← Back **Multivariate chain rule exercise**
Practice Quiz • 20 min • 5 total points

English **Due** Oct 5, 11:59 PM PDT

Congratulations! You passed!

Grade received 80% **To pass** 80% or higher

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1. In this quiz, you will practice calculating the multivariate chain rule for various functions.

1 / 1 point

For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$.

$f(\mathbf{x}) = f(x_1, x_2) = x_1^2x_2^2 + x_1x_2$

$x_1(t) = 1 - t^2$

$x_2(t) = 1 + t^2$

- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [2x_1^2x_2 + x_1, 2x_1x_2^2 + x_2]\begin{bmatrix}-2t\\2t\end{bmatrix}$
- ◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [2x_1x_2^2 + x_2, 2x_1^2x_2 + x_1]\begin{bmatrix}-2t\\2t\end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [2x_1^2x_2 + x_1, 2x_1x_2^2 + x_2]\begin{bmatrix}2t\\-2t\end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [2x_1x_2^2 + x_2, 2x_1^2x_2 + x_1]\begin{bmatrix}2t\\-2t\end{bmatrix}$

Correct
Well done!

2. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2, x_3)$.

0 / 1 point

$f(\mathbf{x}) = f(x_1, x_2, x_3) = x_1^3cos(x_2)e^{x_3}$

$x_1(t) = 2t$

$x_2(t) = 1 - t^2$

$x_3(t) = e^t$

- ◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [3x_1^2cos(x_2)e^{x_3}, x_1^3cos(x_2)e^{x_3}, x_1^3sin(x_2)e^{x_3}]\begin{bmatrix}2\\2t\\-e^t\end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [3x_1^2cos(x_2)e^{x_3}, -x_1^3sin(x_2)e^{x_3}, x_1^3cos(x_2)e^{x_3}]\begin{bmatrix}2\\-2t\\e^t\end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [3x_1^2cos(x_2)e^{x_3}, -x_1^3cos(x_2)e^{x_3}, x_1^3cos(x_2)e^{x_3}]\begin{bmatrix}2\\2t\\e^t\end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{d\mathbf{x}}{dt} = [3x_1^2cos(x_2)e^{x_3}, -x_1^3sin(x_2)e^{x_3}, x_1^3sin(x_2)e^{x_3}]\begin{bmatrix}2\\2t\\e^t\end{bmatrix}$

Incorrect
Be careful when calculating partial derivatives.

3. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{u} = (u_1, u_2)$.

1 / 1 point

$f(\mathbf{x}) = f(x_1, x_2) = x_1^2 - x_2^2$

$x_1(u_1, u_2) = 2u_1 + 3u_2$

$x_2(u_1, u_2) = 2u_1 - 3u_2$

$u_1(t) = cos(t/2)$

$u_2(t) = sin(2t)$

- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [2x_1, 2x_2]\begin{bmatrix}2 & -3\\-2 & -3\end{bmatrix}\begin{bmatrix}-cos(t/2)/2\\2sin(2t)\end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [2x_1, 2x_2]\begin{bmatrix}2 & -3\\2 & 3\end{bmatrix}\begin{bmatrix}sin(t/2)\\2cos(2t)\end{bmatrix}$
- ◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [2x_1, -2x_2]\begin{bmatrix}2 & 3\\2 & -3\end{bmatrix}\begin{bmatrix}-sin(t/2)/2\\2cos(2t)\end{bmatrix}$
- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-2x_1, -2x_2]\begin{bmatrix}-2 & 3\\-2 & -3\end{bmatrix}\begin{bmatrix}-sin(t/2)/2\\2cos(t)\end{bmatrix}$

Correct
Well done!

4. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{u} = (u_1, u_2)$.

1 / 1 point

$f(\mathbf{x}) = f(x_1, x_2) = cos(x_1)sin(x_2)$

$x_1(u_1, u_2) = 2u_1^2 + 3u_2^2 - u_2$

$x_2(u_1, u_2) = 2u_1 - 5u_2^3$

$u_1(t) = e^{t/2}$

$u_2(t) = e^{-2t}$

- ○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-sin(x_1)cos(x_2), cos(x_1)cos(x_2)]\begin{bmatrix}41u_1 & 6u_2 - 1\\2 & -15u_2\end{bmatrix}\begin{bmatrix}e^{t/2}/8\\-2e^{2t}\end{bmatrix}$

◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-sin(x_1)sin(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} 4u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix} \begin{bmatrix} e^{t^2/2}/2 \\ -2e^{-2t} \end{bmatrix}$

○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-cos(x_1)sin(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} u_1 & 6u_2 - 1 \\ 2 & -15u_2^2 \end{bmatrix} \begin{bmatrix} e^t \\ e^t \end{bmatrix}$

○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} = [-sin(x_1)cos(x_2), cos(x_1)cos(x_2)] \begin{bmatrix} u_1 & 6u_2 - 1 \\ 2 & -u_2^2 \end{bmatrix} \begin{bmatrix} e^{t^2/2}/2 \\ -2e^{-2t} \end{bmatrix}$

Correct
Well done!

5. For the following functions, calculate the expression $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt}$ in matrix form, where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{u} = (u_1, u_2)$.

1 / 1 point

$f(\mathbf{x}) = f(x_1, x_2, x_3) = sin(x_1)cos(x_2)e^{x_3}$

$x_1(u_1, u_2) = sin(u_1) + cos(u_2)$

$x_2(u_1, u_2) = cos(u_1) - sin(u_2)$

$x_3(u_1, u_2) = e^{u_1 + u_2}$

$u_1(t) = 1 + t/2$

$u_2(t) = 1 - t/2$

◉ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} =$

$$[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)sin(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] \begin{bmatrix} cos(u_1) & -sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ e^{u_1+u_2} & e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} =$

$$[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)cos(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] \begin{bmatrix} cos(u_1) & sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ e^{u_1+u_2} & -e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} =$

$$[cos(x_1)cos(x_2)e^{x_3}, sin(x_1)sin(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] \begin{bmatrix} -cos(u_1) & -sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ e^{u_1+u_2} & 2e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

○ $\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial \mathbf{u}}\frac{d\mathbf{u}}{dt} =$

$$[cos(x_1)cos(x_2)e^{x_3}, -sin(x_1)^2 sin(x_2)e^{x_3}, sin(x_1)cos(x_2)e^{x_3}] \begin{bmatrix} sin(u_1) & -sin(u_2) \\ -sin(u_1) & -cos(u_2) \\ 3e^{u_1+u_2} & e^{u_1+u_2} \end{bmatrix} \begin{bmatrix} -1/2 \\ -1/2 \end{bmatrix}$$

Correct
Well done!