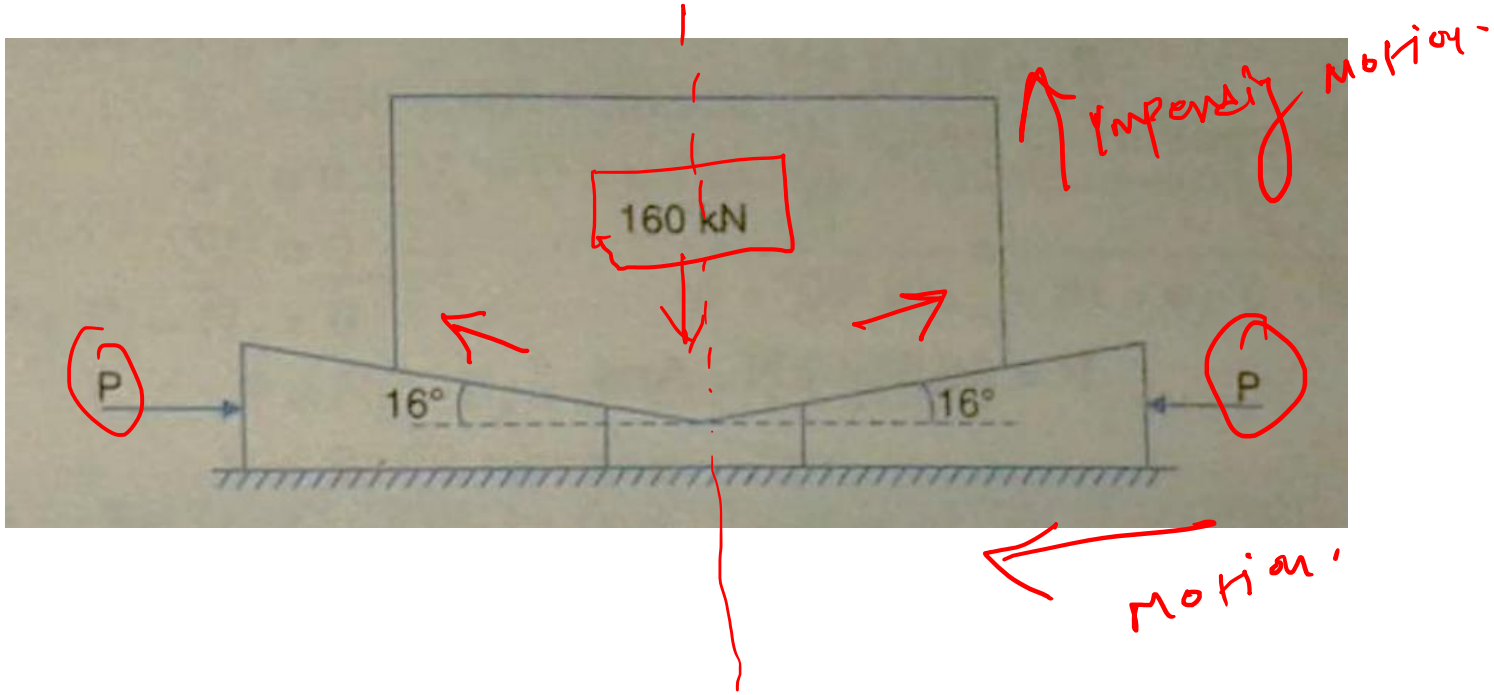
Home work

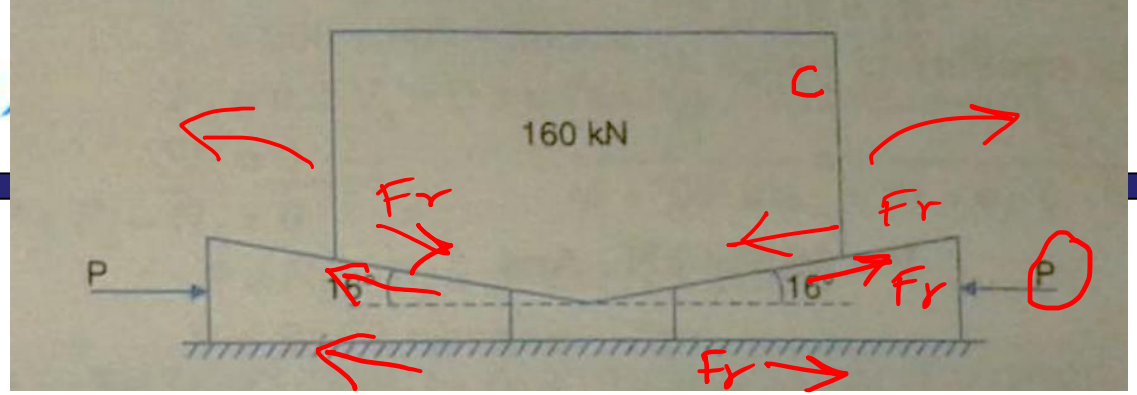
- Determine the force ‘P’ required to move the block A of weight 5000N up the inclined plane. Coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15 degrees.



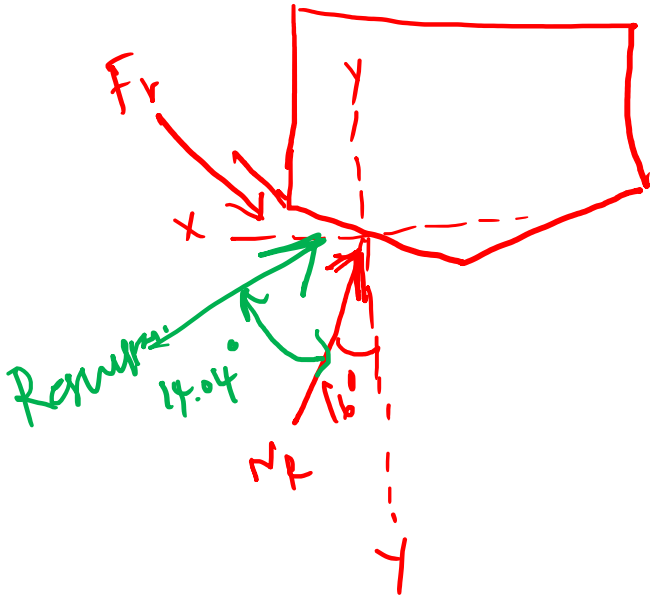
Example

A block weighing 160 kN is to be raised by means dividing wedges as shown in fig. Determine the force ‘P’ required to lift the block. Coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 16 degrees.





Consider block 'C' (160 kN) First

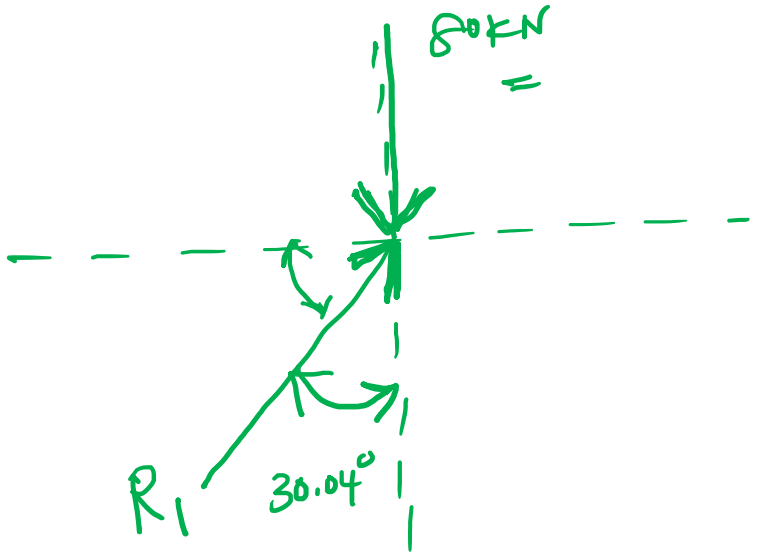


angle of Friction.

