

✔ Congratulations! You passed!

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1. The function

1 / 1 point

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

☒ an inner product

✔ Correct

It's symmetric, bilinear and positive definite. Therefore, it is a valid inner product.

☐ not an inner product

☐ not bilinear

☒ bilinear

✔ Correct

Yes:

- β is symmetric. Therefore, we only need to show linearity in one argument.
- For any $\lambda \in \mathbb{R}$ it holds that $\beta(\mathbf{x} + \lambda \mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda \beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

☐ not symmetric

☒ positive definite

✔ Correct

Yes, the matrix has only positive eigenvalues and $\beta(\mathbf{x}, \mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{0}$ and $\beta(\mathbf{x}, \mathbf{x}) = 0 \iff \mathbf{x} = \mathbf{0}$

☐ not positive definite

☒ symmetric

✔ Correct

Yes: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

2. The function

1 / 1 point

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

☒ symmetric

✔ Correct

Correct: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

☐ not symmetric

☒ not an inner product

✔ Correct

Correct: Since β is not positive definite, it cannot be an inner product.

☒ not positive definite

✔ Correct

With $\mathbf{x} = [1, 1]^T$ we get $\beta(\mathbf{x}, \mathbf{x}) = 0$. Therefore β is not positive definite.

☒ bilinear

✔ Correct

Correct:

- β is symmetric. Therefore, we only need to show linearity in one argument.
- $\beta(\mathbf{x} + \lambda \mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda \beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

☐ not bilinear

☐ an inner product

☐ positive definite

3. The function

0 / 1 point

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

☐ symmetric

☒ not symmetric

✔ Correct

Correct: If we take $\mathbf{x} = [1, 1]^T$ and $\mathbf{y} = [2, -1]^T$ then $\beta(\mathbf{x}, \mathbf{y}) = 0$ but $\beta(\mathbf{y}, \mathbf{x}) = 6$. Therefore, β is not symmetric.

☐ bilinear

- ☐ not bilinear
- ☐ an inner product
- ☒ not an inner product

✔ **Correct**
Correct: Symmetry is violated.

You didn't select all the correct answers

4. The function

1 / 1 point

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{y}$$

is

- ☐ not bilinear
- ☐ not an inner product
- ☒ bilinear

✔ **Correct**
It is the dot product, which we know already. Therefore, it is positive bilinear.

- ☐ not symmetric
- ☐ not positive definite
- ☒ positive definite

✔ **Correct**
It is the dot product, which we know already. Therefore, it is positive definite.

- ☒ an inner product

✔ **Correct**
It is the dot product, which we know already. Therefore, it is also an inner product.

- ☒ symmetric

✔ **Correct**
It is the dot product, which we know already. Therefore, it is symmetric.

5. For any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ write a short piece of code that defines a valid inner product.

1 / 1 point

```
1 import numpy as np
2
3 def dot(a, b):
4     """Compute dot product between a and b.
5     Args:
6     | a, b: (2,) ndarray as R^2 vectors
7
8     Returns:
9     | a number which is the dot product between a, b
10    """
11
12    dot_product = np.dot(a,b)
13
14    return dot_product
15
16 # Test your code before you submit.
17 a = np.array([1,0])
18 b = np.array([0,1])
19 print(dot(a,b))
```

Run

Reset

✔ **Correct**

Good job!