$\tan\phi = \mu$

$\phi = \tan^{-1} 0.25$

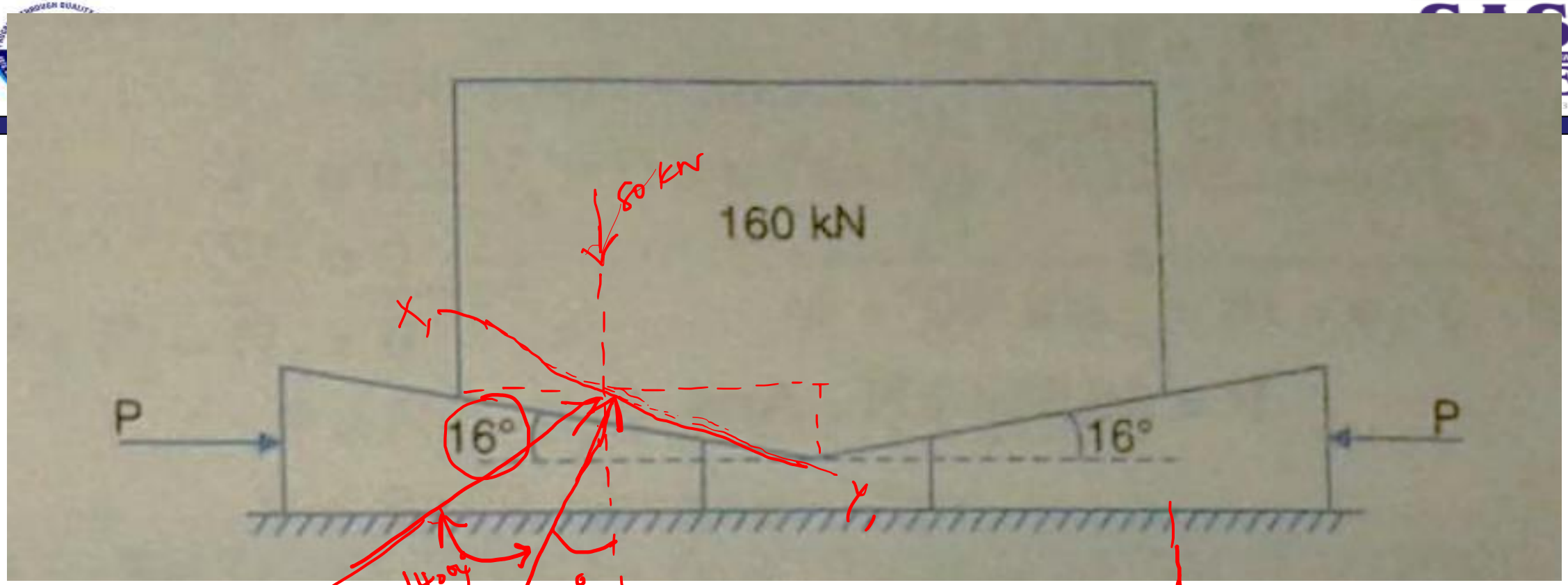
$\phi = 14.04^\circ$

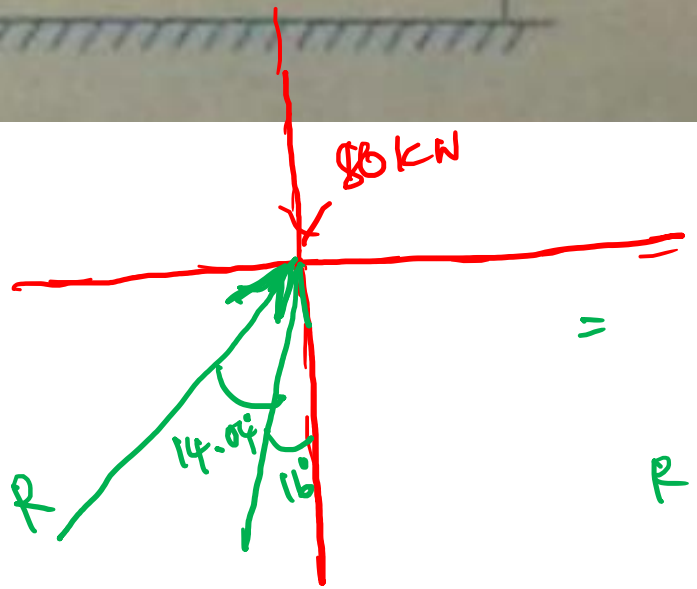


$\Sigma F_y = 0$

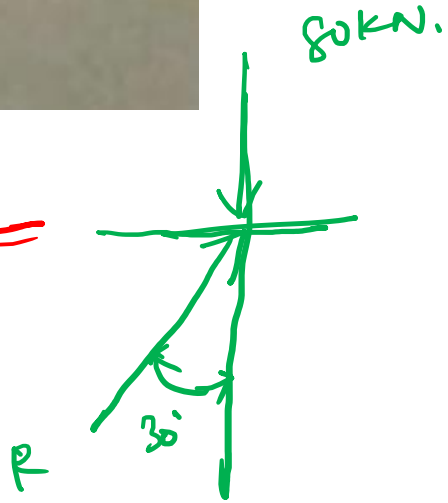
$R_1 \sin 59.96 - 80 = 0$

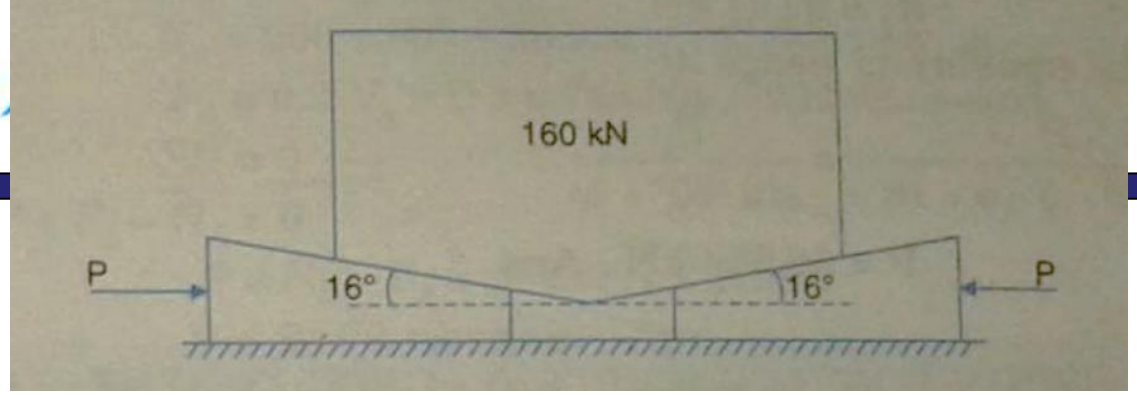
$R_1 = 92.41$ kN


160 kN
P
16°
16°
P
80 kN
X1
Y1
14.04°
16°
NR
Remain

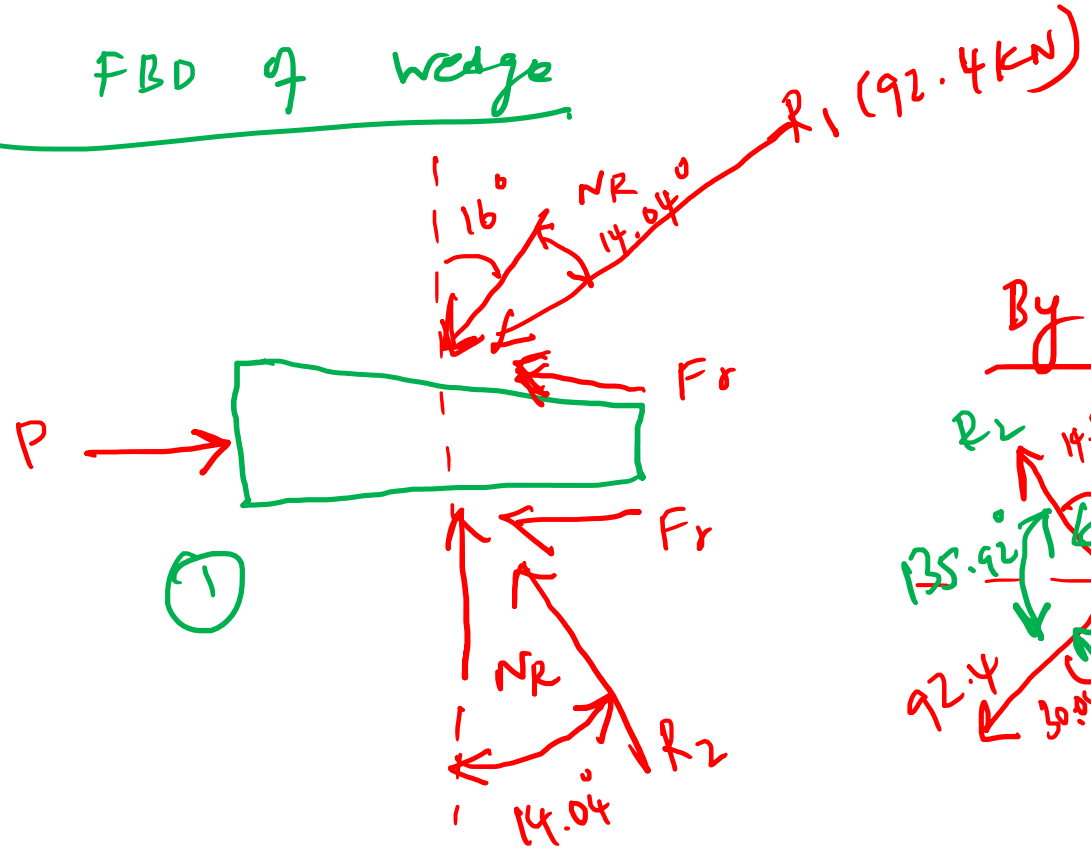

80 kN
R
14.04°
16°

=

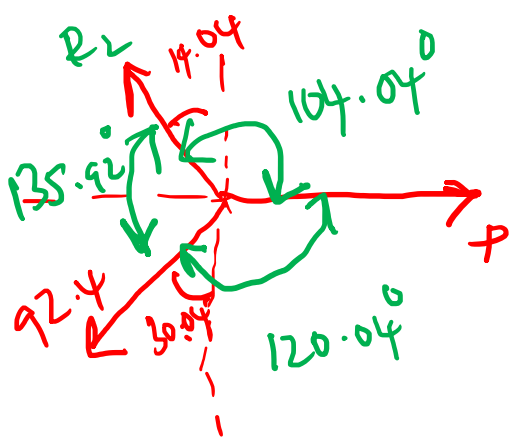

80 kN.
R
30°



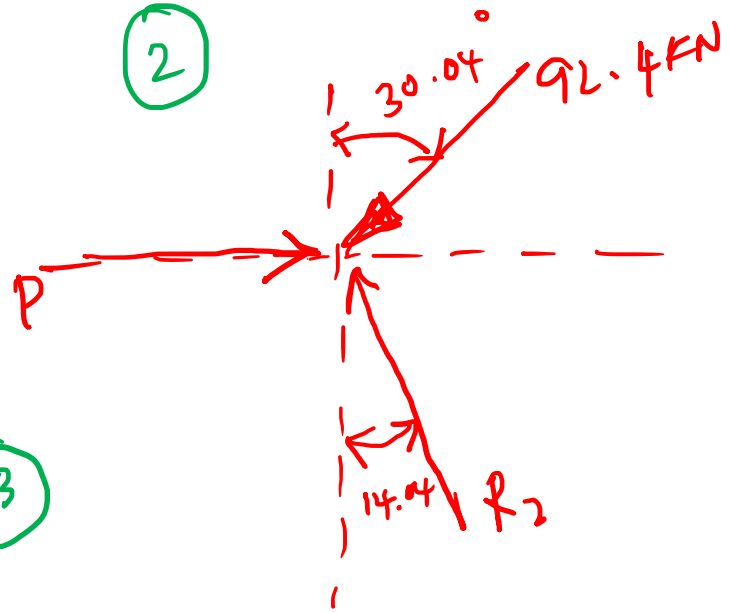
(ii) FBD of wedge



By lamis 3



2



$$\frac{P}{\sin 135.92^\circ} = \frac{92.4}{\sin 104.04^\circ}$$

$$\boxed{P = 66.259 \text{ KN}}$$

Belt friction

- Belt friction is a term describing the friction forces between a belt and a surface, such as a belt wrapped around a bollard. When a force applies a tension to one end of a belt or rope wrapped around a curved surface, the frictional force between the two surfaces increases with the amount of wrap about the curved surface, and only part of that force (or resultant belt tension) is transmitted to the other end of the belt or rope

Application

Capstan

(Use less smaller force to *hold* loads)

Pulley

(Raising or lowering loads)

Torque Transmission

Band brakes

- The equation used to model belt friction is, assuming the belt has no mass and its material is a fixed composition.

Belt Friction (Capstan) Formula

$$T_{load} = T_{hold}\, e^{\mu\phi}$$

T_{load} – applied line tension in N

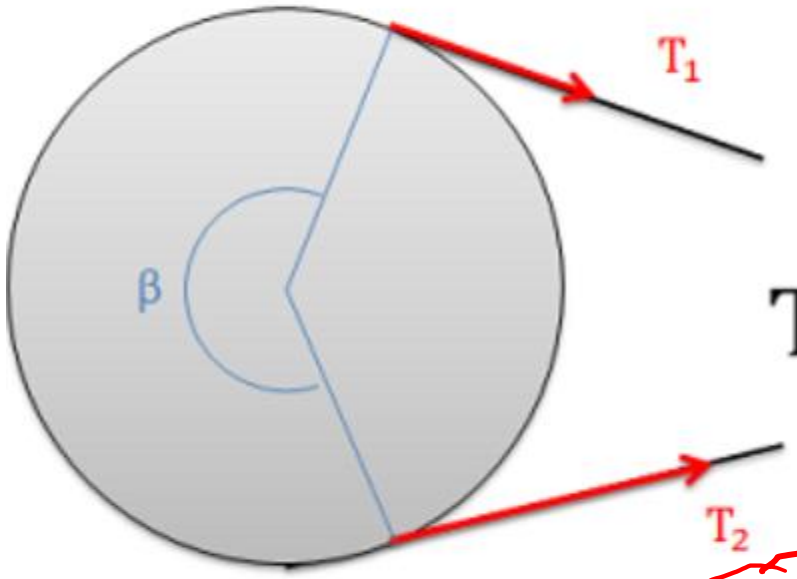
T_{hold} – resulting force at other side in N

μ – coefficient of friction between rope & capstan material

ϕ – total rope angle in radians

angle to rad.

$0^\circ \times \frac{\pi}{180^\circ} = 0$ rad



Coefficient of friction between surface and belt

Euler's constant ≈2.718

Angle of contact in radians

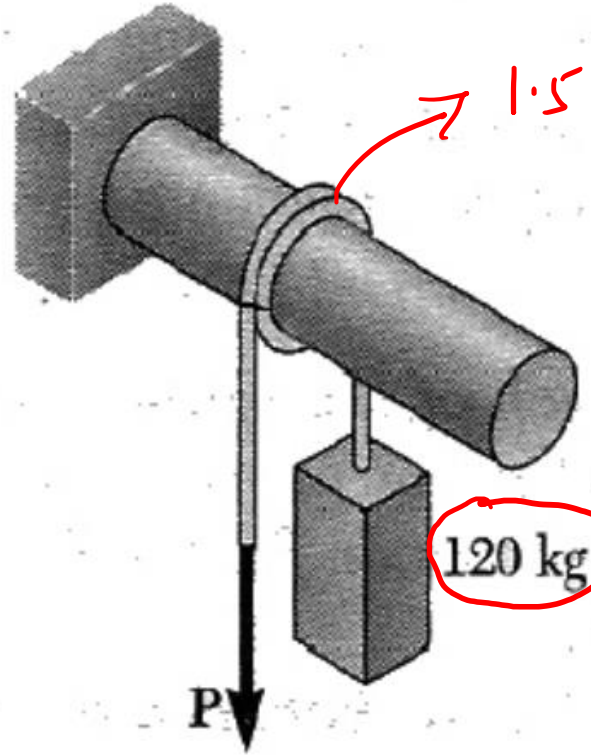
$$T_{2max} = T_1 e^{\mu_s \beta}$$

Always the larger tension force

Always the smaller tension force

Ex - 12

A 120-kg block is supported by a rope which is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.



angle θ

$1.5 \times 360^\circ = 540^\circ$

$540 \times \frac{\pi}{180} = 9.42$ rad

$\mu = 0.15$

varying $P \rightarrow$ Tension by pulling.

$T_2 = T_1 e^{\mu\theta}$

$1177.2 = T_1 \times e^{(0.15 \times 9.42)}$

$1177.2 = T_1 \times 4.108$

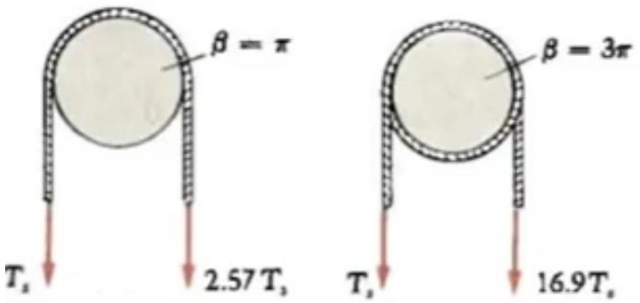
P (or) $= \boxed{T_1 = 286.5\,N}$

T_1

$120 \times 9.81 = 1177.2\,N$

Ex-13

- Determine the tension ratio for rope simply rest on branch and rope having one complete revolution and rest in the tree branch as shown in fig.



$$T_L = T_S e^{\mu\beta}$$

$$T_L = T_S e^{0.3\pi} = 2.57\, T_S$$

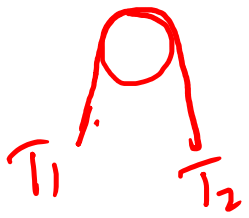
$$T_L = T_S e^{0.3(3\pi)} = 16.9\, T_S$$

Tension ratio increase of 5.6 times!!

$\mu = 0.3$

$180 \times \frac{\pi}{180} = \pi$ rad

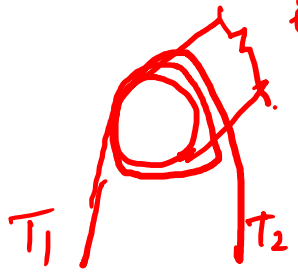
(i)



$T_2 = T_1 e^{(0.3 \times 3.14)}$

$T_2 = 2.56\, T_1$ — (1)

(ii)



$\theta = 180 + 360$

$= 540^\circ$

(or)

$= 9.42$

$T_2 = T_1 e^{(0.3 \times 9.42)}$

$T_2 = 16.9\, T_1$ — (2)

Ratio $\frac{(2)}{(1)} = 5.6$ times

Summary of screw friction

(i) Lifting of heavy weights.

Force Required to lift Heavy weight

$$P = \frac{d}{2R} \cdot W \tan(\theta + \phi)$$

$\phi > \theta$

(Self locked)

Effort $P_1 = \frac{2PR}{d}$

$\theta = \tan^{-1} \frac{\text{Pitch}}{\pi d}$

$R \rightarrow$ distance from c/c of screw to end of lever

$\phi \rightarrow$ Friction angle

$\theta \rightarrow$ angle of inclination

$d \rightarrow$ diameter of shaft (screw)

$W \rightarrow$ weight

(ii) lowering weight

$$P = \frac{d}{2R} W \cdot \tan(\theta - \phi)$$

$\theta > \phi$

Remaining same.

Screw friction

- Screw are used to fasten, lifting or lowering any heavy weight.

- The screw threads are mainly of two types viz. V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. I will be interesting to know that the V-threads are used for the purpose of tightening pieces together (e.g. bolts and nuts etc.). Square threads are used in screw jacks, vice screws etc. which are used for lifting heavy loads.

RELATION BETWEEN EFFORT AND WEIGHT LIFTED BY A SCREW JACK

Force applied → 'P' (Effort)

Weight → W

One complete revolution is → lead of screw head

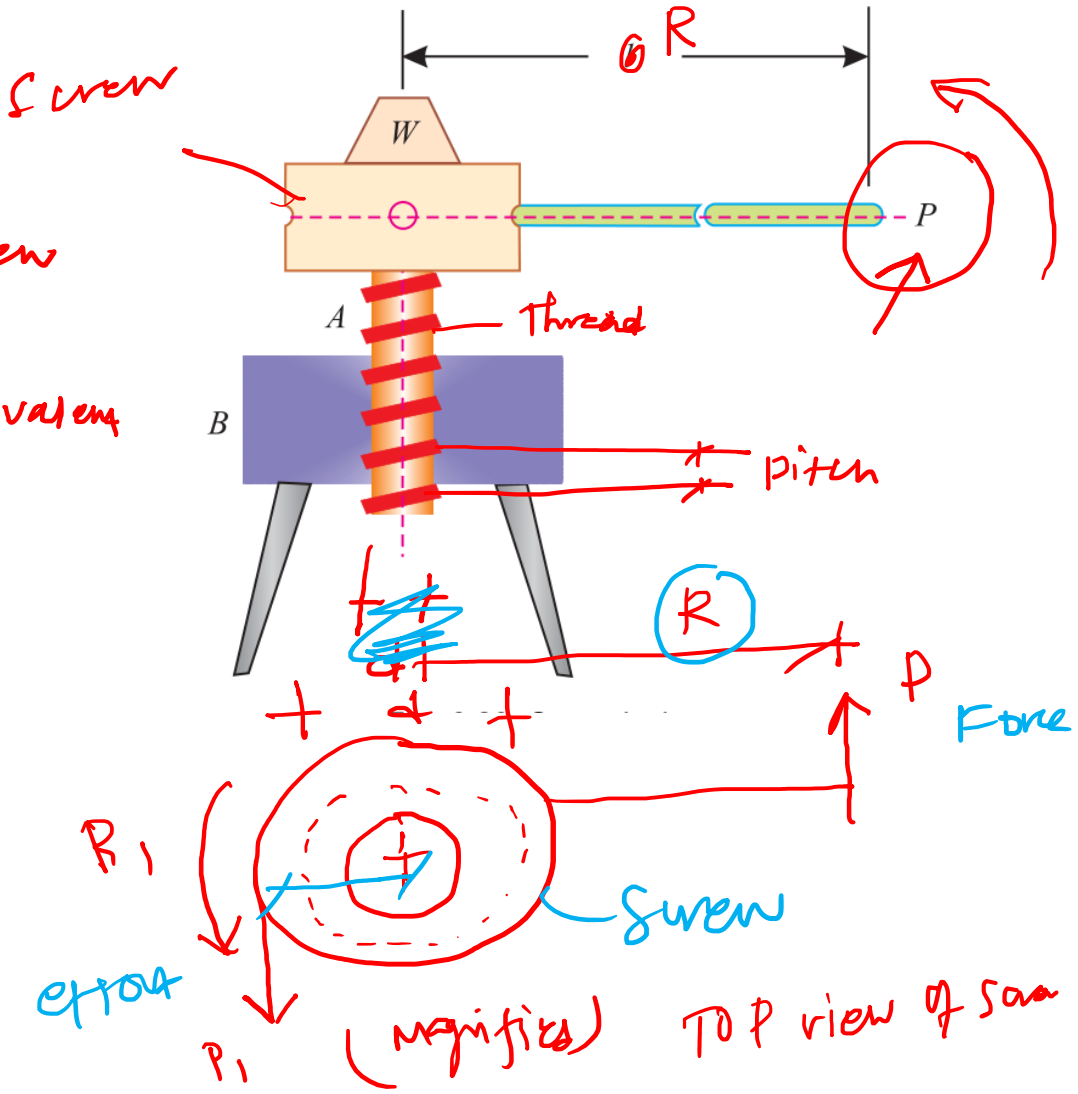
If P is applied at lever end, It is equivalent to an effort 'P_1' at the screw

$$P \times R = P_1 \times d/2$$

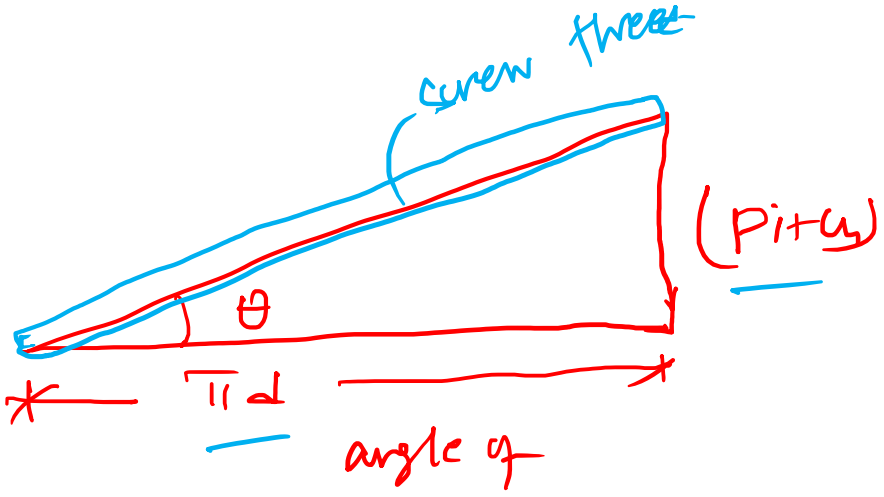
(moment)

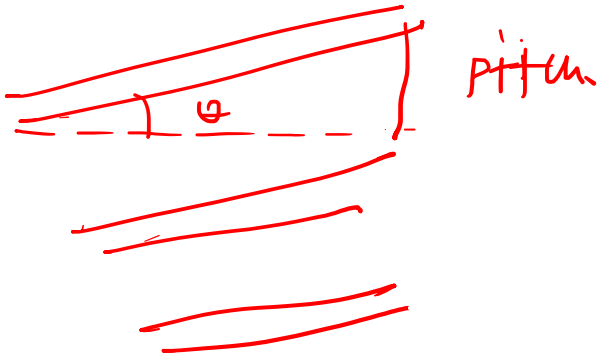
$$\boxed{P_1 = \frac{2PR}{d}}$$

P = ?



Inclination. $\theta = \tan^{-1} \frac{\text{pitch}}{\pi d}$

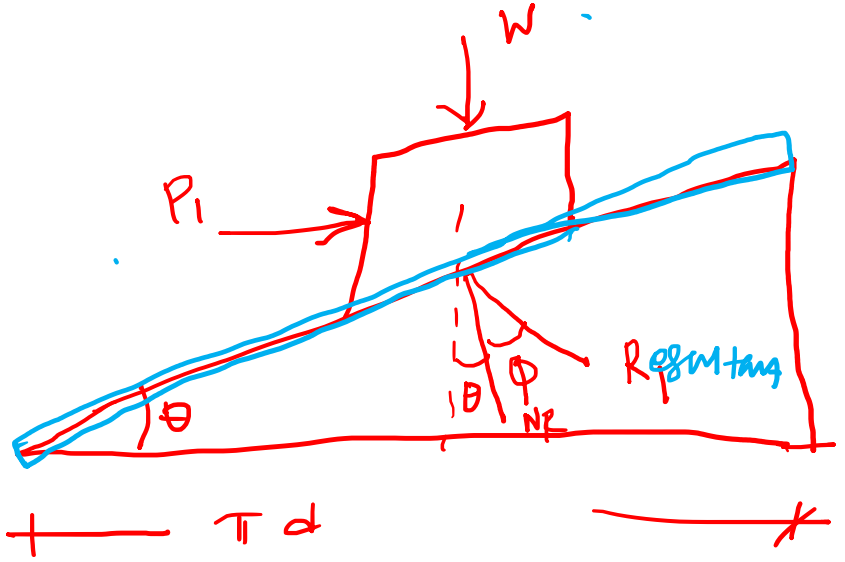




ϕ – angle of friction

θ – angle of inclination

R_1 – Resultant



Resolve horizontally

$\Sigma F_x = 0$

$$P_1 - R_1 \cos(90 - (\theta + \phi)) = 0$$

(or)

$$P_1 - R \sin(\theta + \phi) = 0$$

$$P_1 = R_1 \sin(\theta + \phi) \quad \text{— (1)}$$

Resolving vertically

$\Sigma F_y = 0$

$$W = R_1 \cos(\theta + \phi) \quad \text{— (2)}$$

R_1 – Resultant

Dividing (1) & (2)

$$\frac{P_1}{W} = \tan(\theta + \phi)$$

$$P_1 = W \tan(\theta + \phi) \quad \text{— (3)}$$

W.K.T $P_1 = \frac{2PR}{d}$ Sub in eqn (3)

$$\frac{2PR}{d} = W \tan(\theta + \phi)$$

$(\phi > \theta)$

$\mu = \tan\phi$

$\phi = \tan^{-1}(\mu)$

$$P = \frac{d}{2R} W \tan(\theta + \phi)$$

W.K.T $\tan\phi = \mu$

$$P = \frac{d}{2R} \cdot W \cdot \frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$$

Need not to Remember.

$$P = \frac{d}{2R} \cdot W \frac{\mu + \tan\theta}{1 - \mu \tan\theta}$$

W.K.T $\tan\theta = \frac{P}{\pi d}$

$\theta > \phi$

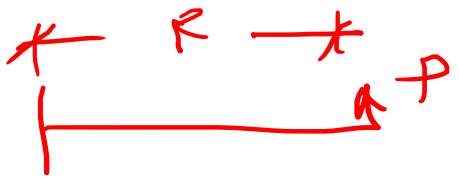
→ If lead is descending, then the friction acting in the reverse direction

$$P = \frac{d}{2R} W \tan(\theta - \phi)$$

Torque required $T = PR$

$$T = \frac{d}{2} W \tan(\theta + \phi)$$

R - levers distance.



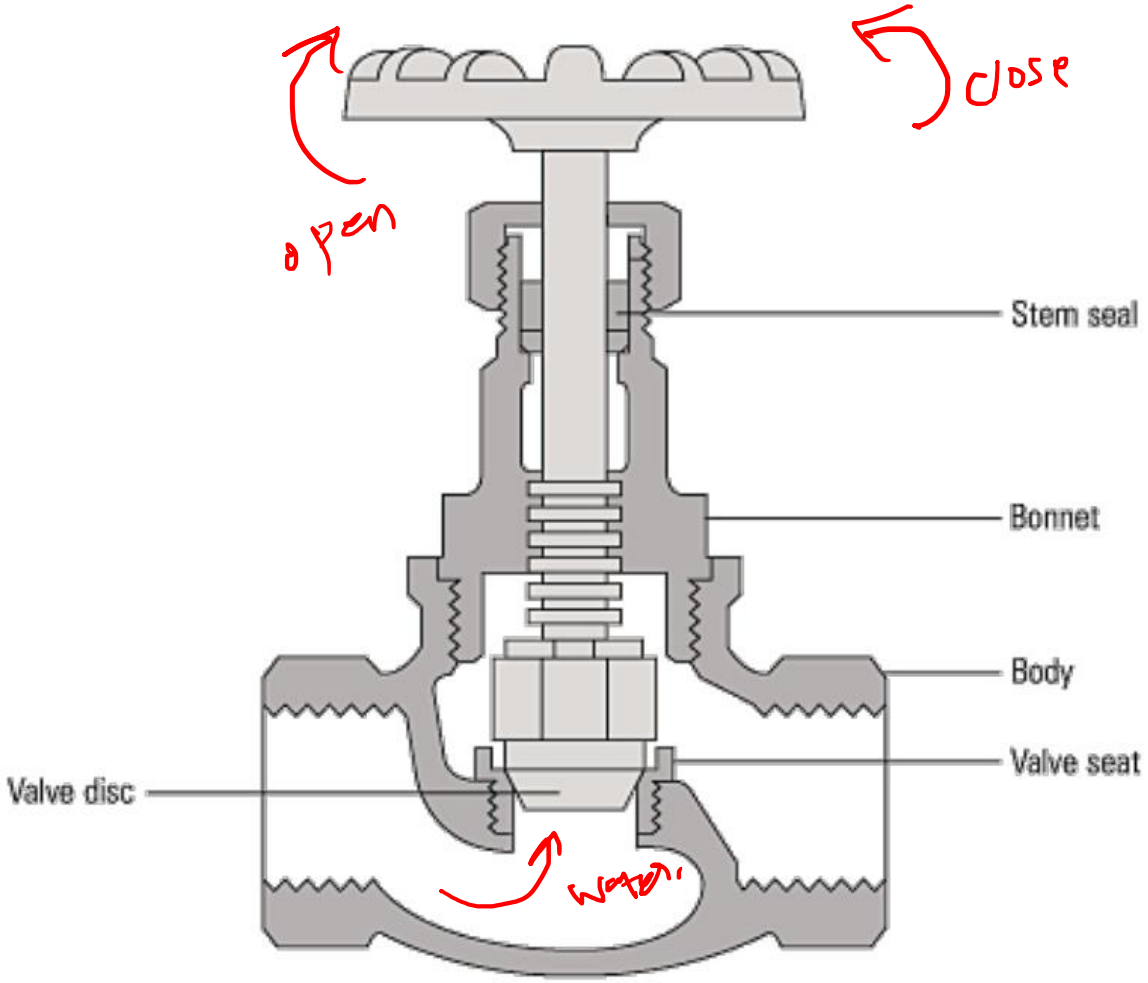
Torque

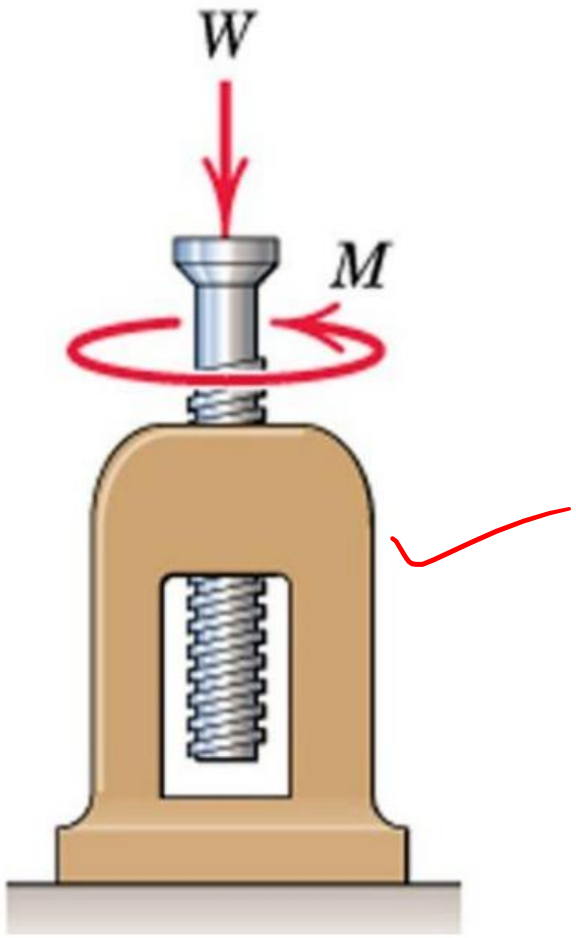
Torque while descending

$$T = \frac{d}{2} W \tan(\theta - \phi)$$

Applications

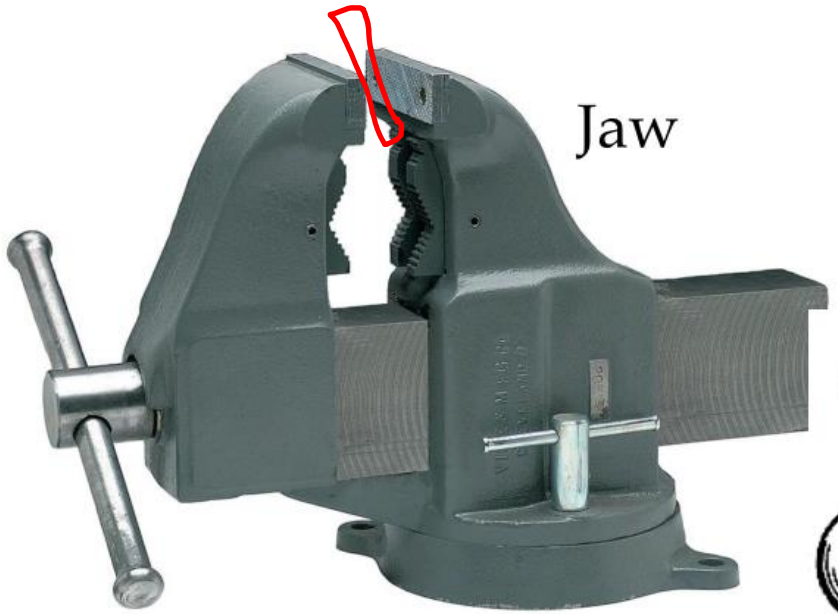
- Vertical direction of motion (*e.g., isolation valve*)

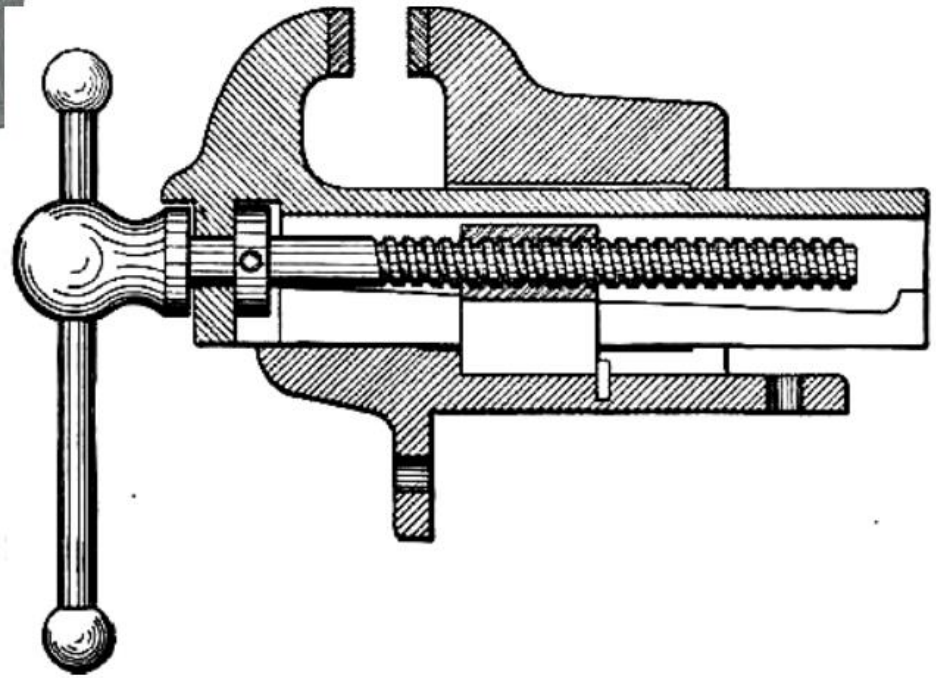




Applications

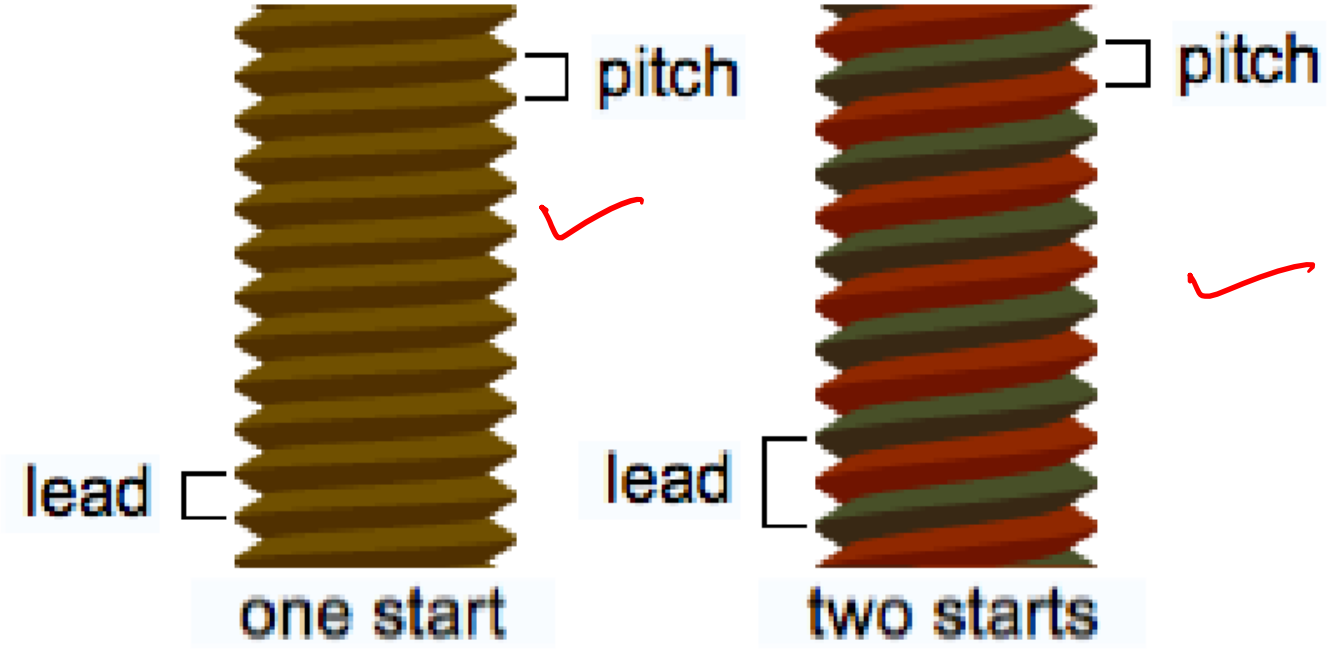
- Horizontal direction of motion (*e.g., vise*)





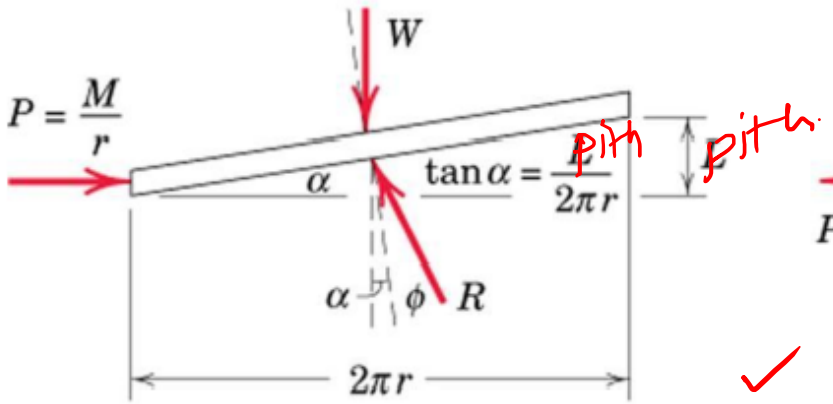
Types of thread

- Thread of a screw



Single thread **Double thread**

three threads



To Raise Load

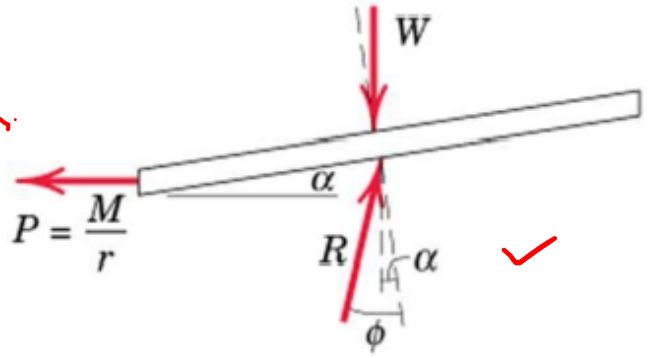
Equivalent force required to push the movable thread up the fixed incline is:

$P = M/r$

From Equilibrium:

$M = W r \tan(\alpha + \phi)$

~~If M is removed: the screw will remain in place and be~~



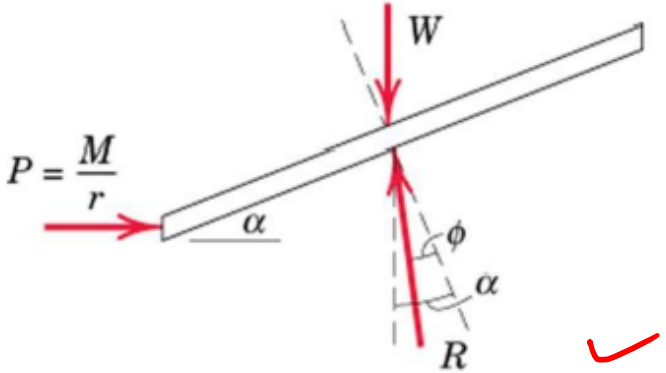
To Lower Load ($\alpha<\phi$)

To lower the load by unwinding the screw, We must reverse the direction of M as long as $\alpha<\phi$

From Equilibrium:

$M = W r \tan(\phi - \alpha)$

→ This is the moment required to unwind the

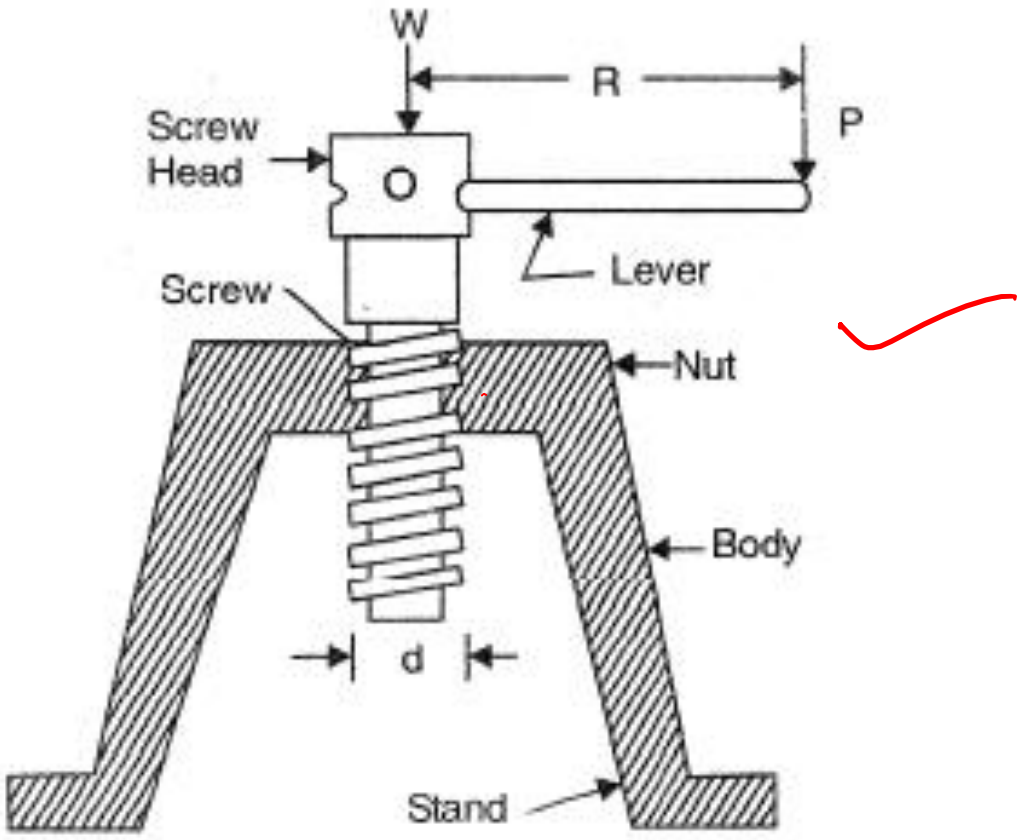


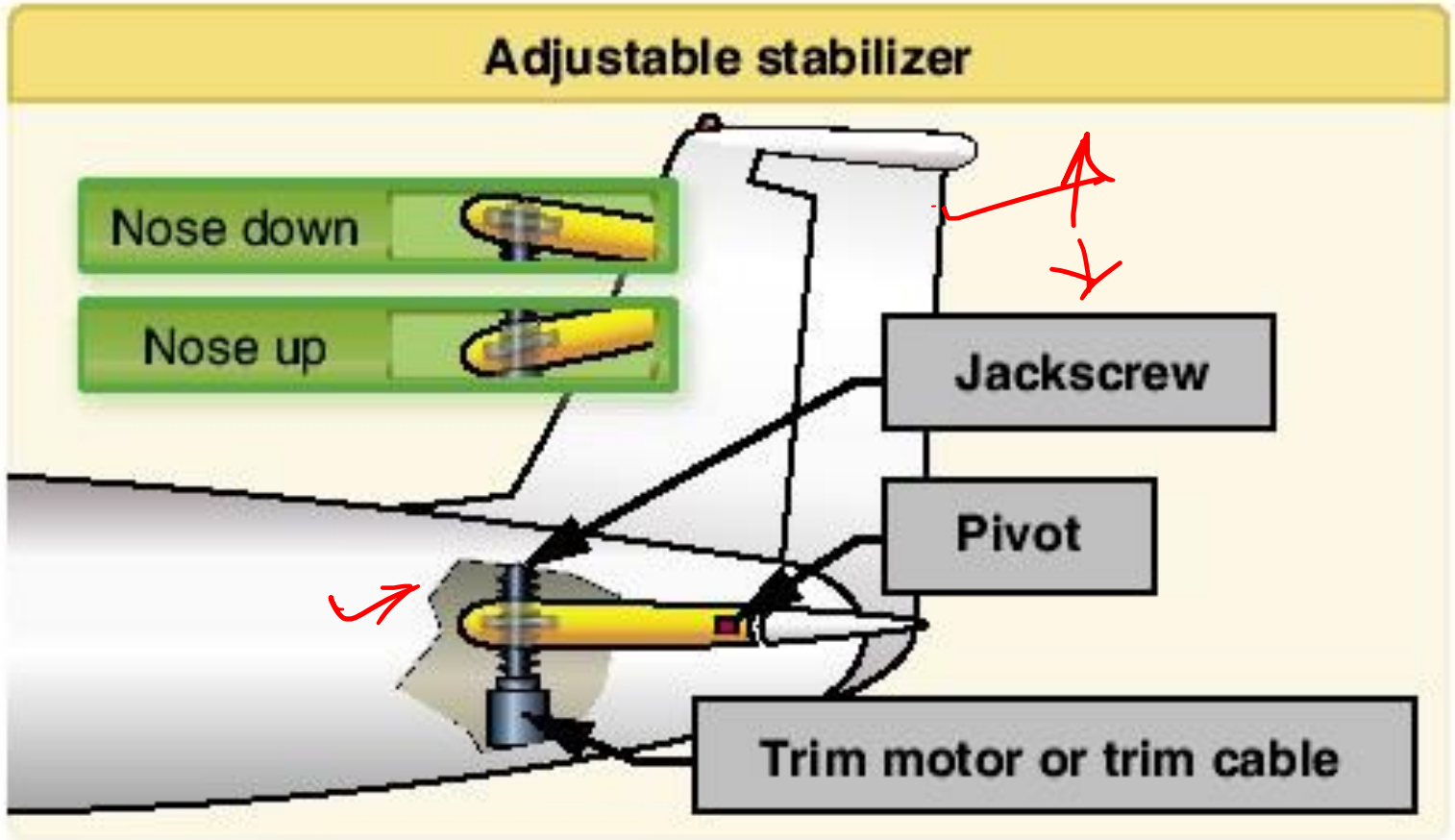
To Lower Load ($\alpha>\phi$)

If $\alpha > \phi$, the screw will unwind by itself. Moment required to prevent unwinding:

From Equilibrium:

$M = W r \tan(\alpha - \phi)$


W
R
P
Screw
Head
O
Lever
Screw
Nut
Body
d
Stand


Adjustable stabilizer
Nose down
Nose up
Jackscrew
Pivot
Trim motor or trim cable

Efficiency and self-locking

- The Efficiency of screw jack

$$\eta = \frac{\tan \theta}{\tan(\theta + \phi)}$$

2 marks.

$\phi > \theta$

Self locked

- The self locking :A **screw** will be **self locking** if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle i.e. μ or $\boxed{\tan \varphi > \tan \theta}$ ... If the efficiency is more than 50%, then the **screw** is said to be overhauling

lowering $\theta > \phi$ $\frac{d}{2R} W(\tan\theta - \phi)$ | lift $\rightarrow \phi > \theta$

Problem A screw jack raises a load of 40 kN (W), square threads three threads 20 mm length & 40 mm in diameter. Calculate the Force (P) Required. The distance of lever $R = 400$ mm. $\mu = 0.12$

$W = 40$ kN

$\phi = \tan^{-1}\mu$

$R = 400$ mm

$d = 40$ mm

Inclination.

lead $= 20/3 = 6.67$ mm. (pitch)

$\tan\theta = \dfrac{6.67}{\pi \times 40} = 0.053$

$\theta = 3.03^\circ$

$\tan^{-1} 0.12 = 6.84^\circ \Rightarrow \phi$

$\phi > \theta$ self locking

$P = \dfrac{d}{2R} W \tan(\theta + \phi)$

$= \dfrac{40}{2 \times 400} \times 40{,}000 \tan(3.03 + 6.84)$

$P = 347.97$ N

Efficiency of Screw $\eta = \dfrac{\tan\theta}{\tan(\theta+\rho)}$

$= \dfrac{\tan 3.03}{\tan(3.03+6.84)}$

$= 30.4\%$

Ex-14

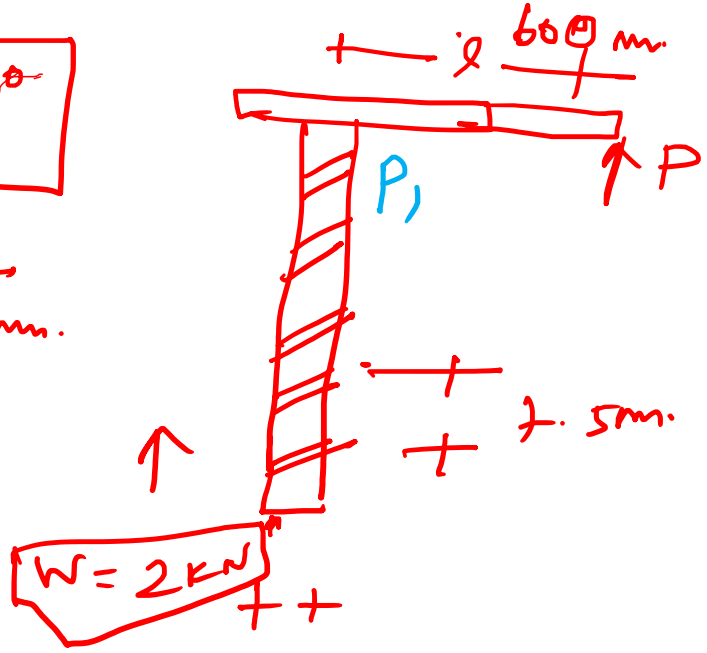
The mean radius of the screw of a square threaded screw jack is 25 mm. The pitch of thread is 7.5 mm. If the coefficient of friction is 0.12, what effort applied at the end of lever 60 cm length is needed to raise a weight of 2 kN

$W = 2000\ N$

$\mu = 0.12$, $\tan^{-1} 0.12 = \boxed{\phi = 6.8^\circ}$

$\tan\alpha = \dfrac{7.5}{\pi \times 50\ mm.}$

$\boxed{\alpha = 2.73^\circ}$



Force $P = \dfrac{d}{2r} W \tan(\alpha + \phi)$

$P = \dfrac{25 \times 2000}{600 \times 2} \tan(2.73 + 6.8) \Rightarrow$ ~~0.37~~ kN $= 6.995\ N$

applying

$P = \cancel{0.24}$

$= \cancel{340\,N}$

$P_1 \times d/2 = P \times R$

$P_1 \times 25 = \cancel{0.24}\ 6.99 \times 600$

ans. $P_1 = \cancel{8.16\,N}\ 167.76\,N$

UNIT – III

Completed.